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Aspects of fine-tuning in theories with extended scalar sectors

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Aspects of fine-tuning in theories with extended scalar sectors

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Chapter 1

Introduction

The field of particle physics is concerned with subatomic particles, describing physics on the smallest scales accessible to experiments and beyond. Over the last century, a huge amount of progress has been made in understanding these subatomic particles. The first half of the century was characterized by a huge wave of discoveries of new particles. In the 1960s, a theoretical picture emerged that could explain this abundance of particles through a much smaller set of underlying elementary particles. By studying these, a theory was established that describes the properties of the elementary particles and how they interact. This theory is called the Standard Model of particle physics (in short Standard Model or SM). In 2012, the ATLAS and CMS experiments at the Large Hadron Collider (LHC) at CERN, Geneva announced the observation of the Higgs boson, the final particle in the Standard Model [1,2].

Although all elementary particles in the Standard Model have been discovered, there are still many open questions. First of all, not all interactions that are predicted by the Standard Model have been experimentally observed. Some interactions, most of them involving the Higgs boson, are so rare and require such high energies that detection has not been demonstrated yet. But also the interactions that have been observed are still being analysed. By increasing the precision at which the Standard Model is tested, the predictions of the Standard Model are put to the test. If deviations from these predictions are discovered, this would indicate the existence of new physics. The search for beyond the Standard Model (BSM) physics comprises a large part of present-day research in particle physics. These searches are done in various different ways. There are high-energy experiments like the ones performed at the LHC that collide particles at ever-increasing energies and intensities, but there are also smaller-scale experiments that can probe similar energy scales through high-precision measurements. In most experiments, energy scales up to about 10 TeV are currently probed. These energies correspond to length scales of 10^{-19} m, which is roughly a factor ten thousand smaller than the radius of a proton. So far, none of these experiments has found any clear evidence for BSM physics.

This is quite a remarkable fact, since there are plenty of reasons to assume the

Figure 1.1 The masses of the Standard Model particles. Based on a figure in Ref. [4], values from Ref. [5].

existence of new physics. First of all, there are astronomical observations that cannot be explained by the Standard Model. An example is the observed distribution of energy in the universe. Measurements done by e.g. the PLANCK satellite have shown that only 5% of the energy content of the universe consists of baryonic matter like we see around us on Earth [3]. The remaining 95% consists of dark matter and dark energy (roughly 25% and 70% respectively). The nature of both these forms of energy is unknown, but we do know some of their properties. For dark matter, it is widely believed that it has a particle nature, but the Standard Model cannot explain its observed abundance. Dark matter is electrically neutral and does not interact via strong interactions. The only stable particles in the Standard Model that meet these requirements are the neutrinos. But these are not massive enough to make up all the dark matter in the universe. So if dark matter has a particle nature, its origin must be beyond the Standard Model.

Another astronomical observation that is at tension with the Standard Model is the asymmetry between matter and antimatter. The planet we live on is completely made up of matter, and observations show that this is also the case for all astronomical objects we can observe. There is a small amount of antimatter present in the visible universe, but the ratio of matter to antimatter is very large. This is remarkable, since one would expect that at the Big Bang, an equal amount of matter and antimatter was produced. While there are SM processes that can generate such an asymmetry during the evolution of the universe, these processes always occur in thermal equilibrium. So the Standard Model cannot explain the matter-antimatter asymmetry, as there will always be an equal rate for processes that generate an asymmetry and processes that negate it again. It is also possible that the visible universe consists only of matter, while other regions of the universe consist mainly of antimatter. But this would lead to emissions from annihilation at the boundaries between these regions. Such emissions have not been observed, so it is presumed that these different regions do not exist, although it is possible that the regions are so large that the emission from the boundary has not reached us yet. If there was indeed no large imbalance between

matter and antimatter in the creation of the universe, the conclusion is that beyond the Standard Model physics is necessary to explain this asymmetry.

In addition to these astronomical observations, there are also experimental measurements that cannot be explained by the Standard Model, like the mass of neutrinos. The masses of the other fermions are determined by Yukawa couplings. But this coupling can only be present when both the left-handed and right-handed fields are present in the model. The Standard Model contains only left-handed neutrinos, so Yukawa couplings are not possible, and neutrinos are massless in the Standard Model. But it is known that neutrinos must be massive due to the existence of neutrino oscillations. The mechanism responsible for these masses is not known. It is possible to generate neutrino masses in the same way as the masses of the other fermions, by adding right-handed neutrinos and introducing Yukawa couplings. However, the smallness of neutrino masses compared to the other fermions (see Figure 1.1) might hint at a different mechanism for generating masses. Different mechanisms exist that address the lightness of the neutrino masses, which will be covered in more detail in Chapter 2. Note that while these arguments point to the existence of some kind of BSM physics, there is no evidence that points to a specific BSM theory. This is true not only for the example of the neutrino masses, but for all examples listed here.

Also the masses of the quarks and the charged leptons are not completely understood. While the mechanism that provides masses to these particles is known, the values of the masses seem arbitrary. Also the fact that there are three generations of fermions is not explained in the Standard Model. Furthermore, it appears that the potential of the SM scalar sector is not stable up to arbitrarily high energies [6,7]. At high energies, the potential might develop a second minimum which is energetically favourable, meaning that the minimum we live in is not stable. The Standard Model does predict that the minimum we live in is meta-stable, with a lifetime predicted to be longer than the age of the universe. But arguably the most striking theoretical shortcoming of the Standard Model is the fact that gravity is not included. Around the Planck scale ($M_{\text{pl}} \approx 10^{19}$ GeV) quantum effects of gravity can no longer be neglected, and a theory of quantum gravity is needed.

So we see that there are many reasons to expect physics beyond the Standard Model. But in the absence of any clear signal for BSM physics, other criteria are being used to try to determine the viability of BSM theories. One such principle is naturalness, which states that dimensionless numbers are expected to have values of $\mathcal{O}(1)$, in the absence of an explanation for the value being much larger (or smaller) than 1. Examples that satisfy this criterion are e.g. the three Standard Model gauge couplings, which have values of $\mathcal{O}(1)$ in the energy range from the electroweak scale ($v_{\text{EW}} = 246$ GeV) all the way up to the Planck scale.

In 1979, 't Hooft proposed a more refined notion of naturalness in the context of quantum field theories [8]. He stated that: “at any energy scale μ , a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ increases the symmetry of the system”. An example of this is the electron mass. The Yukawa coupling of the electron is of the order 10^{-6} , which would be considered unnatural. But setting this coupling to zero increases the symmetry by adding a chiral symmetry for left-handed and right-handed electrons. Therefore the electron Yukawa coupling, and thus also the electron mass, is a natural parameter according to 't Hooft's criterion. This principle is known as technical naturalness. Note that throughout the literature, different definitions of naturalness (and later also fine-tuning and the hierarchy problem) are used. In this thesis, we will use the definitions as given in this chapter and the following chapters.

Due to naturalness arguments, couplings should not be too large or too small. There is another argument that can be used to disfavour large values for couplings. This is the fact that when a coupling is too large, the theory is no longer perturbative. Higher-order effects with a coupling g contribute at each loop order with an additional factor $g^2/4\pi$. So when $g > \sqrt{4\pi}$, every higher order in perturbation theory contributes more than the previous order. In this scenario, perturbation theory can no longer provide valid predictions in the theory.¹ While it is possible that perturbation theory is not a valid approach at all energy scales, this approach works well in describing the high-energy behaviour of the SM. The SM couplings, for example, are all perturbative up to the Planck scale. Therefore, the perturbativity constraint is widely used as a constraint on couplings in theories beyond the Standard Model.

We argued before that we generally expect dimensionless quantities to have values of $\mathcal{O}(1)$. Many exceptions to this principle in a quantum-field-theory context can be explained by the technical naturalness criterion proposed by 't Hooft. However this does not cover all exceptions. In this thesis, we will focus on the situation where naturalness is violated due to fine-tuning. This happens when a quantity has a value that is much smaller than the (independent) contributions it is composed of. To illustrate this, consider a situation where a quantity $z = x - y$ is observed to be much smaller than both individual contributions x and y , even though x and y are completely independent. In that case there needs to be a large cancellation between these contributions in order to accommodate the small value for z . Such a situation is only possible when the parameters of the model (x and y in this case) have very specific values. If one of the values would be slightly different, the cancellation no longer occurs, and the resulting quantity will not be small. A crucial aspect of fine-tuning is

¹For a quartic coupling λ , this happens when $\lambda > 4\pi$.

that there is no underlying reason, like a symmetry, for why the resulting quantity is small. Note that a small amount of fine-tuning is not really a problem, sometimes it simply happens that two contributions cancel each other slightly and result in a smaller value. But when there are cancellations between parameters up to many significant digits, this is generally considered problematic.

An example of a fine-tuning discussion in physics is the fine-tuning-for-life debate. In short, the main question in this debate is: how unlikely is life on Earth? The origin of this discussion is the realization that some processes in nuclear physics, that are essential for providing the observed distribution of elements, are such that if the parameters of the theory would have been slightly different, the distribution of heavy elements would be very different, and it would not be possible for life to emerge [9]. An example of this apparent fine-tuning is the mass difference between u and d quarks, which ensures that the neutron is heavier than the proton, thereby ensuring the stability of the proton. But if the difference between the proton and neutron mass would be too large, the deuteron would be unstable, preventing the existence of complex chemistry as we know it [10]. Another example is the existence of a state in the carbon spectrum which is crucial for the formation of carbon in the universe. The energy level of this Hoyle state [11] depends very sensitively on the masses of the light quarks, with a 0.5% shift in the quark masses already leading to the elimination of carbon-oxygen-based life [12].

One way to circumvent these issues is by the multiverse proposal. This proposal, often used in the context of string-theory models, states that our universe is not unique [13]. There are many other universes, all with different values for the fundamental parameters. The universe we live in just happens to be one with parameters that support the formation of heavy elements and the existence of life. Note that in this proposal there is no real explanation for the fine-tuning, it is just accepted using the reasoning that if there are enough universes, some of them are bound to appear fine-tuned.

Now we will focus more on fine-tuning problems in a beyond the Standard Model context. The most well known fine-tuning problem in that field is probably the hierarchy problem, which is a problem with the lightness of the Higgs boson when the Standard Model is extended with a heavy particle that couples to the Higgs. This problem was first noted in Ref. [14]. A clearer picture emerged later on, under the influence of the seminal papers Ref. [15] by Veltman and Ref. [8] by the 't Hooft. Here we will give a short introduction to the problem, a more detailed explanation is left for Chapter 3.

As mentioned before, there are plenty of reasons to believe that the Standard Model is not the ultimate theory of nature. Since new physics has not shown up so far in

experiments, this new physics probably has to reside at a higher energy scale than the energies currently being probed. But it turns out that the squared mass of the Higgs boson m_h^2 receives corrections which are quadratically dependent on this heavy scale [14]. Therefore, the natural mass scale for the Higgs boson is of the order of the highest scale in the theory, which could be as high as the Planck scale. But with a mass of 125 GeV, the observed Higgs boson is much lighter than the Planck scale. Furthermore, the Higgs mass parameter is not technically natural, since in the limit $m_h^2 \rightarrow 0$, the symmetry of the Lagrangian is not increased.² So accommodating this relatively light mass, while the natural mass scale is much higher, requires fine-tuning of the high-energy parameters. This sensitivity of the Higgs mass to heavy scales is called the hierarchy problem. If there is indeed a contribution of the order of the Planck scale, the discrepancy is as large as M_{pl}^2/v_{EW}^2 . So there would have to be a cancellation between completely unrelated parameters up to 33 digits to ensure a light Higgs boson. Note that the Standard Model by itself does not lead to a hierarchy problem, only by introducing a new heavy mass scale does the hierarchy problem appear.

While it is in principle possible that the high-energy parameters have specific values such that the Higgs boson has a mass much lower than the high-energy scale, this first of all seems very unlikely, but it would also still be problematic. This situation would signal the breakdown of the decoupling approach. In physics, a key concept is that low-energy physics decouples from high-energy physics. So the low-energy effects should not depend on the details of the high-energy theory: one can describe an apple falling from a tree without knowing the details of the individual atoms in the apple. This concept is used across many different energy regions. However the mass of the Higgs boson seems to violate this concept, as the low-energy value is very sensitive to the details of the theory at high scale. If the high-energy parameters would have been slightly different, the mass of the Higgs boson would have a completely different value, indicating the breakdown of the decoupling approach.

We see two options, although there might be other, unknown, alternatives. Either the decoupling approach holds and new physics that mitigates the hierarchy problem appears not too far from the electroweak scale, or fine-tuning is necessary to ensure the lightness of the Higgs boson and the decoupling approach is not valid at high energies. There has been a lot of research on the hierarchy problem, focussed on trying to find mechanisms that provide solutions to the problem. Before discussing a few examples of such mechanisms, we will highlight a few examples of similar situations in particle physics, where new physics did appear before the decoupling approach breaks down. The following examples mostly follow Refs. [16] and [17].

²In a non-interacting theory, a shift symmetry would appear in this limit [8]. However, since the Higgs boson has interactions with other particles, this symmetry does not appear.

One example is the mass difference between neutral and charged pions. Pions are the pseudo-Goldstone bosons originating from the spontaneous breaking of chiral symmetry. There is a mass difference between the neutral and charged pions of $\delta m_{\pi^\pm}^2 = m_{\pi^\pm}^2 - m_{\pi^0}^2 \approx (35.5 \text{ MeV})^2$ [5]. This difference can be explained by QED corrections, which only appear for the charged pions. In the effective theory of pions, a quick estimate of the QED contribution to the charged pion mass gives:

$$\delta m_{\pi^\pm}^2 \approx \frac{3e^2}{16\pi^2} \Lambda_\pi^2, \quad (1.1)$$

with e the electric charge and Λ_π the scale at which the effective theory breaks down. Note that there is a quadratic dependence on the scale of new physics, just like for the Higgs boson. This is the case because pions are scalar particles, so mass corrections have a quadratic dependence on the high-energy scale. But unlike the case of the Higgs boson, we now consider an effective theory, where the particles are not elementary. So Λ_π denotes the scale up until which the effective theory is valid. Above this scale, a more fundamental theory is necessary. While it is possible that new physics only appears at a much higher scale, this would be very unnatural, as this new physics would then have to be fine-tuned in order to explain the pion mass difference. In order to have a natural theory, there needs to be new physics around the scale Λ_π . From Equation 1.1, and the mass difference $\delta m_{\pi^\pm}^2 \approx (35.5 \text{ MeV})^2$, one has $\Lambda_\pi \approx 850 \text{ MeV}$. And indeed, at 775 MeV the ρ meson appears, marking the breakdown of the effective theory of pions. So this is an example where the decoupling approach does work. New physics appears at the scale where the effective theory naturally breaks down.

Another example where the notions of decoupling and naturalness work nicely can be found in the kaon sector. Here the mass difference between long-lived and short-lived kaons can be used to estimate the mass of the charm quark. This estimate preceded the discovery of the charm quark, so this can actually be seen as a prediction of the decoupling approach. This is in contrast to the mass difference in pions, where the decoupling argument was only realized after the discovery of the ρ meson.

In the effective theory of kaons, there is mixing between the states K^0 and $\bar{K}^0$. They mix to form the long-lived K_L^0 and the short-lived K_S^0 . The mass difference between these states can be expressed as [18]:

$$\frac{M_{K_L^0} - M_{K_S^0}}{M_{K_L^0}} = \alpha \frac{G_F f_K^2}{4\sqrt{2}\pi^2} \sin^2 \theta_c \cos^2 \theta_c \frac{m_c^2}{m_W^2 \sin^2 \theta_W}, \quad (1.2)$$

with α the fine-structure constant, G_F Fermi's constant, $f_K \approx 150 \text{ MeV}$ the kaon decay constant, θ_c the Cabbibo angle ($\sin \theta_c = 0.225$), θ_W the Weinberg mixing angle

($\sin^2 \theta_W \approx 0.23$ [5]) and m_c the mass of the charm quark. To explain the observed mass difference between these states of

$$\frac{M_{K_L^0} - M_{K_S^0}}{M_{K_L^0}} = 7 \times 10^{-15}, \quad (1.3)$$

a value of $m_c \approx 1.4$ GeV is necessary. And later in 1974 the charm quark was found with a mass of $m_c = 1.27$ GeV [5], agreeing well with the prediction. So this is another case where the heavy physics could have resided at a larger energy scale, and satisfy the experimental constraints through some fine-tuned behaviour. However, this is not the case, and the natural value as predicted from decoupling is respected by the mass of the charm quark.

On the other hand, there is also a famous example where the decoupling strategy does not seem to work, and which can be considered even worse than the hierarchy problem. This is the problem of the size of the cosmological constant. The observed value is $\Lambda_{cc} \approx (10^{-12} \text{ GeV})^4$ [3]. But when the value of Λ_{cc} is computed in quantum field theory, it is found that quantum corrections to Λ_{cc} are as large as the fourth power of the largest scale in the theory. So if Planck scale physics contributes to the cosmological constant, we would expect $\Lambda_{cc} \approx (10^{19} \text{ GeV})^4$. This results in a discrepancy of 120 orders of magnitude. But even if for some reason Planck-scale physics does not contribute to the cosmological constant,³ there should anyway be corrections of the order of the electroweak scale $v_{EW} \approx 100$ GeV, which still leads to a huge discrepancy.⁴ So independent of the exact prediction for the value of the cosmological constant, the predicted value is many orders of magnitude larger than the observed value.

So we see that decoupling arguments do not work all the time. But we can still apply this reasoning to the mass of the Higgs boson, to see at which energy scale new physics is expected based on decoupling arguments. In the examples of the pion and kaon mass, we could use the measured mass difference between two particles as the starting point of our discussion. This is not the case for the Higgs mass, but we can still write down a constraint that ensures that the theory is natural. This is done by demanding that the quantum corrections to the Higgs mass do not exceed the measured value of the Higgs mass too much. If the quantum corrections are much larger than the measured mass, these corrections would have to cancel either with each other or with the tree-level contribution to obtain the observed value for the Higgs mass. This would imply fine-tuning when the contributions that cancel are

³E.g. when there is no new physics at the Planck scale, or because of supersymmetry which ensures that all contributions to Λ_{cc} above the scale of supersymmetry breaking cancel.

⁴Although the Standard Model does not contain a mechanism to prevent further radiative corrections to Λ_{cc} , so there still has to be new physics that addresses the absence of these corrections.

independent. If we take the Standard Model as an effective theory up to a scale M , where the effective description breaks down, we can write down the correction to the Higgs mass [19]:

$$\delta m_h^2 = \frac{3M^2}{16\pi^2 v_{EW}^2} (m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2) \quad (1.4)$$

Imposing that the size of these quantum corrections should not be much larger than the mass of the Higgs boson ($\delta m_h^2 \lesssim m_h^2$), we find that $M \lesssim 1 \text{ TeV}$.⁵ Note that in order to solve the hierarchy problem, the new physics at the scale M cannot be just any new physics, but it should contain a mechanism to prevent further quadratic divergences at higher energies.

So the problem of reconciling the hierarchy problem with the expected appearance of BSM physics continues to puzzle physicists. But the hierarchy problem is not the only source of fine-tuning in BSM theories. In this thesis, we will discuss several aspects of fine-tuning in a certain class of BSM models, namely the models with extended scalar sectors. These models are relevant for various reasons. First of all, BSM theories often contain extended gauge groups. In order to break such an extended gauge group to the Standard Model gauge group, additional scalar fields are necessary. Furthermore, in some BSM theories like supersymmetric extensions, it is not possible to give mass to all fermions with just one Higgs doublet [20]. So extending the scalar sector is necessary in order to make sure all fermions can get a mass. Also from the experimental side the scalar sector is interesting. There are still some couplings in the scalar sector that have not been observed experimentally, while some of the observed couplings still have large error margins. So there is still room for deviations from Standard Model predictions in the scalar sector, at a much larger level than the heavily constrained fermionic and gauge sectors. This does not mean there are no restrictions on the scalar sector. Observables like the ρ parameter [21] can be used to put constraints on the scalar sector.

But there are also some issues with models with extended scalar sectors. Arguably the biggest issue is that many of these models have an additional fine-tuning problem, which appears at tree-level already. We will discuss multiple aspects of this fine-tuning in this thesis. But before diving into those, we will first give a more detailed introduction to the Standard Model and proposed extensions of the Standard Model in Chapter 2. Since most of our research is focussed on the Higgs sector, we will look in detail at the Higgs sector of the Standard Model, focussing on the properties that will be useful later when discussing models with extended scalar sectors. We will discuss

⁵Note that when the combination $m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2 = 0$, the correction to the Higgs boson would be absent (when the Standard Model is considered as an effective theory). This Veltman condition [15] is not satisfied in the Standard Model and there is no underlying motivation why it should hold. It is still an active area of research however, see e.g. Ref. [19].

why these extended scalar sectors are so important, but also the problems that can arise in such models. Then Chapter 3 will discuss fine-tuning in more detail. The hierarchy problem will be investigated more elaborately, some proposed solutions to this problem are discussed and tree-level fine-tuning is explained in more detail. In this chapter we will also introduce two fine-tuning measures, which quantify the amount of fine-tuning, enabling a comparison of fine-tuning between different model points.

These results are then used in Chapter 4, where we take a close look at how these measures actually work, and what the proper way to apply them is. We will look first at the two-Higgs-doublet model (2HDM), which enables analytical expressions for the quantities of interest. After discussing the fine-tuning in this model in detail, we apply the lessons we learned to a left-right-symmetric model (LRSM), for which it is claimed that the model needs a very large amount of tree-level fine-tuning.

Then in Chapter 5, we look at a Grand Unified Theory (GUT), namely the minimal non-supersymmetric $SU(5)$ GUT. This theory is said to suffer from a tree-level fine-tuning problem: the doublet-triplet splitting problem. We will quantitatively check this argument and analyse the origin of the problem critically. This leads us to proposing an alternative viewpoint on how to approach models with a large hierarchy in scales.

In Chapter 6 we take a look at the effect of loop contributions on the amount of fine-tuning in models with multiple scalars. We will use the formalism of the effective potential to investigate a model with two real scalars, with a large hierarchy in the mass parameters. This hierarchy leads to a problem in the naive implementation of the effective potential, as it is inevitable that large logarithms appear. At every loop order, a higher power of these logarithms appears, thereby invalidating the loop expansion. We will use a proposed solution to this problem in which particles are decoupled by construction when the energy is too low to excite their modes. The model is then analysed by taking into account the one-loop effects using this decoupling approach. We will show that also in this case, fine-tuning of the high-energy parameters is still necessary. Finally, in the last chapter of this thesis, we summarize and conclude.

Chapter 2

The Standard Model and beyond

The theory that currently best describes physics at the smallest scales is called the Standard Model of particle physics (from now on just called the Standard Model or SM). This is a quantum field theory that includes fermionic particles that make up all the matter, and bosonic particles that mediate interactions between the fermions. The Standard Model has been experimentally scrutinized for a long time now, at many different energy scales and through many different kinds of experiments. So far, no conclusive evidence for a specific BSM theory has been found. However there is good reason to believe that the Standard Model is not the final theory of particle physics. In this chapter, we first introduce the Standard Model, highlighting some of the features that are relevant for the remainder of the thesis. Then we focus on the Higgs sector, since this is the sector we will concentrate on most in later chapters. After discussing the effect of loop diagrams, we look at beyond the Standard Model theories. We highlight some theories that provide solutions to problems of the Standard Model, and finish with a discussion on extended scalar sectors.

2.1 The Standard Model

The Standard Model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The SM fermions can be divided in two groups: the quarks and the leptons. The quarks are in the following representations of the SM gauge group:

$$\begin{aligned} Q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in (3, 2, 1/6), \\ u_R &\in (3, 1, 2/3), \\ d_R &\in (3, 1, -1/3), \end{aligned} \tag{2.1}$$

where the numbers in brackets denote the $SU(3)_C$ representation, $SU(2)_L$ representation and $U(1)_Y$ charge respectively. These representations should be understood as triplets in family space, with u denoting the up-type quarks up (u), charm (c) and top (t), while d denotes the down-type quarks down (d), strange (s) and bottom (b).

The charm and top quark have the same properties as the up quark, except that they have a larger mass. In a similar way, the strange and bottom quarks are heavier partners of the down quark. The quarks are all charged under the $SU(3)_C$ group, but they differ in the $SU(2)_L$ and $U(1)_Y$ charge. The $SU(2)_L$ group only couples to left-handed particles, so the right-handed particles are not charged under this group.

The lepton representations are given by:

$$\begin{aligned} L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} &\in (1, 2, -1/2), \\ e_R &\in (1, 1, -1). \end{aligned} \quad (2.2)$$

Again these are triplets in family space, with the only difference being the mass. The particles in the other families are the muon (μ) and tau (τ) and their corresponding neutrinos. The leptons do not have color charge. Note that there are no right-handed neutrinos in the Standard Model, although they can be straightforwardly included as addressed in section 2.2.1.

The interactions between these fermions are mediated by bosonic particles. The forces associated with the three gauge groups are mediated by the gauge bosons. The gauge boson fields and their representations under the Standard Model gauge group are:

$$\begin{aligned} SU(3)_C : \quad G_\mu^a &\in (8, 1, 0), \\ SU(2)_L : \quad W_\mu^i &\in (1, 3, 0), \\ U(1)_Y : \quad B_\mu &\in (1, 1, 0). \end{aligned} \quad (2.3)$$

The gluons G_μ^a and the W bosons W_μ^i also have self-interactions. These self-interactions are included in the kinetic terms of the gauge bosons:

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu,a} - \frac{1}{4}W_{\mu\nu}^i W^{\mu\nu,i} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}, \quad (2.4)$$

where $G_{\mu\nu}^a$, $W_{\mu\nu}^i$ and $B_{\mu\nu}$ are the field strengths:

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \quad (2.5)$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \epsilon^{ijk} W_\mu^j W_\nu^k, \quad (2.6)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (2.7)$$

Here, f^{abc} and ϵ^{ijk} denote the structure constants of $SU(3)$ and $SU(2)$, respectively, and g_s and g denote the coupling strengths of the $SU(3)_C$ and $SU(2)_L$ interactions,

respectively.

The interactions between the fermions and the gauge bosons are introduced in the Lagrangian via the covariant derivative. This object ensures that the kinetic term is gauge invariant. The general covariant derivative for the SM gauge group is given by:

$$D_\mu = \partial_\mu - ig_s G_\mu^a \lambda^a - i\frac{g}{2} W_\mu^i \sigma^i - ig' Y B_\mu, \quad (2.8)$$

where the λ_a and $\sigma^i/2$ are the generators of $SU(3)$ and $SU(2)$, respectively, g' is the coupling strength of the $U(1)_Y$ interaction and Y is the hypercharge. The covariant derivative is different for each particle, as it depends on the representation the particle is in.

The final set of bosonic particles are the scalars. The only scalar field in the Standard Model is the complex Higgs doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \in (1, 2, 1/2). \quad (2.9)$$

This doublet is responsible for the masses of the fermions and gauge bosons, and also mediates scalar interactions through Yukawa interactions.

Using all these ingredients, one can write down the Standard Model Lagrangian:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a} - \frac{1}{4} W_{\mu\nu}^i W^{\mu\nu,i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \bar{Q}_L i \not{D} Q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R \\ & - \bar{Q}_L \phi Y_d d_R - \bar{Q}_L \tilde{\phi} Y_u u_R - \bar{L}_L \phi Y_e e_R + \text{h.c.} \\ & + (D_\mu \phi)^\dagger (D^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \\ & - \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}. \end{aligned} \quad (2.10)$$

Here, Y_i are the Yukawa coupling matrices, $\tilde{\phi}_i = \epsilon_{ij} \phi_j^*$, and $\tilde{G}^{\mu\nu,a} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$. ϵ_{ij} and $\epsilon^{\mu\nu\alpha\beta}$ are the totally antisymmetric tensors, with $\epsilon_{12} = \epsilon^{0123} = +1$. The first two lines of this Lagrangian denote the kinetic terms of the gauge bosons and the fermions. The third line shows the Yukawa couplings, which are the interactions between the fermions and the Higgs boson. After electroweak symmetry breaking, these couplings are responsible for the masses of the fermions. Note that the Yukawa couplings Y_i are matrices in family space, they can mix the quarks between different families. The fourth line describes the Higgs sector, with its kinetic term and the scalar potential. This sector will be discussed in detail later on. The final term is the QCD θ term,

which is an interesting term from the perspective of naturalness. This term is the only one in the QCD sector that can lead to CP violation (more on that in the next section). θ is interpreted as an angle, so it has a value between 0 and 2π . So far, no CP violation has been observed in the QCD sector, leading to the strong limit $\theta < 2.5 \cdot 10^{-10}$ [22]. So this term is either extremely small, or it is actually absent. There is no reason to believe that the term is absent, since there is no symmetry in the Standard Model that could impose this term to be zero. But a very small value for θ is also unexpected. It is not a technically-natural parameter, and there is no mechanism in the Standard Model that can explain such a small value. There are proposed solutions for this problem, most notably the introduction of a new scalar field, called the axion, that relaxes the θ parameter to a small value [23–25]. While searches for axions are being conducted, they have not been detected yet. We will focus on naturalness and fine-tuning issues in the electroweak sector, so this term will not play a role in the remainder of this thesis. Similar terms can also be written down for the $SU(2)$ and $U(1)$ gauge bosons, but these terms can be rotated away by a redefinition of the fields, which is not possible for the $SU(3)$ sector.

2.1.1 Global symmetries of the Standard Model

So far we made extensive use of the gauge symmetries of the Standard Model. But these are not the only symmetries of the SM. We will now look at the global symmetries. They can be further divided in discrete and continuous global symmetries.

Discrete global symmetries

The three discrete symmetry transformations that will be discussed are parity (P), time reversal (T) and charge conjugation (C). Parity and time reversal switch the spatial and temporal direction, respectively, while charge conjugation transforms particles into antiparticles and vice versa.

Focussing on the fermionic sector, and using the Weyl representation for the gamma matrices, the transformations are given by:

$$\begin{aligned} P : \quad \psi(t, \vec{x}) &\rightarrow \eta_P \gamma_0 \psi(t, -\vec{x}), \\ T : \quad \psi(t, \vec{x}) &\rightarrow \eta_T \gamma_1 \gamma_3 \psi(-t, \vec{x}), \\ C : \quad \psi(t, \vec{x}) &\rightarrow \eta_C i \gamma_2 \gamma_0 \bar{\psi}(t, \vec{x})^T, \end{aligned} \tag{2.11}$$

with η_P , η_T and η_C complex phases.

There is an important theorem in quantum field theory, called the CPT theorem, stating that every quantum field theory that is Lorentz invariant and has a Hermitian Hamiltonian is invariant under the combined operation of C , P and T [26]. However,

the individual transformations do not have to be symmetries of the theory. And in the case of the Standard Model it turns out that none of the individual transformations are exact symmetries of all interactions.

While the strong interaction is symmetric under C , P and T separately,¹ this is not the case for the weak interaction. The weak interaction only couples to left-handed fermions and right-handed antifermions. The parity operator flips the handedness of a fermion, and the charge conjugation operator turns fermions into antifermions. So a left-handed fermion (that couples to the weak interaction) is transformed under a parity transformation to a right-handed fermion (which does not couple to the weak interaction). Similarly, under charge conjugation, a left-handed fermion turns into a left-handed antifermion (which again does not couple to weak interactions). Therefore, it is said that parity and charge conjugation symmetry are maximally violated by the weak interaction. The combination CP (and due to the CPT theorem therefore also T) is almost an exact symmetry of the weak interaction, but it is violated by a small phase in the charged-current interactions.

Continuous global symmetries

Apart from Lorentz invariance, the continuous global symmetries are not used as input in constructing the Standard Model, but appear as ‘left-over’ symmetries. They are therefore also known as accidental symmetries. We will focus only on the exact symmetries of the Standard Model here. There are also some approximate symmetries, which are violated only in small amounts, but they will not be discussed further in this thesis.

At the classical level, there are two exact global $U(1)$ symmetries of the Standard Model. These correspond to conservation of baryon number (B) and lepton number (L). In a theory with massless neutrinos, the lepton number of each family would be conserved individually, but massive neutrinos spoil these conservation laws through neutrino oscillations. Therefore only the total lepton number is a conserved quantity.

At the quantum level, these symmetries are no longer exact, since there are non-perturbative effects that violate both baryon and lepton number conservation. But the combination $B - L$ is still conserved in all these processes. This combination is therefore an exact continuous global symmetry of the Standard Model.

2.1.2 Electroweak symmetry breaking

The Standard Model gauge group described so far is only present at high energies. Below an energy of $\mathcal{O}(200 \text{ GeV})$, the $SU(2)_L \times U(1)_Y$ group is spontaneously broken

¹except for the (tiny) θ term

down to $U(1)_Q$. This phenomenon is called electroweak symmetry breaking, and it is triggered by the Higgs sector. The scalar potential of the Higgs doublet ϕ is given by:

$$\begin{aligned} V(\phi) &= \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \\ &= \frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \frac{\lambda}{4} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2. \end{aligned} \quad (2.12)$$

This expression is invariant under the $SU(2)_L \times U(1)_Y$ group. But the crucial point is whether the minimum of this potential is also invariant under this group. If this is not the case, the symmetry is broken. The minimum can be found by setting the first derivative of the potential with respect to the fields equal to zero:

$$\frac{\partial V}{\partial \phi_i} = \mu^2 \phi_i + \lambda \phi_i (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = 0. \quad (2.13)$$

These are the minimum equations of this potential.

The sign of λ is constrained by the restriction that the potential should be bounded from below. So only positive values of λ are allowed. But for μ^2 there is no such restriction. When $\mu^2 > 0$, all contributions to the potential are always positive, and the only way to satisfy the minimum equations for real values of ϕ_i is at the origin:

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (2.14)$$

But there is also the possibility that $\mu^2 < 0$. In that case the situation is different, since now there are both positive and negative contributions to the potential. Now there is another possibility to satisfy the minimum equations, namely by having:

$$\langle \phi_1 \rangle^2 + \langle \phi_2 \rangle^2 + \langle \phi_3 \rangle^2 + \langle \phi_4 \rangle^2 = \frac{-\mu^2}{\lambda} \equiv v_{EW}^2. \quad (2.15)$$

For simplicity we will use v instead of v_{EW} from now on. Using gauge invariance, the vacuum expectation value of ϕ can be rotated such that only one of the components is non-zero. The usual choice is setting:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (2.16)$$

This minimum breaks the $SU(2)_L \times U(1)_Y$ symmetry and leaves only the $U(1)_Q$ subgroup unbroken. This subgroup describes electromagnetic interactions, with the photon (γ) as its force carrier. The electromagnetic charge of a particle is obtained from the original quantum numbers using: $Q = T_3 + Y$, with T_3 the third component of the weak isospin.

The mass spectrum of the scalar sector is found by evaluating the second derivative of the potential at this minimum. This gives the mass matrix:

$$m_{ij}^2 = \left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\langle \phi \rangle} = \begin{pmatrix} \mu^2 + \lambda v^2 & 0 & 0 & 0 \\ 0 & \mu^2 + \lambda v^2 & 0 & 0 \\ 0 & 0 & \mu^2 + 3\lambda v^2 & 0 \\ 0 & 0 & 0 & \mu^2 + \lambda v^2 \end{pmatrix} \quad (2.17)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\lambda v^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We see that there are three massless scalars. These are the three Goldstone bosons resulting from the breaking of three generators of the $SU(2)_L \times U(1)_Y$ group. These massless scalar modes are absorbed by three gauge bosons, resulting in the massive Z and $W^\pm$ bosons. Their masses are given by:

$$m_Z^2 = \frac{1}{4}g^2v^2, \quad m_W^2 = \frac{1}{4}(g^2 + g'^2)v^2. \quad (2.18)$$

From measurements of these masses [5], the value $v = 246$ GeV can be extracted.

2.1.3 Loop effects

So far we have only looked at tree-level effects. While these will be the focus of most of this thesis, there are also some important aspects of the Standard Model that only arise after taking into account loop effects. Here we will briefly highlight one of those effects, which is the running of the coupling constants. In our discussion so far, we regarded the couplings in the Standard Model Lagrangian as constants, their values fixed. But this is only true at the classical level. Upon taking into account loop corrections, the couplings are energy dependent.

In Figure 2.1 (left) a tree-level coupling between a photon and two charged fermions is shown. The coupling is proportional to the electromagnetic coupling e . But when we go beyond tree level, there are new diagrams like Figure 2.1 (right) that also contribute to the coupling. So in this way there are corrections to the tree-level value of the coupling. The energy dependence of the couplings immediately follows from this example. When the fermions have more energy, the probability to emit a virtual particle increases, so there are more corrections to the coupling. This energy dependence of the coupling is called *running*.

Figure 2.1 The tree-level electromagnetic coupling of a charged fermion (left) and the one-loop correction to the coupling (right).

The running of the couplings can be captured in the β -functions. These differential equations describe the running of the couplings with energy scale μ , and they can be derived order by order from the loop diagrams that contribute to the couplings. For the Standard Model gauge couplings, the β -functions at one-loop order are:

$$\begin{aligned}\beta_{g_s} &= \mu \frac{\partial g_s}{\partial \mu} = -\frac{42}{96\pi^2} g_s^3 \\ \beta_g &= \mu \frac{\partial g}{\partial \mu} = -\frac{19}{96\pi^2} g^3 \\ \beta_{g'} &= \mu \frac{\partial g'}{\partial \mu} = \frac{41}{96\pi^2} g'^3\end{aligned}\tag{2.19}$$

Note that the β -function of g' has a different sign than the other two β -functions. So with increasing energy, the value of g' will increase, while the values of g_s and g will decrease. Furthermore, g_s will run more strongly than g . Together with the fact that at the scale $\mu = m_Z$ the hierarchy of the couplings is $g_s > g > g'$, this means that with increasing energy, the couplings move in the same direction. So it is possible that the couplings unify at a certain (high) energy scale. We will look further into this possibility later on.

2.2 Beyond the Standard Model physics

Next we take a look at some of the models that have been proposed to solve some problems of the Standard Model. In the introduction chapter we already noted some problems, now we will highlight a few popular mechanisms and (classes of) theories

that address these problems. We will show how some of these theories provide solutions, but also mention some of the new problems that are introduced by these models.

2.2.1 Neutrino masses

The masses of the neutrinos are one of the clearest reasons for the necessity of beyond the Standard Model physics. As mentioned before, the absence of right-handed neutrinos in the Standard Model prevents the existence of Yukawa couplings for the neutrinos, so they cannot get a mass in the same way as the other fermions. Since neutrino oscillations have been observed, neutrinos masses are necessary. One possibility to accommodate this is by simply adding right-handed neutrinos to the Standard Model.

But when right-handed neutrinos are added, another mechanism for the generation of masses is also possible. Since right-handed neutrinos are gauge singlets, a mass term of the form $\mathcal{L} = -m_R \bar{\nu}_R^c \nu_R$ is allowed. This is a Majorana mass term. While the Dirac masses, originating from the Yukawa couplings, are proportional to the Higgs vacuum expectation value (vev), there is *a priori* no clear mass scale for the Majorana mass of the right-handed neutrinos. When both Dirac masses and Majorana masses are present, the mass eigenstates are obtained by diagonalizing the mass matrix. In the basis $\nu = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$, this mass matrix is given by:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}. \quad (2.20)$$

For simplicity we will work with just one flavour of neutrinos. In that case, and assuming that $m_R \gg m_D$, the diagonalization of this matrix leads to the approximate eigenvalues: $m_1 \approx m_D^2/m_R$, $m_2 \approx m_R$. So one neutrino becomes very light, while the other is heavy. Note that the higher the scale m_R , the smaller the mass of the light neutrino. This behaviour is the reason why it is called the seesaw mechanism [27].

In Equation 2.20, there is a zero entry since there is no $m \bar{\nu}_L^c \nu_L$ term in the Standard Model. This would be a Majorana-type mass term for the left-handed neutrinos. This term violates lepton number conservation by 2 units. But since lepton number conservation is only an accidental symmetry of the Standard Model, violating this symmetry is not a big problem. However, the term in this form also violates the gauge symmetry of the Standard Model before electroweak symmetry breaking, and it is not possible to build a operator with mass dimension ≤ 4 that gives such a term after electroweak symmetry breaking using only SM fields. This can be cured by adding additional fields to the theory. By adding either a scalar triplet or a fermionic triplet, it is possible to generate a Majorana mass for the left-handed neutrinos after spontaneous symmetry breaking. In these scenarios, a right-handed neutrino is not

necessary. However, a seesaw-like behaviour is still present, albeit with a slightly different mass dependence. To distinguish the different seesaw scenarios, the inclusion of right-handed neutrinos is called type I, the usage of a triplet scalar type II, and adding a triplet fermion is type III. It is also possible that multiple mechanisms are present, which work together to generate the neutrino masses. This is the case in e.g. the left-right symmetric models which we will study in Chapter 4. For an overview of the seesaw mechanisms and how they are implemented in various BSM theories, see e.g. Ref. [28] and references therein.

The seesaw mechanisms provide an explanation for the smallness of the neutrino masses. These mechanisms are usually part of a larger theory, often addressing multiple problems of the Standard Model. Here we will list a few examples of such theories (not all containing the seesaw mechanism).

2.2.2 Theories with solutions to SM problems

One of the simplest extensions of the Standard Model is the two-Higgs-doublet model (2HDM) [29]. As the name suggests, the idea of this model is to extend the scalar sector by adding another Higgs doublet. There are several reasons why such a model could be interesting. First of all, the 2HDM allows for CP violation in the scalar sector. With only one Higgs doublet, a phase in the vev can always be rotated away by using a gauge transformation. But when there are two doublets that both get a vev, it is not possible in general to rotate away both phases at the same time. The remaining phase can contribute to CP violation. This is interesting, because additional CP violation is presumably necessary in order to explain the matter-antimatter asymmetry in the universe. Furthermore, the structure of the 2HDM also appears in more extended theories like supersymmetric extensions of the SM. So the 2HDM can provide an easier setting to analyse the Higgs sector of these theories.

Another popular class of BSM theories use supersymmetry (SUSY) [20] to extend the SM. The main idea behind SUSY is to add a symmetry that links bosons and fermions. When extending the SM with SUSY, a whole set of new particles is introduced that are linked to the SM particles. For each boson in the SM there is a fermionic superpartner and vice versa. In the unbroken version of the theory, these supersymmetric particles have the same mass as their partners. Since no supersymmetric particles have been observed so far, supersymmetry can only exist as a broken symmetry (at least at low energies). However, the scale at which this breaking should happen is unknown. For some popular phenomenological applications of SUSY, this breaking scale should not exceed the TeV scale by much, while for other applications, like string theory, this scale can be much higher.

Figure 2.2 The running of the gauge couplings in the Standard Model (left) and in a supersymmetric extension with supersymmetric particles appearing at 1 TeV (right).

One of the key motivations for supersymmetric extensions of the SM, and arguably the reason why it has been so popular over the last few decades, is that supersymmetry provides a solution to the hierarchy problem. This problem, the solution that SUSY provides, and the reason why interest in SUSY has decreased over the last few years will be discussed in more detail in Chapter 3. But (low-energy) SUSY provides solutions to other problems of the Standard Model as well. For example, if the lightest supersymmetric particle is stable, which is the case in many models, SUSY can provide a viable dark matter candidate. Furthermore, SUSY can aid in achieving gauge-coupling unification. When the running of the gauge couplings, as given in Equation 2.19, is plotted up to high energies, one obtains Figure 2.2 (left). In this figure, the following relations are used:

$$\alpha_1 = \frac{g'^2}{4\pi}, \quad \alpha_2 = \frac{g^2}{4\pi}, \quad \alpha_3 = \frac{g_s^2}{4\pi}, \quad (2.21)$$

and the value of $1/\alpha_i$ is plotted. Note that $1/\alpha_i$ is large enough (and therefore α_i is small enough) to have perturbative couplings over the full energy range. The figure shows that while the SM couplings converge at a scale around $10^{13} - 10^{16}$ GeV, they do not unify completely. But if one adds low-scale supersymmetry at the TeV scale, the running of the gauge couplings gets modified. The result of the running in this model is shown in Figure 2.2 (right). It is clear that now there is exact unification

(within experimental uncertainties).²

Another class of proposed BSM theories are the left-right symmetric models (LRSMs) [30,31]. These are models in which a symmetry between left- and right-handed particles is introduced. The gauge group of these models is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. There are numerous advantages to such an approach. First of all, it can explain C and/or P violation as low-energy effects through the spontaneous breaking of a symmetric theory at higher energy. This can be more attractive than the Standard Model, where C and P breaking is explicit. In addition, LRSMs provide a mechanism to allow for naturally light neutrino masses, through a type-I and type-II seesaw mechanism. These models will be studied in more detail in Chapter 4.

Achieving unification of the coupling constants is the main goal of Grand Unified Theories (GUTs). These theories propose that at high energies, there is only a single simple gauge group. So all SM particles have to be part of a representation of this simple group, and there is only one gauge coupling at high energy. In Figure 2.2 (right), the couplings all continue running separately past the point where they intersect. But in a GUT, the three separate couplings would be replaced by a single coupling at energies above this point. Aside from the aesthetic advantage of such a theory, it can also provide an explanation for the phenomenon of charge quantization, which is the observation that all elementary particles have electric charges which are multiples of one third of the electric charge. Common examples of GUTs are theories based on an $SU(5)$ [32] or $SO(10)$ [33] gauge group. We will take a closer look at the minimal non-supersymmetric $SU(5)$ model in Chapter 5.

In the case where low-energy supersymmetry is added around the TeV scale, exact unification seems to be possible. Therefore, supersymmetric GUTs are popular BSM theories. However, from the point of view of gauge-coupling unification, this is not strictly necessary. This is because Figure 2.2 (left) assumes that there are no new particles up to the high-energy scale. Any new particle (which is not a gauge singlet) will modify the running, and in that way can ensure that unification occurs, even without supersymmetry. In addition, heavy particles can lead to threshold corrections, which are jumps in the running of the gauge couplings. These corrections can further aid in achieving unification.

²The normalization of the $U(1)$ coupling g' is arbitrary, so as long as the other two couplings unify, it is always possible to ensure that the $U(1)$ coupling unifies at the same energy. But the choice that is used in this plot ($g' = \sqrt{\frac{5}{3}}g'_{SM}$) can be explained by some popular unification theories, which is not the case when trying to unify the SM couplings.

2.3 Extended scalar sectors

All the examples of BSM theories given above have a shared theme: they all contain extensions of the scalar sector. This is indeed a common feature of many BSM theories. The main reason for this is that many extensions of the Standard Model use extended gauge groups. So the symmetry at some high-energy scale is larger than the Standard Model gauge group. In order to end up with the Standard Model at low energy, this extended gauge group needs to be broken. The common way to do this is to use the Higgs mechanism, just like for electroweak symmetry breaking. But for this a new scalar field is necessary. The inclusion of this new scalar field leads to the extended scalar sector.

Also from the experimental side, extensions of the scalar sector are very interesting. The scalar sector is experimentally the least understood sector of the Standard Model. The Higgs boson has only been discovered as recently as 2012, and many properties of the Higgs are still not determined experimentally. So the scalar sector still allows for large deviations from Standard Model predictions, contrary to the other sectors of the Standard Model.

2.3.1 Solutions to old problems

There are many ways in which theories with extended scalar sectors can help solve some of the outstanding problems of the Standard Model. An example is the amount of CP violation in the Standard Model, as was shown for the 2HDM. So by extending the scalar sector, there could be additional sources of CP violation, which might help explaining the matter-antimatter asymmetry observed in the universe.

And there is another reason why the scalar sector can be important in explaining the matter-antimatter asymmetry in the universe. In the Standard Model, the electroweak phase transition is a continuous cross-over [34]. However, if the Standard Model is extended around the electroweak scale, the transition might turn into a first-order phase transition, where there is an abrupt transition between the (local) minimum at $\phi = 0$ and the (global) symmetry-breaking minimum at $\phi \neq 0$. This is a necessary ingredient for the generation of a matter-antimatter asymmetry through the process of baryogenesis. Since the scalar sector is the least constrained sector at low energy, models with extended Higgs sectors are an interesting candidate for generating this first-order phase transition [35, 36]. Furthermore, it might be possible to observe the remnants of such a transition with the next generation of gravitational-wave detectors [37]

Also in the search for dark matter, theories with extended Higgs sectors can play

an important role. A large class of dark matter models work by means of a Higgs portal [38]. In these models, a new set of particles is introduced that only couple with the Standard Model through the scalar sector. In this way, the scalar sector can be crucial in understanding the nature of dark matter.

Finally, models with extended Higgs sectors can help stabilize the scalar potential. While the Standard Model Higgs potential is only metastable [6, 7], new physics can change the behaviour of the scalar potential. The exact impact is heavily model dependent, but it is possible to increase the stability of the potential by adding fields to the scalar sector.

2.3.2 Appearance of new problems

While extensions of the scalar sector can solve some of the problems of the Standard Model, they can also introduce new problems. For example, while extended scalar sectors can aid in addressing the vacuum stability of the scalar potential, they can also lead to new minima of the potential. The global minimum of the full scalar potential needs to be a neutral vacuum that leads to the correct symmetry breaking pattern. Making sure that the potential of a model satisfies these criteria can be a difficult procedure, and it can put severe constraints on the parameters of the model [39–41].

In theories like GUTs, where the strong and electroweak force are unified, there is the possibility for processes that can mediate proton decay.³ In the Standard Model, there are no operators that can mediate proton decay, which offers an explanation for the fact that the process has never been observed. This lack of observation leads to a very high bound on the lifetime of the proton, of the order of $\tau_p \sim 10^{32}$ years (the exact bound depends on the decay channel). This has led to ever-increasing bounds on the mass scale of the particles that can mediate proton decay. One of the earliest proposed GUTs, namely the minimal model based on the $SU(5)$ gauge group, has even been ruled out by these constraints [42].

Another problem that appears in many GUT-like theories is the doublet-triplet splitting problem. This problem will be analysed in detail in Chapter 5, but the origin of the problem is the combination of two observations. First there is the observation that in GUTs all particles have to be part of a representation of the GUT gauge group. Since the strong and electroweak force are unified, these representations contain particles with both a strong and a weak charge. This is also the case for the representation containing the Higgs boson, which contains both strongly interacting particles and the SM-like Higgs doublet. The second observation is that these strongly interacting

³Although this is not just a problem of the scalar sector. Also vector bosons can mediate proton decay.

particles can mediate proton decay, so they have to be heavy. But the doublet has to remain light. Accommodating both heavy and light particles in a single representation is possible, but might require fine-tuning, as will be discussed in detail later.

2.3.3 Experimental constraints on scalar sectors

While the scalar sector might be the least constrained sector of the Standard Model, that does not mean there are no restrictions on how the scalar sector can be extended. Besides proton decay, one of the most stringent constraints is the ρ -parameter [21]. This electroweak observable is defined as:

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \quad (2.22)$$

with θ_W the weak mixing angle. Alternatively, this parameter can also be written in terms of the vacuum expectation values v_i of the different scalar fields, when written in terms of Standard Model representations [5]:

$$\rho = \frac{\sum_i [T_i(T_i + 1) - Y_i^2] v_i^2 c_i}{\sum_i 2Y_i^2 v_i^2}, \quad (2.23)$$

with $c_i = 1/2$ (1) for real (complex) representations, T_i the weak isospin and Y the hypercharge. A tree-level calculation in the Standard Model gives $\rho = 1$. Quantum corrections give a small deviation from this value, but the experimental result leaves very little room for further deviations: $\rho_{exp} = 1.00038 \pm 0.00020$ [5]. This result can be used to constrain models with extended Higgs sectors. Only representations that satisfy $T(T + 1) - 3Y^2 = 0$ are allowed to get a large vacuum expectation value. Other representations can get a vev, but such a vev is constrained to be much smaller than the electroweak vev.

Another stringent constraint on extended scalar sectors is the observed suppression of flavor-changing neutral currents (FCNCs). In the Standard Model, there are no processes where the flavour of a fermion can be changed by a neutral interaction at tree level. The process can occur in loop diagrams, but even then the interaction is suppressed due to the GIM mechanism [43]. The Standard Model guarantees the absence of these interactions at tree level because there is only one Higgs doublet. This ensures that when the fermionic mass matrices are diagonalized, the Yukawa interactions and the interactions of fermions with the Z boson and photon are diagonalized as well. This is in general not the case when the scalar sector is extended. Already in the 2HDM, the addition of another Higgs doublet spoils the suppression of FCNCs in general. In models with new physics around the TeV scale, often a symmetry has to be imposed to ensure the absence of FCNCs at tree level. For example in the case of

the 2HDM, this can be done by imposing a discrete $\mathbb{Z}_2$ symmetry, where one of the two doublets is charged under the symmetry, while the other one is not. In this way, each fermion can only couple to one of the scalar doublets, and diagonalization of the Yukawa interactions is guaranteed again [44].

The final set of constraints we will mention are the electric dipole moments (EDMs), which put constraints on CP violation in scalar sectors. They are especially useful measurements since the SM values are extremely small, as e.g. the electron EDM only appears at the four-loop level [45]. If new sources of CP violation in extended scalar sectors appear at lower loop level, they would give a clear deviation from the SM result. The strongest EDM constraints on subatomic particles are on the neutron and the electron [5]:

$$\begin{aligned} |d_n| &< 1.8 \times 10^{-26} \text{ e cm}, \\ |d_e| &< 1.1 \times 10^{-29} \text{ e cm}, \end{aligned} \tag{2.24}$$

at 90% confidence level. These limits are still far away from the SM values however, with predictions of $d_n \approx 10^{-31} \text{ e cm}$ for the neutron EDM [46], and $d_e \approx 10^{-38} \text{ e cm}$ for the electron EDM [47]. So there is still a large range available for BSM CP violation to appear in EDM experiments.

Chapter 3

Fine-tuning

In the previous chapters we introduced some arguments why physics beyond the Standard Model is expected. But as we mentioned in Chapter 1, extending the Standard Model with heavy particles can lead to a fine-tuning problem for the mass of the Higgs boson. We also mentioned the existence of other fine-tuning problems at tree-level for theories with extended scalar sectors. In this chapter we will take a closer look at fine-tuning. We first look in more detail at the hierarchy problem, since it is a prime example of fine-tuning in particle physics which highlights some important aspects of the fine-tuning discussion. Another reason to discuss this problem, is that it will be investigated in the context of the effective potential in Chapter 6. After this discussion, we will look into some theories that provide solutions to the hierarchy problem, with supersymmetry as a prime candidate. Then we turn to fine-tuning at tree-level, since this is relevant for many beyond the Standard Model theories with extended scalar sectors. Finally, we discuss how fine-tuning can be quantified, focussing on two measures of fine-tuning, as they will be investigated in detail in the following two chapters.

3.1 The Hierarchy Problem

The hierarchy problem is a problem that arises when a heavy particle that couples to the Higgs boson is added to the Standard Model. To see what the problem is exactly, and to be able to discuss some of the common arguments related to the hierarchy problem, we explicitly calculate the contribution from a relevant Feynman diagram. We will only work through the case where a heavy fermion is added, but the conclusions for adding a heavy scalar or vector boson are the same.

The relevant Feynman diagram for our discussion is the correction to the Higgs mass δm_h^2 shown in Figure 3.1. Evaluating this diagram, with the momenta p and k as indicated in the diagram, y_f the Yukawa coupling of the fermion and m_f the mass of the fermion, gives:

Figure 3.1 The 1-loop diagram contributing to the mass of the Higgs boson mass due to a heavy fermion f .

$$\begin{aligned}
 -i(\delta m_h^2) &= - \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[\frac{iy_f}{\sqrt{2}} \frac{i(\not{p} - \not{k} + m_f)}{((p-k)^2 - m_f^2)} \frac{iy_f}{\sqrt{2}} \frac{i(-\not{k} + m_f)}{(k^2 - m_f^2)} \right] \\
 &= - \frac{y_f^2}{2} \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr}[\not{k}^2 - 2\not{k}m_f - \not{p}\not{k} + \not{p}m_f + m_f^2]}{((k - px)^2 - \Delta)^2} \\
 &= - \frac{y_f^2}{2} \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4} \frac{4(k^2 - p^2 x(1-x) + m_f^2)}{(k^2 - \Delta)^2},
 \end{aligned} \tag{3.1}$$

with $\Delta = -p^2 x(1-x) + m_f^2$. In the last line, the integration variable is shifted from $k^\mu \rightarrow k^\mu - p^\mu x$, terms with an odd power of k are eliminated because the integral is symmetric, and the trace is evaluated in Dirac space. This expression contains both quadratic and logarithmic divergences in the UV region, so we have to choose a regularization and renormalization scheme. We will use dimensional regularization to regularize the integral, where we work in $d = 4 - \epsilon$ dimensions and introduce a mass scale μ to ensure that the mass dimension of the integral remains four. This gives:

$$-i(\delta m_h^2) = - \frac{3iy_f^2}{8\pi^2} \left[\left(\frac{2}{\epsilon} - \gamma + \log 4\pi + \frac{1}{3} \right) \int_0^1 dx \Delta - \int_0^1 dx \Delta \log \frac{\Delta}{\mu^2} \right] + \mathcal{O}(\epsilon). \tag{3.2}$$

Now we are left with a divergent and a finite contribution to the mass of the Higgs boson. Using the $\overline{\text{MS}}$ scheme [48], the divergent part along with some common factors is absorbed by the counterterm:

$$(\delta m_h^2)_{\text{counter}} = - \frac{3y_f^2}{8\pi^2} \left[\left(\frac{2}{\epsilon} - \gamma + \log 4\pi \right) \int_0^1 dx \Delta \right] \tag{3.3}$$

After the divergent part has been absorbed, there are only finite contributions left, so we can take the limit $\epsilon \rightarrow 0$. The renormalized contribution is:

$$(\delta m_h^2)_{\text{ren.}} = -\frac{3y_f^2}{8\pi^2} \int_0^1 dx \Delta \log \frac{\Delta}{\mu^2}. \quad (3.4)$$

The momentum-squared p^2 of the on-shell Higgs boson will be m_h^2 , so we can neglect it with respect to the heavy fermion mass m_f^2 . Therefore, we can take $\Delta \approx m_f^2$, and we find:

$$(\delta m_h^2)_{\text{ren.}} \approx -\frac{3y_f^2}{8\pi^2} m_f^2 \log \frac{m_f^2}{\mu^2}. \quad (3.5)$$

So the finite contribution to the Higgs mass is proportional to m_f^2 . This tells us that at next-to-leading order, there is a correction to the squared mass of the Higgs boson which is proportional to the squared mass of the new heavy particle. While this might seem like a fairly unremarkable observation, it has far reaching consequences. These corrections have been a source of debate for many decades, and have been a prime motivation for many searches for physics beyond the Standard Model.

The important consequence of this observation is that if there is new physics that couples to the Higgs boson at some high-energy scale, the Higgs mass will naturally be pushed up to this scale. And this does not only happen when adding a heavy fermion. Also for heavy scalars and vector bosons that couple to the Higgs, there are corrections to the Higgs mass of the order of the mass of the heavy particle. So for all the BSM theories we have discussed so far, it means that the natural mass scale of the Higgs boson is the highest scale present in the model. However, the observed Higgs boson has a mass of only 125 GeV. To put it in an equation, one can write the physical mass as the sum of the (renormalized) tree-level and one-loop contributions:

$$m_{h,\text{phys.}}^2 = m_{h,\text{tree}}^2 + (\delta m_h^2)_{\text{ren.}} \quad (3.6)$$

So with a value of $m_{h,\text{phys.}} = 125$ GeV on the left-hand side and a large value on the right-hand side ($(\delta m_h^2)_{\text{ren.}} \sim m_f^2$), there needs to be a large cancellation on the right-hand side of the equation in order to result in a small value for the measured Higgs mass. Note that the cancellation can occur either between the tree-level and the one-loop contribution or between different loop contributions, e.g. contributions from different particles. But either way, without any mechanism to explain the small mass, a cancellation to an extremely high precision between independent terms is necessary. Since there is no mechanism in the Standard Model that can explain such a large cancellation, fine-tuning is necessary to explain the small value of the Higgs mass. This is called the hierarchy problem.

3.1.1 Important remarks

Some remarks about the hierarchy problem are in order now. The first one is about the role of the regulator and the renormalization scheme. Sometimes a cutoff regulator is used to illustrate the hierarchy problem, and the problem is then stated as the appearance of a quadratic divergence in the one-loop correction to the Higgs mass. And indeed, when using this method of regularization, an explicit calculation shows that one obtains an expression of the form [49]:

$$\delta m_h^2 \sim \frac{y_f^2}{16\pi^2} \Lambda^2 + \text{subleading terms}, \quad (3.7)$$

with Λ the cutoff scale. But one has to be careful with drawing conclusions on the hierarchy problem with this regulator. The scale Λ can be interpreted in two different ways. On the one hand, we can view Λ as a regulator, so as just a bookkeeping tool used to absorb the divergences later on. In this scenario, Λ is sent to infinity eventually, and the divergences arising in this way are absorbed in the counterterms. After renormalization, the Λ -dependence is completely absent from the expression, so Λ cannot be the origin of the hierarchy problem. The problem purely arises from the quadratic dependence on the heavy scale m_f which is part of the ‘subleading terms’ in Equation 3.7.

However, one can also consider the theory as an effective field theory (EFT), with Λ the scale of some (unspecified) new physics, which can be of the same scale as m_f , or even larger. In the latter case, the heavy fermion f still gives a large contribution through the finite terms which appear in the subleading terms in Equation 3.7. But the even larger scale Λ gives a larger contribution. So now Λ can be seen as the origin of the hierarchy problem, since it is an actual physical scale in the theory.

The next remark is that the Standard Model by itself does not have a hierarchy problem. This is also related to the observation that Λ can only be seen as the origin of the hierarchy problem when it denotes the existence of some new high-scale physics. If there is no new physics that couples to the Higgs boson at high energies, there are no large finite corrections and the fine-tuning problem is not present. When considering only Standard Model particles, the largest contribution comes from the top quark, as it has a Yukawa coupling of $\mathcal{O}(1)$, and is heavier than the Higgs boson. However, the top quark is not much heavier than the Higgs boson, and its contribution to the Higgs mass is suppressed by a loop factor. So the loop contribution from the top quark is smaller than the observed Higgs mass, and there is no large cancellation necessary to get the correct value.

One might wonder why this problem only appears for the Higgs boson, and not for other Standard Model particles. It turns out that only for scalar particles, the

correction to the squared mass depends quadratically on the mass of the heavy particle. The reason why other particles do not have this feature is due to symmetry. For example for the charged fermions in the Standard Model, setting one of their masses to zero increases the symmetry by adding a chiral symmetry. This symmetry is conserved by quantum corrections, which implies that the corrections have to be proportional to the fermion mass. Therefore, there is at most a logarithmic dependence on the heavy scale instead of a quadratic dependence. The hierarchy problem is therefore much less severe for fermions. A similar argument can be made for the gauge bosons, where the gauge symmetry is restored in the limit where the gauge bosons masses are set to zero. But scalars do not have this property. Setting the mass of the Higgs boson to zero does not increase the symmetry,¹ so there can be large corrections due to loop effects.

While the hierarchy problem is considered a problematic situation by many physicists, it must be noted that it is not an internal inconsistency of the theory. It could be the case that there is no mechanism that ensures a light Higgs boson, and that the parameters have very specific values that result in a light Higgs. However, as mentioned in Chapter 1, the hierarchy problem is more than ‘just’ a fine-tuning problem. If there is no mechanism to ensure a light Higgs, this signals the breakdown of decoupling, which is a feature of nature that we are very much used to, since it works so well in e.g. the EFT approach. Without decoupling, the details of the high-energy physics can directly influence the low-energy physics. While it is not guaranteed that decoupling holds at all energy scales, its success so far makes it undesirable to set aside easily.

3.1.2 Proposed solutions

There are many proposed solutions to the hierarchy problem. The most notable of those is supersymmetry (SUSY). As discussed in Chapter 2, extending the SM with supersymmetric particles adds many interesting features to the theory. SUSY adds a bosonic partner for each fermionic particle in the Standard Model and vice versa. At energies where supersymmetry is an exact symmetry, there is a clear connection between the Standard Model particles and their supersymmetric partners: both have the same couplings and the same masses. But the loop contributions to the Higgs mass from bosons and fermions have opposite sign. So the contribution of each particle is cancelled exactly by the contribution of its supersymmetric partner. In this way, SUSY can solve the hierarchy problem.

But if SUSY is broken, the masses of particles and their supersymmetric partners can be different. Then there is no complete cancellation between the contributions

¹In a non-interacting theory the symmetry is increased, since a shift symmetry appears in the limit $m_h \rightarrow 0$ [8]. But since the Standard Model contains interactions with the Higgs boson, this argument does not apply.

of supersymmetric partners, so the hierarchy problem is not solved exactly. Since no supersymmetric particles have ever been observed, we can conclude that SUSY is not an unbroken symmetry at currently accessible energy scales. Following the estimate made in Chapter 1, where the natural breakdown scale of the Standard Model was estimated to be $\Lambda_{SM} \lesssim 1$ TeV, the supersymmetric particles cannot be too heavy in order to be a proper solution to the hierarchy problem. If the scale of supersymmetry breaking is too high, a weaker form of the hierarchy problem appears, since then there is no explanation for the high sensitivity of the Higgs mass to the scale of supersymmetry breaking. The problem introduced in this way is called the little hierarchy problem. It is considered a smaller problem, since further quadratic corrections do not appear above the scale of SUSY breaking. The lack of detection of supersymmetric particles creates some tension with this decoupling bound. However, it has been shown [50] that it is still possible to construct a low-energy theory of supersymmetry that does not introduce a little hierarchy problem and that can explain the observed relic density of dark matter.

The situation becomes more problematic when trying to embed such a low-energy theory into a natural high-energy theory. In doing so, relations are imposed at the high scale (depending on the exact model) that have an effect at low energy. These relations can lead to an increase in the amount of fine-tuning as they link different sectors, so there are more constraints on the high-scale parameters. Furthermore, there will be effects from the RG running from the UV scale to approximately the TeV scale. Although the exact results are very model dependent, it has been shown [51] that this can greatly restrict the applicability of high-energy SUSY models as natural extensions of the SM.

An alternative way to address the hierarchy problem is by using composite Higgs models [52]. In these models, the Higgs boson is not an elementary scalar, but a composite particle consisting of a strongly interacting fermion-antifermion pair. This composite structure should emerge at higher energies. Since fermion masses are only logarithmically sensitive to new mass scales, this can solve the hierarchy problem. However, with current limits these models also suffer from fine-tuning problems of their own [53, 54]. Therefore, the minimal composite Higgs models are considered to be ruled out.

This lack of detection, so far, of the most popular ways to address the hierarchy problem has led to an increased interest in alternative solutions. Examples are clockwork models [55–57] and relaxion models [58, 59]. But also non-minimal versions of supersymmetric extensions and composite Higgs models are being investigated with the goal to find a natural explanation of the hierarchy problem within current constraints.

Note that the idea of the multiverse mentioned in Chapter 1 can also be used to ‘justify’ a small Higgs mass. If there is a multitude of universes, all with different values for the fundamental parameters, there is a chance that one of them has a Higgs mass much smaller than its natural mass scale, and we might live in such a universe. Note that the problem with the breakdown of the decoupling approach is not addressed in this way, but it is simply accepted.

There is another consequence of the lack of detection of new physics that addresses the hierarchy problem. There has been an increasing interest in investigating a whole other class of BSM theories that do not necessarily address the hierarchy problem, like LRSMs, non-supersymmetric GUTs (and GUT-like models such as flipped $SU(5)$ [60, 61]), and trinification models (based on the gauge group $SU(3) \times SU(3) \times SU(3)$) [62, 63]. Note that all these theories have extended scalar sectors. Interestingly, these theories often suffer from fine-tuning problems that appear at tree-level already. Now the question is whether these fine-tuning issues should be discarded, as the hierarchy problem is still present anyway, or whether they should be taken seriously. In this thesis, we will take the second approach, since the origin of the tree-level fine-tuning in these BSM theories is often completely different from the fine-tuning related to the hierarchy problem (as will be discussed below). In our opinion, it is important to understand the origin of this fine-tuning, and that the arguments that are being used to assess the amount of fine-tuning in a theory are well understood.

3.2 Tree-level fine-tuning

So far, we have looked in detail at the hierarchy problem. This is a problem where loop contributions from heavy particles are expected to have a large impact on the mass of the Higgs boson, and fine-tuning might be necessary to keep the Higgs boson light. The central question in that case is: do the parameters of the model (e.g. couplings, mass parameters and/or vevs) need to be fine-tuned in order to reproduce the observed phenomenology? And it is this same question that lies at the origin of tree-level fine-tuning. But there are two major differences compared to the hierarchy problem. First of all, this fine-tuning does not appear at the loop level, but already in constructing the tree-level theory. Secondly, the origin of the fine-tuning is now not the mass of the Higgs boson, but the minimum equations. These equations describe the minimum of the scalar potential and relate the parameters of the model to each other. If large cancellations appear in these equations, this can be a sign of fine-tuning. The Standard Model scalar sector consists of just one scalar doublet, resulting in only one non-trivial minimum equation (cf. Equation 2.15):

$$v^2 = \frac{-\mu^2}{\lambda}. \quad (3.8)$$

This equation cannot lead to fine-tuning, as there is only one term of the right-hand side of the equation, so there is no room for cancellations. But when multiple scalar fields, and more importantly multiple vevs, are present, the minimum equations become a coupled system of equations that can lead to fine-tuning.

There are actually two ways to approach the amount of fine-tuning in such a scenario, both of which will be examined in detail in the remainder of this thesis. In the first approach, which will be covered in Chapter 4, a hierarchy in the vevs of the different scalar fields will be imposed. Taking this hierarchy for granted, the question is then whether the remaining parameters of the model (couplings and mass parameters) are sensitive to this hierarchy, and if fine-tuning is necessary to ensure natural parameters.

The other approach starts with a theory at a high scale, where all the mass parameters of the different fields have a value of the order of the high scale. This theory then has to develop a hierarchy in vevs in order to accommodate both the breaking of the extended gauge group at high energy and then the electroweak symmetry breaking at a much lower scale. The problem of developing such a hierarchy in scales, and then maintaining this hierarchy in the presence of loop corrections, are together called the gauge hierarchy problem [64–66]. In Chapter 5 we will look into the first aspect of this problem in the context of the minimal non-supersymmetric $SU(5)$ GUT.

While fine-tuning arguments have been used often in the construction of BSM theories, the usage of these arguments is not universally accepted. The most common criticism of using fine-tuning arguments is related to the fact that the parameter values of a BSM theory are unknown. So in order to make predictions, one has to make some kind of assumption for these parameters. The way this is usually done is by sampling values from a chosen distribution. Both the predictions of the theory and the amount of fine-tuning will depend on the choice for this distribution [67, 68]. Since we only have our own universe to do measurements on, we cannot make definitive statements on how ‘likely’ certain parameter values are. Therefore some people argue that arguments which depend on the choice of this distribution are not meaningful. In this thesis, we will assume that there are no special values for parameters, and we will use generic values for the parameters, imposing only the constraint that coupling constants should be natural, i.e. have values of $\mathcal{O}(1)$. Most importantly, our results do not depend strongly on the exact distribution that is chosen, as long as the parameters are of $\mathcal{O}(1)$.

Finally we highlight the notion of *technically-natural fine-tuning* that appears in the literature (see e.g. Refs. [69, 70]). While it seems like a contradiction in terms using the definitions in this thesis, this notion describes a situation where fine-tuning has to be imposed at tree level, but once this is imposed, the quantity is stable under radiative

corrections. So the radiative corrections are proportional to the (small) value itself. This is still deemed problematic, but generally considered to be a smaller problem than the situation where radiative corrections introduce an additional fine-tuning problem (like the hierarchy problem).

3.3 Quantifying fine-tuning

In order to compare the amount of fine-tuning between different models, or between different parameter points in a model, one can quantify the amount of fine-tuning. This is done using a number Δ (note that this is a different Δ from the one used in section 3.1). The value of Δ indicates the amount of fine-tuning, with a high value indicating a large amount of fine-tuning. A rough way to interpret Δ is that the (base-10) logarithm of the value denotes up to how many decimal places at least one parameter of the model needs to be specified in order to satisfy the given constraints. So a value of $\Delta = 10^4$ indicates that at least one of the parameters needs to be specified up to 4 decimal places.

In order to calculate Δ , one needs a way to convert a model point into a number that is related to the amount of fine-tuning in the model. This is done using a fine-tuning measure. In this thesis we will investigate two such measures: the Barbieri-Giudice measure [71, 72] and the Dekens measure [73].²

The Barbieri-Giudice (BG) measure was introduced in the 1980s to quantify the amount of fine-tuning in supersymmetric theories. But the measure is more general than that, and can be used to measure fine-tuning in any theory. The idea behind the measure is to look at an observable O (the mass of the Z boson in the original paper by Barbieri and Giudice), and write it in terms of the input parameters p_i of the model. Then the measure is calculated in the following way:

$$\Delta_{BG} = \max_i \Delta_{BG}^i = \max_i \left| \frac{\partial \log O}{\partial \log p_i} \right| = \max_i \left| \frac{p_i}{O} \frac{\partial O}{\partial p_i} \right|. \quad (3.9)$$

By using the logarithmic derivative, two effects are taken into account: both the size of each parameter with respect to the observable is computed, and the effect of a variation in p_i on the value of O is determined. These two contributions are combined to calculate the fine-tuning. It is important to combine these two effects, since together they determine how sensitive the observable is to the parameter.

²There are plenty of other fine-tuning measures in the literature that e.g. try to improve on the BG measure [74, 75] or use Bayesian statistics to measure fine-tuning [76–79]. These measures differ from the measures we investigate in some of the details, but they generally capture the same kind of tuning.

Imagine a situation where a parameter p_i is much larger than the observable O . This could indicate fine-tuning, as a large contribution from p_i needs to be cancelled by another term to result in a small value for the observable. This explains the term p_i/O in the fine-tuning measure. But if the large parameter only has a small effect on O , e.g. by being multiplied by a small parameter, the measure should give a smaller result. Because in that case, the value of O depends less strongly on the exact value of p_i , and the actual contribution of p_i to O is decreased. That is why also the derivative is taken into account, so that when the observable does not feel the effects of a variation in the parameter very strongly, the amount of fine-tuning decreases.

In most cases we will encounter, the observable is a polynomial in the parameters of the theory. So we have for example:

$$O(\lambda_1, \lambda_2, \mu_1, \mu_2) = \lambda_1 \mu_1^2 + \lambda_2 \mu_2^2. \quad (3.10)$$

Then the result of the BG measure would be:

$$\Delta_{BG} = \max \left\{ \left| \frac{\lambda_1 \mu_1^2}{\lambda_1 \mu_1^2 + \lambda_2 \mu_2^2} \right|, \left| \frac{2\lambda_1 \mu_1^2}{\lambda_1 \mu_1^2 + \lambda_2 \mu_2^2} \right|, \left| \frac{\lambda_2 \mu_2^2}{\lambda_1 \mu_1^2 + \lambda_2 \mu_2^2} \right|, \left| \frac{2\lambda_2 \mu_2^2}{\lambda_1 \mu_1^2 + \lambda_2 \mu_2^2} \right| \right\}, \quad (3.11)$$

where the input parameters are taken as $p_i = \{\lambda_1, \mu_1, \lambda_2, \mu_2\}$. So we see that in the case of a polynomial relation, this measure simply compares the size of each contribution to O to the actual value of O . If a contribution is much larger than the value of O , there needs to be a cancellation with the other contribution. This would be fine-tuning. The measure can easily be extended to take into account multiple observables:

$$\Delta_{BG} = \max_{i,j} \Delta_{BG}^{i,j} = \max_{i,j} \left| \frac{\partial \log O_j}{\partial \log p_i} \right|. \quad (3.12)$$

Note that there are also other ways to combine the individual results Δ_{BG}^i or $\Delta_{BG}^{i,j}$ into a final Δ_{BG} . For example, one can define (see e.g. Ref. [80]):

$$\Delta_{BG} = \sqrt{\sum_i (\Delta_{BG}^i)^2}. \quad (3.13)$$

We will not use this definition, as it can artificially increase the measured amount of fine-tuning. Looking at the example in Equation 3.10, note that if there is one term much larger than the observable, the other term has to be large as well. When using this alternative definition of the fine-tuning measure, both these terms are counted, while the first definition only counts the largest contribution. While there are situations where this alternative approach gives more accurate results, taking the maximum value more accurately measures the amount of fine-tuning in the situations we will encounter.

The Dekens measure works in a way that is similar to the BG measure, but it has a different starting point. While the BG measure looks at an observable in terms of the parameters of the theory, the Dekens measure looks at the relation between the parameters. The minimum equations are the starting point for this measure, as they provide such a relation. To apply the measure, first the set of parameters needs to be divided in two: the dependent parameters q_j (as many as there are minimum equations), and the parameters p_i on which they depend. Then the Dekens measure is calculated in a way similar to the BG measure:

$$\Delta_D = \max_{i,j} \left| \frac{\partial \log q_j}{\partial \log p_i} \right| = \max_{i,j} \left| \frac{p_i}{q_j} \frac{\partial q_j}{\partial p_i} \right|. \quad (3.14)$$

There are multiple ways to choose the q_j . In the original paper [73], an additional maximization was used to determine the total amount of fine-tuning: $\Delta_{D,\text{tot}} = \max_{q_j} \Delta_D$. So all combinations for the set of q_j are checked and the combination that gives the maximum result is used as the total amount of fine-tuning in the model. The questions of which kind of fine-tuning is measured in this way, and what the differences are between the BG measure and the Dekens measure, will be investigated in the context of two specific BSM theories in the next chapter.

Finally it is important to understand the way in which a result for the amount of fine-tuning should be interpreted. First of all, there is no clear upper limit for which amount of fine-tuning is acceptable. Barbieri and Giudice used a limit of $\Delta < 10$ in their original paper, but in the literature many different limits are used. It is up to personal taste to determine if a result of e.g. $\Delta \approx 100$ is still considered natural or not. In addition, there can be ambiguity in applying a fine-tuning measure. We already saw that there are different ways of combining the individual Δ^i , which will give a different result. But there are more sources of ambiguity. Using different definitions for the input parameters p_i will lead to different value for Δ . An example of this is the question whether one should use the mass or the squared mass as an input parameter. This change leads to a factor two difference in the value of Δ . And using a different observable can also give a different value for Δ . Furthermore, in some cases the normalization of parameters can affect the amount of fine-tuning. A consequence of this is that it is impossible to compare exact fine-tuning results between different models. It is only possible to draw meaningful conclusions when the amount of fine-tuning is seen as an order-of-magnitude estimate. On the other hand, when comparing the amount of fine-tuning for different points in the same model, it is easier to make a comparison, since the same conventions are used for all points.

Chapter 4

Analysis of fine-tuning measures in models with extended Higgs sectors

The two fine-tuning measures that were discussed in Chapter 3 will be investigated in this chapter¹ in two models with extended Higgs sectors. The main goal of this chapter is to investigate a claim in the literature that several popular left-right symmetric models (LRSMs) need a large amount of tree-level fine-tuning, which can be as high as $\Delta \approx 10^{20}$ [73]. In order to investigate this, we start by investigating fine-tuning in the two-Higgs-doublet model (2HDM). This popular Standard Model extension has many interesting features, like the possibility for CP violation in the scalar sector and the resemblance to the scalar sector of the minimal supersymmetric Standard Model (MSSM). For this study, we use the model because of its relative simplicity: all the relevant expressions can be analysed both analytically and numerically. This makes it easier to investigate the different fine-tuning measures in detail. We later apply the conclusions from this study to the more involved LRSM.

Throughout this chapter, following Ref. [73], we will impose that the vacuum expectation values (vevs) of the scalar fields have a certain hierarchy. Furthermore, our discussion will only concern tree-level fine-tuning. We will not look into the hierarchy problem and possible other fine-tuning problems that can appear due to loop effects.

The structure of this chapter is as follows. First we discuss fine-tuning in the 2HDM in Section 4.1. Here we draw several conclusions on how the Dekens measure should be applied, and how the Dekens measure and BG measure compare. Then we apply those lessons to the more involved LRSMs in Section 4.2. While most of the conclusions from the 2HDM can also be applied to the LRSM, there are a few complications specific to the LRSM that need to be discussed separately. We conclude in Section 4.3.

¹This chapter is based on Ref. [81]

4.1 Fine-tuning study of the Two Higgs Doublet Model

The two-Higgs-doublet model (2HDM) is one of the simpler extensions of the Standard Model. Its relative simplicity makes it possible to find analytic expressions for the quantities of interest, while at the same time having a rich enough structure to be interesting from a fine-tuning perspective. In order to investigate the amount of fine-tuning in the model, we will impose a hierarchy in the vevs. But before analyzing the different fine-tuning measures, we will first discuss the essentials of the model.

4.1.1 The scalar sector of the 2HDM

The scalar sector of the 2HDM is constructed by adding a doublet to the Standard Model scalar sector [44]. The doublets are defined as:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}. \quad (4.1)$$

In order to simplify our discussion, we demand CP invariance and impose a $\mathbb{Z}_2$ symmetry on the potential which transforms the doublets according to: $\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$. Under these constraints, the scalar potential has the form:

$$V = -\mu_1^2 A - \mu_2^2 B + \lambda_1 A^2 + \lambda_2 B^2 + \lambda_3 C^2 + \lambda_4 D^2 + \lambda_5 AB, \quad (4.2)$$

where the invariants A, B, C and D are defined as:

$$\begin{aligned} A &= \Phi_1^\dagger \Phi_1, \\ B &= \Phi_2^\dagger \Phi_2, \\ C &= \frac{1}{2} \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right), \\ D &= \frac{1}{2i} \left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1 \right). \end{aligned} \quad (4.3)$$

We are interested in the minimum of this potential, so we need to introduce vacuum expectation values for the two doublets. In order to avoid charged vacua, we will set:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (4.4)$$

In the most general 2HDM, the two vevs can have an imaginary part. One of those can be rotated away using gauge invariance. The remaining phase is physical, so it can lead to CP violation. To simplify our discussion, we will set this phase to zero.

4.1 Fine-tuning study of the Two Higgs Doublet Model

Using this vacuum structure, we can determine the mass of the W and Z boson in the 2HDM. This gives:

$$m_W^2 = \frac{1}{4}g^2(v_1^2 + v_2^2), \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)(v_1^2 + v_2^2). \quad (4.5)$$

So in order to reproduce the SM masses, the vevs have to satisfy the relation $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$. But we will not impose this constraint, because we are not aiming to do phenomenology with this model. For the sake of studying fine-tuning we will consider the model with a large hierarchy of scales, i.e. we will impose that $v_1 \gg v_2$. This regime is called the decoupling limit [82]. The conclusions are independent of the actual value chosen for v , which we will set to the arbitrarily chosen value of 50 TeV.

After taking derivatives with respect to the fields and inserting the vevs we obtain the minimum equations:

$$\begin{aligned} v_1(-\mu_1^2 + \lambda_1 v_1^2 + \lambda_+ v_2^2) &= 0, \\ v_2(-\mu_2^2 + \lambda_2 v_2^2 + \lambda_+ v_1^2) &= 0, \end{aligned} \quad (4.6)$$

where λ_+ is defined as: $\lambda_+ = \frac{1}{2}(\lambda_3 + \lambda_5)$. Assuming that both v_1 and v_2 are non-zero, we can write:

$$\begin{aligned} \mu_1^2 &= \lambda_1 v_1^2 + \lambda_+ v_2^2, \\ \mu_2^2 &= \lambda_2 v_2^2 + \lambda_+ v_1^2. \end{aligned} \quad (4.7)$$

We can solve this set of equations for any set of two parameters, but we choose to solve for the two μ^2 parameters. We do this since we want to impose constraints on all the other parameters. We want the coupling constants to be of $\mathcal{O}(1)$ (perturbative and natural), so we will consider values in the range $[0.1, 10]$ such that some variation in the values is still possible. Furthermore, we impose that $v_1 \gg v_2$. By solving the minimum equations in this way and imposing the constraints mentioned above, we see that both μ^2 parameters will in general be of $\mathcal{O}(v_1^2)$. So both mass parameters are insensitive to the hierarchy in scales, they will both have a value of the order of the highest scale in the theory.

Masses

Since there are two doublets instead of one, the particle content of the 2HDM is richer than in the Standard Model. There are now five massive scalar bosons after electroweak symmetry breaking. These are the same scalar bosons as in the MSSM [20].

Their masses are:

$$\begin{aligned}
 m_h^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 - \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4\lambda_+^2 v_1^2 v_2^2}, \\
 m_H^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4\lambda_+^2 v_1^2 v_2^2}, \\
 m_{H^\pm}^2 &= -\frac{1}{2}\lambda_3(v_1^2 + v_2^2), \\
 m_A^2 &= \frac{1}{2}(\lambda_3 - \lambda_4)(v_1^2 + v_2^2).
 \end{aligned} \tag{4.8}$$

Since we are using that $v_1 \gg v_2$, we can write approximate expressions for the masses of the neutral CP -even scalars:

$$\begin{aligned}
 m_h^2 &\approx 2\lambda_2 v_2^2 - \frac{(\lambda_3 + \lambda_5)^2}{2\lambda_1} v_2^2, \\
 m_H^2 &\approx 2\lambda_1 v_1^2 + \frac{(\lambda_3 + \lambda_5)^2}{2\lambda_1} v_2^2.
 \end{aligned} \tag{4.9}$$

So we see that there will be one scalar that will be naturally light with a mass of $\mathcal{O}(v_2^2)$, while the heavier scalar will have a mass of the order of the high scale. The other scalars all have masses of the order of the high scale. This is in accordance with the *extended survival hypothesis* [83, 84], which states that at every energy scale, the only light scalars are the ones necessary for symmetry breaking at a lower scale. All other scalars will be heavy [85].

Constraints on the potential

There are two sets of constraints that need to be satisfied by the potential. First of all, the potential must be bounded from below. This can be ensured by demanding (see e.g. Refs. [40, 44, 86]):

$$\begin{aligned}
 \lambda_1 &> 0, \\
 \lambda_2 &> 0, \\
 \lambda_5 &> -2\sqrt{\lambda_1 \lambda_2}, \\
 \lambda_3 + \lambda_5 &> -2\sqrt{\lambda_1 \lambda_2}, \\
 \lambda_4 + \lambda_5 &> -2\sqrt{\lambda_1 \lambda_2}.
 \end{aligned} \tag{4.10}$$

There is also the condition that the squared masses should be positive, to ensure that the extremum of the potential is actually a minimum. This gives the additional

constraints [87]:

$$\begin{aligned}\lambda_3 &< 0, \\ \lambda_4 &> \lambda_3, \\ \lambda_3 + \lambda_5 &< 2\sqrt{\lambda_1\lambda_2}.\end{aligned}\tag{4.11}$$

Having defined the model, we can look at the amount of fine-tuning present in the theory according to the fine-tuning measures discussed in Chapter 3.

4.1.2 Evaluation of the Dekens Measure

In this section we will first determine how we think the Dekens measure should be applied and then conclude how much fine-tuning is found according to this measure. As mentioned in Chapter 3, when using the Dekens measure, we have to split the parameters of the model in two sets. There is no prescription for how this split should be made, since we are using the minimum equations, which can be rewritten in terms of any set of parameters. This is in contrast to the BG measure, where the split is fixed in terms of observables and parameters.

To investigate this, we will look at two different ways to apply the Dekens measure, making different choices for the q_i . Since there are two minimum equations, one has to select two dependent parameters q_i . Throughout this section, we impose the values of the vevs, in order to ensure a hierarchy, and we take $\mathcal{O}(1)$ values for the couplings. The values of the two mass parameters are then determined by the minimum equations.

Case I: $q_i = \{\mu_1, \mu_2\}$

When we choose μ_1 and μ_2 as the dependent variables, we can derive the following expressions for the Dekens measure:

$$\begin{aligned}\Delta_D^I(\mu_1, \lambda_1) &= \frac{1}{2} \frac{\lambda_1 v_1^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2}, & \Delta_D^I(\mu_2, \lambda_2) &= \frac{1}{2} \frac{\lambda_2 v_2^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2}, \\ \Delta_D^I(\mu_1, \lambda_3) &= \frac{1}{4} \frac{\lambda_3 v_2^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2}, & \Delta_D^I(\mu_2, \lambda_3) &= \frac{1}{4} \frac{\lambda_3 v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2}, \\ \Delta_D^I(\mu_1, \lambda_5) &= \frac{1}{4} \frac{\lambda_5 v_2^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2}, & \Delta_D^I(\mu_2, \lambda_5) &= \frac{1}{4} \frac{\lambda_5 v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2}, \\ \Delta_D^I(\mu_1, v_1) &= \frac{\lambda_1 v_1^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2}, & \Delta_D^I(\mu_2, v_1) &= \frac{\lambda_2 v_2^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2}, \\ \Delta_D^I(\mu_1, v_2) &= \frac{\lambda_+ v_2^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2}, & \Delta_D^I(\mu_2, v_2) &= \frac{\lambda_+ v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2}.\end{aligned}\tag{4.12}$$

The total amount of fine-tuning is defined as: $\Delta_D^I = \max_{i,j} |\Delta_D^I(\mu_i, p_j)|$.

In all cases the measure gives a ratio, where one of the contributions to the μ_i^2 parameter is compared to the actual value of μ_i^2 . If a contribution is much larger than the value itself, there has to be a large cancellation between terms, which corresponds to fine-tuning.

Since $v_1 \gg v_2$, there is always one large and one small contribution to μ_i^2 , so we can discard the terms with v_2 in the numerator. In addition, some terms are simply multiples of another. Therefore the relevant terms are:

$$\Delta_D^I = \max \left\{ \left| \frac{\lambda_1 v_1^2}{\lambda_1 v_1^2 + \lambda_+ v_2^2} \right|, \left| \frac{1}{4} \frac{\lambda_3 v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2} \right|, \left| \frac{1}{4} \frac{\lambda_5 v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2} \right|, \left| \frac{\lambda_+ v_1^2}{\lambda_2 v_2^2 + \lambda_+ v_1^2} \right| \right\}. \quad (4.13)$$

But all of these ratios will be $\mathcal{O}(1)$, since the v_1 terms dominates in all the denominators. The only way to get a number much larger than one is by having a large ratio λ_3/λ_+ or λ_5/λ_+ . In that case, the second and third term in Equation 4.13 are large. But in general these ratios will not be very large for natural and perturbative couplings, so we do not expect any fine-tuning. The case where these fractions are large will be discussed separately later.

Case II: $q_i = \{\lambda_1, \lambda_2\}$

Now we look at a different situation. We still determine the values of μ_1^2 and μ_2^2 in the same way as before, but now we calculate the fine-tuning with respect to a different set of parameters, i.e. λ_1 and λ_2 . First we rewrite the minimum equations to obtain expressions for these parameters:

$$\begin{aligned} \lambda_1 &= \frac{\mu_1^2 - \lambda_+ v_2^2}{v_1^2}, \\ \lambda_2 &= \frac{\mu_2^2 - \lambda_+ v_1^2}{v_2^2}. \end{aligned} \quad (4.14)$$

Calculating the different contributions to the fine-tuning measure gives:

$$\begin{aligned} \Delta_D^{II}(\lambda_1, \lambda_3) &= -\frac{1}{2} \frac{\lambda_3 v_2^2}{\mu_1^2 - \lambda_+ v_2^2}, & \Delta_D^{II}(\lambda_2, \lambda_3) &= -\frac{1}{2} \frac{\lambda_3 v_1^2}{\mu_2^2 - \lambda_+ v_1^2}, \\ \Delta_D^{II}(\lambda_1, \lambda_5) &= -\frac{1}{2} \frac{\lambda_5 v_2^2}{\mu_1^2 - \lambda_+ v_2^2}, & \Delta_D^{II}(\lambda_2, \lambda_5) &= -\frac{1}{2} \frac{\lambda_5 v_1^2}{\mu_2^2 - \lambda_+ v_1^2}, \\ \Delta_D^{II}(\lambda_1, v_1) &= -2, & \Delta_D^{II}(\lambda_2, v_1) &= -\frac{2\lambda_+ v_1^2}{\mu_2^2 - \lambda_+ v_1^2}, \end{aligned} \quad (4.15)$$

$$\begin{aligned}\Delta_D^{II}(\lambda_1, v_2) &= -\frac{2\lambda_+ v_2^2}{\mu_1^2 - \lambda_+ v_2^2}, & \Delta_D^{II}(\lambda_2, v_2) &= -2, \\ \Delta_D^{II}(\lambda_1, \mu_1) &= \frac{2\mu_1^2}{\mu_1^2 - \lambda_+ v_2^2}, & \Delta_D^{II}(\lambda_2, \mu_2) &= \frac{2\mu_2^2}{\mu_2^2 - \lambda_+ v_1^2},\end{aligned}$$

And in terms of the input parameters:

$$\begin{aligned}\Delta_D^{II}(\lambda_1, \lambda_3) &= -\frac{1}{2} \frac{\lambda_3 v_2^2}{\lambda_1 v_1^2}, & \Delta_D^{II}(\lambda_2, \lambda_3) &= -\frac{1}{2} \frac{\lambda_3 v_1^2}{\lambda_2 v_2^2}, \\ \Delta_D^{II}(\lambda_1, \lambda_5) &= -\frac{1}{2} \frac{\lambda_5 v_2^2}{\lambda_1 v_1^2}, & \Delta_D^{II}(\lambda_2, \lambda_5) &= -\frac{1}{2} \frac{\lambda_5 v_1^2}{\lambda_2 v_2^2}, \\ \Delta_D^{II}(\lambda_1, v_1) &= -2, & \Delta_D^{II}(\lambda_2, v_1) &= -\frac{2\lambda_+ v_1^2}{\lambda_2 v_2^2}, \\ \Delta_D^{II}(\lambda_1, v_2) &= -\frac{2\lambda_+ v_2^2}{\lambda_1 v_1^2}, & \Delta_D^{II}(\lambda_2, v_2) &= -2, \\ \Delta_D^{II}(\lambda_1, \mu_1) &= \frac{2(\lambda_1 v_1^2 + \lambda_+ v_2^2)}{\lambda_1 v_1^2}, & \Delta_D^{II}(\lambda_2, \mu_2) &= \frac{2(\lambda_2 v_2^2 + \lambda_+ v_1^2)}{\lambda_2 v_2^2},\end{aligned}\tag{4.16}$$

The total amount of fine-tuning is again defined as the maximum of the absolute value of all the terms: $\Delta_D^{II} = \max_{i,j} |\Delta_D^{II}(\lambda_i, p_j)|$. As some of the expressions originating from λ_2 depend on the large ratio v_1^2/v_2^2 , this will result in a large value for the fine-tuning measure.

So we see that when analyzing the model in two different ways, two completely different results for the amount of fine-tuning in the model are obtained. This then raises the questions: is one of these ways to apply the Dekens measure more appropriate than the other? Or does one have to check all cases and find the maximum, as done in Ref. [73]? To investigate this, we will turn to a simplified case where we can easily compare all the different scenarios.

Analysis of the origin of the fine-tuning

We start by looking at the minimum equation for μ_2^2 , since this turns out to be the source of the discrepancy. Dividing both sides by v_1^2 yields:

$$\frac{\mu_2^2}{v_1^2} = \frac{v_2^2}{v_1^2} \lambda_2 + \lambda_+.\tag{4.17}$$

Then we rename the different terms in this equation for clarity. We replace all $\mathcal{O}(1)$ parameters by capital letters, and denote the small fraction v_2^2/v_1^2 by x , so the minimum equation is now written schematically as: $A = xB + C$.

Following the discussion in the 2HDM, we want to impose a small value for x , and $\mathcal{O}(1)$ values for B and C . Then it follows that A will also be $\mathcal{O}(1)$. When taking A as the dependent parameter q_i , and B , C and x as the independent parameters p_j , we obtain for the Dekens measure:

$$\begin{aligned}\Delta_D(A) &= \max \left\{ \left| \frac{xB}{xB+C} \right|, \left| \frac{C}{xB+C} \right|, \left| \frac{xB}{xB+C} \right| \right\} \\ &= \max\{\mathcal{O}(x), \mathcal{O}(1), \mathcal{O}(x)\} = \mathcal{O}(1).\end{aligned}\tag{4.18}$$

This is completely analogous to Case I in the 2HDM discussion.

Now we will look at the scenario similar to Case II, so we take B as the dependent parameter q_i .

$$B = \frac{A-C}{x}.\tag{4.19}$$

The result for the Dekens measure is now:

$$\begin{aligned}\Delta_D(B) &= \max \left\{ \left| \frac{xA}{A-C} \frac{1}{x} \right|, \left| \frac{xC}{A-C} \frac{1}{x} \right|, \left| \frac{x}{B} \frac{A-C}{x^2} \right| \right\} \\ &= \max \left\{ \left| \frac{A}{xB} \right|, \left| \frac{C}{xB} \right|, 1 \right\} = \mathcal{O}(1/x) \gg 1.\end{aligned}\tag{4.20}$$

So in this case we get a large amount of fine-tuning just like in Case II.² But is this large value for the Dekens measure really due to fundamental fine-tuning in the parameters of the theory?

To understand this result, we look at two ways to interpret this situation. We will first compare the size of the terms on the left-hand side of the equation to the right-hand side, and then look at the effect of variations of parameters. These are equivalent ways to approach the situation, but depending on one's intuition, one might be easier to understand than the other.

When comparing the contributions on the left-hand side of the equation to the right-hand side, we see that in the original equation $A = xB + C$, we have two contributions to A : a small one (xB) and a large one (C). So A will be almost exactly the same as C , with only a minor correction due to xB . There is no large cancellation, so no large fine-tuning. But in Case II we set B as the dependent parameter q_i , and then we see that there is a huge cancellation between A and C in order to get the

²In both scenarios the fine-tuning is calculated with respect to x . Writing x as v_2^2/v_1^2 and calculating the fine-tuning with respect to the two vevs separately gives the same result.

4.1 Fine-tuning study of the Two Higgs Doublet Model

$\mathcal{O}(1)$ value for B . But this is not due to fine-tuning, it is due to rewriting an equation where the B term was simply a small correction. We would consider the tuning found in case II as only apparent fine-tuning, but not as actual fine-tuning.

Now we look at variations in the parameters. If a variation of $\mathcal{O}(1)$ in A can only be achieved by varying B , then a large variation in B would be necessary, indicating a large amount of fine-tuning. However, the variation in A can also be achieved by varying C , where only an $\mathcal{O}(1)$ change would be necessary. To demand that the variation in A of $\mathcal{O}(1)$ has to come from a variation only in B , is simply an artificial requirement. Again we would consider this to be apparent fine-tuning.

So we see that in both viewpoints, taking B as the dependent parameter can lead to measuring apparent fine-tuning. We argue that case I is the natural way to approach the problem and therefore that one should use as dependent parameters the same ones that were used to solve the minimum equations. This is in contrast with the method used in Ref. [73] where the maximum value out of all possible combinations for the q_i was used. In Appendix 5.A and in the concluding chapter, we will further discuss how the approach in Ref. [73] compares with our approach in this chapter and what the consequences are when the hierarchy in vevs is not imposed on the theory.

4.1.3 Evaluation of the Barbieri-Giudice measure

To see how the results of the Dekens measure compare with a different fine-tuning measure, we will now evaluate the BG measure. We will use two observables: the masses of the two neutral CP -even scalar bosons h and H . These are the only two interesting masses, since the other masses are all proportional to the combination $v_1^2 + v_2^2 = v^2$. This combination will never lead to fine-tuning, since the contributions from both vevs have the same sign. There is a possibility for fine-tuning in m_A^2 due to a cancellation between λ_3 and λ_4 , however this is not due to a hierarchy in vevs, so we will not consider it here.

We calculated the BG measure numerically for a set of 5000 points. For these points, the different λ parameters are sampled from a combined distribution, consisting of two log-normal distributions: one centred around 1 and another centred around -1^3 . The constraints from Equations 4.10 and 4.11 are used to ensure boundedness of the potential and positive masses. In order to ensure a hierarchy in the vevs, the

³This distribution is used to ensure that values close to one are sampled with a higher probability, and that there is a roughly equal probability to sample values of 10 and 1/10. The exact choice of the distribution will have a small impact on the exact numerical outcomes, but our conclusions do not depend on this, as long as natural and perturbative values are being sampled.

Figure 4.1 Numerical result for the BG measure using the mass of the neutral CP -even Higgs bosons as observables. The results are shown for the light Higgs boson h (left) and the heavier H (right) as a function of v_2 . 5000 points were sampled for these figures, where we have set $v_1^2 + v_2^2 = (50 \text{ TeV})^2$. Note that the left plot uses a logarithmic scale for Δ_{BG} , whereas the right plot uses a linear scale.

parameter $\tan \beta = v_2/v_1$ is sampled from a normal distribution centred at 0.03.⁴

The results of this numerical evaluation of the BG measure with the two masses m_h^2 and m_H^2 as observables are shown in Figure 4.1. We see here that for the lightest Higgs boson, most points have a negligible amount of fine-tuning of $\Delta_{BG} \approx 2$. But there are also some points with a large amount of fine-tuning. We can understand this behavior from the approximate expression for the Higgs mass:

$$m_h^2 \approx \left[2\lambda_2 - \frac{(\lambda_3 + \lambda_5)^2}{2\lambda_1} \right] v_2^2. \quad (4.21)$$

First of all we note that this mass is not sensitive to the large scale v_1 , so the fine-tuning we see is not due to the large hierarchy in the theory. However, there is another source of fine-tuning present. If there is a large cancellation in the term $2\lambda_2 - \frac{(\lambda_3 + \lambda_5)^2}{2\lambda_1}$, we will have $m_h^2 \ll v_2^2$, so the individual contributions are much larger than the final value. This effect is very large in some points, leading to values as high as $\Delta \approx 10^3$. This is simply due to the sampling procedure and the large number of points that are sampled. When enough points are sampled, some are bound to have large accidental cancellations, leading to fine-tuning. Most important is that most points have a low amount of fine-tuning.

For the heavy boson H we see a hard cap at $\Delta_{BG} = 2$. We can again understand this

⁴The important quantity for fine-tuning purposes is v_2^2/v_1^2 , which will be $\mathcal{O}(10^{-3})$. The choice for $\tan \beta$ will have an impact on the numerical results, but the conclusions are independent of the exact distribution as long as a hierarchy in the vevs is imposed.

from the approximate expression for the mass:

$$m_H^2 \approx 2\lambda_1 v_1^2 + \frac{(\lambda_3 + \lambda_5)^2}{2\lambda_1} v_2^2 \quad (4.22)$$

All contributions to this mass are positive, so there is no way to get a cancellation which can lead to fine-tuning. The main contribution to Δ_{BG} is the v_1 term, whose contribution is given by:

$$\Delta_{BG}(v_1) = \left| \frac{v_1}{m_H^2} \frac{\partial m_H^2}{\partial v_1} \right| = \frac{8\lambda_1^2 v_1^2}{4\lambda_1^2 v_1^2 + (\lambda_3 + \lambda_5)^2 v_2^2} \approx 2, \quad (4.23)$$

where we used in the last step that $v_1 \gg v_2$. This value of $\Delta_{BG}(v_1) \approx 2$ explains the cut-off in Figure 4.1 (right). Because of the v_2^2 term in the denominator of Equation 4.23, Δ_{BG} is actually always slightly smaller than 2, as can be seen in the figure.

4.1.4 The case of large Dekens measure

So far we have seen that according to our interpretation, the Dekens measure shows no signs of fine-tuning in the 2HDM, but the BG measure shows that in some cases there can be fine-tuning. One could therefore argue that it might be enough to just use the BG measure, since the Dekens measure does not show any fine-tuning. But we will now highlight a case where the Dekens measure does produce a large value.

We can accomplish this by having large ratios λ_3/λ_+ and λ_5/λ_+ (as discussed below Equation 4.13). This happens when $\lambda_5 \approx -\lambda_3$. This is allowed, since λ_3 has to be negative, while λ_5 can be positive. When these two terms cancel to a large degree, μ_2^2 will no longer be of $\mathcal{O}(v_1^2)$, but can be as small as $\mathcal{O}(v_2^2)$. This will result in a large value for the Dekens measure in Case I, since there is a large cancellation between two terms of $\mathcal{O}(v_1^2)$ that results in a much smaller value.

The question is now whether this fine-tuning also shows up in the BG measure. It turns out that the BG measure is not sensitive to this fine-tuning. We already concluded that the fine-tuning in the BG measure is due to a cancellation between a combination of different λ parameters: $2\lambda_2 - \frac{\lambda_+^2}{2\lambda_1} \approx 0$. But now we have a situation where $\lambda_+ \approx 0$. So with $\lambda_2 \sim \mathcal{O}(1)$, there will never be such a cancellation and the BG measure will not find any fine-tuning. This was confirmed numerically. This means that both measures are necessary in order to capture different sources of fine-tuning in the theory. A possible explanation is that the two measures capture different features of the potential. The Dekens measure uses the minimum equations, which carry information on the first derivative of the potential at the minimum, while the BG measure uses the masses, which stem from the second derivative of the potential.

4.1.5 Independence of results on the choice of basis

Finally, we take a look at the basis dependence of the results for the amount of fine-tuning. Recall that one can choose the two vevs of the Higgs doublets (assuming no charged vacua and no CP violation) as:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (4.24)$$

These vevs can be rotated using a global gauge transformation. In particular, it is possible to make a transformation such that one vev is rotated away completely [88]:

$$\begin{aligned} H_1 &= \cos \beta \Phi_1 + \sin \beta \Phi_2 \\ H_2 &= -\sin \beta \Phi_1 + \cos \beta \Phi_2, \end{aligned}$$

such that

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (4.25)$$

Now one can wonder if such a change of basis has an effect on the amount of fine-tuning, since now there is no hierarchy in vevs anymore. However, one has to realize that this transformation depends on the angle β , defined by $\tan \beta = v_2/v_1$. So this transformation is dependent on the parameters of the model. Therefore we argue that such a transformation does not change the fundamentals of the model. The doublets Φ_1 and Φ_2 are still the fundamental building blocks, including the hierarchy in their vevs. One can in principle rewrite the potential in terms of the new doublets H_1 and H_2 , thereby obtaining new couplings and new minimum equations, however in terms of the original parameters these equations will remain the same, independent of the exact global transformation. The masses will also be independent of the parametrization. So one can always extract the values of v_1 and v_2 from measurements of the masses, thereby showing that these are really fundamental properties of the model. From this, we conclude that a change of basis for the Higgs doublets has no influence on the amount of fine-tuning in the model.

4.1.6 Conclusions on fine-tuning in the 2HDM

Our investigation of the 2HDM has shown that one has to be very careful when determining the amount of fine-tuning in the minimum equations. A large hierarchy between parameters does not automatically mean that there is fine-tuning present. Actually, in the 2HDM it is possible to satisfy the minimum equations in such a way that all coupling constants are natural and perturbative, but without introducing any fine-tuning in the minimum equations.

Interestingly, when evaluating the BG measure, we see that the masses of the scalar bosons are not sensitive to the hierarchy in scales, but there are other sources of fine-tuning that are not visible when using the Dekens measure. On the other hand, there are situations possible where the Dekens measure becomes large for natural and perturbative parameters due to large cancellations, while this is absent in the BG measure. This shows that both measures need to be considered when calculating the amount of fine-tuning in the theory, as they can capture different sources of fine-tuning.

4.2 Fine-tuning in the left-right symmetric model

Now that we understand how fine-tuning works in a 2HDM with a large hierarchy of vevs imposed, we can apply those lessons to a model that is more complex. To that end, we will now look at the (parity-conserving) left-right symmetric model (LRSM) [30, 31, 89, 90]. A previous discussion of fine-tuning in this model claimed that there was tree-level fine-tuning as high as $\Delta \sim 10^{20}$ in this model [73].⁵ We now want to investigate this claim using a similar approach as for the 2HDM. Since we are only interested in the Higgs sector of the model, we will not discuss the full LRSM, but only the scalar sector. We will consider a scalar sector where, in addition to a scalar bidoublet, two triplet scalar fields are used. For a full review of this model, see e.g. Refs. [73, 91, 92].

4.2.1 The LRSM Higgs sector

The gauge group of the Standard Model is $SU(3)_C \times SU(2)_L \times U(1)_Y$. In the LRSM, this gauge group is extended to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at high energies. Since we are only interested in the electroweak sector, the $SU(3)_C$ group will not be considered here. The representations will therefore be given in terms of the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

The Higgs doublet in the Standard Model is now replaced by a bidoublet ϕ in the $(2, 2^*, 0)$ representation. Two additional scalar fields are added, one to give masses to neutrinos, and one to break the LRSM gauge group to the SM gauge group. These fields are $\Delta_L \in (3, 1, 2)$ and $\Delta_R \in (1, 3, 2)$. We can write the scalar fields in terms of complex scalars:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^0 & \phi_2^+ \end{pmatrix}, \quad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}. \quad (4.26)$$

⁵The values of $\Delta \sim 10^{20}$ were found in a slightly more general LRSM, where CP -violating phases were taken into account as well. This does not affect our qualitative conclusions.

From these fields we can construct a potential that is invariant under the gauge group. In addition to this gauge invariance, a discrete symmetry is often imposed on the potential as well. One of the goals of LRSMs is to restore parity and/or charge-conjugation invariance at high energy. This invariance has to be reflected in the high-energy potential. For simplicity we only look at the P -symmetric potential, which is given by [91]:

$$\begin{aligned}
 V_H^P = & -\mu_1^2 \text{Tr}(\phi\phi^\dagger) - \mu_2^2 [\text{Tr}(\phi\tilde{\phi}^\dagger) + \text{Tr}(\tilde{\phi}\phi^\dagger)] - \mu_3^2 [\text{Tr}(\Delta_L\Delta_L^\dagger) + \text{Tr}(\Delta_R\Delta_R^\dagger)] \\
 & + \lambda_1 [\text{Tr}(\phi\phi^\dagger)]^2 + \lambda_2 \left([\text{Tr}(\phi\tilde{\phi}^\dagger)]^2 + [\text{Tr}(\tilde{\phi}\phi^\dagger)]^2 \right) + \lambda_3 \text{Tr}(\phi\phi^\dagger)\text{Tr}(\tilde{\phi}\phi^\dagger) \\
 & + \lambda_4 \text{Tr}(\phi\phi^\dagger) [\text{Tr}(\phi\tilde{\phi}^\dagger) + \text{Tr}(\tilde{\phi}\phi^\dagger)] + \rho_1 \left([\text{Tr}(\Delta_L\Delta_L^\dagger)]^2 + [\text{Tr}(\Delta_R\Delta_R^\dagger)]^2 \right) \\
 & + \rho_2 [\text{Tr}(\Delta_L\Delta_L)\text{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \text{Tr}(\Delta_R\Delta_R)\text{Tr}(\Delta_R^\dagger\Delta_R^\dagger)] \\
 & + \rho_3 \text{Tr}(\Delta_L\Delta_L^\dagger)\text{Tr}(\Delta_R\Delta_R^\dagger) \\
 & + \rho_4 [\text{Tr}(\Delta_L\Delta_L)\text{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \text{Tr}(\Delta_R\Delta_R)\text{Tr}(\Delta_L^\dagger\Delta_L^\dagger)] \\
 & + \alpha_1 \text{Tr}(\phi\phi^\dagger) [\text{Tr}(\Delta_L\Delta_L^\dagger) + \text{Tr}(\Delta_R\Delta_R^\dagger)] \\
 & + \alpha_2 \left(e^{i\delta_2} [\text{Tr}(\phi\tilde{\phi}^\dagger)\text{Tr}(\Delta_R\Delta_R^\dagger) + \text{Tr}(\tilde{\phi}\phi^\dagger)\text{Tr}(\Delta_L\Delta_L^\dagger)] + h.c. \right) \\
 & + \alpha_3 [\text{Tr}(\phi^\dagger\Delta_L\Delta_L^\dagger\phi) + \text{Tr}(\phi\Delta_R\Delta_R^\dagger\phi^\dagger)] + \beta_1 [\text{Tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger) + \text{Tr}(\phi^\dagger\Delta_L^\dagger\phi\Delta_R)] \\
 & + \beta_2 [\text{Tr}(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger) + \text{Tr}(\phi^\dagger\Delta_L^\dagger\tilde{\phi}\Delta_R)] + \beta_3 [\text{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger) + \text{Tr}(\tilde{\phi}^\dagger\Delta_L^\dagger\phi\Delta_R)]
 \end{aligned} \tag{4.27}$$

The vacuum expectation values of the scalar fields are defined as:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix}, \quad \langle\Delta_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \tag{4.28}$$

where the two phases can be sources of spontaneous CP violation. κ and v_R can also develop an imaginary phase, but these phases can be rotated away using global gauge transformations. We will set all phases to zero to simplify the discussion.

In this model, there is a clear hierarchy in the scales: v_R is the highest scale in the theory, since it is responsible for the breaking of the $SU(2)_R \times U(1)_{B-L}$ symmetry. It needs to have a value of at least a few TeV [93–95].⁶ The second to highest scales in the theory are κ and κ' . They are responsible for electroweak symmetry breaking, so they must have a value of $\mathcal{O}(100 \text{ GeV})$. The value of v_L is related to the Majorana

⁶The exact limits depend on the details of the model, like whether $g_L = g_R$ is imposed and at which scale, and the assumed mass of the right-handed neutrinos.

mass of the neutrinos. It gives a mass through a type-II seesaw mechanism, since a triplet scalar generates the neutrino mass. In addition there is also a type-I seesaw, giving mass through the presence of heavy right-handed neutrinos. These two effects combine to give the full neutrino mass. While the neutrino masses are small, in principle it is possible that the contributions from both these seesaw mechanisms are large, and that they cancel to result in a small value [96]. We will assume that this is not the case, that there is no large fine-tuning present in the neutrino sector. So we will assume v_L to be $\mathcal{O}(1 \text{ eV})$.

The minimum equations for the P -symmetric potential are given by:

$$\begin{aligned}
 \frac{\mu_1^2}{v_R^2} &= \frac{\alpha_1}{2} - \frac{\kappa'^2}{2(\kappa^2 - \kappa'^2)}\alpha_3 + \frac{\kappa^2 + \kappa'^2}{v_R^2}\lambda_1 + 2\frac{\kappa\kappa'}{v_R^2}\lambda_4, \\
 \frac{\mu_2^2}{v_R^2} &= \frac{\alpha_2}{2} + \frac{\kappa\kappa'}{4(\kappa^2 + \kappa'^2)}\alpha_3 + \frac{\kappa^2 + \kappa'^2}{2v_R^2}\lambda_4 + \frac{\kappa\kappa'}{v_R^2}(\lambda_3 + 2\lambda_2), \\
 \frac{\mu_3^2}{v_R^2} &= \rho_1 + \frac{\kappa^2 + \kappa'^2}{2v_R^2}\alpha_1 + \frac{2\kappa\kappa'}{v_R^2}\alpha_2 + \frac{\kappa'^2}{2v_R^2}\alpha_3, \\
 2\rho_1 - \rho_3 &= \frac{1}{v_R v_L}(\kappa\kappa'\beta_1 + \kappa^2\beta_2 + \kappa'^2\beta_3),
 \end{aligned} \tag{4.29}$$

where the first three equations are approximate expressions in which terms of order v_L/v_R and higher are neglected. Since we saw in the discussion of the 2HDM that small corrections have no effect on the amount of fine-tuning, we trust that this approximation will not affect our conclusions about the fine-tuning in this model.

We need to ensure that this potential is bounded from below. In general, finding the constraints on the coupling constants to ensure boundedness is a difficult problem (see e.g. Ref. [97]). Therefore we will use a different approach than in the 2HDM to ensure boundedness. Since we impose the values of the vevs, our system of minimum equations is a linear system in the parameters of the potential. Thus it has a unique solution, which means that there cannot be multiple extrema of the potential. Therefore we can ensure boundedness by making sure that the extremum we find is a minimum. We do this by investigating the scalar masses. When all squared masses are positive, we are at a minimum of the potential, and we know that the potential will be bounded from below. The masses will be evaluated numerically, since analytic expressions are not insightful even in the above approximation of dropping terms of order v_L/v_R and higher. In order to speed up the calculation, parameter ranges were determined such that in most cases, all squared masses are positive. The resulting ranges are not completely inclusive, since some valid parameter ranges are discarded, but it was checked numerically that this choice has no effect on our conclusions.

4.2.2 Evaluation of the Dekens measure

We will start our fine-tuning discussion by looking at the Dekens measure in this model. We want to find out if it is possible in this model to impose the hierarchy in vevs as mentioned above, have natural and perturbative coupling constants, but avoid fine-tuning in the parameters of the models.

We can apply most of our conclusions from the 2HDM, at least for the first three minimum equations. If we look at these three equations, we see that we are in nearly the same situation as in the 2HDM. In all of these equations, there is at least one term of $\mathcal{O}(1)$, and the other terms are much smaller than 1. Therefore, all μ_i^2 parameters will be $\mathcal{O}(v_R^2)$, so they will have a value similar to the highest scale in the theory. So for these minimum equations, we can use the μ_i^2 as q_i when applying the Dekens measure. Since in general there are no cancellations appearing in these equations, we again expect to see no fine-tuning.

But the last equation has a different structure than what we have seen so far. There is no mass parameter in this equation, so it is not so clear for which parameter we should solve this equation. Furthermore, there is a clear hierarchy present in this equation. We will therefore have to determine how to apply the Dekens measure in this situation.

If we solve this minimum equation for ρ_1 , we obtain:

$$\rho_1 = \frac{1}{2v_L v_R} (\kappa \kappa' \beta_1 + \kappa^2 \beta_2 + \kappa'^2 \beta_3) + \frac{\rho_3}{2}. \quad (4.30)$$

By taking values of $\mathcal{O}(1)$ for the β parameters, and taking v_R to be $\mathcal{O}(10 \text{ TeV})$, we will get a value for ρ_1 of $\mathcal{O}(\kappa^2/(v_L v_R)) = \mathcal{O}(10^9)$. This is in conflict with our demand that all coupling constants should be $\mathcal{O}(1)$. The only way to avoid this non-perturbative value for ρ_1 is to fine-tune the values of the β 's in such a way that they cancel up to 9 significant digits. When we apply the Dekens measure as argued before, this would give a large amount of fine-tuning, since the individual β parameters would have a much larger contribution to ρ_1 than the actual value of ρ_1 .

However, this is not the only way we can solve this equation. Let us see what happens when we solve for one of the β parameters:

$$\beta_1 = -\frac{\kappa}{\kappa'} \beta_2 - \frac{\kappa'}{\kappa} \beta_3 + \frac{v_L v_R}{\kappa \kappa'} (2\rho_1 - \rho_3). \quad (4.31)$$

Now we can take all coupling constants on the right-hand side of the equation to have values of $\mathcal{O}(1)$, and we will find a value for β_1 which is also $\mathcal{O}(1)$. The two ρ

4.2 Fine-tuning in the left-right symmetric model

parameters are simply small contributions that only contribute at the 10^{-9} level.

In this situation there is no fine-tuning. We see here that solving the minimum equations in a different way can lead to very different conclusions. We argue that a model has no fine-tuning if there is a way to solve the minimum equations such that all coupling constants are $\mathcal{O}(1)$ without introducing fine-tuning. So in this case if we solve for one of the β parameters we can have all coupling constants perturbative and natural without fine-tuning, so we conclude that if the Dekens measure is used in the way that we argued before, one does not find fine-tuning in the LRSM. In practice, one could simply study different cases and if one or more yield a Δ of $\mathcal{O}(1)$, then it means there is a way to solve the system without fine-tuning.

It is important to emphasize that in the approach we use, we treat all parameters in the potential (so the mass parameters, couplings and vevs) on equal footing. In that approach, rewriting this minimum equation is allowed because it is a constraint equation. Such an equation is used to eliminate a parameter from the system, but there is no prescription for which parameter to eliminate. In principle, every way to solve a minimum equation is equally fine, but for some choices it seems like there is fine-tuning, while for other choices this is not the case. If there is a way to obtain a Dekens measure of $\mathcal{O}(1)$ and have natural values for all parameters, then that means there is a way to satisfy the minimum equations without fine-tuning. We argue that that means that the minimum equations are then not a source of fine-tuning.

One might wonder whether it is always possible to solve a minimum equation without mass parameters in a way that avoids fine-tuning and has all parameters natural. But this is not the case in general. Imagine if the seesaw relation has the form:⁷

$$2\rho_1 - \rho_3 = \frac{\kappa^2}{v_L v_R} \beta_1. \quad (4.32)$$

Then there is no way to solve the equation for one of the couplings without the need for unnaturally small parameters. Since if we solve this equation for β_1 , we find:

$$\beta_1 = \frac{v_L v_R}{\kappa^2} (2\rho_1 - \rho_3) \sim 10^{-9} (2\rho_1 - \rho_3). \quad (4.33)$$

So with $\mathcal{O}(1)$ values for the ρ parameters, β_1 will have an unnaturally small value. While this is not due to fine-tuning, it is still an undesirable situation. On the other

⁷We are not aware of any symmetry that arises in the case where two out of the three β terms are absent, or when all three β terms are equal (which leads to a similar situation). Therefore, this scenario might not seem like a realistic option for the LRSM we consider here. But such a structure could appear in e.g. LRSMs with a different scalar sector or other BSM theories.

hand, when the equation is solved for one of the ρ parameters, the relation is:

$$\rho_1 = \frac{\kappa^2}{2v_L v_R} \beta_1 + \frac{\rho_3}{2} \sim 10^9 \beta_1 + \frac{\rho_3}{2}. \quad (4.34)$$

In this case, $\mathcal{O}(1)$ values for β_1 and ρ_3 will result in $\rho_1 \sim 10^9$. This is unacceptable from a perturbativity perspective. In order to avoid this, β_1 would have to be extremely small, which is again undesirable. Therefore, the conclusion is that a minimum equation of this form cannot be solved in a satisfactory way where all parameters have natural values.

4.2.3 Evaluation of the Barbieri-Giudice Measure

Just like in the 2HDM, we can see how the results of the Dekens measure compare to the BG measure. The difference is that in this case it is not possible to get analytic expressions for the masses. While approximate expressions exist in certain limits [98–101], we will only use numerical results. The derivatives in the BG measure have to be evaluated numerically as well.

The BG measure will now be computed as the maximum of the contributions of the individual masses:

$$\Delta_{BG} = \max_{i,j} \left| \frac{p_j}{m_i^2} \frac{\partial m_i^2}{\partial p_j} \right|. \quad (4.35)$$

We show in Figure 4.2 (left) the result of the BG measure for 5000 sampled points. Whereas there were only two masses with non-trivial expressions in the 2HDM case, now we have 14 masses that all depend non-trivially on the input parameters. Therefore, we expect that the amount of fine-tuning spreads over a larger range, since the chance is now higher that there is some cancellation in one of the masses. One should also keep in mind that the more points are sampled, the higher the chance to find some case that happens to have some cancellation and hence fine-tuning. This is why we see some points with a large amount of fine-tuning in the figure. It is clear though that the generic model will have a fine-tuning below 100. We see a lower bound of $\Delta_{BG} \approx 2$ in this figure, which has the same origin as the lower bounds in the 2HDM results in Figure 4.1. Note that the y-axis in Figure 4.1 (right) is very different from the other two plots (Figure 4.1 (left) and Figure 4.2 (left)). Keeping this in mind, all three figure indeed have the same lower bound $\Delta_{BG} \approx 2$, which originates from the quadratic dependence of the masses on the vevs. Since we take the vevs (and not the squared vevs) as the independent parameters, the derivative in the BG measure produces a factor 2.

In this analysis the dimensionless parameters have been sampled from a combined

Figure 4.2 Numerical result of evaluating the BG measure of 5000 points in the LRSM with no constraint imposed on the Higgs mass (left) and the constraint $100 \text{ GeV} < m_h < 150 \text{ GeV}$ (right).

distribution consisting of two log-normal distributions, one centered around 1 and the other centered around -1. Constraints were imposed on some parameters to ensure boundedness of the potential using the method described in Section 4.2.1. The values of the vevs are sampled according to the expected hierarchy: $v_L \sim 1 \text{ eV}$, $\kappa^{(\prime)} \sim 100 \text{ GeV}$, $v_R \sim 10 \text{ TeV}$.

Imposing the Higgs mass constraint

In order to see the impact of phenomenological constraints on the fine-tuning, we impose that the mass of the lightest Higgs boson is close to 125 GeV [1, 2]. There are much more constraints in the LRSM that could be investigated, but that is beyond the scope of this work. Because the computation of a single model point takes a relatively long time, we will not impose a very strict constraint. We require that the mass is between 100 GeV and 150 GeV. The result of calculating the BG measure in model points with this constraint is shown in Figure 4.2 (right).

There are two observations we can make based on this figure. First of all, we see that most points have a larger amount of fine-tuning than in the case without constraint on the Higgs mass. Even though we do not have analytic expressions for the masses, we can still make observations on the cause for this increase in fine-tuning. This moderate amount of fine-tuning is not due to the hierarchy between κ , κ' and v_R , since increasing the value of v_R has no effect on the mass of the lightest scalar boson in the bulk of the points (and therefore no effect on the fine-tuning). Our explanation for this increase in fine-tuning is that the mass of the lightest Higgs boson is a sum of multiple terms, all of which are $\mathcal{O}(\kappa^{(\prime)2})$. If there are a lot of these terms, the sum can be significantly larger than $\kappa^{(\prime)2}$, such that some cancellations are necessary in order to get a Higgs mass in the correct range.

The other observation is that while the bulk of the points are uniformly distributed across the v_R range, there is another set of points showing a clear line in the (v_R, Δ_{BG}) -plane. These points are concentrated around small v_R values. When we investigate these points, we see that they have small values for both v_R and the combination $2\rho_1 - \rho_3$. The term $(2\rho_1 - \rho_3)v_R^2$ appears in entries of the scalar mass matrix. For large v_R values, and $2\rho_1 - \rho_3 \sim \mathcal{O}(1)$, this term is large, and it only contributes to the heavy scalars. But for smaller v_R values and/or when $2\rho_1 - \rho_3$ is small, this term can contribute to the light scalar mass as well. But in order to obtain the correct Higgs mass, there needs to be a large cancellation either between this term proportional to v_R^2 and other terms proportional to $\kappa^{(\prime)2}$, or between the ρ_1 and ρ_3 term in this contribution. For large v_R values, the cancellation between the two ρ terms needs to increase in order to keep this contribution small. This explains that for larger v_R values the value of Δ_{BG} increases and the density of points that show this behaviour decreases. The expression for Δ_{BG} is dominated by the ρ_1 contribution in this case (the ρ_3 contribution is a factor 2 smaller), and it is approximately given by:

$$\Delta_{BG} = \frac{\rho_1}{m_h^2} \frac{\partial m_h^2}{\partial \rho_1} \approx \frac{\rho_1}{m_h^2} 2v_R^2 \approx \frac{v_R^2}{m_h^2}. \quad (4.36)$$

This formula nicely explains the line in the plot.

4.2.4 Large Dekens measure

Just like in the 2HDM, it is also possible in the LRSM to get a large value for the Dekens measure while keeping the BG measure low. We will consider two different ways to achieve a large Dekens measure. First we will see what happens when we artificially introduce a large Dekens measure in one of the equations for a μ_i^2 parameter, which is very similar to our discussion in the 2HDM. Then we will investigate the case where there is a large cancellation in the seesaw relation.

If we look at the equation for μ_1^2 (Eq. (4.29)), we see that it is possible to have a large cancellation while keeping all coupling constants $\mathcal{O}(1)$. We can do this by having a value of α_3 such that it (nearly) cancels the α_1 contribution. Since a perfect cancellation between the terms would be unrealistic, we add a small correction to α_3 :

$$\alpha_3 = \frac{\kappa^2 - \kappa'^2}{\kappa'^2} \alpha_1 + \mathcal{O}(\kappa^2/v_R^2). \quad (4.37)$$

After imposing this constraint, μ_1^2 can be as small as $\mathcal{O}(\kappa^2)$. So just like in the 2HDM case, one of the μ^2 parameters will be much smaller than the other ones. And just like in the 2HDM case, this cancellation does not have an influence on the BG measure, as

Figure 4.3 Numerical result of evaluating the BG measure of 2000 points in the LRSM, where a cancellation has been imposed in the formula for μ_1^2 according to Equation 4.37.

can be seen in Figure 4.3.

The reason that we do not see any signs of this cancellation is that the relevant terms in the mass matrix all look like $\mathcal{M}^2 \sim \alpha_1 v_R^2 - 2\mu_1^2$. So when we fill in the formula for μ_1^2 (Eq. (4.29)) the α_1 dependence completely drops out of the equation, and the cancellation between α_1 and α_3 is no longer visible. The same happens when introducing fine-tuning in one of the other μ^2 relations. So fine-tuning in any of the mass parameters of the potential does not show up in the BG measure.

Now we will see what happens when we introduce a cancellation in the seesaw relation. Like argued before, when we solve this relation for one of the β_i parameters, we do not get any fine-tuning in general. However, when there is some numerical coincidence such that the two other β_j parameters add up to a very small contribution to β_i , there might be fine-tuning present.

When solving the seesaw relation for β_1 (Eq. (4.31)), and we have:

$$\beta_3 \approx -\frac{\kappa^2}{\kappa'^2} \beta_2, \quad (4.38)$$

the value of β_1 will be much smaller than the contributions of $\beta_{2,3}$, and the Dekens measure will give a large value. Once again the BG measure remains small. The reason for this lack of sensitivity is that the scalar masses are not very dependent on the value of the β parameters. The results for Δ_{BG} are very similar to Figure 4.3 in this case.

One may wonder whether the large Dekens measure due to large cancellations in the seesaw relation could be avoided by solving the minimum equations in yet another way. This is indeed possible, but requires at least one other parameter to be unnaturally small. In our numerical analysis we have not allowed unnaturally small parameters by considering the range $[0.1, 10]$ as $\mathcal{O}(1)$. Finding a large Dekens measure may thus indeed indicate large fine-tuning if one requires naturalness and perturbativity of the parameters. The same reasoning holds in our discussion of a large Dekens measure in the 2HDM (section 4.1.4).

4.3 Conclusions

We have investigated two different measures of fine-tuning in two beyond the Standard Model theories, with the condition that a hierarchy in vevs is imposed on the theory. We note that there are multiple ways to apply the Dekens measure, and argue how we think it should be applied. We compare our results with the results for the BG measure, which could perhaps aid in determining how to apply the Dekens measure, as it is expected that the two measures give similar results. And indeed we find that the two measures agree in most cases using our approach for the Dekens measure. However, there are situations where the measures disagree, which can be understood from the fact that they use different properties of the potential to determine the fine-tuning. The Dekens measure looks at the minimum of the potential, whereas the BG measure captures features of the second derivative of the potential. Therefore, we argue that both the Dekens measure and the Barbieri-Giudice measure should be evaluated when determining the amount of fine-tuning present in a theory. Of course, we do not exclude that there may be other more effective measures of fine-tuning or there may be other observables besides scalar masses that could be sensitive to fine-tuning.

Furthermore, we conclude that it is possible to impose a large hierarchy of scales in theories like the left-right symmetric models without having fine-tuning of the other parameters, at least at tree-level. The conclusion on the absence of fine-tuning is reached based on our argument that if it is possible to solve the minimum equations in a way that results in a low amount of fine-tuning, it means that the theory is not fine-tuned in any region of the parameter space where the parameters are both natural and perturbative.

Concerning fine-tuning in left-right symmetric models, we conclude that although in a generic LRSM there is no tree-level fine-tuning, the BG measure indicates that imposing the mass of the lightest Higgs boson to be $m_h = 125$ GeV may require a moderate amount of fine-tuning.

Chapter 5

Fine-tuning and the doublet-triplet splitting problem in the minimal $SU(5)$ GUT

In the search for beyond the Standard Model (BSM) physics, Grand Unified Theories (GUTs) have been a prime candidate for a long time. These theories can provide a common origin for the different forces we observe at low energy, and give an explanation for the quantization of charge. The first GUT put forward was based on $SU(5)$ symmetry [32]. The minimal non-supersymmetric model based on $SU(5)$ is currently considered ruled out, because it leads to a too short lifetime of the proton compared to its experimental lower limit [42]. Besides this lack of phenomenological viability, it is also considered to suffer from a theoretical problem common to many supersymmetric and non-supersymmetric GUTs: the doublet-triplet splitting problem (DTSP) [102,103]. This is the problem that fine-tuning seems necessary to produce masses of very different scales for the Higgs triplet and doublet, which originate from the same representation at high scale. Such fine-tuning seems required already at leading order (classically). For specific GUTs there exist solutions to the DTSP, e.g. the Dimopoulos-Wilczek solution for $SO(10)$ [104], the missing partner mechanism [105,106], the sliding singlet mechanism [107,108] and GIFT (Goldstones instead of fine-tuning) [109,110]. These solutions often rely on including additional representations in the model. In this chapter,¹ we will reanalyse the minimal non-supersymmetric $SU(5)$ GUT and address the problem of fine-tuning at tree-level quantitatively by investigating both the equations that define the minimum of the potential and the masses of the scalar bosons. While the minimal model is ruled out by experimental constraints, non-minimal models are still being investigated. By extending the scalar and/or fermionic sector with additional representations, the constraints from proton decay can be avoided (see e.g. Refs. [112–114]). We will focus on the minimal model since it provides a nice framework to study the doublet-triplet splitting problem. We first investigate which symmetry breaking pattern is required for a phenomenologically viable theory, with

¹This chapter is based on Ref. [111]

both high-scale $SU(5)$ breaking and low-scale electroweak symmetry breaking. Then we determine the fine-tuning, taking the vacuum expectation values (vevs) as the output parameters of the model. After concluding that the model indeed needs a large amount of fine-tuning, we analyse a similar situation in the two-Higgs-doublet model (2HDM) for which one can analyse the problem analytically. This analysis shows that it is actually a general problem to obtain a large hierarchy in vevs in a model with coupled scalar sectors. We then argue for a different viewpoint where, on top of the orientation of the vevs, also the values of the vevs (or at least the hierarchy in vevs) are considered input parameters of the theory. We emphasize that if one views the vevs as output parameters, the orientation of the vevs has to be selected in order to obtain the desired breaking pattern. Whether a particular orientation corresponds to the global minimum depends on the parameters of the couplings. Therefore, requiring a specific orientation at low energies implies implicitly selecting the parameter ranges for which the desired symmetry breaking pattern emerges as the energetically favoured minimum [115–118]. In the viewpoint we propose, we impose on top of this the range of scales at which each symmetry breaking occurs. This viewpoint is similar to how we discussed fine-tuning in the previous chapter. We evaluate the fine-tuning using this approach, and find that the general theory with a large hierarchy in vevs will not require fine-tuning of the remaining free parameters. It must be pointed out that our discussion concerns leading order and does not pertain to the standard hierarchy problem arising from loop corrections to the scalar boson masses nor to the gauge hierarchy problem [64, 66, 119]. The latter problem originates from the observation that the couplings will get corrections at every loop order. So in order to fix the couplings and ensure the hierarchy, one first needs to compute perturbative corrections to the couplings up to a sufficiently high loop order. This is determined by the size of the hierarchy. With a coupling $\alpha = g^2/4\pi$, L loops need to be computed, with L determined by $\alpha^L = v_{\text{EW}}^2/m_{\text{GUT}}^2$. At the end of the chapter we include an appendix where we look back at the 2DHM and LRSM, and investigate the amount of fine-tuning when we view the vevs as output parameters in those models. We leave a discussion of these findings to the concluding chapter.

5.1 The doublet-triplet splitting problem

In order to discuss the doublet-triplet splitting problem (DTSP), we will start by recalling the minimal Higgs potential of $SU(5)$. This potential contains two fields: the Φ field in the adjoint (24) representation, and the H field in the vector (5) representation [66, 120]. In terms of real scalar fields we write:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} H_1 + iH_2 \\ H_3 + iH_4 \\ H_5 + iH_6 \\ H_7 + iH_8 \\ H_9 + iH_{10} \end{pmatrix}, \quad \Phi = \sum_{i=1}^{24} \frac{\lambda_i}{2} \phi_i, \quad (5.1)$$

with H_i and ϕ_i real scalar fields, and λ_i the traceless generators of $SU(5)$ satisfying $\text{Tr} \lambda_i \lambda_j = 2\delta_{ij}$.

We will impose a $\mathbb{Z}_2$ symmetry that transforms $\Phi \rightarrow -\Phi$. This will be justified later. When imposing this symmetry, the potential is given by:

$$V(H, \Phi) = -\frac{\nu^2}{2} H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 - \frac{\mu^2}{2} \text{Tr} \Phi^2 + \frac{a}{4} (\text{Tr} \Phi^2)^2 + \frac{b}{2} \text{Tr} \Phi^4 \\ + \alpha H^\dagger H \text{Tr} \Phi^2 + \beta H^\dagger \Phi^2 H. \quad (5.2)$$

This $SU(5)$ symmetric potential is broken to the Standard Model gauge group by selecting the vacuum expectation value of the Φ field to be $\langle \Phi \rangle = \text{diag}(v, v, v, -\frac{3}{2}v, -\frac{3}{2}v)$, where the factors $-\frac{3}{2}$ are necessary to ensure the tracelessness of Φ . This form can be achieved by setting $\langle \phi_{24} \rangle = \sqrt{15}v$. Using this vev, we can write down one condition defining the minimum of the potential (a minimum equation):

$$\mu^2 = \frac{15a + 7b}{2} v^2. \quad (5.3)$$

Now we can look at the masses of the doublet and triplet components of the H field. If we look at the potential at the minimum of Φ , we can write down the purely H -dependent part of the potential, and write it in terms of the doublet $\vec{H}_2$ and the triplet $\vec{H}_3$:

$$V(\vec{H}_2, \vec{H}_3) = \left[-\frac{\nu^2}{2} + \frac{15\alpha}{2} v^2 + \frac{9\beta}{4} v^2 \right] \vec{H}_2^\dagger \vec{H}_2 + \left[-\frac{\nu^2}{2} + \frac{15\alpha}{2} v^2 + \beta v^2 \right] \vec{H}_3^\dagger \vec{H}_3 \\ + \frac{\lambda}{4} \left[(\vec{H}_3^\dagger \vec{H}_3)^2 + (\vec{H}_2^\dagger \vec{H}_2)^2 + 2\vec{H}_3^\dagger \vec{H}_3 \vec{H}_2^\dagger \vec{H}_2 \right]. \quad (5.4)$$

From this we can read off the masses of the doublet and triplet:

$$m_{\vec{H}_2}^2 = -\frac{\nu^2}{2} + \frac{15\alpha}{2} v^2 + \frac{9\beta}{2} v^2, \\ m_{\vec{H}_3}^2 = -\frac{\nu^2}{2} + \frac{15\alpha}{2} v^2 + \beta v^2. \quad (5.5)$$

In order to discuss the amount of fine-tuning in this model, we need to write the masses in terms of the input parameters. These are usually considered to be the mass

parameters μ^2 and ν^2 and the couplings in the Lagrangian. The vev is generally not considered an input parameter, as it arises when the energy of the system is lowered and is determined by the other parameters using the minimum equation (Equation 5.3). This is in contrast to the approach used in Chapter 4, where we considered the vevs as input parameters. After replacing the vev we find:

$$\begin{aligned} m_{\tilde{H}_2}^2 &= -\frac{\nu^2}{2} + \frac{15\alpha + 9\beta}{15a + 7b}\mu^2, \\ m_{\tilde{H}_3}^2 &= -\frac{\nu^2}{2} + \frac{15\alpha + 2\beta}{15a + 7b}\mu^2. \end{aligned} \tag{5.6}$$

The mass parameters μ^2 and ν^2 originate from the Φ and H sector respectively. When we consider these sectors separately, there is no reason that the mass parameters are of the same order of magnitude. We would expect them to be quite different. The interactions between the Φ and H sector do not change the fact that the mass parameters can be different, at least at tree-level. Applying this to Eq. 5.6, we see that both masses will in general be of the order of the highest scale, whether this is ν^2 or μ^2 . This also implies that the masses of the doublet and the triplet are of the same order of magnitude.

However, phenomenological constraints require a large hierarchy between the triplet and doublet masses. The triplet can mediate proton decay. A rough estimate for the proton lifetime in minimal $SU(5)$ gives [121]:

$$\tau_p \approx \frac{m_X^4}{m_p^4}, \tag{5.7}$$

with m_X the mass of a particle X that can mediate proton decay, and m_p the proton mass. Using the limit $\tau_p/\text{Br}(p \rightarrow e^+\pi^0) > 1.67 \cdot 10^{34}$ yr [122], this gives a (rough) bound on the triplet mass of $m_{\tilde{H}_3} > 10^{16}$ GeV. This is also the typical scale at which the coupling constants become of the same order and could unify. When we assume that the couplings are all $\mathcal{O}(1)$, i.e. natural and perturbative, and there are no cancellations between a and b , this means that at least one of ν^2 and μ^2 has to be of the order of this high scale. But the doublet should behave like the Standard Model Higgs doublet, giving masses of $\mathcal{O}(100)$ GeV to the electroweak gauge bosons and the Higgs boson. Therefore we expect $m_{\tilde{H}_2}^2 \approx 100$ GeV. Assuming that there is no large cancellation between α and β , it means that the large contribution which is necessary for a heavy triplet now has to cancel with the other contribution to a very high degree in order to result in a doublet that is much lighter than the triplet. This is the essence of the doublet-triplet splitting problem. So in order to have the large hierarchy between the triplet and doublet masses, one needs either a cancellation between α and β if $\mu = \mathcal{O}(v) \gg \nu$, or a cancellation between the μ^2 and ν^2 term,

in which case both have to be of the high scale v . In either case there is fine-tuning required at tree-level. In conclusion, the generic case will not lead to the desired hierarchy in masses.

The discussion so far does not take into account electroweak symmetry breaking (EWSB), but that is expected to play a role as it must be compatible with the $SU(5)$ symmetry breaking and allow for a phenomenologically viable scenario. In the next section we will investigate the role of EWSB, and see which possibilities there are to realise EWSB in a satisfactory way.

5.2 Including electroweak symmetry breaking

The most straightforward implementation of EWSB is by giving the H field a vev of the form: $\langle H \rangle = \frac{1}{\sqrt{2}}(0, 0, 0, 0, v_0)^T$, in addition to the previously defined vev of Φ . In that case we obtain three minimum equations:

$$\begin{aligned}\nu^2 &= (15\alpha + \frac{9}{2}\beta)v^2 + \frac{1}{2}\lambda v_0^2, \\ 0 &= -\frac{3}{2}\beta v v_0, \\ \mu^2 &= (\frac{15}{2}a + \frac{7}{2}b)v^2 + (\alpha + \frac{3}{10}\beta)v_0^2.\end{aligned}\tag{5.8}$$

Note that the structure of the $\beta H^\dagger \Phi^2 H$ term gives rise to a term proportional to $\phi_{23}\phi_{24}$. This term leads to a non-zero contribution in the right-hand side of the second minimum equation, originating from taking the derivative with respect to ϕ_{23} . Since this is the only term in the potential with this kind of mixing, this term must be equal to zero. So this implies the additional constraint that $\beta = 0$. If we continue analyzing the model, and calculate the masses of the scalars, we see that we get 21 massless modes, whereas one only expects 15 Goldstone bosons (12 from the breaking of $SU(5)$ and 3 from EWSB). The six additional massless scalars arise since the potential has a larger symmetry when we set $\beta = 0$. In that case the H and Φ sector no longer mix, and there is an additional global $SU(5)$ symmetry for the H field. This symmetry remains unbroken when breaking the local $SU(5)$ symmetry. But when H gets a vev, it breaks from a global $SU(5)$ to a global $SU(4)$. This results in 9 massless modes. Three of these are eaten by the electroweak gauge bosons, the other six remain present in the theory as massless modes. This is phenomenologically not an acceptable scenario.

Besides phenomenological considerations, another reason to discard this scenario is that the input parameter β needs to have a specific value to ensure a certain low-energy outcome. The constraint $\beta = 0$ only arises from the electroweak symmetry

breaking, it is not present when only breaking the $SU(5)$ symmetry. So the theory at the GUT scale has to ‘know’ already about the low-energy physics in order to satisfy the minimum equations and have $\beta = 0$.

Beyond leading order, gauge boson loops will induce a $H^\dagger \Phi^2 H$ term, leading to a non-zero value for β . This can be seen from the $SU(5)$ effective potential [123].² Hence, beyond leading order the above argument may not apply, because the minimum equations will be different and the Goldstone bosons may be absent. But as mentioned before, we restrict ourselves to a discussion of the DTSP at tree level. Beyond leading order one also has to deal with the standard hierarchy problem due to loop corrections.

The restriction on β can be lifted at leading order already by realizing that the Φ field can also contribute to EWSB, see e.g. Ref. [124]. This is done by adding a vev v_1 according to $\langle \phi_{23} \rangle = -v_1$, which leads to $\langle \Phi \rangle = \text{diag}(v, v, v, -\frac{3}{2}v - \frac{1}{2}v_1, -\frac{3}{2}v + \frac{1}{2}v_1)$. The H field has the same vev as before: $\langle H \rangle = \frac{1}{\sqrt{2}}(0, 0, 0, 0, v_0)^T$. When we write down the minimum equations in this case, we obtain:

$$\begin{aligned} \nu^2 &= (15\alpha + \frac{9}{2}\beta)v^2 - 3\beta vv_1 + (\alpha + \frac{1}{2}\beta)v_1^2 + \frac{1}{2}\lambda v_0^2, \\ v_1\mu^2 &= \frac{1}{2}(15a + 27b)v^2v_1 + \frac{1}{2}(a + b)v_1^3 + \frac{1}{2}(2\alpha + \beta)v_0^2v_1 - \frac{3}{2}\beta vv_0^2, \\ v\mu^2 &= (\frac{15}{2}a + \frac{7}{2}b)v^3 + (\frac{1}{2}a + \frac{9}{10}b)v_1^2v + (\alpha + \frac{3}{10}\beta)v_0^2v - \frac{1}{10}\beta v_0^2v_1. \end{aligned} \quad (5.9)$$

As mentioned before, in the standard viewpoint the vevs are regarded as the output parameters. Therefore we should solve the minimum equations with respect to the vevs. However, doing so will in general result in a theory without a large hierarchy. If both mass parameters μ^2 and ν^2 are $\mathcal{O}(10^{16})$ GeV, which is the simplest way to have the triplet heavy enough and the doublet light enough, then generally the vevs will also be of this scale. In order to avoid this situation, we will not solve the minimum equations for the vevs, but for three other parameters, so we can ensure that a hierarchy in vevs is present. Afterwards the fine-tuning necessary to obtain this hierarchy can be determined by solving the system for the vevs.

To see whether all three vevs are really necessary, we can check the situation where $v_0 = 0$, so only the Φ field gets a vev. In that case the minimum equations are given

²Note there must be an error in Eq. 2.4 of Ref. [123] because there is a non-zero contribution to the $H^\dagger \Phi^2 H$ term from Higgs loops even when $\beta = 0$. This is impossible, since there are no terms present in the Higgs potential that mix individual components of H (the λ interaction can mix squared terms H_i^2 , but for the β term one needs mixing of individual H_i terms). It was checked that the gauge-boson loop really does give rise to an effective β parameter.

by:

$$\begin{aligned}\mu^2 &= \frac{3}{2}(5a + 9b)v^2 + \frac{1}{2}(a + b)v_1^2, \\ \mu^2 &= \frac{15a + 7b}{2}v^2 + \left(\frac{1}{2}a + \frac{9}{10}b\right)v_1^2.\end{aligned}\tag{5.10}$$

Solving this system for the two vevs (assuming that $b \neq 0$) shows that both vevs will be $\mathcal{O}(\mu)$, which makes sense since this is the only scale present in the system. The exact solutions show that $v_1 = \pm 5v$,³ This situation is clearly unwanted, and this option is therefore discarded.

Finally, it is also possible to break $SU(5)$ directly to the Standard Model by setting $v = 0$ and having both v_1 and v_0 non-zero. However this situation again leads to the constraint $\beta = 0$, so this option is also discarded. Therefore we conclude that the only viable option is to have all three vevs v , v_0 and v_1 non-zero.

So we are forced to set both $v_1 \neq 0$ and $v_0 \neq 0$ and have the hierarchy $v_0 \ll v$, with $v_0 \sim \mathcal{O}(100)$ GeV and $v \sim \mathcal{O}(10^{16})$ GeV. However for v_1 it is not clear how large it should be. Therefore we will use the minimum equations to figure out how large v_1 is. This is possible since we have three minimum equations, which we can solve for the two mass parameters and for v_1 . Doing this gives rise to three separate solutions, with the result for v_1 for each of these solutions given by:

$$1) \quad v_1 = -5v,\tag{5.11}$$

$$2) \quad v_1 = \frac{20bv^2 + \beta v_0^2 - \sqrt{400b^2v^4 - 8b\beta v^2v_0^2 + \beta^2v_0^4}}{8bv},\tag{5.12}$$

$$3) \quad v_1 = \frac{20bv^2 + \beta v_0^2 + \sqrt{400b^2v^4 - 8b\beta v^2v_0^2 + \beta^2v_0^4}}{8bv}.\tag{5.13}$$

We can approximate the second and third expression by expanding in small v_0 . Depending on the sign of b , these expressions switch places, but the two results are:

$$v_1 \approx \frac{3\beta}{20b} \frac{v_0^2}{v},\tag{5.14}$$

$$v_1 \approx 5v + \frac{\beta}{10b} \frac{v_0^2}{v}.\tag{5.15}$$

³Note that the sign of v_1 is irrelevant, it can always be changed by defining $\langle \phi_{23} \rangle = v_1$ instead of $\langle \phi_{23} \rangle = -v_1$ so the two vevs will always be of the same order of magnitude and there is not even a possibility to fine-tune the parameters in order to obtain a large hierarchy in the vevs. Furthermore, this value of v_1 will lead to $\langle \Phi \rangle = \text{diag}(v, v, v, v, -4v)$ (or with the last two elements interchanged for the other sign of v_1). So the $SU(5)$ symmetry is actually not broken to $SU(3) \times U(1)$, but to $SU(4)$.

So we see that in two out of the three cases v_1 will be of the order of the high scale, while in the other case v_1 is very small. We will show that using phenomenological constraints from the ρ parameter, the solutions with large values for v_1 can be discarded.

The ρ parameter was introduced in Chapter 2, as a way to constrain vevs of additional scalar fields. In terms of Standard Model representations, it is given by:

$$\rho = \frac{\sum_i [T_i(T_i + 1) - Y_i^2] v_i^2 c_i}{\sum_i 2Y_i^2 v_i^2}, \quad (5.16)$$

with T_i the weak isospin, Y_i the hypercharge, v_i the vev and $c_i = 1/2$ (1) for real (complex) representations. The experimental value is $\rho = 1.00038 \pm 0.00020$ [5], while the Standard Model (tree-level) result is $\rho = 1$, so only very small deviations are still allowed. Explicitly, the decompositions of H and Φ are given by:

$$\begin{aligned} 5 &\rightarrow (3, 1, -1/3) + (1, 2, 1/2), \\ 24 &\rightarrow (8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (3, 2, -5/6 \, q) + (3^*, 2, 5/6 \, q). \end{aligned} \quad (5.17)$$

The only contributions to ρ arise from representations that get a vev. So the $(1, 2, \frac{1}{2})$ of H contributes (just like in the Standard Model) with vev v_0 . The vev v is part of the $(1, 1, 0)$ and has $T = 0$ and $Y = 0$, so it does not contribute to ρ . The only remaining contribution comes from v_1 . It is part of the $SU(2)$ triplet field $(1, 3, 0)$. Therefore ρ is given by:⁴

$$\begin{aligned} \rho &= \frac{\sum_i [T_i(T_i + 1) - Y_i^2] v_i^2 c_i}{\sum_i 2Y_i^2 v_i^2}, \\ &= \frac{v_0^2/2 + 2v_1^2}{v_0^2/2} = 1 + 4 \frac{v_1^2}{v_0^2}. \end{aligned} \quad (5.18)$$

So to keep $\rho \approx 1$, we need to have $v_1 \ll v_0$. Therefore we can justify using only the $v_1 \sim \frac{v_0^2}{v}$ solution. This solution satisfies the bounds on ρ .

Now we can also justify our choice of imposing a $\mathbb{Z}_2$ symmetry on the potential. When we consider the potential without this symmetry, there are two additional terms: $\Lambda_1 \text{Tr}(\Phi^3)$ and $\Lambda_2 H^\dagger \Phi H$, with Λ_1 and Λ_2 couplings with mass dimension equal to one. Now we want to see if the inclusion of these terms can change the reasoning about the necessary structure in the vevs that we discussed above. And indeed, we can now obtain a viable mass spectrum even when we set $v_1 = 0$. However, this situation introduces another problem. In the case $v_1 = 0$, the minimum equations fix $\Lambda_2 = 3\beta v$.

⁴Cf. Refs. [125, 126] for similar discussions in non-minimal models, which give the same result in the limit where only the **5** and **24** representations get a vev.

This is independent of the inclusion of the Λ_1 term. The existence of the Λ_2 term allows for a tree-level contribution to the H^4 coupling with an exchange of a Φ . This term would contribute with a factor:

$$\delta\lambda = \frac{\Lambda_2^2}{m_\Phi^2} \sim \frac{(3\beta v)^2}{v^2}. \quad (5.19)$$

So this contribution from high-scale physics contributes to the low-energy quartic H coupling at the $\mathcal{O}(1)$ level [127]. This is called tree-level non-decoupling. Since we do not want the Φ field to have a large effect at low energies, we want to avoid such a term, so we still have to discard the scenario where $v_1 = 0$. So also in the case without the $\mathbb{Z}_2$ symmetry, we need all three vevs. In order to simplify the discussion, and since this is the case most commonly discussed in the literature, we chose to impose the $\mathbb{Z}_2$ symmetry.

5.3 Scalar masses

Now that we have a model where both $SU(5)$ breaking and EWSB are properly taken into account, with the hierarchy $v \gg v_0 \gg v_1$, we can take another look at the scalar masses. We will write these masses in terms of the vevs, since this most clearly illustrates the hierarchy in masses. Since we are interested in the doublet-triplet splitting problem we will focus on the masses of the components of H . But these components will mix with components of Φ , so this mixing also needs to be taken into account. The expressions for the entries of the mass matrices before substituting the expressions for μ^2 and ν^2 can also be found in Ref. [120].

First we look at the triplet components of H . The real scalar field H_1 mixes with ϕ_{11} . After inserting the minimization conditions for μ^2 and ν^2 we get:

$$M^2(\phi_{11}, H_1) = \begin{pmatrix} \frac{1}{10v}[-4bv v_1(v_1 + 5v) + \beta v_0^2(2v + v_1)] & -\frac{\beta v_0(v-v_1)}{2\sqrt{2}} \\ -\frac{\beta v_0(v-v_1)}{2\sqrt{2}} & -\frac{1}{4}\beta(5v^2 - 6vv_1 + v_1^2) \end{pmatrix}. \quad (5.20)$$

Using that $v_1 \sim v_0^2/v$, we can approximate these expressions, keeping only terms of $\mathcal{O}(v_0^2)$ or larger:

$$M^2(\phi_{11}, H_1) \approx \begin{pmatrix} -2bv_1v + \frac{1}{5}\beta v_0^2 & -\frac{\beta v_0 v}{2\sqrt{2}} \\ -\frac{\beta v_0 v}{2\sqrt{2}} & -\frac{1}{4}\beta(5v^2 - 6vv_1) \end{pmatrix}. \quad (5.21)$$

So the H - H component has mass of $\mathcal{O}(v^2)$, while the ϕ - ϕ component has mass of $\mathcal{O}(v_0^2)$. After diagonalizing this mass matrix we get one massless mode and a mode with mass squared of $\mathcal{O}(v^2)$. This massless mode is a Goldstone boson arising from the

breaking of the $SU(5)$ symmetry. The same result holds for the other components of the H triplet. This is an exact result, it is not due to the approximation that was made.

For the doublet components, we separately consider the Goldstone modes that do not get a vev, and the massive mode. Each Goldstone mode will mix with one component of Φ . As an example we can look at the H_7 component, which mixes with ϕ_{21} :

$$M^2(\phi_{21}, H_7) = \begin{pmatrix} \frac{1}{2}[20bv^2 - \frac{4}{5}bv_1^2 + \frac{2}{5}\beta v_0^2 + \frac{v_1 v_0^2}{5v}\beta] & -\frac{3vv_0}{2\sqrt{2}}\beta \\ -\frac{3vv_0}{2\sqrt{2}}\beta & 3vv_1\beta \end{pmatrix}. \quad (5.22)$$

We see that now the ϕ - ϕ component has mass $\mathcal{O}(v^2)$, while the H - H component is much lighter with mass $\mathcal{O}(v_0^2)$. After diagonalizing we again get a massless mode and a mode with mass $\mathcal{O}(v^2)$. This massless mode arises from the breaking of the electroweak symmetry. The expressions for the H_8 and H_{10} components are exactly the same.

To finalise our discussion on the masses of the scalars, we look at the H_9 component, which gets a vev in EWSB. This component mixes with ϕ_{23} and ϕ_{24} :

$$M^2(\phi_{23}, \phi_{24}, H_9) = \begin{pmatrix} \frac{1}{4}(a+b)v_1^2 + \frac{3\beta vv_0^2}{8v_1} & \frac{1}{8}\sqrt{\frac{3}{5}}(\beta v_0^2 - 2(5a+9b)vv_1) & -\frac{1}{4}v_0((2\alpha+\beta)v_1 - 3\beta v) \\ \frac{1}{8}\sqrt{\frac{3}{5}}(\beta v_0^2 - 2(5a+9b)vv_1) & \frac{3}{4}(5a-b)v^2 + \frac{1}{10}bv_1^2 - \frac{1}{20}\beta v_0^2 + \frac{3\beta vv_0^2}{8v_1} & \frac{1}{4}\sqrt{\frac{3}{5}}v_0((10\alpha+3\beta)v - \beta v_1) \\ -\frac{1}{4}v_0((2\alpha+\beta)v_1 - 3\beta v) & \frac{1}{4}\sqrt{\frac{3}{5}}v_0((10\alpha+3\beta)v - \beta v_1) & \frac{1}{4}\lambda v_0^2 \end{pmatrix}. \quad (5.23)$$

After inserting the expression for v_1 and expanding in small v_0 , we find that there is one eigenvalue of $\mathcal{O}(v_0^2)$ and two eigenvalues of $\mathcal{O}(v^2)$.

All in all, we find that after breaking $SU(5)$ to the Standard Model and then to $SU(3) \times U(1)$, we get 15 massless modes: 12 from breaking $SU(5)$ to the Standard Model and 3 more from EWSB. The other scalars will be massive, with masses of $\mathcal{O}(v^2)$, except for one scalar which takes the role of the SM Higgs boson, with a mass of $\mathcal{O}(v_0^2)$. This is exactly as desired. Note that there are no scalars with a mass of $\mathcal{O}(v_1^2)$. This is again in line with the *extended survival hypothesis*, as discussed in section 4.1.

5.4 Amount of fine-tuning in the minimal $SU(5)$ GUT

Now we have all the ingredients to quantitatively assess the amount of fine-tuning in the minimal non-supersymmetric $SU(5)$ GUT in the standard viewpoint. In this section we

will compare the results of the fine-tuning measures that were introduced in Chapter 3 and analysed in the context of the two-Higgs-doublet model and the left-right symmetric model in Chapter 4. These are the Dekens measure [73] and the BG measure [71,72].

In the original description of the Dekens measure, there is no prescription for which parameters should be used as the dependent parameters q_j . In the original paper, all possibilities are checked, and the set of q_j which gives the maximum amount of fine-tuning is selected. But following the logic of section 5.1, the vevs are usually regarded as output parameters, so we use these as the q_j . As mentioned in Section 5.2, in order to guarantee the proper hierarchy in vevs, we start with the values of v and v_0 , and use the minimum equation to determine the corresponding values of μ^2 , ν^2 and v_1 . To determine the fine-tuning in the vevs, we then need to solve the minimum equations for the vevs.

Because the minimum equations in our model are a complicated coupled system of equations in terms of v , v_0 and v_1 , we will do a numerical study. We start from sampling the vevs v and v_0 in the ranges $v = 10^{15} - 10^{17}$ GeV, $v_0 = 200 - 300$ GeV. Just like in Chapter 4, the coupling constants are sampled from a combined distribution consisting of two log-normal distributions (one centered around 1, the other centered around -1) in order to ensure natural and perturbative couplings. In addition we also take into account some constraints to ensure that the potential is bounded from below [120]:

$$\begin{aligned}
 a + \frac{7}{15}b &> 0, \quad a + \frac{7}{3}b > 0, \quad \lambda > 0, \\
 \min \left[\alpha, \alpha + \frac{1}{2}\beta \right] &> \max \left[-\frac{1}{2}(\lambda(a+b))^{\frac{1}{2}}, -\frac{1}{2} \left(\lambda \left(a + \frac{3}{2}b \right) \right)^{\frac{1}{2}} \right], \\
 \min \left[\alpha + \frac{3}{10}\beta, \alpha + \frac{2}{15}\beta \right] &> -\frac{1}{2} \left(\lambda \left(a + \frac{7}{15}b \right) \right)^{\frac{1}{2}}.
 \end{aligned} \tag{5.24}$$

The final constraint we impose is that the SM gauge group provides the energetically favourable minimum. So the breaking to the SM should result in the lowest energy compared to the other possible breakings. As worked out in Ref. [116], this can be achieved by setting $b > 0$ and $\beta < 0$, in addition to the constraint $a + 7/15 b > 0$ which was already in place from demanding that the potential is bounded from below. Note that the breaking from the $SU(5)$ gauge group down to $SU(3) \times SU(2) \times U(1)$ can also be realized with $\beta > 0$. However, then electroweak symmetry breaking does not give the desired breaking pattern, as breaking to $SU(2) \times SU(2) \times U(1)$ is more favourable in that scenario. For a discussion on this issue for general $SU(N)$ models, see Refs. [115, 117, 118]. Interestingly, for N odd and $N > 5$, the $b > 0$ and $\beta < 0$ region consists of different sectors that lead to different symmetries at low scale. So

Figure 5.1 The result of calculating the Dekens measure in a minimal $SU(5)$ GUT using the vevs as dependent parameters. 1000 points were sampled using the parameter ranges as mentioned in the text, with the bounded-from-below constraints 5.4 and the constraints to ensure that the SM breaking pattern is the global minimum.

for large N one would have to choose the parameters in a very specific way in order to get the correct breaking pattern.

The parameter values that satisfy all these constraints are used to determine μ^2 , ν^2 and v_1 . Then these values are used in the formula for the Dekens measure, where we take the mass parameters and coupling as input parameters, and the vevs (including v_1) are taken as the q_j . This gives the result shown in Figure 5.1. This figure shows that the Dekens measure consistently indicates a large amount of fine-tuning of the order $\frac{v^2}{v_0^2}$. This result agrees with estimates in the literature on the amount of fine-tuning in this model [66].

The other fine-tuning measure we will investigate is the Barbieri-Giudice (BG) measure Δ_{BG} [71, 72], which determines the amount of fine-tuning in an observable with respect to the input parameters p_i of the model. We will use the scalar masses as observables, so our expression for the BG measure is:

$$\Delta_{BG} = \max_{i,j} \left| \frac{p_i}{m_j^2} \frac{\partial m_j^2}{\partial p_i} \right|. \quad (5.25)$$

Note that in order to calculate the fine-tuning, we need to write all the masses in terms of the input parameters p_i . These expressions are too elaborate to show here,

Figure 5.2 The result of calculating the BG measure in a minimal $SU(5)$ GUT in the standard viewpoint. 1000 points were sampled using the parameter ranges as mentioned in the text, with the bounded-from-below constraints 5.4 and the constraints to ensure that the SM breaking pattern is the global minimum.

and calculating the derivatives has to be done numerically. The results of calculating Δ_{BG} in this model are shown in Figure 5.2. We see that the results for the Dekens measure and the BG measure are extremely similar. This can be explained from the observation that the masses are proportional to a vev. So since the values of the vevs are the main source of fine-tuning, this same amount of fine-tuning will show up in the BG measure. There will only be a small difference due to numerical factors in the conversion from the vevs to the masses.

This concludes the quantitative discussion of the doublet-triplet splitting in the standard viewpoint. Let us summarise the findings thus far. We have first considered how to break the $SU(5)$ symmetry, and later also the ways to subsequently break the electroweak symmetry. We described all possible ways to achieve this in the minimal $SU(5)$ model and reached the conclusion that in the standard viewpoint all options should be discarded because either they require fine-tuning of the parameters in the Lagrangian or they lead to phenomenologically unacceptable side effects, like additional Goldstone bosons or large ρ parameter corrections. It is clear that in none of these scenarios, a phenomenologically viable hierarchy of vevs will result from the generic theory with natural and perturbative parameters. The DTSP is an expression of this problem. But as we will show in the next section this problem is not unique to the $SU(5)$ GUT. Similar problems arise in simpler extensions of the SM with two coupled scalar sectors like the two-Higgs-doublet model (2HDM), and in e.g. the LRSM, as we

discuss in Appendix 5.A.

5.5 Hierarchies in the 2HDM

As mentioned in Chapter 4, the Higgs sector of the 2HDM is constructed by adding an additional doublet to the Standard Model Higgs sector [44, 81]. Recall that the two doublets are defined as:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}. \quad (5.26)$$

In order to simplify our discussion, we demand CP invariance and impose a $\mathbb{Z}_2$ symmetry on the potential. Under these constraints, the Higgs potential has the form:

$$V = -\mu_1^2 A - \mu_2^2 B + \lambda_1 A^2 + \lambda_2 B^2 + \lambda_3 C^2 + \lambda_4 D^2 + \lambda_5 AB,$$

where the invariants A, B, C and D are defined as:

$$\begin{aligned} A &= \Phi_1^\dagger \Phi_1, & C &= \frac{1}{2} (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1), \\ B &= \Phi_2^\dagger \Phi_2, & D &= \frac{1}{2i} (\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1). \end{aligned} \quad (5.27)$$

Following the discussion in Chapter 4, we will avoid charged and CP -violating vacua. So the vevs of the two doublets are given by:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (5.28)$$

This breaks the $SU(2)_L \times U(1)_Y$ symmetry to the $U(1)_Q$ symmetry, leading to the appearance of three Goldstone bosons that get absorbed by the W and Z bosons. Note that there is a difference with the discussion on symmetry breaking in the $SU(5)$ GUT. In the $SU(5)$ model, one has to impose constraints on the parameters in order to ensure that the SM breaking pattern leads to the global minimum of the potential. In the 2HDM, when a minimum exists that conserves charge and CP , it is automatically the global minimum [128]. It is possible however that there are multiple minima of the potential that conserve charge and CP [129], so tunneling between these minima is possible. We will not look further into this possibility, as it is not relevant to our discussion.

In the standard phenomenological analyses of the 2HDM, the two vevs are required to satisfy the relation $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$, which precludes a large hierarchy except for one of the vevs extremely close to zero. Here we will not impose this constraint in order to see if a large hierarchy of vevs can result from the general theory

with parameters of $\mathcal{O}(1)$.

Contrary to the $SU(5)$ GUT, we can analytically solve the minimum equations of the 2HDM for the vevs without having to make an approximation. Assuming that both v_1 and v_2 are non-zero, we find:

$$\begin{aligned} v_1^2 &= \frac{\lambda_2 \mu_1^2 - \lambda_+ \mu_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\ v_2^2 &= \frac{\lambda_1 \mu_2^2 - \lambda_+ \mu_1^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \end{aligned} \quad (5.29)$$

where $\lambda_+ \equiv \frac{1}{2}(\lambda_3 + \lambda_5)$. To investigate how natural a large hierarchy in vevs is, we will investigate two scenarios for the mass parameters μ_i^2 : the case where there is a large hierarchy in the mass parameters, say $\mu_1^2 \gg \mu_2^2$, and the case where they have very similar values, $\mu_1^2 \approx \mu_2^2$.

For a large hierarchy in the mass parameters $\mu_1^2 \gg \mu_2^2$, we see from Eq. 5.29 that in the generic model both vevs will have values of $\mathcal{O}(\mu_1)$. So in general a large hierarchy in the mass parameters will not translate into a large hierarchy in the vevs. In that case there will not be any fine-tuning, neither in the BG measure nor in the Dekens measure with the vevs taken as output parameters. It is possible though to obtain a large hierarchy in vevs in the special case that $\lambda_5 \approx -\lambda_3$, since then there are two contributions of $\mathcal{O}(\mu_1^2)$ to v_2 that cancel to give a much smaller value. The Dekens measure will show a large amount of fine-tuning in this case and moreover, a slight variation in one of the parameters will lead to the disappearance of the large hierarchy.

When the two mass parameters have similar values, again the generic model will have two vevs with similar values. There is no fine-tuning present then. But also in this case it is possible to obtain a large hierarchy, although there is a different cancellation necessary to achieve this. When one of the two numerators in Eq. 5.29 is very small, there will be a large hierarchy in vevs. This again comes at the cost of fine-tuning, due to a large cancellation between the contributions of the mass parameters. In Appendix 5.A, we discuss in more detail the amount of fine-tuning that is necessary to obtain a hierarchy in vevs, using both the Dekens and the BG measure.

So we can say in general that independent of the values one takes for the mass parameters, the generic theory will have no large hierarchy in vevs. The reason for this is the coupling between the two scalar sectors. If the two sectors had been completely decoupled, a hierarchy in mass parameters would translate directly into a hierarchy in vevs. But due to the coupling λ_+ , which relates the two sectors, any hierarchy in

the μ_i^2 gets nullified. This is also a key fine-tuning aspect of the DTSP in the $SU(5)$ GUT and a general problem when trying to achieve a large hierarchy in vevs in BSM theories with extended scalar sectors.

The example with a cancellation between λ_3 and λ_5 shows that if the coupling between the scalar sectors is very small, a large hierarchy in the mass parameters can still translate into a large hierarchy in vevs. We note that small λ_+ is also possible when $\lambda_3, \lambda_5 \ll 1$. If these small couplings are technically natural, for instance in case they arise effectively from an intermediate (portal) channel between the scalar sectors, the resulting hierarchy can be considered natural.

An example of such a situation is the phenomenon of superconductivity [130]. Inside a superconductor, electrons form Cooper pairs. These Cooper pairs form a scalar condensate, that breaks the $U(1)_Q$ symmetry. Due to this breaking, photons obtain a mass inside a superconductor. This additional step of symmetry breaking introduces a hierarchy between the mass of the photon and the electroweak scale. And it turns out that the photon mass is much smaller than the electroweak scale, but there is a natural explanation for this. The coupling between the two sectors can only happen indirectly, since the Cooper pairs do not couple directly to the Higgs field. The only way they interact is via the electron Yukawa coupling, leading to a coupling proportional to m_e^2/v_{EW}^2 . This coupling is small but technically natural. So the small coupling between the different sectors makes it possible for the scalar condensate to develop a small vev in a natural way.

But in the case of the $SU(5)$ GUT, and other GUTs, this does not work, because in order for the doublet and triplet fields to have masses of different scales, a sufficiently large coupling between the Φ and H fields is necessary. So α and β cannot both be small. This is another aspect of the DTSP in the $SU(5)$ GUT, which makes it harder to obtain a viable model in a natural way, and which usually does not receive much attention.

To summarize, our discussion of the 2HDM has given us some insights in the origin of the DTSP. We have found that the problem consists of two main aspects: first of all, a hierarchy in mass parameters will not translate in general to a hierarchy in vevs. Secondly, it is not possible in the $SU(5)$ GUT to have a small coupling between the two scalar sectors. These aspects are in general present in all models with coupled scalar sectors with natural and perturbative couplings. Therefore, we conclude that it is not possible to achieve a viable hierarchy in both vevs and scalar masses in a model like the minimal $SU(5)$ GUT with natural parameters and without fine-tuning.

The analysis of the $SU(5)$ GUT so far has assumed that the scales at which the

different symmetries break emerge from the Lagrangian. In the next section we will argue for a different viewpoint, where the hierarchy in vevs is used as input.

5.6 Alternative viewpoint on hierarchy and fine-tuning

We showed in section 5.2 and 5.3 that in order to have a large hierarchy of triplet and doublet masses, without introducing additional Goldstone bosons or large corrections to the ρ parameter, it is necessary to introduce a small vev in the Φ field that breaks the electroweak symmetry. This means that already at the GUT breaking scale a (tiny) EWSB term is introduced, so the electroweak symmetry is actually never present below the $SU(5)$ breaking scale. This means that the two breaking mechanisms are not independent, since there is EWSB simultaneously with the breaking of the GUT. Therefore the theory at the GUT scale already has to ‘know’ about the implementation of EWSB at low energy. To incorporate this, we advocate a viewpoint where besides the orientation of vevs ($\langle\Phi\rangle = \text{diag}(v, v, v, -\frac{3}{2}v - \frac{1}{2}v_1, -\frac{3}{2}v + \frac{1}{2}v_1)$ and $\langle H\rangle = \frac{1}{\sqrt{2}}(0, 0, 0, 0, v_0)^T$ in this case) also the scale at which each breaking occurs is specified. In other words, the values of the vevs, or at least their hierarchy is imposed.

As discussed in Section 5.4, one has to choose the parameter ranges of some of the parameters in the Lagrangian, in order to ensure that the obtained global minimum is the desired minimum (ignoring metastable vacua for simplicity). In the case of the minimal $SU(5)$ GUT, this is done by having $b > 0$, $\beta < 0$.

So the most general theory will not necessarily lead to the desired breaking pattern. If one were to consider a generic theory with $SU(5)$ breaking at a high scale, then this will not lead to EWSB automatically and certainly not EWSB at a much lower scale. So why not simply accept the fact that one has to start from the subset of theories that have the desired breaking pattern and hierarchy of scales built in from the start? Instead of imposing symmetry A at scale a and expecting that generically symmetry B will result at scale b with $b \ll a$, we simply accept that we need to restrict to theories that have symmetry A at scale a and symmetry B at scale b .

Just like imposing a symmetry will result in less free parameters, imposing a hierarchy on the vevs that break the symmetry will result in less free parameters. So in this viewpoint we are considering a specific subset of theories with less free parameters than the fully generic theory. Still all terms allowed by the symmetries will be present, but not all terms will be required to be completely independent from each other. Full independence appears to be too much to ask if one wants the SM to arise at low energy scales. We will therefore adopt this alternative viewpoint next and ask the more restricted question whether there is particular fine-tuning needed within the

subset of theories that lead to the desired hierarchy. While this question may seem to have the obvious answer ‘no’, we showed in the previous chapter that even when one imposes the values of the vevs and views them as input parameters, there are still cases where fine-tuning appears in either the BG measure or the Dekens measure. Therefore we will now investigate the amount of fine-tuning in the minimal $SU(5)$ GUT in this scenario.

Since we propose to consider the vevs as input parameters, for illustration purposes we make them part of the Lagrangian via Lagrange multipliers, although this is not needed. Using Lagrange multipliers,⁵ we can impose the values of the vevs in the following way (for simplicity we restrict to the case $v_1 = 0$):

$$\mathcal{L} = \mathcal{L}_0 + \xi \left(\text{Tr}(\Phi^2) - \frac{15}{2}v^2 \right) + \xi_0 \left(H^\dagger H - \frac{v_0^2}{2} \right), \quad (5.30)$$

where v and v_0 are to be understood as input parameters and ξ and ξ_0 are Lagrange multipliers. These constraints are imposed in an $SU(5)$ invariant way.⁶ At the GUT breaking scale, we minimise the potential for both Φ and ξ , which ensures that the GUT breaking minimum is located at the scale v . Then at the EWSB scale, we minimise with respect to H and ξ_0 , ensuring the proper scale for v_0 . In this way, the vevs are already present at the level of the Lagrangian.

Alternatively, we could write the constrained Lagrangian as:

$$\mathcal{L} = \mathcal{L}_0 + \xi \left(x \text{Tr}(\Phi^2) - H^\dagger H \right), \quad (5.31)$$

where x quantifies the hierarchy in vevs: $x = v_0^2/15v^2$. In this case we minimise with respect to ξ only at the EWSB scale. The difference between the two approaches is that when we only impose the hierarchy, there is only one free parameter less, as opposed to two free parameters when we impose the value of both vevs. Since the doublet-triplet splitting problem only depends on the hierarchy, and not on the individual scales, it is enough to add a constraint for the hierarchy. From now on we will work with this approach, but in practice there is no difference with imposing the values of the vevs, since the value of v does not require fine-tuning, and imposing both v and x is equivalent to imposing the value of v and v_0 .

⁵Alternatively, one could consider constraint effective potentials [131]. The two methods are not completely equivalent. The method with Lagrange multipliers only holds on-shell, since it enforces the constraint when minimizing the potential. But the constraint effective potential adds a restriction to the path integral, so the constraint also holds off-shell.

⁶With $v_1 \neq 0$ the first constraint becomes $\text{Tr}\Phi^2 = \frac{15}{2}v^2 + \frac{1}{2}v_1^2$. So in order to fix both the value of v and v_1 , one needs to impose a second independent constraint on Φ , e.g. $\xi_1(\text{Tr}\Phi^4 - \frac{1}{8}(105v^4 + 54v^2v_1^2 + v_1^4))$.

For any given x value, the Lagrangian will thus contain one free parameter less, which is fully determined by the other parameters and the vevs. As our earlier analysis shows, variation of just a single parameter will result in a change of x that generically will tend to be of $\mathcal{O}(1)$. In that sense small x values are not stable under variation of a single parameter. But it can be stable when variation of one parameter is accompanied by variation of another one. The question then arises whether that should be considered fine-tuning. As argued in the previous chapter, we would not call this fine-tuning, since changes in one natural parameter require changes in another one of exactly the same order. The fact that there are also parameters that need very large or very small changes is irrelevant, as long as the variation in the first parameter can be accommodated by a natural change in some parameter. This reasoning can only be applied when the values of the vevs (or the hierarchy in vevs) are imposed, so when their values are no longer considered as output parameters.

This statement can then quantitatively be checked in an analysis of the Dekens measure, where the vevs are now considered input parameters. Now the question arises again what the proper way is to apply the Dekens measure: which choice should we make for the dependent parameters q_j ? As argued in Chapter 4, we will adopt the approach that we first solve the minimum equations for a certain set of parameters, and then apply the Dekens measure using these same parameters as the dependent parameters. If there is a set of parameters that leads to little or no fine-tuning, we argue that there is no fine-tuning necessary to maintain the large hierarchy in vevs.

For the minimal $SU(5)$ case, this means that we can take e.g. μ^2 , ν^2 and v_1 as dependent parameters. Taking these parameters as the q_j , and imposing the hierarchy on the vevs as described above, the results for the Dekens measure are shown in Figure 5.3. This shows that there is no fine-tuning present now, meaning that it is possible to maintain the large hierarchy in vevs by adjusting parameters by the same order of magnitude.

The BG measure will also not show any fine-tuning in this scenario. All scalar masses are proportional to a vev, and the mass spectrum is exactly like it should be if the vevs have the proper values. So by imposing the vevs we ensure the correct mass spectrum and there will be no large amount of fine-tuning. This is not automatically guaranteed, as we showed in the previous chapter. However we only looked at the generic model now, and we did not do an analysis of e.g. the amount of fine-tuning to ensure the correct Higgs mass, as in the discussion on the LRSM. But with the hierarchy in vevs fixed, this can only lead to minor fine-tuning, not nearly as much as in the standard viewpoint.

Figure 5.3 The result of calculating the Dekens measure in a minimal $SU(5)$ GUT using μ^2 , ν^2 and v_1 as dependent parameters. 1000 points were sampled using the distributions for the parameters mentioned in Section 5.4.

5.7 Remarks on a large hierarchy in vevs

In the previous section, we imposed the values of the vevs or their hierarchy, and concluded that there is no fine-tuning necessary to maintain this hierarchy under variations of the remaining free parameters. We advocated for the approach where the hierarchy in vevs is imposed in order to fix the symmetry breaking pattern at high energy already, so the GUT scale physics ‘knows’ about the symmetry breaking pattern at low energy. But this leaves open the question whether any hierarchy in vevs is acceptable, as this will always lead to a small ratio. If one includes the hierarchy in vevs as a parameter in the Lagrangian, one may wonder if such a hierarchy should not be considered unnatural to begin with. The x values in the numerical study of the previous section are of the order 10^{-26} to 10^{-30} . For such small values one might wish to have an explanation in terms of an enhanced symmetry when $x \rightarrow 0$, in accordance with the technical naturalness argument proposed by ‘t Hooft [8]. Typically setting a vev to zero indeed enlarges the symmetry from a subgroup H of a group G to the full group G . However, in the case considered in this chapter, setting v_0 or v_1 to zero does not enhance the symmetry, unless one sets them both to zero, since both vevs break electroweak symmetry. So these two vevs are not technically natural individually. But even in the case where one sets both vevs to zero, one may wonder whether the symmetry is really enhanced, since the full group G is still nonlinearly realised in the broken phase [132]. In this sense the full symmetry is still present in the underlying description of the theory and no enhanced symmetry ever arises if vevs are set to zero.

If the symmetry argument can then not be applied to vevs, it is also not required to apply 't Hooft's naturalness criterion to them. The options in this model are then either to only allow ratios of $\mathcal{O}(1)$ when imposing the values of the vevs, which will never result in a theory with a difference in scales, or to accept a large hierarchy in the vevs, which can result in a viable model.

5.8 Conclusions

We have studied the symmetry breaking pattern in the minimal non-supersymmetric $SU(5)$ GUT, and confirmed in line with the DTSP that it is not possible to obtain the desired hierarchy of vevs without fine-tuning in a theory with all natural and perturbative parameters. We emphasize that this is actually a general problem of coupled scalar sectors: the coupling between the two sectors will prevent a large hierarchy in mass parameters to translate into a large hierarchy in vevs. And since a (technically-natural) small coupling between the different scalar sectors is not possible in the minimal $SU(5)$ GUT (either α or β needs to be large to ensure the splitting between the doublet and the triplet), fine-tuning is always necessary. We show a different viewpoint where we restrict ourselves to the subset of theories that contain a large hierarchy in vevs. By imposing the hierarchy in vevs, the amount of free parameters is reduced. We highlight this different viewpoint, since it is not possible to view the breaking of the GUT symmetry separate from the EWSB, which implies that the theory at the GUT scale has to be adjusted to the low-energy physics by imposing the orientations of the vevs. This orientation has to be selected by restricting to a subset of the parameter space such that the proper global minimum is reached. We argue that to avoid fine-tuning, the hierarchy in vevs can be imposed on the theory. In that case, the vevs cannot be considered output parameters anymore. For illustration purposes we incorporated this idea by imposing the hierarchy in vevs by using a Lagrangian multiplier. We find that the generic theory with a large hierarchy in vevs will not need any fine-tuning of the remaining free parameters in order to produce the correct mass spectrum. This means that changes in one natural parameter can be countered by changes of the same order of magnitude in another natural parameter. This conclusion is not automatically guaranteed, as we showed in Chapter 4. Finally, we emphasize once more that our discussion only applies to the classical level and does not shed any light on issues due to loop corrections, which will be the main focus of the next chapter.

Appendix

5.A Fine-tuning in vevs in the 2HDM and LRSM

In Chapter 4 we discussed the amount of fine-tuning in the 2HDM and the LRSM when we impose the values of the vevs and consider them as input parameters. In Chapter 5, we discussed fine-tuning when viewing the vevs as output parameters. In this appendix, we first consider the same situation as in Section 5.5, where we argued that in general, the 2HDM with the vevs as output parameters will not generate a hierarchy in vevs. Now we will determine the amount of fine-tuning, according to the Dekens measure and the BG measure, that is necessary to obtain a hierarchy in vevs. Next we turn to the LRSM, and determine how much fine-tuning is necessary in order to have the desired hierarchy in vevs. We leave this discussion for an appendix because it does not add much to the discussion of the standard viewpoint in Chapter 5, it is too involved to be discussed there. We do want to come back to this point, however, since it is relevant to the discussion in Chapter 4. In this appendix, we only discuss the numerical results, a discussion on the interpretation of these results is left for the concluding chapter.

5.A.1 Fine-tuning in the 2HDM

For the 2HDM, we can determine the amount of fine-tuning analytically. Solving the minimum equations (Equation 4.7) for the vevs gives:

$$v_1^2 = \frac{\lambda_2 \mu_1^2 - \lambda_+ \mu_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \quad v_2^2 = \frac{\lambda_1 \mu_2^2 - \lambda_+ \mu_1^2}{\lambda_1 \lambda_2 - \lambda_+^2}. \quad (5.32)$$

We can now determine the contributions to the Dekens measure. We write all contributions in terms of the vevs so we can interpret the expressions more easily.

$$\begin{aligned}
 \Delta_D(v_1, \mu_1) &= \frac{\lambda_2}{v_1^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, & \Delta_D(v_2, \mu_1) &= -\frac{\lambda_+}{v_2^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_D(v_1, \mu_2) &= -\frac{\lambda_+}{v_1^2} \frac{\lambda_2 v_2^2 + \lambda_+ v_1^2}{\lambda_1 \lambda_2 - \lambda_+^2}, & \Delta_D(v_2, \mu_2) &= \frac{\lambda_2}{v_2^2} \frac{\lambda_+ v_1^2 + \lambda_2 v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_D(v_1, \lambda_1) &= -\frac{\lambda_1 \lambda_2}{2(\lambda_1 \lambda_2 - \lambda_+^2)}, & \Delta_D(v_2, \lambda_1) &= \frac{\lambda_1}{2v_2^2} \frac{\lambda_+ v_1^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_D(v_1, \lambda_2) &= \frac{\lambda_2}{2v_1^2} \frac{\lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, & \Delta_D(v_2, \lambda_2) &= -\frac{\lambda_1 \lambda_2}{2(\lambda_1 \lambda_2 - \lambda_+^2)}, \\
 \Delta_D(v_1, \lambda_3) &= \frac{\lambda_3}{4v_1^2} \frac{\lambda_+ v_1^2 - \lambda_2 v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, & \Delta_D(v_2, \lambda_3) &= -\frac{\lambda_3}{4v_2^2} \frac{\lambda_1 v_1^2 - \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_D(v_1, \lambda_5) &= \frac{\lambda_5}{4v_1^2} \frac{\lambda_+ v_1^2 - \lambda_2 v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, & \Delta_D(v_2, \lambda_5) &= -\frac{\lambda_5}{4v_2^2} \frac{\lambda_1 v_1^2 - \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}.
 \end{aligned} \tag{5.33}$$

The result for the Dekens measure is then determined by taking the maximum of the absolute values of these expressions: $\Delta_D = \max_{i,j} |\Delta_D(v_i, p_j)|$. From the individual expressions we can already see that the main source of fine-tuning will be the fraction v_1^2/v_2^2 . The origin of this fine-tuning is that in order to have the hierarchy $v_1 \gg v_2$, we need a cancellation between the μ_1^2 and μ_2^2 contributions to v_2^2 when both mass parameters have similar values, or between different contributions of $\mathcal{O}(\mu_1^2)$ when $\mu_1^2 \gg \mu_2^2$. Since these contributions need to be of $\mathcal{O}(v_1^2)$ in both scenarios, the fine-tuning we find will be of $\mathcal{O}(v_1^2/v_2^2)$.

This behaviour can also be seen in the upper plots of Figure 5.A.1, where the result for the distribution of the ratio $\tan^2 \beta = v_1^2/v_2^2$ (left-hand side) is compared with the result for the Dekens measure (right-hand side). The difference between the two distributions (roughly a factor 4) can be understood from the expressions in Equation 5.33. All leading contributions (so the contributions proportional to v_1^2/v_2^2) contain a suppression factor. For $\Delta_D(v_2, \lambda_3)$ and $\Delta_D(v_2, \lambda_5)$ this is exactly a factor 4. For $\Delta_D(v_2, \mu_1)$, $\Delta_D(v_2, \mu_2)$ and $\Delta_D(v_2, \lambda_1)$ this is the factor λ_+ , which is generally smaller than one, since it is the sum of a number of $\mathcal{O}(-1)$ and a number of $\mathcal{O}(1)$. In combination with the factor $1/2$ in the definition of λ_+ , this factor is generally small and will lead to a suppression.

The reasoning for the BG measure is very similar. The small value of v_2 will again be the main source of fine-tuning, which now shows up in the mass of the lightest Higgs m_h^2 . Using the approximation Equation 4.9, the individual contributions to the BG

Figure 5.A.1 The distribution of 5000 parameter points in the 2HDM, showing the ratio $\tan^2 \beta = v_1^2/v_2^2$ in the upper left, the result for the Dekens measure in the upper right and the result for the BG measure in the lower panel. The fine-tuning measures are calculated considering the vevs as output parameters. The BG measure is calculated without using the approximation of the hierarchy in vevs in the expressions for the masses.

measure are given by:

$$\begin{aligned}
 \Delta_{BG}(m_h^2, \mu_1) &= -\frac{2\lambda_+}{v_2^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_{BG}(m_h^2, \mu_2) &= \frac{2\lambda_1}{v_2^2} \frac{\lambda_+ v_1^2 + \lambda_2 v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_{BG}(m_h^2, \lambda_1) &= \frac{\lambda_+}{v_2^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_{BG}(m_h^2, \lambda_3) &= -\frac{\lambda_3}{2v_2^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}, \\
 \Delta_{BG}(m_h^2, \lambda_5) &= -\frac{\lambda_5}{2v_2^2} \frac{\lambda_1 v_1^2 + \lambda_+ v_2^2}{\lambda_1 \lambda_2 - \lambda_+^2}.
 \end{aligned} \tag{5.34}$$

The result for the BG measure is then given by: $\Delta_{BG} = \max_i |\Delta(m_h^2, p_i)|$. It is clear from these expressions that the main source of fine-tuning is again the fraction v_1^2/v_2^2 .

This can also be seen in Figure 5.A.1, where the lower panel shows the result for the BG measure. This distribution is very similar to the distribution of $\tan^2 \beta$ (upper-left plot), indicating that this is indeed the main source of fine-tuning. Whereas there was roughly a factor 4 difference between the distribution of $\tan^2 \beta$ and the Dekens measure, this is now only a factor 2 for the BG measure. The origin of this difference is the fact that the Dekens measure uses the vevs, and not the squared vevs as the q_j , whereas we use the squared Higgs mass in the BG measure. This square produces a factor 2 in the derivative in the BG measure, leading to the observed difference.

So we see that the Dekens measure and the BG measure in general agree on the amount of fine-tuning, which can be understood from the fact that all the scalar masses are proportional to a vev. The fine-tuning in the vevs that is needed to get $v_2 \ll v_1$ will also show up in the BG measure when considering a mass that is proportional to v_2 (m_h^2 in this case).

5.A.2 Fine-tuning in the LRSM

In the LRSM, the situation is more complicated because of the more involved minimum equations (Equation 4.29). We did not find analytical solutions when solving the minimum equations for the vevs. Therefore, we will just show numerical results for the amount of fine-tuning that is necessary to obtain the desired hierarchy with $v_L \sim \mathcal{O}(1 \text{ eV})$, $\kappa, \kappa' \sim \mathcal{O}(100 \text{ GeV})$ and $v_R \sim \mathcal{O}(10 \text{ TeV})$. First we will discuss the Dekens measure, where following Ref. [73] we will argue that the main source of fine-tuning in the minimum equations is now the see-saw relation, which needs a large cancellation between the different β_i terms in order to result in a small value for v_L . This results in the same amount of fine-tuning that was noted in Section 4.2.2 when solving the seesaw relation for one of the ρ parameters. Next we investigate the BG measure, which interestingly highlights a different kind of fine-tuning. We will highlight why the Dekens and BG measure give different results, and discuss the origin of the fine-tuning that is present in the BG measure.

Let us first explain how we solve the minimum equations and calculate the amount of fine-tuning using the Dekens measure. In order to realize the desired hierarchy in vevs, we start with the values of the vevs and solve the minimum equations for four other parameters: μ_1^2 , μ_2^2 , μ_3^2 and β_1 . Note that we could have solved for a ρ parameter as well, but this would result in a value much larger than one. By solving for one of the β parameters, we avoid this problem. Having obtained values for all parameters such that the minimum equations are satisfied, we can determine how much fine-tuning is necessary to realize this hierarchy. To this end, we solve the minimum equations for

the vevs and apply the Dekens measure according to:

$$\Delta_D(v) = \max_{i,j} |\Delta_D(v_j, p_i)| = \max_{i,j} \left| \frac{p_i}{v_j} \frac{\partial v_j}{\partial p_i} \right|, \quad (5.35)$$

with $v_j = \{v_L, v_R, \kappa, \kappa'\}$, and the p_i include all mass parameters and couplings (so the μ_i^2 and β_1 are included in this set). The derivatives in this expression cannot be calculated analytically, so we vary the p_i one by one. After each variation, the minimum equations are solved for the vevs,⁷ and the derivative $\partial v_j / \partial p_i$ can be calculated from the difference between the initial value of the vev and the value after variation. In these calculations, we no longer make the approximation from Equation 4.29 where we dropped terms of order v_L/v_R and higher.

We will discuss the impact of the different vevs on the fine-tuning in two steps. First we discuss the effect of the size of v_L and v_R , as these have a clear effect on the amount of fine-tuning. Then we discuss κ and κ' , as their effect is slightly less clear. For both analyses we will use a fixed set of values of $\mathcal{O}(1)$ for the coupling constants in order to simplify the discussion. We will only consider the effect of varying the vevs, we do not consider the effects of all the couplings on the amount of fine-tuning.

For these fixed values of the couplings, and fixed vevs $\kappa = 100$ GeV and $\kappa' = 150$ GeV, we vary the value of v_L in the range $[10^{-9}, 10^{-11}]$ GeV and v_R in the range $[10^4, 10^6]$ GeV. The results for the Dekens measure, with the vevs as dependent parameters, are shown in Figure 5.A.2. This figure shows a clear dependence on the values of v_L and v_R . Closer inspection reveals that the dependence is $\Delta_D \propto 1/(v_L v_R)$. Looking at the minimum equations (Equation 4.29), we can anticipate that the seesaw relation might be the main source of fine-tuning. And indeed, when we estimate the amount of fine-tuning to be $\Delta \sim \kappa \kappa' / v_L v_R$, using the hierarchy in vevs described above, we find a value $\Delta \sim 10^{10}$, which agrees with the results in Figure 5.A.2. To further investigate this, we have to look into the dependence on κ and κ' .

We can make a similar plot as for the v_L, v_R dependence, but now fixing $v_L = 10^{-10}$ GeV, $v_R = 10^5$ GeV and varying κ and κ' . The result for the Dekens measure in this scenario is shown in Figure 5.A.3.

This figure shows three features we want to discuss. First of all, on the diagonal where $\kappa = \kappa'$, the system cannot be solved, as the fraction $1/(\kappa^2 - \kappa'^2)$ which appears in the minimum equations diverges. This explains the band of distinct values on the diagonal in Figure 5.A.3. Next we see a slight gradient from the lower-left to the upper-right

⁷The minimum equations do not provide a unique solutions for the vevs. We always select the solution where the vevs have the desired hierarchy. We use a small enough variation in the p_i to ensure that the hierarchy is not spoiled by the variation.

Figure 5.A.2 The result for the Dekens measure, using the vevs as dependent parameters, as a function of v_L and v_R . The couplings are fixed at values of $\mathcal{O}(1)$, and the other vevs are fixed at $\kappa = 150$ GeV and $\kappa' = 200$ GeV.

corner, indicating that, in line with the expectation from the seesaw relation, larger values of κ and κ' give a larger amount of fine-tuning.

The third feature we see is that when increasing the ratio κ'/κ (and to a lesser extent also for the inverse), the fine-tuning increases. The exact mechanism behind this is unclear, and hard to determine because of the system of four coupled non-linear equations, which does not have an analytical solution in terms of the vevs. There are several ways in which the values of κ and κ' appear in the minimum equations. There are terms $\beta_i \kappa^{(\prime)2}$ in the seesaw relation, but also terms $\lambda_i \kappa^{(\prime)2}$ and $\alpha_i \kappa^{(\prime)2}$ in the other three minimum equations. Furthermore, there are factors $\kappa'^2/(\kappa^2 - \kappa'^2)$ and $\kappa \kappa'/(\kappa^2 + \kappa'^2)$ that appear in the minimum equations. This results in a quite complicated dependence on the values of κ and κ' , in addition depending strongly on the exact values of the coupling constants. While this effect is quite large, it appears to be an additional effect on top of the seesaw relation, which still provides the main source of fine-tuning for this hierarchy of vevs. The results on fine-tuning in the seesaw relation agree with the literature [73, 91, 133].

Figure 5.A.3 The result for the Dekens measure, using the vevs as dependent parameters, as a function of κ and κ' . The couplings are fixed at values of $\mathcal{O}(1)$, and the other vevs are fixed at $v_L = 10^{-10}$ GeV and $v_R = 10^5$ GeV.

So the seesaw relation is the main source of fine-tuning. One way to understand this (at the order-of-magnitude level) is by solving this relation for v_L :

$$v_L = \frac{\kappa\kappa'\beta_1 + \kappa^2\beta_2 + \kappa'^2\beta_3}{(2\rho_1 - \rho_3)v_R}. \quad (5.36)$$

When we have the desired hierarchy between v_R and $\kappa^{(\prime)}$, we see that if we take natural and perturbative value for the couplings, v_L will have a value of $\mathcal{O}(1)$ GeV. So in order to have v_L ten orders of magnitude smaller (as desired to have viable neutrino masses), the β parameters need to cancel up to about ten significant digits, resulting in the observed fine-tuning. Note that there is a viable way to get rid of this tree-level fine-tuning in the seesaw relation, which is by increasing the value of v_R up to a scale $\mathcal{O}(10^{14})$ GeV. But this would make the new particles in the LRSM undetectable in the foreseeable future, and will induce other tree-level fine-tuning problems, as we will see in the BG measure, which we will discuss next.

The calculation of the BG measure has to be done numerically as well. The procedure we use is very similar to the determination of the Dekens measure described above. We first take the desired values for the vevs and use the same set of fixed couplings.

Figure 5.A.4 The result for the BG measure, using the vevs as dependent parameters, as a function of v_R and κ (left) and as a function of v_R and κ' (right). The couplings are fixed at values of $\mathcal{O}(1)$, the value of v_L is fixed at $v_L = 10^{-10}$ GeV, and the vev that is not varied in the plot is fixed at 190 GeV in both cases.

These are used to determine the masses. But to calculate the derivatives in the BG measure, we need to write the masses in terms of the mass parameters and all the couplings. So, using the minimum equations, we determine the values of the μ_i^2 and a parameter in the seesaw relation (we again choose β_1 , this value is hence not included in the set of fixed couplings). Now we can vary the parameters (including the μ_i^2 and β_1) one by one, determine the new values for the vevs and from this calculate the new masses. The difference between the original masses and the new masses is then used to calculate a numerical derivative, which is used in the BG measure.

The result for calculating the BG measure in this way is shown in Figure 5.A.4, where in the left (right) plot we fixed $v_L = 10^{-10}$ GeV and $\kappa' = 190$ GeV ($\kappa = 190$ GeV), and varied v_R and κ (κ') in appropriate ranges. The first observation from this figure is that the amount of fine-tuning is much lower than in the Dekens measure. We see $\Delta_{BG} \sim 10^3$ to 10^4 compared to $\Delta_D \sim 10^{10}$. This is an indication that the seesaw relation is no longer the main source of fine-tuning. This can be understood from the fact that there are no scalar masses which are proportional to v_L , whose value is the main source of fine-tuning in the Dekens measure. It was checked numerically that the value of v_L indeed has no significant effect on the BG measure. The scale v_L does show up in the neutrino masses. So, if we would include those masses in the set of observables in the BG measure, the fine-tuning from the seesaw relation would again appear. This shows that it is important to have a full range of observables to assess the amount of fine-tuning with the BG measure.

Then the question is what the source of the remaining fine-tuning is. Since the seesaw relation no longer plays a role, we are in a situation that is quite similar to the 2HDM. We have two scales, v_R and $\kappa^{(\prime)}$, so we expect the amount of fine-tuning to be proportional to $v_R^2/(\kappa\kappa')$, where the combination $\kappa\kappa'$ should be understood as a set of terms $a_1\kappa^2 + a_2\kappa'^2 + a_3\kappa\kappa'$.⁸ And this is indeed what we see in Figure 5.A.4. The v_R -dependence indeed seems to follow $\Delta_{\text{BG}} \sim v_R^2$. For κ , the effect is less clear, but this was expected. First of all, the range of κ values is much smaller, so its effect is less pronounced. But we do see in general that for increasing values of κ , the amount of fine-tuning decreases. This is in line with our expectation. There is a clear exception to that rule, which is visible in the figures for values of $\kappa \approx 200$ GeV (left figure) and $\kappa' \approx 180$ GeV (right figure). For these values, the amount of fine-tuning increases significantly. The exact origin of this feature is not clear, it is probably hidden somewhere in the diagonalization of the mass matrix, more specifically in the components that contribute to the light scalar mass. An effect that probably plays a role is the factor $1/(\kappa^2 - \kappa'^2)$ which appears in the minimum equations. This term blows up for $\kappa \approx \kappa'$, which indeed happens around the indicated values because in both cases the other vev is fixed at 190 GeV. This does not explain the fact that this feature appears at different values when varying κ or when varying κ' , so the actual effect is not completely symmetric. Discussing all the different effects that play a role in this system of equations is beyond the scope of this thesis. The fine-tuning of order $v_R^2/(\kappa\kappa')$ was also noted in Ref. [134].

Concluding our discussion of fine-tuning in the LRSM with the vevs as output parameters, we can say that the desired hierarchy will not arise generally from natural parameters in the P -conserving LRSM we considered. Considerable fine-tuning of the model parameters is necessary to obtain this hierarchy.

⁸Just like in the discussion of the Dekens measure, the exact dependence on κ and κ' is even more complicated than that.

Chapter 6

The hierarchy problem and fine-tuning in a decoupling approach to multi-scale effective potentials

While the previous chapters were concerned only with fine-tuning at tree-level, in this chapter we will focus on fine-tuning originating from loop effects.¹ In Chapters 1 and 3, we already discussed the hierarchy problem as an example of fine-tuning due to loop effects. To briefly recap, we saw in the first chapters of this thesis that there are many reasons to believe that physics beyond the Standard Model is necessary to answer some of the unresolved problems in particle and astrophysics. However, we saw that a problem appears when the Standard Model is extended with new physics, that couples to the Higgs boson, at energies much larger than the electroweak scale. This new physics leads to corrections to the squared mass of the Higgs boson that are of the order of the square of the scale of new physics. Therefore, the natural mass scale of the Higgs boson is not the electroweak scale, but the highest energy scale in the theory. Explaining the small value of the Higgs mass in the presence of new physics at high energy needs a large amount of fine-tuning of the high-energy parameters.

In this chapter, we do not consider specific theories that aim to solve the hierarchy problem, but we will take a closer look at various aspects of the hierarchy problem from the perspective of the effective potential.² This is the proper quantity to determine the true minimum of a scalar field theory and its stability at the quantum level. In addition, it allows to quantitatively address the effect of including loop corrections on the properties of scalar particles (masses, couplings, vacuum expectation values). Here, we are especially interested in the scenario where there is a large hierarchy of mass scales in the model, so as to resemble a high-energy extension of the Standard Model. When new mass scales are introduced, the sensitivity of the low-energy parameters to the high-energy UV physics is reflected in different observables obtained

¹This chapter is based on Ref. [135]

²See Ref. [136] for an extensive review of the effective potential both at zero and finite temperature.

from the effective potential, such as the vacuum expectation value (vev) and mass of the low-energy scalar field.

There is, however, an additional difficulty when a multi-scale effective potential is considered. The loop expansion of the effective potential contains logarithms of the ratio m_i^2/μ^2 , with m_i being the mass eigenvalues of the model, and μ the renormalization scale. In a model with a hierarchy in mass scales, it is impossible to choose a value for μ such that all logarithms containing the ratio m_i^2/μ^2 are small at the same time. The remaining large logarithms will appear with higher powers at higher orders, invalidating the loop expansion. The non-convergence of the perturbative expansion of the effective potential in the presence of a multi-scale system will hamper its predictiveness. At the level of the effective potential, various approaches to handling a multi-scale system have been put forward: (i) multi-scale renormalization methods [137–139]; (ii) a decoupling method in mass-independent renormalization schemes [140–142]; (iii) a more recent implementation via a single-scale renormalization-group improvement for multi-scale effective potentials [143]. In Ref. [142] it was shown within the Higgs-Yukawa model that the decoupling method is equivalent to the multi-scale method used in [138, 139]. Conversely, the single-scale improvement is not equivalent to the other two methods, as it does not always resum the leading logarithms [143]. For these reasons, we choose to focus on the decoupling method of [142] in this paper. We will also comment on two other methods: first a more general implementation of the decoupling prescription [144] and then a method using effective field theory (EFT) techniques [145, 146].³

Here, we consider a rather simple model with two scalar fields, where we take the mass parameters with a large scale separation. Moreover, the light scalar field undergoes spontaneous symmetry breaking (SSB) at tree-level, whereas the additional heavy scalar field, that mimics the new physics, does not (see Refs. [144, 145] for a similar model). The two-scalar model contains the main ingredients to inspect fully the hierarchy problem and it may resemble the Standard Model Higgs sector extended with an unknown high-energy completion. Our goal is to address two main aspects of the hierarchy problem when implementing the decoupling approach: (i) checking whether the SSB still occurs at the quantum level in the presence of high-energy degrees of freedom, and to which extent the minimum of the potential and the mass of the light scalar are affected; (ii) inspecting the fine-tuning of the model parameters. To the best of our knowledge, these two issues have not been discussed in detail in the framework of the effective potential in a scenario with well-separated mass scales. We shall compare the results of the standard implementation of the effective potential with the decoupling prescription for handling a multi-scale system.

³The latter reference [146] reproduces the decoupling prescription for the Higgs-Yukawa model, though the derivation is more rigorous.

Note that in this chapter, the fine-tuning is studied in a slightly different way compared to the discussion in Chapter 3, although the underlying physics is exactly the same. Here, we focus on the effect of the renormalization group (RG) evolution on the low-energy mass parameter. The β -function of this parameter includes quadratic contributions from the high-scale mass parameter. Hence, the mass parameter of the low-energy field depends very sensitively on the initial values at high energy. The Feynman diagrams, and therefore the underlying physics, that contribute to this effect are exactly the same ones as described in Chapter 3, only the interpretation is slightly different.

In contrast to the previous two chapters, we will not use fine-tuning measures to quantify the amount of fine-tuning in this chapter. We chose to do this because we are not interested necessarily in quantifying exactly the amount of fine-tuning, but we rather investigate the general behaviour of the model parameters with respect to fine-tuning. In our opinion, this is captured well by the analysis we present in this chapter. Furthermore, the model studied in this chapter is considered more of a toy model, and not so much a realistic BSM theory.

This chapter is organized as follows. In Section 6.1 we introduce the two-scalar model and highlight the different aspects of the hierarchy problem in the context of the effective potential. Then in Section 6.2, after the decoupling method is introduced, we address the numerical evaluation of the vev of the low-energy scalar field, its stability under radiative corrections and its mass in Section 6.2.1, while in Section 6.2.2 the connection between fine-tuning and the effective potential is studied. In Section 6.3 we comment on different implementations of the decoupling method, one of which is an EFT approach for handling the effects of heavy degrees of freedom on the effective potential. Finally, the conclusions are offered in Section 6.4.

6.1 A large hierarchy in the effective potential

In order to set a reference for the decoupling approach, we discuss the different problems that appear when we naively apply the effective potential formalism to a theory with a hierarchy in the mass parameters (referred to as the non-decoupling approach). To this end, we work with a simple model, which is a theory with two real scalar particles. The model Lagrangian can be written as follows:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu S \partial^\mu S - V(\phi, S), \quad (6.1)$$

where the potential with bare parameters reads:

$$V(\phi, S) = \frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{2}\mu_S^2 S^2 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_S}{4}S^4 + \alpha\phi^2 S^2. \quad (6.2)$$

On the one hand, the parameters of the field ϕ are chosen to achieve spontaneous symmetry breaking at tree level, so we take $\mu_\phi^2 < 0$ with the corresponding vev $v_\phi = \pm\sqrt{-\mu_\phi^2/\lambda_\phi}$. On the other hand, the mass parameter for the S field already corresponds to a physical mass and we take it to satisfy $\mu_S^2 \gg |\mu_\phi^2|$. As mentioned, the scalar field S plays the role of a high-energy degree of freedom and it belongs to the UV energy domain in our model (the highest scale we shall consider when running the parameters will be μ_S). We remark that the heavy field does not develop a vev. This will still hold under radiative corrections to the heavy scalar mass, since the corrections from both μ_S and μ_ϕ are not large enough to flip the sign of μ_S^2 . To ensure that the potential Eq. 6.2 is bounded from below, there are some constraints on the quartic couplings (see e.g. Ref. [147]):

$$\lambda_\phi > 0, \quad \lambda_S > 0, \quad \alpha > -\frac{1}{2}\sqrt{\lambda_\phi\lambda_S}. \quad (6.3)$$

Next we turn to the effective potential for the model. The RG-improved effective potential in any mass-independent renormalization scheme can be organized in a loop expansion $V_{\text{eff}} = V^{(0)} + V^{(1)} + \dots$, where $V^{(0)}$ is the RG-improved tree-level potential and $V^{(1)}$ is the one-loop correction after renormalization which reads (using the $\overline{\text{MS}}$ scheme):

$$V^{(1)} = \frac{1}{4(4\pi)^2} \left\{ m_\phi^4 \left[\log \frac{m_\phi^2}{\mu^2} - \frac{3}{2} \right] + m_S^4 \left[\log \frac{m_S^2}{\mu^2} - \frac{3}{2} \right] \right\}. \quad (6.4)$$

The masses in this expression are the tree-level masses, as a function of the classical field value ϕ_c . They are found to be:

$$m_\phi^2 = \mu_\phi^2 + 3\lambda_\phi\phi_c^2, \quad m_S^2 = \mu_S^2 + 2\alpha\phi_c^2. \quad (6.5)$$

According to our assumptions, the heavy scalar does not develop a nonzero vev by construction, so that we can set $S_c = 0$. Therefore, the masses in Eq. 6.5 are independent of S_c . When extracting the counterterms of the effective potential, one can see that μ_S^2 , λ_S and α do not run, in contrast to λ_ϕ and μ_ϕ^2 . The resulting

Figure 6.1 The RG running of the mass parameter μ_ϕ^2 (orange) at 1-loop level using the benchmark point described in the text, with $\alpha > 0$ (solid line) and $\alpha < 0$ (dashed line). The value of μ_S^2 is constant (solid blue), as explained in the text.

β -functions for λ_ϕ and μ_ϕ^2 are given by:⁴

$$\beta_{\lambda_\phi} = \mu \frac{\partial \lambda_\phi}{\partial \mu} = \frac{1}{8\pi^2} (9\lambda_\phi^2 + 4\alpha^2), \quad (6.6)$$

$$\beta_{\mu_\phi^2} = \mu \frac{\partial \mu_\phi^2}{\partial \mu} = \frac{1}{8\pi^2} (3\lambda_\phi \mu_\phi^2 + 2\alpha \mu_S^2). \quad (6.7)$$

In order to illustrate the behaviour of this model, we will use a benchmark point for the parameter space throughout the paper. The parameter values are:

$$\begin{aligned} \mu_\phi^2(\mu_{\min}) &= -(10^2 \text{ GeV})^2, & \mu_S^2(\mu_{\min}) &= (10^5 \text{ GeV})^2, \\ \lambda_\phi(\mu_{\min}) &= 0.8, & \lambda_S(\mu_{\min}) &= 1.3, & \alpha(\mu_{\min}) &= \pm 0.5. \end{aligned} \quad (6.8)$$

where $\mu_{\min} = 100 \text{ GeV}$. Furthermore, we take μ_S as the maximum value for the renormalization scale, so $\mu_{\max} = 10^5 \text{ GeV}$. As we will see, changing the sign of α has a large effect on some of the results. Hence, we will use both a positive and negative value for α in our benchmark point. We adopt this set of parameters for all numerical results presented in this chapter, unless explicitly stated otherwise. The running of

⁴We checked that, when letting S_c have a varying field value, we recover the full set of RGEs, see e.g. Ref. [148].

Figure 6.2 The shape of the scalar potential in the non-decoupling case. The left figure shows the tree-level result, evaluated at $\mu = \mu_{\min}$. The right plot shows the results including the 1-loop contributions for $\alpha > 0$ (blue solid line) and $\alpha < 0$ (orange solid line). In the right figure, a constant term c is subtracted as explained in the text to normalize the potential to zero at $\phi = 0$. The solid lines in the right plot show the result for $\mu = \mu_{\min}$, and a band is shown for the range $\mu_{\min} \leq \mu \leq \mu_{\max}$.

the mass parameter μ_ϕ^2 (and the comparison with the constant μ_S^2) in this model is shown in Fig. 6.1. One observes that μ_ϕ^2 receives large corrections from the high-energy modes already for small deviations from the initial renormalization scale. This is an expression of the hierarchy problem. When $\alpha > 0$, the μ_S^2 contribution to the running of μ_ϕ^2 is positive, and μ_ϕ^2 flips sign already at $\mu = 100.008$ GeV (solid orange line). At high energy a large positive value of $\mu_\phi^2 \approx 10^9$ GeV is reached. For $\alpha < 0$, there are only negative contributions to the running of μ_ϕ^2 . So the value at high energy will have the opposite sign as in the case $\alpha > 0$, but the absolute value is essentially the same, since the small initial value of μ_ϕ^2 at $\mu_{\min}$ has practically no effect on the high-energy value. Therefore, in the following we will often select only a particular sign of α to illustrate some point.

6.1.1 Properties of the light scalar from the effective potential

In this section we address the shape of the potential and the corresponding mass eigenvalue of the scalar field ϕ when we include the 1-loop corrections. Let us begin with the study of the shape of the potential for different values of the renormalization scale, and for both $\alpha > 0$ and $\alpha < 0$, that we show in Fig. 6.2. The left plot shows the

tree-level result evaluated at the scale $\mu_{\min}$. This potential exhibits the well-known Mexican-hat behaviour. In the right plot, the 1-loop contributions are added and the behaviour is quite different from the tree-level result. First of all, as can be seen from Eqs. 6.4) and 6.5, there is a large constant term present in the potential ($\propto \mu_S^4$), which leads to a shift in the potential. In order to normalize the potential to zero at $\phi = 0$, and better compare the shape of the potential with the tree-level case, we subtract from the one-loop potential the following term

$$c = \frac{\mu_S^4}{64\pi^2} \left(\log \frac{\mu_S^2}{\mu^2} - \frac{3}{2} \right) + \frac{\mu_\phi^4}{64\pi^2} \left(\log \frac{\mu_\phi^2}{\mu^2} - \frac{3}{2} \right). \quad (6.9)$$

We note that the μ_ϕ^4 term in Eq. 6.9 is always subdominant. However, for large μ values it does give a sizable contribution as in that case μ_ϕ^2 is only an order of magnitude smaller than μ_S^2 (see Fig. 6.1). Looking at the shape of the potential, there is a clear difference between $\alpha > 0$ and $\alpha < 0$. When $\alpha > 0$, there is no non-trivial minimum anymore, hence no spontaneous symmetry breaking can occur (solid blue line). This happens because the contributions proportional to m_S^4 dominate the potential, and they are all positive (recall the expression for m_S^2 in Eq. 6.5). In the case $\alpha < 0$, the m_S^4 term in the potential gives both positive ($\alpha^2 \phi^4$) and negative ($\alpha \mu_S^2 \phi^2$) contributions. The competition between these terms results again in a Mexican-hat shape, as can be seen in the right panel of Fig. 6.2 (solid orange line). However, the location of the vev is very different from the tree-level case, with a value for the 1-loop vev of $v_\phi^{(1)} \approx 2.5 \cdot 10^4$ GeV much larger than the tree-level vev $v_\phi^{(0)} \approx 112$ GeV. In the figure, a band is shown for the range $\mu_{\min} \leq \mu \leq \mu_{\max}$ which demonstrates that the results are largely independent of the chosen scale. We also note that for small values of $\phi = \phi_c$, both lines stop, which happens because of the appearance of imaginary values of the potential. This is a well-known feature that does not imply a breakdown of the effective potential approach, but rather signals a decaying state. For details we refer to [149].

Next, we want to see the effect of the 1-loop contributions on the mass of the light boson, as this is important in understanding the hierarchy problem in this model. In general, the mass of ϕ is given by the second derivative of the potential with respect to ϕ . The full expression contains the tree-level contribution and 1-loop contributions from the ϕ and S sectors. We single out the dominant term, which is the one that depends on the heavy scale μ_S^2 :

$$m_\phi^{2,(1)}(\mu) \sim \frac{\alpha}{16\pi^2} (\mu_S^2 + 2\alpha\phi_c^2) \left[\log \frac{\mu_S^2 + 2\alpha\phi_c^2}{\mu^2} - 1 \right]. \quad (6.10)$$

For $\mu \not\approx e^{-1/2} m_S$, this term is large and pushes the mass of ϕ to large values. Notably, there is a large shift in m_ϕ^2 already for $\mu = \mu_{\min}$. This is how the hierarchy problem

Figure 6.3 (Left) Diagrammatic representation of the prescription for determining the amount of fine-tuning (p_{LE} and p_{UV} stand for the parameters at low-energy and at the ultraviolet scale respectively). There is running of the parameters from low to high energy, a shift of the parameters through $\mathcal{F}_{ft}$ and running back to low energy. (Right) The result of running the mass parameters up from low energy (dotted line) and down from high energy after changing the high-energy parameters with $X = 0.01$ (solid line), with $\alpha < 0$.

of the Higgs mass manifests itself in this model.

Finally, we want to make contact between the effective potential and the amount of fine-tuning that is necessary at high energy ($\mu_{\max}$) in order to obtain a small value for μ_ϕ^2 at low energy ($\mu_{\min}$). To this end, we start with the model defined at the initial scale $\mu_{\min}$. Then, we run the parameters up to the high-energy scale $\mu_{\max}$ using the RG equations 6.6 and 6.7. Here, we implement a variation by multiplying the parameters with $\mathcal{F}_{ft} = 1 + X$, with $X \in [0, 1]$, and then run the parameters back down to $\mu_{\min}$. A diagrammatic representation of the prescription is given in Fig. 6.3 (left). An example of the μ_ϕ^2 -running from high energy down to low energy, after a variation at high energy is implemented, is shown in Fig. 6.3 (right). With only a 1%-level variation at the high-energy scale, we see that the low-energy value of μ_ϕ^2 shifts from -10^4 GeV^2 to $+10^7 \text{ GeV}^2$. Hence, a small variation at the high scale has an enormous effect at low energy. This indicates that in order to obtain the desired low-energy value for μ_ϕ^2 , a large amount of fine-tuning is necessary.

Let us summarize the findings so far. The effective potential has been used to study the shape of the potential, the vev of the low-energy field ϕ , the mass of ϕ and the fine-tuning of the model parameters. As for the features related to the

shape of the potential, we see that the effect of including the 1-loop corrections is large. Even at the scale $\mu = \mu_{\min}$ there are large corrections to the location of the vev and the mass of the light scalar. For the fine-tuning we see that already for small deviations from the initial scale $\mu_{\min}$, the high-energy degrees of freedom are active and provide large radiative corrections. The high-scale theory therefore needs a large amount of fine-tuning in order to accommodate the large hierarchy at low energy.

As mentioned in the beginning of this chapter, the perturbative expansion of the effective potential will involve large logarithms when a large hierarchy in scales is present. As can be seen from the expression for the 1-loop potential in Eq. 6.4, ratios of scales appear in the two logarithms: m_ϕ^2/μ^2 and m_S^2/μ^2 with $m_\phi^2 \ll m_S^2$. Because of this hierarchy it is not possible to find a value of μ such that both logarithms in Eq. 6.4 are small at the same time. This problem invalidates the loop expansion, because such logarithms will appear with higher powers at higher orders. The non-convergence of the perturbative expansion of the effective potential in the presence of a multi-scale system hampers its predictiveness. The decoupling method has been proposed as a possible solution to this problem and in the next section we will quantitatively study this approach.

6.2 The decoupling method

The main idea behind the decoupling method is to decouple *by hand* any particle in the theory as soon as the energy is too low to excite its modes [140–142]. In practice, one introduces step functions in the effective potential, such that the contribution of particles with a mass larger than the decoupling scale μ_{dec} is switched off. Using this prescription, the full effective potential can be safely evaluated down to the energy scale of the lightest particle. For the model at hand, the one-loop contribution can then be written as follows:

$$V_{\text{dec}}^{(1)} = \frac{1}{4(4\pi)^2} \left\{ \theta(\tilde{\mu} - m_\phi) m_\phi^4 \log \frac{m_\phi^2}{\tilde{\mu}^2} + \theta(\tilde{\mu} - m_S) m_S^4 \log \frac{m_S^2}{\tilde{\mu}^2} \right\}, \quad (6.11)$$

where we write $\tilde{\mu}^2 = \mu^2 e^{3/2}$, in order to absorb the usual numerical factor, which is $3/2$ for scalars. We can determine the β -functions by taking the derivative with respect to the renormalization scale μ of $V_{\text{dec}}^{(1)}$.⁵ In order to avoid some clutter, we define $\theta_i \equiv \theta(\tilde{\mu} - m_i)$. The decoupling is inherited by the two parameters λ_ϕ and μ_ϕ^2

⁵We note that the step functions do not contribute to this derivative, since they give a factor $m_i^4 \delta(\tilde{\mu} - m_i) \log \frac{m_i^2}{\tilde{\mu}^2}$. This factor is always zero, because for the single value where the δ -function is non-zero, the logarithm vanishes. Also note that one could replace the step functions by smooth versions, but this will not affect our conclusions.

Figure 6.4 (Left) The running of the mass parameters, with μ_S^2 in blue, and μ_ϕ^2 in solid orange for the case without decoupling, and in dashed green for the case with decoupling, using $\alpha > 0$. (Right) A comparison between the shape of the tree-level potential evaluated at $\mu = \mu_{\min}$ (blue line) and the tree-level + 1-loop potential using the decoupling method (orange line). The potential is plotted in units of $V/\mu_{\min}^4$.

so that the β -functions now read:

$$\beta_{\lambda_\phi} = \mu \frac{\partial \lambda_\phi}{\partial \mu} = \frac{1}{8\pi^2} (9\lambda_\phi^2 \theta_\phi + 4\alpha^2 \theta_S), \quad (6.12)$$

$$\beta_{\mu_\phi^2} = \mu \frac{\partial \mu_\phi^2}{\partial \mu} = \frac{1}{8\pi^2} (3\lambda_\phi \mu_\phi^2 \theta_\phi + 2\alpha \mu_S^2 \theta_S). \quad (6.13)$$

In our setting, when we run down from high energy, we obtain first a decoupling of the heavy S field. For $\tilde{\mu} < m_S$, i.e. $\mu \approx \mu_{\max}/2$, the effect of the heavy degree of freedom on the running parameters is absent and the change with the renormalization scale is only due to the lighter scalar ϕ . Going further down $\tilde{\mu} < m_\phi$, the couplings are frozen and do not run anymore. So the decoupling method provides a prescription for the renormalization scale μ [142], which is setting it equal to or smaller than the lowest decoupling scale $\mu^* \equiv m_\phi e^{-3/4}$ (which in our setting we also take larger than $\mu_{\min}$). In this way, the loop contributions are absent, so the problematic logarithms disappear from the potential. Only the RG-improved tree-level contribution $V^{(0)}(\mu < \mu^*)$ remains. This prescription ensures that higher-loop contributions also vanish, restoring

the predictivity of the perturbative expansion.⁶

In Fig. 6.4 (left) one finds the mass parameter μ_ϕ^2 evolved with and without decoupling, in order to show the strongly different behaviour. The same benchmark point is used as in the case without decoupling, with $\alpha > 0$. This figure shows that when using the decoupling method, μ_ϕ^2 remains small for most of the considered energy range since the high-energy modes only start contributing at $\mu \approx \mu_{\max}/2$. Moreover, although the high-energy modes do contribute to the RG running for $\tilde{\mu} > m_S$, the final value of μ_ϕ^2 at $\mu_{\max}$ is one order of magnitude lower than in the case without decoupling. The results are essentially the same for $\alpha < 0$, even though it changes the value of m_S (cf. Eq. 6.5) and therefore the value of the decoupling scale. This effect will be small, because we only consider small values for ϕ_c , so it will not change the fact that the high-scale modes contribute above $\mu \approx \mu_{\max}/2$.

6.2.1 Minimum and shape of the potential

Let us now use the prescription for the renormalization scale in the decoupling method to evaluate the effective potential. A comparison between the tree-level result and the one-loop result in the decoupling method is shown in Fig. 6.4 (right). It is worth stressing that the shape of the 1-loop effective potential (solid-orange line in Fig. 6.4) is independent of the sign of α (whereas this affects severely the 1-loop effective potential in the naive implementation, see Fig. 6.2). This is due to the absence of α -dependent terms at scales $\tilde{\mu} < m_S$. In contrast to the case without decoupling, we now see that the 1-loop effective potential only introduces small corrections, signalling a good perturbative expansion at least in the low-energy domain and at this loop order. Moreover, one can see that a non-trivial minimum is realized and spontaneous symmetry breaking can occur, also for $\alpha > 0$. The truncation of the orange solid line is due to the squared mass eigenvalue m_ϕ^2 turning negative for values $\phi_c \leq 68$ GeV for this choice of the parameters. This makes it impossible to define a meaningful decoupling scale, and leads to the aforementioned decaying state [149], but we will not address this further here.

We have checked numerically that the good agreement between the tree-level potential and the decoupling potential is not due to the specific choice of parameters in the benchmark point. For different low-energy values of λ_ϕ and μ_ϕ^2 , the shift in the value of the vev $v^{(1)}$ is at the few-percent level.

⁶In order to find an appropriate renormalization scale, the authors of Ref. [142] suggested two conditions: $V_{\text{dec}}^{(1)}(\mu) = 0$ (best perturbative convergence) and $d(V^0 + V_{\text{dec}}^{(1)})/d\mu=0$ (least μ -dependence). When choosing the renormalization scale smaller than the smallest mass eigenvalues, the two prescriptions are fully equivalent.

Figure 6.5 The effect of small variations in the high-energy value of the parameters on the low-energy value $\mu_{\phi,LE}^2 = \mu_{\phi}^2(\mu_{min})$, using the benchmark point with $\alpha > 0$. The results for the decoupling (orange line) and non-decoupling (blue line) cases are compared. A field value of $\phi_c = 300$ GeV is used.

Let us now look at what happens to the mass of the ϕ scalar when the decoupling method is used. Since the 1-loop contributions are absent from the potential in this case ($V_{dec}^{(1)} = 0$), the term that previously pushed m_{ϕ} to large values no longer contributes. The mass of the ϕ boson is now given by:

$$m_{\phi}^2(\mu < \mu^*) = \mu_{\phi}^2 + 3\lambda_{\phi}\phi_c^2, \quad (6.14)$$

where all parameters are understood to be evaluated at the same scale $\mu < \mu^*$. In contrast to Eq. 6.10, there are no μ_S^2 contributions in Eq. 6.14, and the decoupling method ensures the lightness of the ϕ boson.

6.2.2 Fine-tuning

In this section we repeat the fine-tuning analysis carried out in Section 6.1 for the decoupling approach. In contrast to Section 6.2.1, the whole range of energy scales is now directly involved, since we have to use the RGEs up to the high-energy domain. Accordingly, above the decoupling value $\tilde{\mu} > m_S$, the heavy degree of freedom is active and affects the running considerably. We use the same procedure as in the case without decoupling, where we run the low-energy parameters up to the high scale, then vary them at the high scale, and run them down to low scale. In this case we use the RG equations 6.12 and 6.13 that govern the decoupling.

Figure 6.6 The low-energy value $\mu_{\phi,LE}^2 = \mu_{\phi}^2(\mu_{\min})$ as a function of X for different model parameter values. The fixed parameters have the value of the benchmark point in Eq. 6.8 with $\alpha > 0$. The solid lines show the results in the non-decoupling case, the dashed lines show the decoupling results.

In order to compare the sensitivity to the high-energy parameters with and without the decoupling prescription, we show in Fig. 6.5 the value of μ_{ϕ}^2 evaluated at $\mu = \mu_{\min}$ as a function of X , respectively in the solid orange and solid blue line. One observes that $\mu_{\phi}^2(\mu_{\min})$ is much less sensitive to variations at the scale $\mu_{\max}$ when using the decoupling method. However, there is still quite some fine-tuning necessary even when

using the decoupling method, since for a percent-level variation of the high-energy parameters, $|\mu_\phi^2|$ changes more than one order of magnitude. To illustrate the robustness of these findings, we explore the behaviour of μ_ϕ^2 when varying the model parameters λ_ϕ , α , μ_ϕ^2 and μ_S^2 . We aim to study how the fine-tuning depends on these parameters, and to show that the choice for our benchmark point is not special. In Fig. 6.6, the value of μ_ϕ^2 at $\mu = \mu_{\min}$ is given as a function of X . In each plot, a different parameter is varied one at a time, while the remaining ones are kept at the same value as in the benchmark point, with $\alpha > 0$ in all plots (negative α yielding qualitatively similar results). Both the decoupling (dashed lines) and non-decoupling (solid lines) results are shown. In the upper-left panel, there is almost no dependence on the value of λ_ϕ in the decoupling case (solid lines). Therefore, the three lines overlap almost completely, and only the result for $\lambda_\phi = 1.2$ is visible.

We end this section by noting that the large sensitivity at low energy to changes of the parameters of the model at high energy is very different from the analogous sensitivity to the standard logarithmic RG running one has in any theory. Running the SM parameters up to the GUT scale for instance and then changing the parameters at the GUT scale by a little and then running down, will also generate a substantial difference at low energies. The difference is that in the present case the effects depend quadratically on the large scale as opposed to logarithmically.

6.3 Alternative implementations of the decoupling method

The decoupling theorem suggests that heavy degrees of freedom cannot be excited, and hence affect the physics, at energies smaller than their masses. However, it is not an exact recipe that specifies at which particular scale the decoupling happens. In the former section, we chose perhaps the simplest prescription [142], where the decoupling scale is fixed completely by the (scalar) particle masses, the exact definition is $\mu_i = m_i e^{-3/4}$. In this section, we elaborate on the fine-tuning and the shape of the potential when considering the different decoupling approaches in Refs. [144,145].

In Ref. [144], a formal generalization of the effective potential that we adopted is put forward. The main modification is the appearance of decoupling scales, μ_{m_i} , that are in general different from the mass thresholds m_i , so that the one-loop potential in our model reads

$$V_{\text{dec}}^{(1)} = \frac{1}{4(4\pi)^2} \sum_{i=\phi,S} m_i^4 \left[\log \frac{m_i^2}{\mu_{m_i}^2} + \theta(\mu - \mu_{m_i}) \log \frac{\mu_{m_i}^2}{\mu^2} - \frac{3}{2} \right]. \quad (6.15)$$

One can easily check that for $\mu_{m_i} = m_i e^{-3/4}$ the expression in Eq. 6.11 is recovered. When μ is larger than all the μ_{m_i} , Eq. 6.15 corresponds to the effective potential Eq. 6.4 with no decoupled particles. As the authors in Ref. [144] suggest, a reasonable choice for the decoupling scales is $\mu_{m_i} \simeq m_i$, as this improves the validity of the perturbative expansion and follows the reasoning of the decoupling theorem [150, 151]. As mentioned, there is no exact recipe that specifies at which particular scale the decoupling happens, but we find that choosing slightly different scales can have a large effect on the shape of the potential. In order to make our point, we consider two different choices for the decoupling scales to be inserted in Eq. 6.15, namely $\mu_{m_i} = m_i e^{-3/4}$, which is the value of the decoupling scale in the potential 6.11, and $\mu_{m_i} = 1.01 m_i e^{-3/4}$. In the first case, as shown in Fig. 6.4 (right), we obtained a very good agreement between the tree-level and one-loop potential. In the latter case, despite the decoupling scales being changed at the percent level, the one-loop correction shifts the minimum far away from the tree-level value (more than one order of magnitude), as clearly shown in Fig. 6.7 (left).⁷ This is traced back to the residual log-term (the first term in the squared brackets in Eq. 6.15), which is multiplied by the large scale $m_S^4 \approx \mu_S^4$. This term gives a large contribution to the potential for $\mu_{m_S} \neq m_S e^{-3/4}$. It is worth mentioning that if one takes decoupling scales $\mu_{m_i} < m_i e^{-3/4}$ (even slightly smaller), the minimum of the one-loop corrected potential disappears because the overall sign of the residual large logarithm in Eq. 6.15 is negative. This leads to an effective ϕ^2 term in the potential which is dominated by a term $\sim \alpha \mu_S^2$, resulting in a positive value of the ϕ^2 term. This applies when $\alpha > 0$, while the situation is reversed when taking $\alpha < 0$, so in that case symmetry breaking will only occur when $\mu_{m_i} < m_i e^{-3/4}$. Hence, just like the non-decoupling approach discussed in Section 6.1.1, the effective potential in Eq. 6.15 leads to a strong dependence of the low-energy physics on the sign of α , i.e. on the interactions involving the high-energy scalar. This behaviour was not present when using the decoupling implementation discussed in Section 6.2 according to Eq. 6.11.

Our conclusions on the large sensitivity at low energy to changes of the parameters of the model at high energy still hold when considering the more general decoupling approach 6.15, since the RG running of the parameters is only slightly altered compared to the RG running described in Section 6.2.

As a second comparison, we consider the EFT approach as described in Ref. [145], which uses the same model Lagrangian as we use 6.1. In this case, a low-energy field theory is explicitly constructed by integrating out the heavy degree of freedom. In doing so, the scalar S does not appear in the effective Lagrangian, though μ_S enters the matching conditions between the effective potentials of the low-energy and high-energy

⁷The change is not so dramatic if one compares pairs of scales different from $\mu_S e^{-3/4}$, say $\mu_{m_i} = m_i$ and $\mu_{m_i} = 1.01 m_i$. However, in that case the vevs are both much larger than the tree-level result.

Figure 6.7 Left: The shape of the potential in the tree-level case at $\mu_{\min}$ (blue) and using the decoupling method Eq. 6.15 (orange) for $\alpha = 0.5$. The decoupling scales are set to $\mu_{m_i} = 1.01 \cdot m_i e^{-3/4}$. Right: illustration of the running affected by the heavy scalar S in the decoupling method as in Section 6.2 (red-solid curve) and the EFT method (orange-solid curve), both tuned to give a low-energy ϕ scalar mass. Despite the solid lines just sketching the difference between the two methods, the value for $m_\phi^2(\mu_S)$ resembles our finding in Fig. 6.4 and, in general, $m_\phi^2(\mu_S)$ is going to be much smaller than μ_S^2 for $\alpha \sim \mathcal{O}(1)$ (though $m_\phi^2(\mu_S) \gg \mu_{\min}^2$).

theory. These matching conditions provide the relation between the corresponding masses and couplings (and vacuum energy) in the two theories. In Ref. [145], the logarithms vanish when the matching scale is equal to the mass of the heavy scalar μ_S . So this choice minimizes higher-order corrections. Below this scale, the mass parameter m_ϕ runs very modestly, because the heavy degree of freedom is decoupled. Once the smallness of the ϕ -scalar mass in the low-energy theory is imposed, there is little impact on the shape of the potential when one considers one-loop corrections, since the latter are minimized and the heavy degree of freedom does not enter the running. Contrary to the potential in Eq. 6.15, and similarly to the implementation in Eq. 6.11, the shape of the one-loop effective potential stays very close to the tree-level result. However, fine-tuning is still present in this approach. The fine-tuning can be explicitly noted in the matching condition, where one has to require a very fine cancellation between two large quantities when imposing the lightness of the ϕ scalar mass [145]:⁸

$$m_{\phi,\text{LE}}^2(\mu_S) = m_\phi^2(\mu_S) - \frac{\alpha}{8\pi} \mu_S^2, \quad (6.16)$$

where $m_{\phi,\text{LE}}$ is the ϕ -scalar mass in the low-energy theory, whereas m_ϕ is the mass in the high-energy theory. Slightly changing the value of the high-energy parameter m_ϕ

⁸We adapted the expression to our different notation for the couplings and mass parameters.

at $\mu > \mu_S$, and running it down to $\mu = \mu_S$, would have a significant impact on Eq. 6.16 and it would spoil the lightness of $m_{\phi, \text{LE}}$. When comparing this EFT approach to the decoupling method in Eq. 6.11, one notices that the decoupling scale is different in the two cases: μ_S against $m_S e^{-3/4} \simeq \mu_S/2$. However, the source of the fine-tuning is the same, as the difference between the approaches is only in the energy window where the heavy degree of freedom has an effect, as sketched in Fig. 6.7 (right). So the amount of fine-tuning that appears in the m_ϕ^2 observable is again very similar in both cases.

To avoid this fine-tuning problem, one could supplement the decoupling method or the EFT matching method by the requirement that all parameters of the high-energy theory should be fixed at the decoupling scale of the high-energy scalar. In that way, there will be no large sensitivity to changes of the parameters at the decoupling scale. Running downward will not lead to large changes, while running upward will also not be very dependent on variations of parameters at the decoupling scale (we have numerically checked this statement). The fine-tuning problem that we have addressed arises from running upward beyond the decoupling scale, then modifying the parameters at some higher scale and running down again. In the case of one decoupling scale at high energies, this prescription that all parameters of the high-energy theory should be fixed at the decoupling scale offers a way to avoid the fine-tuning problem. However, when there are multiple decoupling scales, e.g. in case the sum in Eq. (6.15) includes a third particle with a mass that is substantially higher than m_S , there will be large sensitivity to changes of the parameters of the theory at the highest of the decoupling scales (in practice a factor 2 higher will already give rise to a fine-tuning problem). It is very unlikely that if there is new physics, there will be only new particles present in a very narrow range of high-energy scales. So we conclude that the prescription that all parameters of the theory should be fixed at the decoupling scale, will not in general offer a way to avoid the fine-tuning issue we have pointed out in this paper.

6.4 Conclusions

In this chapter we have carried out an analysis of the hierarchy problem from the viewpoint of the effective potential. Our study does not propose any solution to the hierarchy problem, rather it aims to show that this issue is present in the extraction of the observables from the effective potential, even when using a decoupling approach. For this purpose we have mainly concentrated on the decoupling method of [142], which freezes the effect of heavy particles on the running at low energies. Working at the one-loop level, and within a simple two-scalar theory, we disentangled the effects of the high-energy degree of freedom on the shape of the potential and on the fine-tuning of the model parameters. We find that while the decoupling method

leads to an acceptable and convergent effective potential, the method does not solve the fine-tuning problem that is inherent to the hierarchy problem of multiple-scale theories. Compared to the non-decoupling approach, the amount of fine-tuning is reduced. Nevertheless, the mass of the low-energy scalar turns out to be still very sensitive to small changes of the parameters at the scale of the high-energy sector.

We considered two other implementations of the decoupling method which lead to different conclusions on the impact of quantum corrections on the shape of the potential, in particular on the vev of the low-energy scalar field. For the decoupling potential in Eq. 6.15, we find that small deviations from the ‘reference’ decoupling scale $m_i e^{-3/4}$ make the minimum shift far away from the tree-level value if $\mu_{m_i} > m_i e^{-3/4}$, whereas the minimum is absent whenever $\mu_{m_i} < m_i e^{-3/4}$, when $\alpha > 0$. The situation is reversed when negative values for α are used. Therefore, the decoupling method outlined in Eq. 6.15 reintroduces the dependence of the shape of the potential, and the existence of a non-trivial minimum, on the sign of the coupling α (as it was for the non-decoupling approach described in Section 6.1). On the contrary, the EFT approach guarantees fairly small loop corrections for the low-energy parameters at scales below the decoupling scale (which is set to the high-scale mass parameter μ_S). The fine-tuning problem is very similar in the different implementations, however. In case there is only one decoupling scale at large energy, one can avoid the fine-tuning by requiring that all parameters of the high-energy theory should be specified at the decoupling scale. But in the case of two or more high-energy decoupling scales the fine-tuning problem resurfaces. There appears to be no straightforward way to adjust the decoupling method to avoid this problem.

Chapter 7

Conclusions and outlook

In this thesis, we have investigated several aspects of fine-tuning in models with extended scalar sectors. While discussions around the hierarchy problem and fine-tuning have been around for quite some time already, this field is still an active area of research, see e.g. Refs. [152–160] for some papers that appeared in 2020 alone. After giving some background and introduction in the first three chapters, the main focus of Chapters 4 and 5 was on tree-level fine-tuning in models with extended scalar sectors. The most important realization in these chapters was that there are (at least) two distinct ways to approach the parameters in the Lagrangian and the minimum equations. The first one is what we called the standard viewpoint, in which the couplings and mass parameters in the Lagrangian are considered to be the only input parameters of the model. When the energy is lowered from the high scale where the Lagrangian is defined, the vevs emerge and lead to symmetry breaking. The values of the vevs are then determined by the mass parameters and couplings in the Lagrangian through the minimum equations.

In Chapter 5, we investigated this viewpoint in the context of the minimal non-supersymmetric $SU(5)$ GUT. We gave an overview of all possibilities for symmetry breaking and discussed different aspects of the doublet-triplet splitting problem, including a quantitative analysis of the problem. To the best of our knowledge such an extended overview has not been given in the literature before. We showed that in the minimal model, there is no way to combine the breaking of the GUT symmetry with electroweak symmetry breaking in a phenomenologically viable way, without the need for fine-tuning or unnatural parameters in the Lagrangian. In addition, we found that the DTSP consists of two main features. First of all, a hierarchy in mass parameters will generally not translate into a hierarchy in vevs when there is a substantial coupling between the different sectors. This is actually a general problem occurring in many beyond the Standard Model theories, which we demonstrated using the two-Higgs-doublet model. Secondly, we saw that in the minimal $SU(5)$ GUT, the couplings between the two scalar fields cannot both be small, as they are needed to split the mass scales of the triplet (GUT scale) and doublet (electroweak scale). This aspect need not hold in general for BSM theories, such as in popular theories with a portal coupling.

By analyzing the different aspects that play a role in the different steps of symmetry breaking in this GUT, we found that there are multiple reasons to reconsider the standard viewpoint. We highlighted that the different symmetry breaking steps are in fact not independent, since in the phenomenologically viable options, the electroweak symmetry is already broken at the GUT breaking scale. Furthermore, in order to obtain the correct symmetry breaking pattern, some of the parameters need to be chosen in specific ranges. For theories based on large symmetry groups, these constraints can greatly restrict the allowed parameter space. So the general theory at high scale will in general not lead to the desired symmetry breaking pattern, and certainly not to a pattern with widely separated energy scales. These arguments led us to propose an alternative viewpoint, where next to the symmetries before and after the different breaking steps, one imposes the scales at which the breakings occur (or the hierarchy in scales) on the theory. In this viewpoint, the number of free parameters in the theory is reduced, which is similar to how imposing a symmetry reduces the number of free parameters. The minimum equations now provide additional relations between the parameters of the theory, and can be used to fix some of their values. We showed that, in this alternative viewpoint, there is no fine-tuning necessary in the remaining free parameters. This is not guaranteed, as our analysis in Section 4.2.2 showed. While our analysis was focussed on the minimal non-supersymmetric $SU(5)$, and on the two-Higgs-doublet model and the left-right-symmetric model in Chapter 4, we believe these conclusions are quite general, and can be applied to many other BSM theories as well. Using the alternative viewpoint could be a way to avoid the troubling fine-tuning issues at tree-level that plague many of these theories.

Chapter 4 was mainly concerned with investigating both qualitatively and quantitatively two measures of fine-tuning when using this alternative viewpoint. We investigated both the Dekens measure and the Barbieri-Giudice measure, first analyzing the 2HDM with a hierarchy in vevs imposed. We determined how we think the Dekens measure should be applied in this viewpoint, and compared these findings with the results for the BG measure. We showed that the two measures use different features of the potential, the first derivative and the second derivative, respectively, and can therefore give different results for the amount of fine-tuning in the model. We concluded that, in general, a hierarchy in vevs can be accommodated in the 2HDM without fine-tuning in the remaining free parameters of the model.

We also investigated the amount of fine-tuning in a specific left-right-symmetric model in Chapter 4. Using the results from our discussion on the 2HDM, we concluded that three out of the four minimum equations will generally not lead to fine-tuning. The remaining minimum equation, the seesaw relation, is fundamentally different from the minimum equations in the 2HDM and the other minimum equations in

the LRSM because of the absence of a mass parameter. After discussing how we think the Dekens measure should be applied in this case, we concluded that also this minimum equation can be solved with natural and perturbative values for the parameters, without fine-tuning.

While the hierarchy in vevs is no longer a source of fine-tuning in the alternative viewpoint, this does not mean that fine-tuning is completely absent. In Chapter 4, we considered the BG measure in a specific LRSM, using the alternative viewpoint. While we found no large amount of fine-tuning in the general model, we did find that imposing a constraint for the Higgs mass introduces a moderate amount of fine-tuning. An interesting possibility for future research would be to investigate the effect on the amount of fine-tuning when including additional experimental constraints.

In appendix 5.A, we discussed the results for the amount of fine-tuning in the LRSM when using the standard viewpoint. And we saw that, while Ref. [73] uses an alternative method to determine the amount of fine-tuning, it gives the same results for the amount of fine-tuning in the seesaw relation as other sources in the literature. It is unclear if the approach from Ref. [73] is also valid when there is a system of equations, with possibly multiple sources of fine-tuning. This raises the question if the different hierarchies present in models like the LRSM ($v_L \ll \kappa^{(\prime)} \ll v_R$) should be viewed as independent sources of fine-tuning, and if so, how a fine-tuning measure can accurately measure this combination of different sources of fine-tuning. We leave an analysis of these kinds of questions to future research.

We finish our discussion of tree-level fine-tuning by discussing the question whether the appearance of a hierarchy in vevs in theories like the LRSM or the $SU(5)$ GUT should really be considered problematic in the standard viewpoint. We note that many BSM theories, like e.g. the LRSM, are generally not considered to be a fundamental theory. Instead, they are seen as a ‘stepping stone’ towards a more fundamental theory. Hence, it could be the case that the hierarchy in vevs in the LRSM we considered, which by itself does not have a natural explanation in the standard viewpoint, can be explained by the more fundamental theory. This raises the question whether one demands every single step of new physics to be natural by itself, or that one only demands that the most fundamental theory is natural, and that all low-energy theories emerge from this theory in a natural way. This could e.g. be realised by some dynamical mechanism in the high-energy theory which provides a natural explanation for hierarchies in the theories at low energy. Following this reasoning, one could argue that an explanation for a hierarchy in vevs in the LRSM is not necessary, while it is necessary in a GUT such as the one based on $SU(5)$. It must be noted that such a reasoning makes it hard to judge the viability of BSM theories based on fine-tuning arguments, as there is (currently) no clear way to determine if a theory is fundamental

or not. Similar arguments can be used to argue that a large hierarchy in vevs in the alternative viewpoint does not need an explanation, and should therefore not be considered problematic, even if one would consider the hierarchy unnatural. In this reasoning, there should still be a mechanism in the fundamental theory that explains the hierarchy in scales at lower energy.

In Chapter 6, we turned to fine-tuning due to loop effects. We investigated different aspects of the hierarchy problem using the effective potential. We looked at a model with two real scalars, with a large hierarchy in their mass parameters, which resembles an extension of the Standard Model with some new high-scale physics. The naive implementation of the effective potential in this model leads to a large amount of fine-tuning, but it also leads to a non-convergent loop expansion. We investigated a proposed solution to this non-convergence, namely the decoupling method, in which the renormalization group running of heavy particles is frozen when the energy is too low to excite their modes. While the convergence of the effective potential is indeed guaranteed by this method, we found that it does not solve the fine-tuning issue. There is still significant fine-tuning necessary in the high-energy parameters of the model, although less than in the naive implementation. We also commented on two other ways to implement decoupling, first through a more general method to do the decoupling, and secondly through an effective field theory approach. We showed that the EFT approach leads to very similar results for the shape of the potential, but the more general decoupling method can give different results, depending on the exact choice of the decoupling scale in this method. Both alternative implementations showed a similar amount of fine-tuning in the high-energy parameters. We noted that in a theory with only two distinct mass scales, it is possible to avoid fine-tuning by defining the model parameters at the scale where the high-energy particle decouples. In this way, there is only a small effect in the RG running, both when running downwards and upwards from this scale. However, as soon as a third scale is introduced, which does not even need to be very far away from one of the other scales, the argument no longer holds. An interesting avenue for future work would be to investigate the decoupling method described in this chapter in a more realistic scenario, in a viable BSM theory with e.g. multiple particles at high energy and including experimental constraints. But we expect that the conclusions found in this chapter generally carry over to other models.

Samenvatting

De deeltjesfysica is het deel van de natuurkunde dat zich bezighoudt met de eigenschappen van de allerkleinste deeltjes in de natuur, en de interacties die zij met elkaar hebben. De eigenschappen van deze deeltjes worden beschreven door het Standaard Model van de deeltjesfysica (ook wel Standaard Model of SM genoemd). Dit model bevat fermionen, die de bouwstenen zijn voor de materie om ons heen, en bosonen, die zorgen voor interacties tussen de fermionen. De fermionen kunnen verder onderverdeeld worden in twee groepen: de quarks, dit zijn deeltjes die de sterke kernkracht voelen, en de leptonen, die de sterke kernkracht niet voelen. De bosonen bestaan ook uit 2 groepen. De ijkbosonen zijn verantwoordelijk voor het uitoefenen van krachten. Zo zijn er de gluonen die de sterke kernkracht uitoefenen, fotonen voor de elektromagnetische kracht, en Z - en W -bosonen voor de zwakke kernkracht. De tweede groep bosonen zijn de scalairen. Het Standaard Model bevat maar een scalar, namelijk het higgsboson, wat verantwoordelijk is voor de massa van de elementaire deeltjes. Het Standaard Model kan de resultaten van de meeste experimenten goed beschrijven, maar toch zijn er verschillende redenen om aan te nemen dat deze theorie niet volledig is.

Er zijn bijvoorbeeld astronomische observaties die suggereren dat de bekende fysica niet voldoende is om alle geobserveerde fenomenen te verklaren. Een voorbeeld hiervan is het feit dat we in het heelal bijna alleen maar materie zien, en bijna geen antimaterie, terwijl we een ongeveer gelijke hoeveelheid van beide verwachten. Ook is er een probleem met de verdeling van energie in het heelal. De normale materie kan maar zo'n 5% van de totale hoeveelheid energie in het heelal verklaren. De overige 95% bestaat uit donkere materie ($\approx 25\%$) en donkere energie ($\approx 70\%$). Voor zowel de verdeling van energie in het heelal als de onbalans tussen materie en antimaterie geldt dat het Standaard Model geen verklaring biedt, maar het is aannemelijk dat de deeltjesfysica een rol speelt bij deze fenomenen. Het wordt daarom aangenomen dat er nieuwe fysica nodig is die het Standaard Model uitbreidt om onder andere deze fenomenen te verklaren.

Deze astronomische observaties zijn niet de enige redenen om aan te nemen dat het Standaard Model niet volledig is. Zo is er geen verklaring voor het feit dat neutrino's veel lichter zijn dan alle andere deeltjes, zijn er wellicht problemen met de stabiliteit van de scalaire sector bij hoge energie, en wordt een kracht zelfs helemaal niet beschreven door het Standaard Model, namelijk de zwaartekracht. Dit geeft aan

dat het Standaard Model waarschijnlijk niet de meest fundamentele theorie is. In dit proefschrift onderzoeken we verschillende modellen voorbij het Standaard Model (in het Engels *beyond the Standard Model* oftewel BSM genoemd), en in het bijzonder modellen waarbij de scalaire sector wordt uitgebreid ten opzichte van het Standaard Model. Deze uitgebreide scalaire sectoren zijn populair, want veel BSM theorieën zijn gebaseerd op een ijkgroep die groter is dan de ijkgroep van het Standaard Model. Om deze grotere ijkgroep te breken naar het Standaard Model wordt meestal gebruik gemaakt van een higgsmechanisme, waarvoor nieuwe scalaire deeltjes nodig zijn.

Er zijn verschillende eisen waaraan deze uitgebreidere scalaire sectoren moeten voldoen. Zo mogen ze niet tot te veel CP -schending leiden, en zijn er limieten op de vacuüm verwachtingswaarden van de nieuwe scalaren, bijvoorbeeld via de waarde van de ρ -parameter. Dit proefschrift richt zich op een ander aspect van deze modellen, namelijk het probleem dat veel van deze modellen een grote hoeveelheid *fine-tuning* nodig hebben om de fysica op lagere energie correct te beschrijven. Fine-tuning betekent dat verschillende bijdragen aan een observabele of parameter elkaar met een grote nauwkeurigheid moeten opheffen. Als deze bijdragen onafhankelijk van elkaar zijn, betekent dit dat het model de metingen alleen kan beschrijven als de parameters heel specifieke waardes hebben. Hoe nauwkeuriger de parameters gekozen moeten worden, hoe meer fine-tuning het model heeft. Een grote hoeveelheid fine-tuning wordt gezien als een ongewenste eigenschap van een model. Het zou namelijk betekenen dat als de waardes van de parameters maar een klein beetje anders zouden zijn, de fysica er compleet anders uit zou zien.

In dit proefschrift kijken we naar verschillende aspecten van fine-tuning, met name naar fine-tuning in niet-supersymmetrische uitbreidingen van het Standaard Model. In hoofdstuk 4 kijken we in detail naar twee fine-tuning maten. Dit zijn manieren om de hoeveelheid fine-tuning in een model te kwantificeren. Door deze maten te gebruiken is het makkelijker om de hoeveelheid fine-tuning in verschillende modellen met elkaar te vergelijken. In dit hoofdstuk nemen we als uitgangspunt dat de vacuüm verwachtingswaarden (vevs) van de verschillende scalaire velden worden opgelegd. Met dit uitgangspunt onderzoeken we vervolgens de Barbieri-Giudice (BG) maat en de Dekens maat. De BG maat gebruikt de relatie tussen een observabele en de input parameters van het model om de hoeveelheid fine-tuning te berekenen, terwijl in de Dekens maat de minimumvergelijkingen het uitgangspunt zijn. Dit zijn de vergelijkingen die volgen uit het minimaliseren van de scalaire potentiaal, en deze geven een verband tussen de verschillende parameters van het model.

In hoofdstuk 4 onderzoeken we vervolgens of een hiërarchie in vevs automatisch fine-tuning impliceert. Het is bekend dat kwantumcorrecties een fine-tuning-probleem opleveren, maar deze BSM theorieën kunnen ook op laagste orde al fine-tuning-

problemen hebben. Daar focussen we op in dit hoofdstuk. Daarnaast bekijken we hoe de Dekens maat (volgens ons) gebruikt dient te worden. We onderzoeken daarvoor eerst het twee-Higgs-doublet-model (2HDM), waarbij het Standaard Model wordt uitgebreid met een extra scalair doublet. Door dit tweede doublet is er ook een tweede vev aanwezig, waardoor een hiërarchie in vevs mogelijk is. Het model is wel nog zo eenvoudig dat alle uitdrukkingen voor bijvoorbeeld massa's en de minimumvergelijkingen analytisch kunnen worden bestudeerd. We concluderen dat in het 2HDM een hiërarchie in vevs niet automatisch betekent dat er fine-tuning nodig is.

De volgende stap is dan om een ingewikkelder model te bekijken. We onderzoeken daarom een links-rechts-symmetrisch-model (LRSM), wat meer scalaire deeltjes bevat, en als gevolg een veel complexere potentiaal heeft. Dit model bevat vier vevs, en de potentiaal leidt dan ook tot vier gekoppelde minimumvergelijkingen. Door onze conclusies uit de analyse van het 2HDM te gebruiken concluderen we dat de Dekens maat geen fine-tuning vindt in drie van de vier minimumvergelijkingen. Voor de laatste vergelijking ligt het lastiger, maar na een analyse van deze vergelijking concluderen we dat ook deze vergelijking geen fine-tuning bevat. Als we de BG maat berekenen in dit model vinden we eveneens geen fine-tuning, maar dit verandert zodra we een eis opleggen op de massa van het higgsboson. In dat geval vinden we dat er enige fine-tuning nodig is om te zorgen dat dit deeltje niet te zwaar is.

In hoofdstuk 5 bekijken we vervolgens een ander model, namelijk een *Grand Unified Theory* (GUT). Dit is een klasse van modellen waarbij wordt aangenomen dat de drie krachten in het Standaard Model bij hoge energie voortkomen uit een enkele kracht. Wij kijken in het bijzonder naar de minimale niet-supersymmetrische GUT gebaseerd op de ijkgroep $SU(5)$. Dit model bevat een bekend fine-tuning probleem op laagste orde, namelijk het doublet-triplet splitting probleem (DTSP). Een uitgangspunt van dit probleem is dat de vevs niet worden gezien als inputparameters (zoals in hoofdstuk 4), maar dat ze volgen uit de overige parameters in de potentiaal, wat wellicht een redelijke aanname is voor een fundamentele theorie. In dit hoofdstuk geven we eerst een uitgebreid overzicht van de verschillende mogelijkheden voor symmetriebreking in deze GUT en geven we een kwantitatieve analyse van het DTSP. Deze analyse levert een aantal inzichten op die ertoe leiden dat we een ander standpunt introduceren, namelijk het standpunt waarbij de vevs als extra inputparameters worden opgelegd op het systeem. Dit levert dan een situatie op die vergelijkbaar is met het uitgangspunt van hoofdstuk 4. We laten zien dat in dit geval er geen fine-tuning nodig is in de overige parameters om een grote hiërarchie in de vevs te behouden.

In het laatste hoofdstuk kijken we niet langer naar fine-tuning op laagste orde, maar onderzoeken we het effect van kwantumcorrecties. Er blijkt namelijk een fine-tuning probleem te ontstaan door deze correcties in vrijwel alle BSM theorieën die op hoge

energie nieuwe fysica introduceren. Dit wordt het hiërarchie probleem genoemd, en de oorzaak van het probleem is dat de massa van het higgsboson zeer gevoelig is voor hoge massaschalen in de theorie. De natuurlijke massa van het higgsboson is daardoor van de orde van de hoogste schaal in de theorie. Maar de waarde van de higgsmassa is ‘slechts’ 125 GeV. Er is daarom een grote mate van fine-tuning nodig van de parameters op hoge energie om zo’n licht higgsboson te krijgen, met name als er nieuwe fysica bestaat op een schaal die vele orders van grootte hoger is dan de massa van de deeltjes die tot dusver bekend zijn.

In dit hoofdstuk gebruiken we de effectieve potentiaal om dit effect te onderzoeken. De effectieve potentiaal is een manier om op systematische wijze de effecten van kwantumcorrecties op de scalaire sector te berekenen. We zijn opnieuw geïnteresseerd in een situatie met een hiërarchie in de vevs. Het blijkt alleen dat er in dat geval een probleem optreedt. Het is namelijk onvermijdelijk dat de effectieve potentiaal dan grote logaritmes bevat. Als de effectieve potentiaal tot hogere orde wordt berekend, zijn deze grote logaritmes met hogere machten aanwezig, wat ervoor zorgt dat de resultaten van de lusexpansie niet betrouwbaar zijn. Er zijn verschillende voorstellen gedaan om dit probleem op te lossen. In dit hoofdstuk gebruiken we het voorstel waarbij zware deeltjes ontkoppeld worden van de theorie zodra de energie te laag is om deze deeltjes aan te slaan. Vervolgens kijken we wat de invloed van deze aanpak is op het hiërarchie probleem. We laten zien dat deze methode leidt tot een betrouwbare expansie van de effectieve potentiaal, maar ook dat het ontkoppelen van de zware deeltjes geen oplossing biedt voor het hiërarchie probleem, hoewel het er wel toe leidt dat de hoeveelheid fine-tuning afneemt.

Concluderend geeft dit proefschrift enkele nieuwe inzichten in de discussie over fine-tuning. We laten zien dat verschillende aspecten een rol spelen bij het berekenen van de hoeveelheid fine-tuning. Er kan zowel op klassiek niveau als op het niveau van kwantumcorrecties fine-tuning aanwezig zijn, maar daarnaast is ook de zienswijze van invloed. Deze zienswijze bepaalt mede of een opgelegde conditie redelijk is, of dat deze moet worden afgewezen omdat deze onnatuurlijk is. De hoeveelheid fine-tuning is dus geen vast gegeven, maar afhankelijk van het uitgangspunt.

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Curriculum Vitae

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