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Casimir and electrostatic forces from Bi₂Se₃ thin films of varying thicknessZ. Babamahdi,¹ V. B. Svetovoy², D. T. Yimam,¹ B. J. Kooi,¹ T. Banerjee,¹ J. Moon³,
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We explore here the decisive role of film thickness on Casimir and electrostatic forces from topological insulating Bi₂Se₃ films. The forces were measured for the Au-Bi₂Se₃ system under ambient conditions to simulate realistic applications. It is shown that the electrostatic forces associated with a nonzero contact potential of $V_0 \sim 200$ mV played a dominant role, and could not be compensated for thinner films (≤ 20 nm thick). However, for thicker films the measured force, after voltage compensation, started to approach the genuine Casimir force in very good agreement with Lifshitz theory. Our results point out that any gradual deviation from theory for thinner films underlies the presence of strong electrostatic effects.

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The Casimir force that originates from quantum fluctuations of electromagnetic vacuum fluctuations is today still a topic of intense research, though it had already been proposed in 1948 by Casimir and Polder [1]. The interest stems from a multitude of research fields ranging from fundamental physics in search of forces beyond the standard model to micro/nanodevices for technology applications [2–4]. In the 1950s, Lifshitz and co-workers [5] considered the general case of interaction between dielectric parallel plates by exploiting the fluctuation-dissipation theorem, which relates fluctuations of the electromagnetic field (EM) with the dissipative properties of the plates due to field absorption by many microscopic dipoles. The theory describes the attractive interaction due to quantum and thermal fluctuations for all separations covering both the Casimir (long-range) and van der Waals (short-range) regimes [3]. Therefore, as devices enter the submicron range within the realm of nanoelectromechanical systems, a deep understanding of Casimir forces at nanoscale separations between real materials is inevitable, requiring extensive studies also in relation to remnant electrostatic potential contributions.

Understanding the Casimir force between conductive materials provides accessibility to micrometer- and submicrometer-range physics, which has been of strong theoretical and experimental interest for the last two decades [1,5–7]. On the other hand, in recent years three-dimensional (3D) topological insulators (TIs), possessing a conductive surface and an insulator bulk with surface states located within the bulk band gap, have been massively studied [8–10]. Although these challenging systems are not yet thoroughly understood, they provide specific properties that can be highly promising for future micro/nanoelectromechanical

systems [11,12]. Therefore, a natural question to ask is how the Casimir force, which is omnipresent between condensed media, would manifest itself in these novel materials as they interact with other materials. So far, there has been a number of theoretical works predicting the Casimir force exerted by TIs [8,9]. In fact, it was predicted only theoretically that the Casimir force can be tuned and, in some cases becomes repulsive, by controlling the sign of the topological magnetoelectric polarizability (TMEP) and the distance between the surfaces [13]. Moreover, how a TI would influence the Casimir force, taking into account possible electrostatic contributions in the nanoscale vicinity of a metal surface, still remains unexplored. Hence, the absence of Casimir experiments with TIs does not allow one to draw meaningful conclusions about how this intriguing class of materials interacts with other materials. Indeed, it has been observed both theoretically and experimentally that metals in the proximity of TIs give rise to potential variations, which are of great importance for the preservation of the topological surface state [14–16]. Consequently, and in view of future applications, experimental studies are necessary to shed light not only on the electrostatics of the TI surface when approaching a metal, but also more interestingly on the associated Casimir force [9,10].

Therefore, we investigated Casimir forces from thin films of Bi₂Se₃, a well-known 3D topological insulator, when they are exposed to ambient conditions leading to nonideal behavior—something which is very likely to occur in applications. Our measurements show that when the Bi₂Se₃ surface is approaching an Au surface, it acts as a poor conductor most likely due to its surface oxidation upon exposure to air as shown by surface analysis using x-ray photoelectron spectroscopy. This is an important insight on how to balance electrostatic effects in submicron devices made of such materials, although under ideal circumstances 3D TIs have surface

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conductivity [11]. Accordingly, due to surface oxidation upon exposure to air the studied system cannot be considered as a perfect TI with a conductive surface caused by the surface states. More interestingly, our study revealed that the measured Casimir force is dependent on the thickness of the Bi_2Se_3 film resulting in a dramatic deviation from Lifshitz theory predictions when the thickness of Bi_2Se_3 is as low as 10 nm. Consequently, we measured the forces for Bi_2Se_3 samples with different thicknesses of 10, 20, 30, and 100 quintuple layers (QLs, where the thickness of each QL ≈ 1 nm).

The Bi_2Se_3 films were fabricated by molecular beam epitaxy (MBE) on $\text{Al}_2\text{O}_3(0001)$ substrates in a custom-designed SVTA MOS-V-2 MBE system at Rutgers University (USA) following the methods as described in [17]. Furthermore, in order to measure the contact potential V_0 , which is the electrostatic potential existing between electrically conductive materials due to work function differences and trapped surface charges, we performed electrostatic calibration using atomic force microscopy (AFM) in contact mode. The knowledge of V_0 is of crucial importance, since it should be applied as the compensation voltage to suppress electrostatic contributions, which in turn enables a more precise measurement of the genuine Casimir force. In the force setup, the plate is the Bi_2Se_3 film, and the spherical probe was made by attaching a borosilicate sphere with a $10\ \mu\text{m}$ radius at the end of a tipless cantilever. The process was followed by coating the probe with 60 nm Au using a Cressington 208 HR sputter coater. The Au layer is sufficiently thick to be optically opaque (infrared skin depth for Au is ~ 22 nm) and simultaneously has sufficiently low roughness to allow short-range force measurements. By applying various voltages in the range -1.5 to 1.5 V to the plate making direct electrical contact with Ag paste to the surface and the sphere being grounded, we can estimate the minimum deflection of the cantilever corresponding to the contact potential V_0 between Au and Bi_2Se_3 (see Fig. 1).

The contact potential for the Au/ Bi_2Se_3 system was $V_0 \approx -200$ mV for all samples with different Bi_2Se_3 thicknesses. Moreover, repetitive measurements showed that V_0 remained constant and stable over a period of 6 months. Figure 1 shows the three sets of measurements plotted together to stress the consistency of the contact potential independent of the age of the samples. In addition, the electrostatic measurements yielded similar results at different locations on the surface of the Bi_2Se_3 samples (for all thicknesses studied). However, translating the measured contact potential V_0 between the two surfaces into the work function difference between them is not trivial, since the work function difference between materials always consists of a bulk contribution, a surface contribution, and an image charge contribution. In equilibrium, very small surface charges can build up a voltage of the order of the work function difference [18].

In any case, to better interpret the measured contact potential, the chemical composition of the surface of Bi_2Se_3 and the work function of the sample were studied by performing x-ray photoelectron spectroscopy (XPS) and ultraviolet photoelectron spectroscopy (UPS), respectively (see Fig. 2). As reference sample, Au (111) was used ($E_F = 26.26$ eV). The measured work function of Bi_2Se_3 using UPS [see Fig. 2(2)] yielded a value of 3.42 eV, which is smaller than those re-

FIG. 1. Cantilever deflection versus applied voltage in the range -1.5 to 1.5 V. Different colors correspond to different sets of measurement, and within the same color different data points at a specific voltage correspond to different separations (more than 100 nm) between the interacting surfaces, where the electrostatic force is the dominant force and the Casimir effect can be neglected. The minimum of all the data points meet at -200 mV which is considered as the contact potential between Bi_2Se_3 and Au. The inset shows the height topography of the Bi_2Se_3 surface having a nominal thickness of 100 nm. The triangular features indicate the high-quality growth, and the rms roughness of this sample is 2.85 nm.

ported in previous studies with values of 5.40 eV [19] and 5.60 eV [20]. Nevertheless, a decrease of the work function of Bi_2Se_3 with 1.4 eV due to air exposure was previously reported [19], which is in good agreement with our measurement. Figure 2(1) shows the O 1s core level spectrum of the 10 nm thick Bi_2Se_3 sample, which was exposed to air (see Supplemental Material, Sec. I) [21].

Figure 1 shows also the surface topography of the 100 nm thick Bi_2Se_3 sample (measured by AFM in TM mode, Bruker Multimode 8), where the triangular features indicate the high-quality growth of the Bi_2Se_3 films. The topography of the thinner Bi_2Se_3 samples is shown in the Supplemental Material, Fig. SM2 [21]. Using also reverse AFM (with a dedicated TGT1 grating from NT-MDT spectrum instruments), we can measure the morphology of the sphere surface that interacts with the Bi_2Se_3 surfaces during force measurements. From the topography of the involved surfaces the root mean square (rms) roughness and the height of the peaks can be derived, which yield the actual distance between surfaces. The measured distance upon contact d_0 , which is strongly affected by the roughness of the sphere, can be estimated as $d_0 \approx d_{0\text{ sph}} + \omega_{\text{plf}}$, where $d_{0\text{ sph}}$ is the maximum positive full width of the height distribution and ω_{plf} is the rms surface roughness of the Bi_2Se_3 surface (see Supplemental Material, Fig. SM2 [21]) [22]. We obtained $d_0 = 28 \pm 1.7$ nm, 30 ± 2.1 nm, 27.5 ± 1.8 nm, and 28 ± 2.3 nm for the 10, 20, 30, and 100 nm $\text{Bi}_2\text{Se}_3/\text{Au}$ systems, respectively.

Furthermore, the optical properties of the interacting materials are necessary as an input for the Casimir force calculations via the Lifshitz theory. Therefore, the dielectric

FIG. 2. (1) Oxygen 1s core level spectra for the (a) Bi sample, (b) Se sample, and (c) Bi₂Se₃ sample (film thickness 10 nm). Bi and Se samples are used as a reference. The O 1s envelopes look different for all the samples. The peak positions for the Se sample are shifted towards higher binding energies compared to the other two samples. (2) (d) UPS spectra (He I, photon energy 21.2 eV) for the Bi₂Se₃ sample (film thickness 10 nm) to determine the work function (sample was biased with -5 V). The black arrow shows the secondary electron cutoff (SECO). (e) UPS spectra for the Au (111) single crystal which was used as a reference. The measurement conditions were the same as for the Bi₂Se₃ sample.

functions of the interacting bodies were measured by ellipsometry in a wide frequency range (corresponding to wavelengths from 140 nm to 33 μ m) at J. A. Woollam Co. Inc (USA) (see Fig. 3). The UVV-VASE (variable angle spectroscopic ellipsometer) and IR-VASE measurements were performed at incidence angles of 50° and 70° with respect to the surface of the samples. Since the thickness of the Bi₂Se₃ films are in the range 10, 20, 30, and 100 nm, the optical response contribution from the substrate is not negligible. For this reason, we performed the same optical measurements for an Al₂O₃ substrate similar to that used for the film growth. The approach to analyze the optical data and calculate the Casimir force is to consider the Bi₂Se₃ samples as a two-layer system where the contribution of the substrate is taken into account [23]. Accordingly, we extracted the data corresponding to Bi₂Se₃ from the raw ellipsometry data using the WVASE software by performing multiple fitting steps (point by point and then multiple Gaussian dispersion relations, where the final fits for the whole spectrum are shown in the Supplemental Material, Fig. SM3) [21]. The calculated dielectric functions at imaginary frequencies $\epsilon(i\xi)$ for both the Bi₂Se₃ and Al₂O₃, which are the necessary inputs in Lifshitz theory, are illustrated in the inset of Fig. 3. Since the measured optical data do not cover the whole frequency range 0– ∞ that is used in the Lifshitz theory, we have extrapolated the data at low frequencies using the Drude model for Au (see Supplemental Material, Sec. IV) [21].

In order to compare the theoretical force calculation with the experimental force measurement, the knowledge of the

spring constant of the cantilever-sphere system is necessary; it can be also obtained from the electrostatic calibration of the system (see Supplemental Material, Sec. V) [21,24,25]. The

FIG. 3. Imaginary part of the frequency dependent dielectric function for Bi₂Se₃ and the substrate Al₂O₃. The peak at (1.96 eV) for Bi₂Se₃ is associated with transitions from the valence band (v) to the conduction band (c) and the strong peaks in Al₂O₃ are the phonon polariton absorption peaks. The inset shows the calculated dielectric function of Bi₂Se₃ and Al₂O₃ at imaginary frequencies, which is the necessary input for the calculation of the Casimir force with Lifshitz theory.

FIG. 4. Experimental Casimir force data together with the Lifshitz theory calculation for separations below 100 nm for different thicknesses of Bi_2Se_3 . The experimental data for the shortest separation are restricted by surface roughness leading to a deviation from the power law behavior. The lower inset shows the variation of the power law exponent versus film thickness.

values of the spring constant k for the probes used for the force measurements of the Bi_2Se_3 samples with thicknesses 10, 20, 30, and 100 nm were $k = 1.62 \pm 0.03$, 1.48 ± 0.007 , 1.58 ± 0.04 , and 1.89 ± 0.02 N/m, respectively. Force measurements between metallic and nonmetallic materials were also performed previously using the same apparatus to ensure proper operation [22,25]. Moreover, the separation dependent hydrodynamic drag force has a negligible effect as compared to the Casimir force for the separations of interest between the interacting surfaces (<100 nm) [24,26]. In addition, the thermal correction at $T = 300$ K can also be neglected for separations <100 nm. The calculated and measured Casimir forces for the different thicknesses of Bi_2Se_3 are shown in Fig. 4, where for consistency both the 10 and 100 nm thick Bi_2Se_3 samples were characterized optically and the data were used for the force calculations. From Fig. 4 it is evident that there is a significant discrepancy between theory and experiment, especially for the case of the 10 nm thick Bi_2Se_3 sample. As the film thickness increases, the measured force data approach the theoretical prediction as is the case for the 100 nm thick Bi_2Se_3 . The strong deviations at the shorter separations (<30 nm) can be explained by the roughness effect on the Casimir force due to the high peaks [25,27].

The deviation of the force measurements from theory for thinner films is related to electrostatic contributions, which cannot be nullified by applying the compensating voltage. Therefore, as the film thickness decreases, we observe a clear trend going from the genuine Casimir force to a state

where the force is dramatically influenced by electrostatics. Beyond the shortest separations where the roughness plays a significant role (>30 nm), the force curves show a power law behavior $F_C \sim z^{-m}$ with exponent $m < 3$, which is anticipated for the sphere-plate geometry in the separation range <1 μm . The obtained power law values were $m \approx 1.87$, 2.03, 2.6, and 2.68 for the 10, 20, 30, and 100 nm $\text{Bi}_2\text{Se}_3/\text{Au}$ interacting systems, respectively (Fig. 4, inset). The value of the exponent from the Casimir force calculations is $m \approx 2.7$ for all the $\text{Au}/\text{Bi}_2\text{Se}_3$ systems, which is close to the experimental values for Bi_2Se_3 samples with film thicknesses ≥ 30 nm. For thinner films, the exponents start to approach values closer to that for electrostatic dominating forces (with $\sim z^{-1}$ in the sphere-plane geometry) [3,7,25].

One would expect for thinner Bi_2Se_3 films that the conductive properties of the surface will be more prominent, but the force measurements rather show the opposite trend. This behavior can be explained by an increase of the electron density and a decrease of the electron mobility as the thickness decreases [28]. This implies that the electrical properties of Bi_2Se_3 are thickness dependent and consist of bulk and surface-interface contributions. The latter contribution diminishes as the film thickness increases, becoming negligible for thick films such as 100 nm [28]. The charge transport in the surface-interface layer can lead to accumulation of electrons and the formation of a kind of patch potential effect on the surface [29]. In addition, since the roughness of all TI films is significantly less than the film thickness, there is no exposure of the underlying Al_2O_3 substrate supporting the fact that the potential/charging effects are due to the thickness of the sample. Our analysis indicates that the effect of electrostatics from TIs could vary with film thickness and from having a more significant signature for thinner films, though more detailed analysis is necessary. This is because surface-interface states become more dominant for thinner Bi_2Se_3 films. Instead of being conductive, they are insulating, which is probably due to surface oxidation.

In conclusion, we have explored the role of film thickness on Casimir and electrostatic forces for the topological insulator Bi_2Se_3 . The measurements were done under ambient conditions to simulate realistic conditions in applications. In fact, the electrostatic forces associated with a nonzero contact potential V_0 played a dominant role and could not be compensated for the thinner films (<20 nm), while above 30 nm thickness the obtained force after voltage compensation was closer to the genuine Casimir force. Hence, the measurements for the thickest film (100 nm) showed very good agreement with Lifshitz theory predictions, while there is a trend of deviation for thinner films underlying the significance of strong electrostatic effects.

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[1] H. B. G. Casimir and D. Polder, The influence of retardation on the London-van der Waals forces, *Phys. Rev.* **73**, 360 (1948).

[2] A. W. Rodriguez, F. Capasso, and S. G. Johnson, The Casimir effect in microstructured geometries, *Nat. Photon.* **5**, 211 (2011).

- [3] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, Oxford, 2009).
- [4] G. Palasantzas, M. Sedighi, and V. B. Svetovoy, Applications of Casimir forces: Nanoscale actuation and adhesion, *Appl. Phys. Lett.* **117**, 120501 (2020).
- [5] E. Lifshitz, The theory of molecular attractive forces between solids, *Zh. Eksp. Teor. Fiz.* **29**, 94 (1955) [*Sov. Phys. JETP* **2**, 1 (1956)].
- [6] F. Chen, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, Demonstration of the Difference in the Casimir Force for Samples with Different Charge-Carrier Densities, *Phys. Rev. Lett.* **97**, 170402 (2006).
- [7] U. Mohideen and A. Roy, Precision Measurement of the Casimir Force from 0.1 to 0.9 μm , *Phys. Rev. Lett.* **81**, 4549 (1998); H. C. Chiu, C. C. Chang, R. Castillo-Garza, F. Chen, and U. Mohideen, Experimental procedures for precision measurements of the Casimir force with an atomic force microscope, *J. Phys. A: Math. Theor.* **41**, 164022 (2008).
- [8] W. Nie, R. Zeng, Y. Lan, and S. Zhu, Casimir force between topological insulator slabs, *Phys. Rev. B* **88**, 085421 (2013).
- [9] L. Chen and S. Wan, Casimir interaction between topological insulators with finite surface band gap, *Phys. Rev. B* **84**, 075149 (2011).
- [10] J. C. Martinez and M. B. A. Jalil, Tuning the Casimir force via modification of interface properties of three-dimensional topological insulators, *J. Appl. Phys.* **113**, 204302 (2013).
- [11] L. Fu, C. L. Kane, and E. J. Mele, Topological Insulators in Three Dimensions, *Phys. Rev. Lett.* **98**, 106803 (2007).
- [12] S.-C. Zhang, X. Dai, X.-L. Qi, C.-X. Liu, H. Zhang, and Z. Fang, Topological insulators in Bi_2Se_3 , Bi_2Te_3 and Sb_2Te_3 with a single Dirac cone on the surface, *Nat. Phys.* **5**, 438 (2009).
- [13] A. G. Grushin and A. Cortijo, Tunable Casimir Repulsion with Three-Dimensional Topological Insulators, *Phys. Rev. Lett.* **106**, 020403 (2011).
- [14] M. Brahlek, N. Koirala, N. Bansal, and S. Oh, Transport properties of topological insulators: Band bending, bulk metal-to-insulator transition, and weak anti-localization, *Solid State Commun.* **215–216**, 54 (2015).
- [15] E. Frantzeskakis, N. de Jong, B. Zwartsenberg, T. V. Bay, Y. K. Huang, S. V. Ramankutty, A. Tytarenko, D. Wu, Y. Pan, S. Hollanders, M. Radovic, N. C. Plumb, N. Xu, M. Shi, C. Lupulescu, T. Arion, R. Ovsyannikov, A. Varykhalov, W. Eberhardt, A. de Visser *et al.*, Dirac states with knobs on: Interplay of external parameters and the surface electronic properties of three-dimensional topological insulators, *Phys. Rev. B* **91**, 205134 (2015).
- [16] E. K. de Vries, S. Pezzini, M. J. Meijer, N. Koirala, M. Salehi, J. Moon, S. Oh, S. Wiedmann, and T. Banerjee, Coexistence of bulk and surface states probed by Shubnikov–de Haas oscillations in Bi_2Se_3 with high charge-carrier density, *Phys. Rev. B* **96**, 045433 (2017).
- [17] N. Bansal, Y. S. Kim, E. Edrey, M. Brahlek, Y. Horibe, K. Iida, M. Tanimura, G.-H. Li, T. Feng, H.-D. Lee, T. Gustafsson, E. Andrei, and S. Oh, Epitaxial growth of topological insulator Bi_2Se_3 Film on Si(111) with atomically sharp interface, *Thin Solid Films* **520**, 224 (2011).
- [18] P. Hofmann, *Surface Physics Book*, <https://philiphofmann.net/surface-physics-book/>.
- [19] M. T. Edmonds, J. T. Hellerstedt, A. Tadich, A. Schenk, K. Michael O'Donnell, J. Tosado, N. P. Butch, P. Syers, J. Paglione, and M. S. Fuhrer, Stability and surface reconstruction of topological insulator Bi_2Se_3 on exposure to atmosphere, *J. Phys. Chem. C* **118**, 20413 (2014).
- [20] F. T. Takahashi, T. Sagawa, and H. Hamanaka, Photoemission (XPS and UPS) study of amorphous Bi_2Se_3 , *J. Non-Cryst. Solids* **65** 261 (1984).
- [21] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.103.L161102> for XPS data analysis, AFM images of Bi_2Se_3 , optical data analysis, Lifshitz theory, and electrostatic calibration of the cantilever spring constant.
- [22] W. Broer, G. Palasantzas, J. Knoester, and V. B. Svetovoy, Roughness correction to the Casimir force at short separations: Contact distance and extreme value statistics, *Phys. Rev. B* **85**, 155410 (2012).
- [23] F. Zhou and L. Spruch, van der Waals and retardation (Casimir) interactions of an electron or an atom with multilayered walls, *Phys. Rev. A* **52**, 297 (1995).
- [24] B. Ohler, Cantilever spring constant calibration using laser Doppler vibrometry, *Rev. Sci. Instrum.* **78**, 063701 (2007).
- [25] V. B. Svetovoy and G. Palasantzas, Influence of surface roughness on dispersion forces, *Adv. Colloid Interface Sci.* **216**, 1 (2015); Z. Babamahdi, V. B. Svetovoy, M. Enache, M. Stöhr, and G. Palasantzas, Comparison of Casimir Forces and Electrostatics from Conductive SiC-Si/C and Ru Surfaces, *Phys. Rev. B* **100**, 245422 (2019).
- [26] O. I. Vinogradova, Drainage of a thin liquid film confined between hydrophobic surfaces, *Langmuir* **11**, 2213 (1995).
- [27] W. Broer, G. Palasantzas, J. Knoester, and V. B. Svetovoy, Significance of the Casimir force and surface roughness for actuation dynamics of MEMS, *Phys. Rev. B* **87**, 125413 (2013).
- [28] J. Suh, D. Fu, X. Liu, J. K. Furdyna, K. M. Yu, W. Walukiewicz, and J. Wu, Fermi-level stabilization in the topological insulators Bi_2Se_3 and Bi_2Te_3 : Origin of the surface electron gas, *Phys. Rev. B* **89**, 115307 (2014).
- [29] J. L. Garrett, J. Kim, and J. N. Munday, Measuring the effect of electrostatic patch potentials in Casimir force experiments, *Phys. Rev. Res.* **2**, 023355 (2020).