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WHEN WILL THEY EVER MAKE UP THEIR MINDS? THE SOCIAL STRUCTURE OF UNSTABLE DECISION MAKING

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French (1977), Harary (1959), and Abelson (1964) initiated a prominent line of social influence models to explain social norms or collective decisions from the structure of influence networks. These models fail to generate unstable decision dynamics, a phenomenon that can be observed in collective decision-making. To capture instability, we assume that decision-makers raise their level of salience to reduce expected losses from decision-outcomes. Our model generates persistently unstable outcome patterns under conditions related to the social network and to intolerance for expected losses. A 6-actor example reveals stable outcomes for low intolerance, complex oscillations for intermediate levels of intolerance, and simple and regular oscillation for high intolerance. We discuss implications for the predictability of collective decision-making.

Keywords: Opinion formation, Social networks, Collective decision-making, Non-linear behavior, Chaos theory

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1. INTRODUCTION

Formal studies of collective decision-making and opinion formation address the question how networks of social influence relations affect outcomes and dynamics of collective decision making (French, 1977; Harary, 1959; Abelson, 1964; Stokman and Van den Bos, 1992; Friedkin, 1999, 2001; Kobayashi, 2001). The models in this line of research draw on two common core assumptions. First, they assume that decision-makers try to influence each other's preferences or opinions about substantive issues. Second, they emphasize that the social structure of decision-making defines restrictions and opportunities for influence.

Social influence models have been applied to many situations of opinion formation and collective decision-making. For example, Abelson and Bernstein (1963) developed a simulation model to study community referendum controversies. They applied the model to fluoridation issues in local American communities to detect cleavage patterns resulting from the social structure of the communities. Stokman and Van den Bos (1992) extended the social influence model into a two-stage model of collective decision-making and applied it to decision-making in the U.S. energy policy domain. Friedkin (2001) further developed the model to re-analyze. Roethlisberger and Dickson's (1939) data on the development of output-norms in work groups.

The classical versions of social influence models (French, 1977; Harary, 1959; Abelson, 1964; Stokman and Van den Bos, 1992) yield two robust results. The first result is that decision outcomes and positions¹ of decision-makers display *stable* patterns over time.² After some initial influence rounds, positions and decision outcomes always converge to a stable position on the issue scale. The second result is that in a large range of social structures, characterized by the weak restriction of fully connected networks, positions and decision-outcomes converge to a *unanimous*

¹We use the concepts of "group norm" and "decision-outcomes" as well as the concepts of "opinions" and "policy positions" interchangeably throughout this paper. Group norms and opinions refer to the interpretation of influence models in terms of opinion formation, whereas policy positions and decision-outcomes refer to the interpretation of influence models in terms of collective decision making.

²With the exception of particular degenerate cases, for example, the case in which at least two group members can influence each other, but none of them is able to influence himself. This would imply that they permanently cycle forth and back between each other's opinions. To avoid this complication, we exclude throughout this paper the case that group members have no self influence.

position on the issue scale.³ Apparently, according to social influence models, convergence and stability seem to be the basic features of collective decision-making. This inexplicable homogeneity in fully connected networks puzzled Abelson as inconsistent with the results of empirical studies on community decision-making. It lead him to ask, "What on earth one must assume in order to generate the bimodal outcome of community cleavage studies?" (1964). Also, to Friedkin, the strong and recurrent appearance of unanimous group norms was a remarkable, counterintuitive result (2001).

Indeed, one may wonder why collective decision-making should always produce stable and unanimous decision-outcomes. Prominent conflicts in the world display irregularities that hardly seem to follow the patterns predicted by the social influence models. For example, the Israeli-Palestinian dispute displays a pattern that oscillates irregularly between peace proposals and war threats. Such a pattern has been present for many years, and the outcome of the dispute seems hard to predict. In a similar vein, empirical analyses of collective decision-making confirm unstable outcome patterns. Richards (1990) analyzes two data sets of strategic decision-making processes in which she indicates deterministically chaotic dynamics. One data set describes fluctuations of U.S. defense spending as a percentage of total federal spending in the period 1885–1985. The second data set measures written communications within and between governments during the Cuban missile crisis. Other studies in the field of political science have revealed the existence of non-linear outcome patterns in elections (Brown, 1993).

In this paper, we propose a simple explanation of instability in collective decision-outcomes. We draw on Stokman and Stokman (1995) who assume that interdependent decision-makers: (a) are able to manipulate the level of salience they attach to an issue and (b) are to some degree intolerant for a discrepancy between the expected outcome and their own preference. Stokman and Stokman (1995) showed that it is a Nash equilibrium strategy of decision-makers to raise their level of salience—and thus their effort invested in influencing—if the discrepancy between their preference and the expected outcome increases. Stokman and Stokman's analysis does not incorporate effects of influence networks. We adapt their result and integrate it into a social influence model based on the model of French and others. Our new model thus advances the dynamic salience model by

³This is different for the more recent influence model proposed by Friedkin (1999, 2001), who assumes that in adopting a new opinion, group members with self-influence take into account their initial opinion throughout the entire influence process. As a consequence, the Friedkin model predicts that some level of disagreement will remain as long as group members have self-influence and disagree initially, even in a fully connected network.

identifying specific network-structural conditions under which unstable patterns of preferences and outcomes occur.

In the following section, we describe the social influence model and the dynamic salience model in more detail. In section 3, we present the formal integration of these two models. In section 4, we use the integrated model to deduce necessary and sufficient conditions for unstable outcome patterns to emerge in a minimal size, three-person group. Subsequently, we explore the boundary conditions under which our model produces different unstable outcome patterns in a more complex, six-person group example. The paper concludes with a discussion of model implications.

2. SOCIAL INFLUENCE MODELS, UNANIMITY AND STABILITY IN COLLECTIVE DECISION MAKING

The core assumption of social influence models is that participants in decision making gradually adapt their opinions, or policy positions, over time in response to social influences from other group members. Opinions are represented on a unidimensional scale where influence imposes a “pull” on the actor who is the target of influence to shift his opinion towards the position of the actor who exerts the pressure. The extent to which a particular group member can impose influence on others is described by an exogenous influence network, which models the structure of power relations in the group. It is assumed that this network reflects the institutional or social structure of influence. More precisely, following French’s initial contribution influence models interpret group decision-making in terms of the operation of a “force field” between group members. The models implicitly assume that all group members have full information on the opinions and the relative influences in the group. The force field induces changes in the opinions of group members during a sequence of time steps. In the classical models, in each time step a group member adopts a new opinion, which is a compromise between his current opinion and the opinions of other group members who influence him. The Friedkin model is somewhat more complex: Friedkin assumes that the initial position of a group member reflects interests that are exogenous to the influence process. Accordingly, in his model the initial position bears a weight upon the new opinion throughout the entire influence process. This weight depends upon the group members’ susceptibility to other opinions (Friedkin, 2001, p. 171).

If group members i and j disagree, the strength of the force between them is proportional to the level of their disagreement at the outset of the influence round and to their relative influences on each other as described in the influence matrix. Moreover, the force is directed towards the more

influential one of the two group members. Technically, the magnitude of the pull between i and j is generally modeled as a linear function of *disagreement* (in terms of the difference between positions on the opinion scale) and *relative influence* (in terms of the difference of the influence of i on j and the influence of j on i).

French (1977), followed by Harary (1959) and Abelson (1964), used their models of social influence to identify the necessary and sufficient conditions for stability of decision outcomes and for convergence towards unanimity in opinion formation with divergent initial opinions. In this literature, unanimity is defined as an outcome where all opinions eventually converge to the same position. French showed that the structure of the influence network is crucial for unanimity. He found that higher group cohesiveness, i.e., higher average strength of influence relations in the group, fosters convergence towards unanimity. Abelson (1964, p. 145) analyzed a time-continuous reformulation of French's originally time-discrete model. He further specifies the conditions for unanimity in terms of "matrix compactness." The matrix representing the influence network is called *compact* if at least one group member is able to influence all others directly or indirectly. When the weight matrix is compact, Abelson shows, a unanimous equilibrium outcome is the result. Clearly, compactness is a weak condition for the emergence of global uniformity. When a matrix is not compact (or fully connected), this implies that there is at least one subgroup in the influence network that is completely isolated from outside pressures. This is a network property that seems rather unlikely, at least in small groups of decision makers.

Long-term stability of decision outcomes is the second pervasive characteristic of social influence models. Group unanimity in social influence dynamics by itself does not necessarily imply stability of individual opinions. Group members may converge in opinions while these opinions still change over time. However, Harary (1959) showed that regardless of unanimity, the stability of individual opinions is a general long-term property of the underlying social influence models. Using the theory of Markov chains, Harary proved that "[i]n every group, regardless of its power structure or its initial opinion distribution, each member reaches a stable final opinion" (Harary, 1959, p.180). This individual stability in opinions is a consequence of the inherent compromise character of the influence process. The average disagreement between all pairs of actors i and j only can decline in the course of the influence process. There is just one way how this decline may come to a halt. This is when subgroups are present in the network such that neither subgroup can influence its counterpart. In that case, opinions will stabilize at some constant level of group disagreement.

Clearly, traditionally formal models of social influence in collective decision making unequivocally suggest that opinion dynamics converge on

stable decision outcomes sooner or later. At least when networks are compact in the sense of full connectedness and agents are fully susceptible to outside influences, the equilibrium outcome is sustained by global uniformity in the opinions of all group members. We consider the model property of an inherently stable equilibrium as unsatisfactory. Although convergence may often occur in reality, there is nothing to *guarantee* stability in group decision making. Accordingly, we argue that models of social influence need to be extended such that they provide the theoretical tools necessary to identify conditions for instability.

Stokman and Stokman (1995) moved towards integrating unstable dynamics into a model of a political decision making process. Their work draws on an earlier model of issue bargaining. In this logrolling model, a quick convergence on an equilibrium outcome arises. In equilibrium, all actors involved have found an optimally profitable exchange of political positions (for a similar approach, see Coleman, 1990). The authors then add to this model the notion of strategic interest manipulation. They assume that actors aim to anticipate rationally the expected political influence and interests of other parties. Decision makers use this expectation to strategically choose the level of "leverage," or salience, they bring to bear on an issue in order to optimize the overall outcome of decision making relative to their own opinion. Subsequently, actors learn from the past outcome and adapt their predictions of other's salience, so that they choose a new strategic salience level for the next round of influence. Stokman and Stokman show how these additional assumptions suffice to generate unstable decision dynamics. In particular, actors may alternate in shifting back and forth their salience between different issues, generating highly irregular long-term oscillations in opinion outcomes.

Although Stokman and Stokman's analysis of strategic salience manipulation points out a promising avenue, their approach does not address effects of social structure. Social structure is the central variable that explains the dynamics and outcomes of decision-making in social influence models. In Stokman and Stokman's logrolling model, influence relations only arise from the distribution of positions and interests across a range of issues without exogenous structural restrictions. To integrate structural effects into the analysis of instability in decision making, we will incorporate into a conventional social influence model Stokman and Stokman's assumption that actors change their salience in order to avoid undesired decision outcomes. Because we wish to concentrate on the effects of social structure, we simplify Stokman and Stokman's *multi-issue* bargaining approach and model decision making with respect to one single issue. It should be emphasized that our adaptive salience model can be readily extended to collective decision-making in a multi-issue domain.

Integration of strategic salience into social influence models also requires a modification in the behavioral assumptions that underlie the notion of strategic manipulation of salience. Social influence models generally assume a model of behavior that may be characterized as “backward-looking” (Macy, 1990), gradual adaptation of positions to the social pressures that a decision maker faces. By contrast, the strategic salience model assumes sophisticated rational anticipation and perfect maximization of expected utility when it comes to choosing the right level of salience for the present round of influence. The authors derive strategic salience levels as a Nash equilibrium outcome in which all players are aware of, and respond optimally to, the salience levels that other parties bear upon the issue. The social influence models do not incorporate this notion of maximization of expected utility in their conceptualization of opinion change. Accordingly, a combination of strategic salience manipulation with social influence dynamics in decision-making would result in a hybrid model that simultaneously incorporates two fundamentally different behavioral mechanisms. To make our new model compatible with previous research on social influence, we simplify the assumption of strategic interest manipulation. Instead, we assume that in the choice of salience levels decision-makers follow a simple backward-looking heuristic: they increase their salience by some amount if things went badly, and reduce their salience if things went well⁴. Consistent with the strategic salience model, we furthermore assume that the larger the difference between expected outcome and the present opinion of an actor, the larger the influence weight that this actor will impose on other decision makers.

3. THE ADAPTIVE SALIENCE MODEL OF SOCIAL INFLUENCE

The Baseline Model

French’s and Harary’s original formulations of the social influence model form the baseline for our formalization and are a special case of the adaptive salience model.⁵ Following French, we express the initial opinion

⁴A second motivation to move towards a more backward-looking model of opinion change is the recent criticism of game theoretic “reflexive rationality of actors anticipating each other’s choices” (Scharpf 1990, 71). Authors such as Simon contend that this is a model of cognition that is at best a prescriptive model of how choices ought to be made, but one that bears little resemblance to actual decision-making, even by business firms where this forward-looking calculated rationality seems most plausible (Simon 1992, 36).

⁵Substantively, the central assumptions of our model are also compatible with Abelson’s approach. However, there is a technical difference. Following French and Harary, we conceive of social influence as a time-discrete process. Abelson instead assumes a continuous process, using a differential equation model.

of a group member i at time t as a real number $x_i(t)$ that varies on a one-dimensional scale. For convenience, we assume in our implementation that opinions are bounded in the unity interval, i.e., $0 \leq x_i(t) \leq 1$. A group member adapts his opinion in every round t under the influence of the opinions of other group members j . A weight matrix $\mathbf{Y}$ defines the strength of the influence imposed by every group member ego on every other member alter.

The social influence model formalizes the new opinion of actor i as a weighted average of the opinions of those members by whom the focal actor is influenced, including the “self influence” imposed by his own opinion. Formally,

$$x_i(t+1) = \sum_{j \in N} y_{ij} x_j(t). \quad (1)$$

In Equation (1), the influence weights y_{ij} scale the strength of the influence that j imposes on i . To scale relative influences between actors, the model normalizes the rows of the weight matrix. Accordingly, for member i the total amount of incoming influence is equal to unity, i.e., $\sum_{j=1}^N y_{ij} = 1$, where N denotes group size. It is important to note that in this *baseline model* influence weights y_{ij} are static, i.e., they do not change over time. The self-weight y_{ii} scales an actor's openness to the influence of other group members. In our analyses we exclude the special case that group members have no self influence ($y_{ii} = 0$). Moreover, we assume that the new opinion $x_i(t+1)$ is not directly influenced by the initial opinion, $x_i(1)$, following the French–Abelson–Harary tradition, and Doreian and Hummon's (1976) model of the influence processes in organizational change.

The weight matrix affects the behavior of group members in a straightforward way. The most simple case arises when all group members exert equal influence on all other group members, where $y_{ij} = 1/N$ for all pairs i and j . In that situation, the group will converge to unanimity in one influence round, because the new opinion $x(t+1)$ is equal for all actors. The unanimity opinion of the group is the average opinion of all group members, i.e., $x(t+1) = \frac{1}{N} \sum_{j \in N} x_j$.⁶ In this simple case, the location of the unanimous group opinion on the opinion scale solely depends upon the initial distribution of opinions. In the more general case, with unequal

⁶If the opinions in the standard model are interpreted as policy positions, the model would predict an outcome of group decision making which is the average position of all group members, of course under the condition that the influence weights y_{ij} all are equal. In fact, the average policy position is used in models of collective decision making as a variant to Duncan Black's median voter theorem. For example, the two-stage model of collective decision making developed by Stokman and Van den Bos (1991) is functionally equivalent to French's standard model of opinion formation.

weights y_{ij} , the value of the unanimous group opinion additionally depends upon the structure of the weight matrix $\mathbf{Y}$. In any case, the group opinion will be in between the values of the opinions of the two most extreme group members.

To define the group opinion under persistent disagreement, we follow previous models of political decision making. We assume that in every influence round there is a final “voting phase,” in which individual opinions are aggregated in a group opinion, i.e., a norm or collective outcome (Stokman and van den Bos, 1992). We assume that the group opinion is a weighted average of the opinions of all group members, where the weights indicate the constant relative voting power v_i of an actor i . The sum of these weights is constant at unity. More precisely, we compute the group outcome $o(t)$ at the end of every influence round t according to Equation [2].

$$o(t) = \sum_{i=1}^N v_i x_i(t) \quad (2)$$

Expected Frustration and Salience Adaptation

At the outset of a decision round, actors myopically assume that the outcome of the next round will be identical to the final outcome of the voting phase of the preceding round. Accordingly, actors expect that they will experience at least some frustration after the round, because their current opinion diverges to some extent from the prior and expected decision outcome. In other words, actors make a myopic prediction of the utility loss they will suffer. In a process of opinion formation, this loss arises because decision-makers may have a deviant position in the group. In a process of collective decision-making, utility loss arises because the expected outcome differs from the most preferred outcome. Formally, the expected frustration in round $t + 1$ of group member i is the absolute difference $|x_i(t) - o(t)|$ between the group outcome expected at the outset of $t + 1$, $o(t)$, and the final opinion of i of the previous round t , $x_i(t)$.

We assume that the expectation to get frustrated by the group outcome will drive actors to invest some effort in changing their relative weights in the influence process. More precisely, group members change their weights in the following influence round proportional to their expected level of frustration. In terms of the strategic salience model, we interpret this change in relative weights as a change in issue salience.⁷ If a group member

⁷In fact, Abelson incorporates into the standard model the concepts of (a) actor preferences and (b) their interests in substantive policy issues. Abelson acknowledges the personal advice of James Coleman who, at that moment, was working on models of political exchange.

expects a satisfactory outcome, he will put no additional effort in achieving this outcome. At the same time, the group member will raise his level of salience for the issue if the expected outcome deviates from his opinion or preference.

The underlying assumption is that frustration of decision makers is caused by a deviation of the expected group outcome from the *previous opinion* of the group member, and not by a deviation from the *initial opinion* of the group member. In processes of social norms formation, this reflects group members' need for behavioral confirmation from other group members. They seek to obtain this confirmation by adopting an opinion that reflects the group norm as much as possible. In processes of collective decision-making, group members need to obtain consensus to arrive at legitimate compromises that solve political issues. If decision-makers would fall back on their initial policy positions after a compromise has been made, this would make them less credible during consensus formation. However, other than in norm formation, political decision makers have the additional goal of achieving a specific outcome that reflects their political interest. Nevertheless, we argue that in the course of consensus formation, political decision makers need to commit themselves to a group compromise. If the compromise deviates from their initial policy positions, they cannot easily revoke this compromise without damaging their reputation in their constituency (Torenvlied, 2000; Torenvlied and Thomson, 2003).

Our assumption of changing influence weights introduces a self-correcting feedback-loop into the baseline model. The effort group members invest to influence others increases with the difference between the decision outcome and the present individual opinion. The larger this difference, the more likely it is that sufficient effort is invested to move the outcome closer to the position of the focal actor. This, in turn, may increase future investments in effort by other decision makers, etc. Clearly, the resulting feedback effect might induce endogenous instability.

To model the dynamics of influence weights technically, we assume that the effective weights for the new influence round $t+1$, $y_{ij}^*(t+1)$, partially depend on expected frustration and partially on a latent static influence structure y_{ij} . Equation (3) formalizes the weight dynamics.

$$y_{ij}^*(t+1) = \frac{y_{ij}|x_j(t) - o(t)|^\tau}{\sum_{k \in N} y_{ik}|x_k(t) - o(t)|^\tau} \quad (3)$$

The numerator in Equation (3) expresses our central substantive assumption. The larger the difference between the expected outcome and the present opinion of actor j , the larger the influence weight that actor j

imposes on actor i . This effect is scaled with an *intolerance parameter* τ .⁸ Broadly, the larger τ , the more difference it makes for an actor when the discrepancy between expected group opinion and his own opinion increases. When $\tau = 0$, our dynamic influence model is identical with the classic French model of social influence. Furthermore, we assume that $\tau \geq 0$. The resulting influence weight is subsequently modified by the static (structural) component of j 's power over i , y_{ij} . We assume that $y_{ij} \in [0, 1]$. This implies in particular that if it is structurally impossible for j to impose influence on i ($y_{ij} = 0$), then j exerts no influence over i , no matter how large is the frustration of j . Finally, the denominator in Equation (3) guarantees that the row-sum of all influence weights is kept constant at one.

We combine our model of adaptive salience with the baseline model as follows. At the outset of every decision round, actors calculate the expected group outcome of round t on the basis of the final opinions of the preceding round t , as given by Equation (2). Subsequently, actors adapt influence weights according to Equation (3) in order to reduce expected frustration in round t . This results in a new weight matrix $\mathbf{Y}^*(\mathbf{t})$ that drives the influence dynamics in round $t + 1$. The influence dynamics as such are equal to the baseline model, as expressed by Equation (4) below.

$$x_i(t+1) = \sum_{j \in N} y_{ij}^*(t+1) x_j(t). \quad (4)$$

The influence dynamics in the model of adaptive salience imply that frustration increases influence, and that success follows effort. However, the effect of an increased level of frustration of group member i is conditional upon the level of frustration of other group members j . Moreover, the static (structural) component of influence places an upper boundary on the effective influence that a frustrated actor can impose on other group members, because $y_{ij}^*(t+1) \leq y_{ij}$.

4. RESULTS

To explore necessary and sufficient conditions for unstable outcome patterns in collective decision-making, we analyze the dynamic salience model in two steps. In the first step, we derive analytically the network structures for instability in the smallest possible social system in which unstable outcome patterns can emerge at all. In the second step, we use computer

⁸The intolerance parameter corresponds with Stokman and Stokman's (1995) parameter r for a "risk taking propensity" (294).

simulation to explore the boundary conditions under which different, more complex outcome patterns emerge in a larger group.

4.1 Necessary Conditions for Persistent Instability

Collective decision-making is persistently unstable if there is at least one decision-maker whose opinion changes between two subsequent time steps over an infinite length of time. In our dynamic salience model, the following mechanism induces persistent instability. A group member adjusts his influence weights as soon as he discovers that the expected outcome differs from his present opinion. If the discrepancy is large, larger adjustments will be made. Consequently, the outcome will move towards his opinion but may move away from the opinions of other group members. Accordingly, other group members may adjust their influence weights in the following influence round, and hence the expected outcome moves back. On a larger time-scale, the group outcome would bounce back and forth on the opinion scale.

We identified two network conditions that are necessary but not sufficient to induce persistent instability in a group with initially heterogeneous opinions. The first condition is that the group contains *at least two “stubborn” actors*. These are actors who have access to at least one other group member, but who are insulated from outside pressures in the influence network and therefore do not change their opinions.⁹ However, stubborn actors may still adjust their influence weights proportional to their relative frustration. Without the presence of at least two stubborn actors, the distance between actor opinions will ultimately approach zero and, accordingly, an equilibrium is attained even with dynamic salience. Inspection of Equations [3] and [4] shows why Equation [4] guarantees that *if* positive influence is imposed by at least one other group member on each of the two most extreme actors at opposing ends of the opinion scale, then in every time step these actors will move closer towards the rest of the group. This inevitably results in convergence and stability. The only exception is that the extreme actors at both ends of the opinion spectrum are stubborn. Equation [3] makes sure that without two stubborn actors and without unanimity there is always at least one other group member who imposes positive influence on at least one of the extreme positions. For every member who has a positive static influence weight $y > 0$ on the extreme positions, even the slightest discrepancy between expected

⁹Our analysis generalizes to “stubborn subgroups” which are fully connected subgraphs that can not be accessed from their complement in the network but who contain at least one member that has influence upon a node in the complement. For simplicity, we identify in the following “stubborn subgroups” with stubborn actors.

outcome and actual position results in an effective “pull” $y^* > 0$ shifting the target closer to the source of influence.¹⁰ As a consequence, a group with exactly one stubborn actor will eventually converge on the opinion of this actor, whereas a group with no stubborn actor inevitably agrees on some compromise between the initial positions. In both cases, persistent instability is impossible.

The second necessary social structural condition for persistent instability is that *at least one “bouncing” actor* is present in the network. A bouncing actor is a group member who can be influenced by both stubborn actors. This group member is subjected to the changing influence weights of the stubborn actors. If a bouncing actor were absent, every group member’s opinion would eventually collapse into one of the opinions of the stubborn actors, which generates a stable compromise group outcome.

Finally, our model suggests a third necessary condition for persistent instability that is unrelated to network structure. The level of intolerance for a discrepancy between expected group opinion and the own opinion of decision-makers needs to be sufficiently high, i.e., the intolerance parameter τ should exceed a critical level τ^* . The reason is that with intolerance approaching zero, the adaptive salience model turns into the conventional social influence models where the long-term outcome is a stable equilibrium.

4.2 The Smallest Group Exhibiting Persistent Instability

The smallest group that can contain two stubborn actors and one bouncing actor is a three-actor network. Clearly, the two social structural conditions for persistent instability can also be met for groups with a size larger than two. However, to illustrate how intolerance for relative frustration shapes the qualitative dynamics of collective decision-making we will first discuss this minimal network. Figures 1 through 4 show how opinion formation with dynamic influence weights develops over time for the minimal 3-person group. The weight matrix $\mathbf{Y}$ in the figures formalizes the exogenously imposed social structure of influence. In Figure 1, as a baseline we use the level of intolerance $\tau = 0$ for which the adaptive salience model becomes equivalent to the classical social influence models. In Figures 2 through 4 we introduce the feedback effect of expected frustration and gradually increase the level of intolerance. Initial opinions and

¹⁰The only exception occurs when every actor with access to the extreme positions is simultaneously fully satisfied with the outcome. In that case, they have either already adopted the extreme positions or at least some of these actors will be pulled towards the extreme positions, which in turn changes the outcome and increases their frustration in the next round.

FIGURE 1 Stable convergence of the average opinion $o(t)$ in a polarized 3-person group (intolerance level $\tau = 0$).

the network structure are kept constant such that differences in patterns between figures are strictly the result of endogenous processes.

Figure 1 illustrates the effect of the presence of two stubborn actors in a three-person group without salience adaptation. This is the standard social influence model. For simplicity, we assume equal voting power $v = 1/3$ for all actors. The weight matrix indicates that members 1 and 3 are stubborn, i.e., they cannot be reached by any other group member. The influence weights for access of actor 2 to actors 1 and 3 are set to zero in the weight matrix. The figure shows that opinions do not converge to unanimity in this situation. From the first influence round on, only actor 2 gradually adjusts his opinion towards the average of the opinions of actors 1 and 3. His opinion shifts from 0 to 0.50 over time, until virtual stability is reached in iteration 5. As a result, the group opinion o shifts from the initial group

FIGURE 2 Unstable convergence of the average opinion $o(t)$ in a 3-person group (intolerance level $\tau = 2$; weight matrix as in Figure 2).

average (0.33) towards the average of the opinions of the stubborn actors 1 and 3 (0.50).

Figures 2 and 3 clearly display a pattern of unstable convergence. The outcome $o(t)$ bounces back and forth on the opinion scale, but in due time the process stabilizes towards one single outcome. The difference between Figures 2 and 3 is the presumed level of intolerance of the group members: In Figure 3 we assumed actors to be less tolerant for discrepancies between their opinion and the expected outcome. As a consequence, the opinion formation process takes more time to converge. With $\tau = 2$, convergence occurs within approximately 10 iterations. With $\tau = 3$, the group opinion has still not stabilized after about 50 iterations. Further simulations showed that convergence required about 1500 iterations under $\tau = 3$. Note that in both cases the two stubborn actors, with initial opinions $x_1(t_1) = 0.9$ and $x_3(t_1) = 0.1$, firmly maintain their opinions. Only the bouncing actor 2, with his initial position of $x_2(t_1) = 0$, exhibits an unstable behavioral pattern.

Figure 4 shows the persistent instability that arises if the intolerance parameter τ is sufficiently large. The bouncing group member 2 oscillates between the opinions of the two stubborn group members 1 and 3. Their opinions $x_1(t_1) = 0.9$ and $x_3(t_1) = 0.1$ define the amplitude of a cyclical oscillation that fails to converge.

To test whether this instability is persistent, we replicated the simulation with 10.000 influence rounds instead of 50. We found no indication that even with this large number of influence rounds, changes in group-opinion between two subsequent rounds get smaller. Consequently, the pattern displayed in Figure 4 with genuine and persistent instability. This

FIGURE 3 Unstable convergence of the average opinion in a 3-person group (intolerance level $\tau = 3$; weight matrix as in Figure 2).

FIGURE 4 Persistent instability of the average opinion $o(t)$ in a 3-person group (intolerance level $\tau = 4$; weight matrix as in Figure 2).

does not preclude that convergence into a stable outcome may occur on a much larger time scale.¹¹ However, for all practical purposes this number of iterations suggests that for empirical situations consistent with the specification of our model a stable outcome will not be reached.

Figure 4 shows that after some time the dynamics stabilize into a simple oscillation that repeats itself after two influence rounds. We expect that this pattern is a consequence of the simple structure of the three-person weight matrix rather than a fundamental characteristic of our non-linear model of opinion formation. In this weight matrix, there is only one group member who changes his position. With only one bouncing actor present, every position of the group outcome exactly corresponds to one position on the opinion scale that the bouncing actor can take. If, by contrast, a bouncing subgroup is present in the weight matrix, one and the same group outcome may be generated by different distributions of opinions in the group. Then, it is not only the previous group outcome that shapes the next influence round, but it is also the internal state of the bouncing group that matters. Clearly, more complex influence dynamics may result if members of a bouncing *subgroup* change their potentially heterogeneous opinions rather than only one bouncing *actor*.

¹¹For this it would be necessary to obtain an analytical proof that for the given parameters the system will never converge. A more general analytical proof would shed light on the question whether indeed a discontinuity occurs between $\tau = 3$ and $\tau = 4$ despite the fact that our system is very simple. Such a result still eludes us.

4.3 Example: Complex Instability in a Six-person Group

To study the possible complexities that arise from a bouncing subgroup, we analyzed a hypothetical six-actor group. In the weight matrix, two stubborn actors and a bouncing subgroup are defined. The example we study here is derived from a political decision setting as it may typically occur in a city. The group is now interpreted as a policy network and its members as corporate actors representing organized interests (Laumann and Knoke, 1987). Figure 5 presents the social structure of the group: the static weight matrix $\mathbf{Y}$ and its network representation, together with the assumed initial values of voting power v and initial positions $x(t=1)$.

In the policy network, city council (2) faces a decision about imposing environmental regulations on the local plant of a multinational enterprise (1). The international company is one of the two stubborn actors in this process. Because it is extremely powerful it cannot be influenced by any of the other actors in the policy network. The other stubborn actor has opposed interests: a national pro-environmental organization (6), which is likewise insulated from local political pressures. Both stubborn actors have no formal voting power in the city council. Voting power over the decision accrues to two local political parties (3) and (5) who form the enacting coalition. Both coalition parties influence each other. Who furthermore influence the political parties? The more conservative party (3), with most voting power, interacts with organized business interests in the city (2) and is influenced by the multinational enterprise. The more liberal party (5) is being influenced by the environmentalists and by the mayor of the city (4). In this example, the environmentalists and the multinational enterprise (the stubborn actors) generally exert their vote in this process via indirect influence channels. The multinational enterprise (1) influences the decision indirectly by the mayor and the business interests lobbyists, and directly via its influence channel to the more conservative party. The environmental lobby only influences the smaller, liberal coalition party.

FIGURE 5 The 6-actor group: initial values of the weight matrix and its network representation, voting power and initial positions.

4.4 Complex Patterns of Persistent Instability

To demonstrate the effect of intolerance on this multi-actor decision process, we replicate our analysis of the 3-person group. We compare the dynamics for three qualitatively different levels of intolerance: *low* intolerance ($\tau = 2$), *moderate* intolerance ($\tau = 3.5$), and *high* intolerance ($\tau = 5$). Figures 6 through 8 show simulation results of the first 50 iterations of the corresponding dynamics. Figure 6 shows that the behavioral pattern under *low intolerance* is qualitatively similar to the corresponding dynamics of the 3-member group in Figure 1. After a relatively short period of initial instability, opinions stabilize into persistent polarization around approximately round 15 of the influence process.

Figure 7 shows that with *moderate intolerance* the six-actor group in Figure 7 displays persistent instability, as did the three-member group in Figure 3b. In addition, Figure 7 reveals that moderate intolerance entails a qualitatively new form of influence dynamics. Comparison of Figures 3 and 7 shows that the dynamics are considerably more complex in the six-actor group. Such a complex non-linear pattern could represent two fundamentally different types of dynamics; a complex oscillation or deterministic chaos. It is well known that even relatively simple non-linear systems may generate deterministic chaos. The defining characteristic of deterministic chaos is that the trajectory of the system is bounded in space, but in contrast to an oscillation it never repeats itself (Schuster, 1988).

Visual inspection of Figure 7 shows that after about 10 iterations, the influence dynamics seem to approach a pattern of changing outcomes that repeats itself around iteration 30. A more appropriate analysis than visual inspection is required to determine whether this pattern represents

FIGURE 6 Unstable polarization in a 6-actor group with *low* intolerance level ($\tau = 2$).

FIGURE 7 Complex persistent instability in a 6-actor group with *intermediate* intolerance level ($\tau = 3.5$).

an oscillation or perhaps deterministic chaos. Further below, we discuss this in more detail. At this point it suffices to note that the dynamics in Figure 7 are persistently unstable and exhibit much less regularity in the opinion pattern than does any of the other opinion dynamics we found earlier.

Figure 8 finally shows that there is no simple relation between the level of intolerance and the complexity of behavior. *High intolerance* suppresses the complex oscillation, which arose under moderate intolerance. The influence dynamics narrow down to a simple oscillation between the most extreme positions the bouncing subgroup possibly can adopt.

How can we explain the transition to a completely different pattern of behavior when we increase the value of intolerance τ ? The highly intolerant stubborn actors allow members of the bouncing group to adopt only a narrow range of opinions. Under moderate intolerance, stubborn actors allowed at least some discrepancy between their own position and the decision outcome. As a consequence, members of the bouncing group shifted their positions only gradually towards the positions of the stubborn actors through a sequence of intermediate steps. When the expected frustration of at least one of the stubborn actors becomes too high, he imposes pressure to gradually pull the bouncing group back towards his position, again in a number of intermediate steps. Under high intolerance, the stubborn actors now mobilize so much effort that members of the bouncing group are pulled back and forth between the most extreme positions that they can possibly adopt, given the influence of other group members.

Our analysis of the effects of intolerance in the 6-actor social structure suggests that at least three qualitatively different types of non-linear

FIGURE 8 Simple persistent instability in a 6-actor group with *high* intolerance level ($\tau = 5$).

behavior occur under different values of intolerance. We might even observe phase-transitions: dramatic changes in behavior resulting from only a marginal increase in the level of intolerance. Under low intolerance, opinions rapidly converge to one stable group opinion. Under moderate intolerance, a complex oscillation or even deterministic chaos in the decision outcome over time may arise. Under high intolerance instability reduces to a simple oscillation.

To test this conjecture, we replicated the simulation of the six-actor group with conditions as displayed in Figure 5 for 200 influence rounds. In this replication, we varied the level of intolerance, τ , systematically between $\tau = 2$ and $\tau = 5$ in small increments of 0.01. To display the complexity of the dynamics in 200 influence rounds we plotted all group opinions that occur within one simulation after iteration 20 on a horizontal line representing the opinion space $[0, \dots, 1]$. Consequently, on each horizontal line 180 dots are plotted, but dots may overlap and there is not necessarily a dot on every position of the line. To demonstrate how a marginally changing value of τ affects long-run dynamics, we display the level of intolerance on the vertical dimension. Figure 9 shows the results.

Figure 9 shows that indeed three ranges of intolerance levels exist with qualitatively different dynamics. Below approximately $\tau = 2.8$, intolerance is so low that a stable group opinion emerges.¹² All 180 outcomes are plotted on one position. In this region of the parameter space there exists

¹²More precisely, below $\tau = 2.8$ the dynamics of opinion formation are characterized by unstable polarization, where opinions stabilize into an stable individual opinion within 20 influence rounds.

FIGURE 9 Bifurcation diagram. Effect of intolerance τ on the long-term distribution of group opinion $o(t)$ in the 6-actor group of Figure 4.

one unique group opinion after iteration 20 at all levels of τ . Above approximately $\tau = 4.5$, intolerance is so high that decision dynamics bounce back and forth between only three different outcomes. These outcomes correspond to a cycle of length 3 that passes through: (a) two group opinions maximally close to the positions of the two stubborn actors, and (b) one outcome in between the positions of the stubborn actors.

In the intermediate range, between $\tau = 2.8$ and $\tau = 4.5$, Figure 6 reveals a complicated dependency of decision dynamics on intolerance. From the value of $\tau = 2.8$ bifurcation arises: two and rapidly more different outcomes obtain. From the overall pattern in this region we draw two observations.

Firstly, the group opinion passes through cycles with a large number of different outcomes. For example, at $\tau = 4$, the group opinion varies across 9 different opinion levels, ranging between approximately $o = 0.4$ and $o = 0.8$. Second, small changes in the level of intolerance may have enormous effects on the influence dynamics. For illustration, a slight increase in intolerance from $\tau = 4$ to only $\tau = 4.01$ dramatically increases the number of different group opinions from 9 to 21.

4.5 Deterministic Chaos or Complex Oscillation?

The effects of intolerance in the intermediate range suggest the possibility that decision dynamics exhibits properties of deterministic chaos. There are a number of statistical methods that allow to identify the qualitative pattern of the dynamics (cf. Schuster, 1988). For our purposes, we apply the method of a Fourier transformation of the time series of group opinions. The Fourier transformation $F(f)$ indicates the degree to which there is an oscillation with frequency f in a time series. If there are a few clear peaks in the Fourier transformation F , this indicates that the time series is generated by overlapping oscillations rather than by real deterministic chaos. By contrast, real deterministic chaos generates an unstable pattern of opinions that is completely non-periodic. Correspondingly, the Fourier transformation of a chaotic time series shows a broad spectrum of noise that mostly occurs at low frequencies of opinions (Schuster, 1988, p. 9).

Figure 10 shows the Fourier transformation of a time series of group opinions for one of the cases that are most disordered according to

FIGURE 10 Fourier transformation of the complex cycle of the group opinion $o(t)$ in 6-actor group of Figure 4 with intolerance level $\tau = 3.5$.

simulations of Figure 9, the case of $\tau = 3.5$ shown in Figure 7. We ran a simulation with 1000 iterations. The Fourier transformation shows that, despite its complexity, the opinion dynamic is not chaotic. Instead, the time series consists of overlapping oscillations that generate main frequencies of about 390 and 222, corresponding to cycles of length 2.69 and 3.85, respectively. For reliability, we repeated this analysis for a number of other levels of intolerance in the intermediate range of Figure 9. For all of the intolerance levels we inspected, this analysis confirms that the dynamics consist of complex oscillations rather than deterministic chaos.

5. CONCLUSION AND DISCUSSION

We proposed a simple modification of social influence models of collective decision-making. We added to these models one assumption. Decision-makers adapt their level of salience in order to avoid expected losses from decision-making. Our analysis showed that this simple assumption implies non-linear, persistently unstable behavioral patterns in a social influence model that was known to be inherently stable in previous research. Our model yields two general results. First, the social structure of decision-making has a profound impact on the likelihood of persistent instability. We found that at least two stubborn actors need to be present in the network. These are actors who are able to influence other group members but unable or unwilling to be influenced themselves. We found that at least one bouncing actor should be present. A bouncing actor is an actor who is directly or indirectly influenced by two or more stubborn actors. Second, we demonstrated that instability arises only if decision-makers are sufficiently intolerant for expected losses.

The specific pattern of persistent instability in collective decision-outcomes appeared to depend on the social structure and the level of intolerance in a non-trivial, complicated way. In a three-actor social structure, the most complex outcome pattern is a simple cyclic pattern, which emerges at *high* levels of intolerance. In a more complex six-actor social structure, the most complex outcome pattern is an irregular oscillation with a large periodicity, which emerges at *intermediate* levels of intolerance. For high levels of intolerance, a simple cyclic pattern emerges. We found no evidence for the presence of deterministic chaos. These results strongly suggest instability in a broader class of both simple and complex network structures that include two stubborn actors and one bouncing actor. Further research should employ a combination of analytical and simulation methods to yield more general results on the network structures that are conducive to instable behavior.

The detection of complex oscillations with large periodicity may have implications for the predictability of collective decision making. The exact value of a predicted decision outcome becomes highly dependent upon the time step at which a collective decision is made. Unforeseen variation of only one time-step can make a large difference in the value of the predicted outcome. Moreover, the range of possible outcomes becomes highly sensitive to very small changes in the level of intolerance. Correct predictions would require an estimation of the level of intolerance, more precise than imaginable.

Yet we should not jump to conclusions about the unpredictability of collective decision making too rapidly. According to our model, unpredictability due to complex oscillations only exists under very specific conditions: a social structure characterized by the presence of at least two stubborn actors and one bouncing subgroup, as well as intermediate levels of intolerance. This raises the question how likely it will be that such conditions will be met in empirical reality. Although one may wonder whether stubborn actors exist in the real world, it is quite plausible that some actors in a social system can *hardly* be influenced. In many respects, the position of the United States in international relations appears to be an example of an actor who is weakly stubborn because his relative power makes him hardly accessible for outside pressures. Another example might be countries, such as North Korea, largely but not entirely isolated from outside influences due to ideologically hardened positions. Under such weaker conditions, the dynamics of the adaptive salience model remain *qualitatively* the same, at least in the intermediate term. Therefore, temporary unpredictability of decision outcomes may occur in real world situations that resemble the structural conditions of our examples.

Our model builds upon the strong assumption that frustration increases effort, which results in success in influencing other group members. From group dynamics research one might also expect that frustrated group members who are not willing to accept a group compromise, at some point will be excluded from the social influence network (Homans, 1951). To incorporate this social mechanism in our model would require that we make the static weight component endogenous, and thus assume that group members can cut themselves off from deviant influences. It seems reasonable that with this extension a higher level of stability in decision-making would be predicted than by our analyses. The scope of our analysis is thus confined to situations in which deviant group members cannot easily be excluded from the influence process, as for example in political decision-making or in work groups.

A further possible limitation of scope results from the assumption that group members simultaneously influence each other and adapt their salience in discrete time-steps. We deem this a natural assumption for

situations in which influence occurs in influence rounds, for example, in repeated committee meetings or in project teams where members come together in recurring meetings. Future work needs to explore the extent to which the introduction of alternative regimes, such as asynchronous influence, or continuous influence and salience adaptation, may increase the stability of decision making even under the conditions for which we predict persistent instability.

The instability and non-linear dynamics we generated clearly also depends upon our assumptions about the rationality of social actors. We drew upon the result of Stokman and Stokman, who found that it is Nash-equilibrium strategy of rational decision-makers to increase salience on issues on which they expect a discrepancy between their own opinion and the outcome. In order to integrate this result in the social influence model of collective decision-making, we assumed that social actors are relatively blind in past and future. With this, we follow the recent trend in game theory towards low rationality (Erev and Roth, 1998) models of adaptive or myopic decision optimization (Fudenberg and Levine, 1998). Further research should show how sensitive our results are for changes in the micro-foundations of the model. If social actors are indeed more rational than we assume, they would be able to foresee future unstable behavior and have an incentive to reach stable agreements. If actors were more capable of learning from the past, they would optimize on a longer time frame, which might increase the likelihood that an equilibrium is reached (Fudenberg and Tirole, 1991) even under the structural conditions that we identified as conducive for unstable decision-making. For example, we expect that to the extent that the level of frustration would be also based upon a comparison of the group outcome with the *initial* opinion, more stability might result.

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