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Angle Rigidity Graph Theory and Multi-agent Formations

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DOI:
[10.33612/diss.169592252](https://doi.org/10.33612/diss.169592252)

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2021

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):
Chen, L. (2021). *Angle Rigidity Graph Theory and Multi-agent Formations*. [Thesis fully internal (DIV), University of Groningen]. University of Groningen. <https://doi.org/10.33612/diss.169592252>

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Angle Rigidity Graph Theory and Multi-agent Formations

Liangming Chen

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The research described in this dissertation has been carried out at the Faculty of Science and Engineering, University of Groningen, The Netherlands.

This work was supported by the Ubbo Emmius Scholarship.

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Angle Rigidity Graph Theory and Multi-agent Formations

PhD thesis

to obtain the degree of PhD at the
University of Groningen
on the authority of the
Rector Magnificus, Prof. C. Wijmenga
and in accordance with
the decision by the College of Deans.

This thesis will be defended in public on

Friday 21 May 2021 at 9.00 hours

by

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I would like to dedicate this thesis to my loving parents.

Acknowledgments

My journey in Groningen is coming to an end soon, which could not have been so memorable without the support and help from my colleagues, friends, and family. First, I would like to express my sincerest gratitude to my supervisor, Prof. Ming Cao, for giving me this opportunity to conduct the Ph.D. program in Groningen. I still remember that when I was first at his office discussing future research, he drew a geometric shape with many interior angles, which guides me to think deeper and deeper about formation control and finally comes to the contents of this thesis. I really appreciate his patient guidance, strict criticism and high standard on writing and publishing. I also wish to show my great thanks to him for the career suggestions and the numerous effects of revising papers and this thesis. Then, I want to thank my second supervisor Prof. Jacquélien M.A. Scherpen who provided a very pleasant and nice research environment for our group. I appreciate her reading and commenting on this thesis and providing support when I was looking for a postdoc position.

I would like to give my special thanks to Prof. Brian D. O. Anderson from Australia National University. I really appreciate the opportunities of talking with him and his help in many aspects. Sincere thanks to Prof. Chuanjiang Li from Harbin Institute of Technology who is always there when I meet some difficulty. I want to thank Dr. Hector G. de Marina, Dr. Zhiyong Sun and Dr. Qingkai Yang for many discussions about formation control problems. I also want to express my thanks for the helpful discussion with my collaborators Dr. Yuri Kapitanyuk, Dr. Xiaodong Cheng, and Dr. Mingming Shi.

My sincere thanks also go to all my dear colleagues. I gratefully thank Prof. Bayu Jayawardhana for our academic discussions, his delightful humor in our group activities, and his organization of the Nexus meetings. I would like to thank Dr. Ashish Cherukuri for the nice talking and travelling together to attend American Control Conference. Thank Prof. Claudio De Persis for the short time but very nice talking many times in the lunch time. I also want to thank Simon Busman

and Martin Stokroos for the experimental support in the development of robotic formation platforms. I am grateful to our secretary Frederika Fokkens for countless help in many aspects. Thanks also go to my officemates and all colleagues in DTPA, SMS, ODS, and SCAA groups.

Special thanks to my reading committee members, Prof. Hyo-Sung Ahn, Prof. Claudio De Persis, and Prof. Iman Shames, for spending time reading and commenting this thesis.

I thank Weijia Yao and Ningbo Li for being my paranymphs. Also, many thanks go to Nelson Chan for translating the summary of this thesis into Dutch and Matthijs de Jong for proofreading it. I also want to thank so many of my friends in Groningen and my life in Groningen could not have been so joyful without you. Last but not least, I would like to thank my parents for the endless love and support. I also thank my girlfriend for her support and care.

Liangming Chen
Groningen
27-01-2021

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