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The design and pricing of hybrid debt

Vullings, Daniël

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Daniël Vullings

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The design and pricing of hybrid debt

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Promotor

Prof. dr. P. A. Bekker

Copromotores

Dr. D. Ronchetti

Dr. L. Dam

Beoordelingscommissie

Prof. dr. K.F. Roszbach

Prof. dr. J.J.A.G. Driessen

Prof. dr. F. Fecht

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Chapter 1

Introduction

The impact of the 2008 financial crisis is still being linked to many aspects of both political and daily life (Tooze, 2018). The GDP level in the EU still has not recovered to the level of 2007 (World Bank, 2019) and the surge of populism in the western hemisphere has been partially ascribed to the crisis. Risky investments of major financial institutions and the lack of capital buffers to absorb losses from these investments have been identified as two important causes of the financial crisis. Close to a decade after the crisis, policy reforms in the form of Basel III (Basel Committee on banking supervision, 2017) and MiFID II (European Securities and Markets Authority, 2019) were still being implemented to mend the weaknesses that have proven to be so disastrous in 2008. Proposed solutions potentially have unforeseen consequences that need to be carefully examined. This thesis considers the potential of hybrid financial instruments as tools to increase capital buffers and limit risk taking.

Firms are traditionally financed with debt and equity. In the seminal paper Modigliani & Miller (1958), the authors show that under some strict assumptions the value of the firm is independent of its capital structure. However, these assumptions are violated in practice and more recent papers focus on the optimal capital structure while relaxing these assumptions. In this setting, both equity and debt financing have advantages and shortcomings and many firms choose a mix of the two. Whereas equity yields more volatile payouts and is more susceptible to asymmetric information regarding its value, firms cannot default on equity. Debt has relatively constant payouts, is considered to discipline management and its issuance provides a better signal regarding the fundamental value of the firm. However, firms can default on debt which can potentially lead to high social costs. This

feature of debt played an important role in the financial crisis. Other discussions on the choice between debt and equity for the capital structure include pecking order theory, where the preference of firms for internal financing and debt over equity is explained (Myers & Majluf, 1984), and trade-off theory, where the use of debt as a tax shield is considered (Kraus & Litzenberger, 1973). Hybrid debt instruments aim to combine features of debt and equity to retain the advantages of both while mitigating the weaknesses. The traditional capital structure consisting of debt and equity can then be complemented with these hybrid instruments to increase the firm value and, in the case of major financial institutions, increase financial stability.

After the financial crisis one hybrid debt contract in particular received much attention. The so called contingent convertible bonds (CoCos) are issued as debt, but convert to equity when the firm is in financial distress. The idea is to retain the advantages of debt in good states, but reduce the probability of default in bad times. To achieve this, CoCos are designed to recapitalize firms in distress. As one of the major events in the financial crisis was the default of Lehman Brothers and the near collapse of several other major banks that held insufficient capital buffers (Vallascas & Hagendorff, 2013), both scientists and regulators are interested in CoCos as a tool to prevent future crises.

The main advantage of CoCos is that they allow firms to recapitalize in times when it is relatively costly to do so with regular equity. Firms typically default when the value of equity is wiped out. Due to the limited liability of equity, the equity holders are not accountable for further losses and debt holders will typically seize the firm in a bankruptcy. When a firm is near bankruptcy, the value of the debt decreases as there are usually costs attached to bankruptcies which are borne by the debt holders. When the firm decides to raise capital in these circumstances, part of the value of the new equity will increase the value of the debt. This means that the equity value increases by less than the funds that flow into the firm. The costs for this are borne by the original equity holders, who therefore can find it unattractive to issue equity in times of distress. CoCos are issued in times when a firm has sizable capital buffers. When the financial health of the firm deteriorates, the CoCos automatically convert to equity under predetermined conditions. As the recapitalization occurs automatically, the friction where equity holders want to avoid recapitalization is no longer a problem. CoCos can therefore increase firm resilience.

The automatic conversion is crucial to ensure that CoCos are effective. Although CoCos can theoretically be issued by any firm, they are mostly used by banks. The

literature on CoCos therefore discusses its traits in the context of banks, which is also done throughout this thesis. In practice there are three trigger mechanisms discussed in the literature and by policy makers. The first is a regulatory trigger, meaning that regulators decide when a bank needs to be recapitalized. Currently a CoCo is required to have a regulatory trigger to be counted as additional Tier 1 (AT1) capital according to the Basel III regulations (Basel Committee on banking supervision, 2017). As banks mainly issue CoCos to satisfy regulatory requirements, most currently issued CoCos have this feature (Avdjiev, Bolton, Jiang, Kartasheva & Bogdanova, 2015). The second trigger is a book trigger. The CoCos convert when the book value of equity drops below a specific threshold. Both the regulatory and the book value trigger are scrutinized by the scientific literature (Calomiris & Herring, 2013). In 2017 Banco Popular was bought by Santander after the European Central Bank declared that the bank was “failing or likely to fail” (Smith & Kahn, 2017). However, the CoCos of Banco Popular did not convert. Both the regulatory and book trigger failed to intervene timely. Regulatory triggers are criticized as regulators are considered to not be able to foresee and act on signs of financial distress in a timely manner. One of the major criticisms of book values is that they might be subject to manipulation as the management of the firm could prefer to prevent costly CoCo conversions. The third trigger, called a market trigger, uses the market capitalization of a bank to determine whether CoCos should convert. The CoCos convert when the equity value of a bank drops below a prespecified value. The market is considered to be a better judge of financial distress than regulators and more reliable than book values (Sundareshan & Wang, 2015). In line with the recent literature on CoCos, this thesis focuses on CoCos with market triggers.

In practice, CoCos with a market trigger have not been issued. One of the main problems of these instruments is the possibility for multiple or no competitive equilibrium prices (Sundareshan & Wang, 2015). As a result there is the risk of manipulation and uncertainty in CoCo markets, making them less suitable as a tool to bring stability to financial markets. When CoCos convert to equity, the balance sheet of the firm changes as debt is written off and more equity shares are issued. As this change depends on the trading price of the shares of equity, the probabilistic structure of the cash flows of the equity and CoCo changes endogenously. CoCos convert when the trading price of the equity drops below a certain threshold. This change affects the value and consequently the trading price of the equity. When conversion leads to a transfer of value from the original equity holders to the CoCo holders, the anticipation of conversion will lead to a decrease in the trading price of

the equity. This decrease can trigger the conversion. Hence, the anticipation of conversion becomes a self-fulfilling prophecy. Alternatively, when conversion leads to a transfer of value from the CoCo holders to the original equity holders, the anticipation of conversion pushes the trading price of the equity up, which decreases the probability that this conversion occurs. This leads to a self-defeating prophecy.

Glasserman & Nouri (2016) show that in a continuous time setting, when conversion favors the CoCo holders, there is a unique competitive equilibrium price. Pennacchi & Tchisty (2018) find that in a continuous time setting for perpetual CoCos, when the volatility of the assets of the firm is below a given threshold, there is a unique competitive equilibrium price when the conversion favors the equity holders as well.

Another major branch in the CoCo literature considers the effect of CoCos on the risk taking of banks. A transfer of value between the equity holders and the CoCo holders when the CoCos convert from a debt like instrument to equity might change the firm strategy in order to change the probability that this transfer of value occurs. There is no consensus on what design of the CoCo leads to the lowest risk taking incentives (see Hilscher & Raviv 2014, Berg & Kaserer 2015 and Martynova & Perotti 2018).

Excessive risk taking is a classic problem in finance and has been addressed for over 40 years (Jensen & Meckling, 1976). As equity holders have limited liability, they do not have an incentive to limit losses if a firm is going to default. Consequently, in classic models such as Black & Scholes (1973) the value of the equity increases in the variance of the underlying asset value. Leverage is therefore associated with risk-shifting (Admati, DeMarzo, Hellwig & Pfleiderer, 2018). Moreover, in the case of major banks, deposit guarantee schemes have given a further incentive to increase risk taking (Acharya & Yorulmazer 2007 and Dam & Koetter 2012). Even before the financial crisis this problem received much attention from the scientific community and the crisis has further emphasized the importance of the topic.

An early proposal that aimed to limit the risk-taking incentives of equity holders was by Green (1984), who proposed to use convertible bonds to limit the upside of equity holders. There is little evidence that firms use convertible bonds for this purpose though (Dutordoir, Lewis, Seward & Veld, 2014). More recently, several proposals, for example T. A. John & John (1993), K. John, Saunders & Senbet (2000) and Eufinger & Gill (2017), have considered the management compensation as a tool to reduce excessive risk taking. Firm decisions are made by managers, so dis-

couraging management to increase risk taking might be effective. Early proposals consider rewarding management with long-term debt, while more recent proposals propose to combine this with regulation and capital requirements. By making management compensation negatively related to risk taking, debt holders understand that the risk taking will be limited and firms can acquire cheaper debt and increase the value of the firm. As a result, it is attractive for firms to reward management with long-term debt. A downside is that as soon as the debt has been bought, equity holders have an incentive to change the incentives of management. As part of the management compensation is discretionary (Eufinger & Gill, 2017) this could be a realistic threat. Even when this is not a possibility, the issuance of long-term debt to management increases the probability of default of the firm. Furthermore, family firms, where the owner is the manager, cannot use this method. Debt covenants are a third proposed alternative that is often considered to limit risk-taking incentives. However, Leland (1994) shows that debt covenants can increase the probability of default and decrease the value of the debt holders.

As CoCos have become popular among the scientific community and been issued by banks relatively recently, there is little data to work with. Furthermore, it is unclear how to price CoCos and how these instruments affect different stakeholders and financial stability. All chapters are therefore of a theoretical nature. By abstracting from reality, the mechanisms through which CoCos strengthen or adversely affect the issuing firms and the financial sector can be highlighted. The results cannot directly be applied in practice. However, valuable insights about CoCos, that future research can use when empirically investigating them and when implementing future varieties in practice, can be gained.

Chapter 2 proposes the design of a CoCo with floating coupons that mitigate the multiple pricing problem. It is shown that this design leads to a unique price independent of the discreteness of the model and the volatility of the firm its assets. By resolving the main criticism on CoCos with a market trigger, the proposed solution can inspire a new generation of CoCos.

To find a unique competitive equilibrium price, the coupons paid to the CoCo holders are adjusted in such a way that there is no transfer of value between the equity holders and the CoCo holders. The required coupon payments depend on the distribution of the CoCo value upon conversion. To obtain this distribution, the dynamics of the asset value of the bank are simulated. Several commonly used models are compared and the robustness of the coupon payments to different model and parameter choices is considered. In general the coupon payments are

robust to the modeling choice. When the bank is close to conversion, the coupon payments increase to compensate for the fewer expected future coupon payments and the possibility of a conversion where the CoCo holders get little value in equity. When the bank is far away from default, the coupon payments are similar to the coupon payments of senior debt. Close to conversion the coupon payments increase to levels comparable to CoCo coupon payments used in practice (see Avdjiev et al. 2015). In practice it can be hard to precisely determine the required coupon payments. However, the proposed CoCo can serve as a starting point for a future generation of CoCos that approximate the floating coupon payments to reduce the problem of multiple prices.

Chapter 3 considers a Bayesian Nash equilibrium instead of a competitive equilibrium to see whether the multiple pricing problem addressed in the literature will be a problem in practice. A competitive equilibrium assumes that all investors are price takers whose actions do not influence the price of an asset. In reality investors could have some power to influence prices. Furthermore, investors will be heterogeneous in their market power and have imperfect information regarding the market power of other investors. To accommodate this feature, we consider a Bayesian Nash equilibrium. Whereas most conventional asset pricing models consider a single representative investor whose behavior summarizes the aggregate market outcome of all investors, see e.g. Constantinides & Duffie (1996a), this chapter considers several heterogeneous representative agents. This highlights that, upon conversion of the CoCos, the transfer of value affects stakeholders differently and these stakeholders have as a result different preferences for conversion. It is shown that there exists a unique Bayesian Nash equilibrium under mild assumptions for the utility of the representative investors, for any kind of discretization of the trading, and any direction of the transfer of value.

The market power of investors depends on their portfolio holdings, which in turn depends on the risk factors that the investors are exposed to. The investors that trade in the asset with an endogenous change in the probabilistic structure of the cash flows might not be exposed to all risk factors. Only the risk factors that the investors of the asset of interest are exposed to are relevant for the pricing of the asset. By considering these risk factors, the probability of a change in the probabilistic structure of the cash flows of the asset can be determined. In a replication of the numerical analysis in Sundaresan & Wang (2015), multiple competitive equilibrium prices and, by accounting for the additional information, a unique Bayesian Nash equilibrium are found.

Due to the mild assumptions and the nonparametric setting, the findings are general. The findings can therefore be applied to other settings where there is an endogenous change in the probabilistic structure of the cash flows. For example, several indexes are based on market capitalization. When a firm is included in these indexes, the demand for these assets from institutional investors will typically change. Chang, Hong & Liskovich (2014) show that this leads to an index inclusion effect that affects the value of the included equities and an index deletion effect for the equities excluded. As a result, equities that are eligible to be included in an index based on market capitalization and with an index inclusion or deletion effect do not have a unique competitive equilibrium price. Firms whose management bases its actions on the market capitalization of the firm suffer from the same phenomenon (Bond, Goldstein & Prescott, 2009).

In an experimental paper by Davis et al. (2014) it is shown that there is a regularity in what equilibrium occurs for CoCos with a market trigger. Whereas the findings cannot be explained by existing theories, they line up closely to the predictions made by the theoretical model in Chapter 3. As predicted, when the conversion favors CoCo holders, there is a unique equilibrium when investors agree on whether it is possible that conversion can be averted. If all investors believe that there exists a no conversion competitive equilibrium, this equilibrium will realize instead of a conversion equilibrium. Chang et al. (2014) observe that investors do not anticipate the inclusion in or exclusion out of indexes based on market capitalization and with an inclusion or deletion effect. The inclusion and deletion effect lead to the same endogeneity problem as CoCos with market triggers and the findings in this chapter can therefore be applied to this phenomenon as well. This illustrates the relevance of the model for financial markets.

The findings in Chapters 2 and 3 have important policy implications. CoCos with a market trigger are, except for the pricing problem, considered to be the most promising type of CoCo for banks as a tool to recapitalize in times of distress. It is shown that by either adjusting the CoCo design or by using the pricing model of Chapter 3, the main downside of CoCos with a market trigger can be overcome. This can give regulators confidence in the use of these CoCos as tools to recapitalize banks in distress. Furthermore, it can lead to banks issuing this type of CoCo.

Chapter 4 proposes a new hybrid debt contract designed to mitigate excessive risk-taking incentives. CoCos appear not to be a suitable hybrid debt instrument to solve this problem. However, as excessive risk taking is largely due to the division of the firm cash flows between the equity and the debt holders of the firm, other

hybrid debt instruments that alter this division could be effective. In Chapter 4 Risk incentive compatible (RIC) bonds are proposed. These instruments pay fixed coupon payments similar to debt. When the cash flow of a firm is high, which is more likely when the firm engaged in excessive risk taking, random payments, next to the regular fixed payments, are made to the bond holders. The upside of more risky investments for equity holders is, as a result, limited. When properly designed, it is undesirable for the equity holders of a firm that has issued RIC bonds to increase the firm risk taking. Because of this, issuing RIC bonds sends a credible signal that the firm will not increase the risk taking.

RIC bonds can be desirable to issue for firms as the cost of debt is reduced due to the limited risk taking while the payoffs are still relatively constant, which is desired by risk averse investors (Hébert, 2017). When a firm wants to issue regular debt, the debt holders are aware of the incentive for risk shifting. In order to be protected against this, they demand higher coupon payments or principal values. However, Leland (1994) shows that these additional payments can increase the probability of default and as a result decrease the value of the debt. Ex ante the equity holders of a firm prefer to limit the risk taking and obtain cheaper debt. However, as soon as the debt contract has been signed, the cost of debt is fixed, while shifting risk does still yield benefits for the equity holders. Consequently, equity holders are able to increase the risk taking without having to pay additional coupon payments. As the coupons of RIC bonds adjust, based on the expected future cash flow of the firm, to make it preferable for the equity holders to maintain a given level of risk over increasing the risk, RIC bonds provide the credible signal needed to convince investors that there will be no risk shifting.

Chapter 4 shows in an $(n + 1)$ -period discrete time model, where a firm can issue regular debt, CoCos, equity and RIC bonds, that there are scenarios where RIC bonds are the optimal financing strategy. The model features two frictions: excessive risk taking and differences in risk aversion between the original equity holders and investors. In a numerical example it is illustrated that when both frictions increase in severity RIC bonds become desirable over equity and regular debt. Convertible bonds use a mechanism similar to RIC bonds to limit risk taking. However, whereas convertible bonds can mitigate the risk-taking incentives in the first period, in the next period the firm could be in a financially weaker situation which magnifies the risk-taking incentives for original equity holders of the firm. In the subsequent period, the original equity holders have no incentive to issue additional convertible bonds. Hence, the firm will not issue new convertible bonds to limit

the increased risk-taking incentive and instead try to increase the risk taking. RIC bonds are designed to overcome this shortcoming. As a result, issuing RIC bonds can be the optimal financing strategy for firms.

Through Chapters 2 to 4, this thesis considers the design and pricing of hybrid debt. First, a new variety of CoCos is introduced that resolves the problem of multiple competitive equilibrium prices. Second, a new pricing framework is introduced that shows that there is a unique Bayesian Nash equilibrium price for assets with endogenous changes in cash flows. Third, a new hybrid debt contract is proposed that is effective in mitigating risk-taking incentives of equity holders. By addressing these topics, this thesis contributes to the scientific literature that considers capital buffers and excessive risk taking. Furthermore, the findings can contribute to financial stability and help resolve frictions in firm financing.

Chapter 2

Fixing the equilibrium problem: Contingent convertible bonds with floating coupon payments

2.1 Introduction

During the crisis, bankers and researchers have started to look for new ways to help banks meet the stricter capital requirements as defined in Basel III. Contingent Convertible bonds (CoCos) are becoming increasingly popular among banks to serve this purpose, resulting in a sharp increase in the volume of CoCos being issued. Due to being a hybrid between debt and equity, CoCos are considered to be a promising instrument to help meet these requirements. In their current form, CoCos offer coupon payments similar to regular bonds. However, when a certain trigger event takes place, the bonds are either converted to equity or written down. The goal of this conversion is to quickly and efficiently recapitalize banks in distress, which might at that time not be able to obtain new capital by issuing more shares. Since the recent crisis, both practitioners and academics have engaged in

This chapter is based on Vullings (2016).

a heated debate on the proper design of CoCos and the risks that arise from using these kinds of instruments.

Due to the Basel III regulation and banks issuance of more CoCos every year, it is crucial to design CoCos that enhance the stability of the financial system. In this chapter I consider a new, hypothetical, design of a CoCo contract with a market trigger. Crucially, I propose floating coupon payments that incorporate market information and are such that the market value of the CoCos should equal the face value. Upon conversion, the number of shares obtained is such that the total value of equity in possession of the new shareholders is equal to the face value of the original CoCos. This design resolves the multiple equilibria or no equilibrium problem, from now on referred to as the multiple equilibria problem (see Sundaresan & Wang 2015). This design could, furthermore, reduce the potential increasing risk taking incentives addressed in Berg & Kaserer (2015) and Chan & van Wijnbergen (2016) and the problems with pricing CoCos due to the hybrid structure as in Wilkens & Bethke (2014). The lack of a unique equilibrium price for the CoCos and equity could cause high volatility in these prices and lead to discomfort among investors. As CoCos are designed to bring more stability to the financial sector, this is an undesirable feature that should have a high priority to be resolved. The new design proposed here is of a hypothetical nature and depends on assumptions on the stochastic process underlying the asset value and the probability of default. Simulation techniques are required to find the appropriate coupon values. The trigger value is not considered, but I assume it is high in order to make sure that the CoCo coupon payments, that increase when a bank faces more financial distress, cannot cause the bank to run into trouble.

The floating coupon payments may give the impression that the proposed CoCos are similar to equity, with coupon payments resembling dividend payments. The distinction between our CoCo design and equity is sizeable though. The crucial difference between the proposed CoCos and equity is that the market value of the CoCos is constant, and equal to the face value, at all times that coupon payments are made, until conversion has taken place. When conversion takes place, CoCo holders will get the number of shares that ensures that the value of the shares held by the former CoCo holders equals the original face value of the CoCos. The former CoCo holders can, therefore, get their original face value back upon conversion by selling the newly obtained shares of stock. This structure of the CoCo has several implications that make it crucially different from equity. First, the value of the equity varies over time both before and after the conversion of the CoCo.

Second, the equity holders are the first to suffer when the bank faces financial distress. When the bank suffers from financial distress, the debt remains unaffected and the value of the CoCos is constant at given times before conversion. Hence, the share price will drop and equity holders take a loss. CoCos are less risky, as the price is guaranteed to have a certain value at given times before conversion. The third difference is that whereas dividends paid to equity holders will typically decrease when the return on investments disappoints, the coupon payments for the CoCo holders will increase to compensate the CoCo holders for the increased probability of default. Hence, there are clear distinctions between equity and CoCos with the proposed design. The perpetual nature of the CoCos, the juniority, and the floating coupon payments make the CoCos different from bonds as well. The coupon payments of the CoCos will be higher than the coupons for senior debt due to the additional risk incurred by investing in CoCos and this higher expected return will make the CoCos interesting to investors.

The coupon payments to CoCo holders are determined by market prices of both equity and debt. As this contains all the information from the market, investors have little reason to believe that the value of the CoCos differs from the face value when coupon payments are made. The proposed design, standing in between debt and equity, makes the instrument interesting for investors. When investors consider equity to be too risky, while they aim for higher expected returns than bonds have to offer, the proposed CoCos are an alternative to equity and debt. The multiple equilibria problem is resolved in our design, as there is no transfer of value between equity holders and CoCo holders upon conversion. The proposed design is mostly hypothetical. As the design depends crucially on the probability of default, the volatility of the asset value, and the equity value, a good measure for the probability of default and the volatility of the asset value should be developed before the instrument can be introduced in the financial sector. However, market-based proxies for the asset value are derived from the CDS market. Credit default swaps can be used to estimate the probability of default and the volatility of the asset value. Although this will result in a rough estimate, it is based on the perception of the market of the probability of default and therefore the best indicator I can find. Future research should look into the use of credit default swaps to design instruments that are inspired by the one I propose here. The aim of this chapter is to show what the theoretic design of the proposed CoCo, with its desirable characteristics, should look like and how this differs for different stochastic processes that govern the asset value. This will give valuable insights in the value of CoCos, its

dependency on the default probability, and the equity value. It could, furthermore, form the basis for the design of a new class of CoCos that do incorporate some of the advantages, while relaxing the exact requirements for the CoCos as proposed in this chapter.

The main contribution of this chapter is the proposal of a new hypothetical instrument that resolves the multiple equilibria problem. The traditional desirable features of CoCos, namely to recapitalize banks in distress and reduce potential bail out costs, are meanwhile retained. I am, to the best of my knowledge, the first to consider a CoCo with floating coupon payments that equalize the market value of CoCos to the face value while allowing for jumps in the asset value to occur. Other problems addressed in the literature, such as the increased risk taking incentives (see Berg & Kaserer 2015 and Chan & van Wijnbergen 2015) and the threat of a death spiral (see Maes & Shoutens 2012) are potentially reduced as well. The reason for this is that, upon conversion, there is no longer a transfer of value between equity and CoCo holders. As this transfer of value was the cause of these issues, the new design will likely be much less sensitive to these problems. A downside of the proposed approach is that it relies on having a good measure of the total asset value. As the total value of debt is estimated only at low frequency, this can form a problem for the estimation of the floating coupons. In order to apply CoCos with floating coupons in practice, future research is necessary to obtain better estimates of the total asset value.

Although the designed CoCo is of a hypothetical nature, valuable insights in the value of CoCos and its sensitivity to different stochastic processes that govern the asset value of a bank, will be obtained. This can inspire the development of a new class of CoCos that are less sensitive to the problems that CoCos are currently vulnerable to. Due to the advantages of the new design, the proposed CoCo could be the inspiration for the development of a new class of CoCos and contribute to the stability of the financial system. Another important contribution of this research is the insight gained in the value of CoCos. The value of traditional market triggered CoCos is strongly linked to the coupon payment. When a higher coupon payment is necessary to keep the value of the CoCo constant, a CoCo with a constant payment will decrease in value. Hence, this chapter will demonstrate how CoCos behave under different circumstances and how stable the value of CoCos is.

The structure of the chapter is as follows: In Section 2.2 I will consider the literature on CoCos; Section 2.3 describes the multiple equilibria problem; the desired coupon payments are determined in Section 2.4; Section 2.5 considers the simula-

tion of the stochastic process that governs the asset value and gives a numerical illustration of the characteristics of the proposed CoCo; the policy implications and conclusions are provided in Section 2.6.

2.2 Literature review

Since the financial crisis much research has focussed on CoCos. In this section I will discuss the basic properties of CoCos and give a short overview of the most crucial debates in the literature. See Flannery (2014) for a more complete summary of the literature.

An early mention of what is now called a CoCo is in Flannery (2005), where the author introduces so-called “Reverse convertible debentures”. These instruments convert when the equity to debt ratio falls below a pre specified value to allow banks to recapitalize in a timely manner and prevent financial distress. Due to the lower costs of debt compared to equity, there is a strong incentive for large banks to operate under high leverage. Debt is less costly as it is tax deductible. Furthermore, it is expected that governments will bail out systematically important banks, which makes debt safer and therefore less costly. A natural solution to this problem could be to force banks to hold more equity. However, bankers argue that this will make banks uncompetitive as equity is expensive, even though this claim is rejected by, e.g., Admati, Demarzo, Hellwig & Pfleiderer (2013). It is clear that this argument is, at least to some extent, considered to be valid, as CoCos have become an accepted instrument to reinforce capital buffers as stated in Basel III.¹ After the crisis both regulators and banks became more interested in CoCos due to the new regulatory standards banks needed to live up to. This has led to substantial growth of academic articles on this topic.

Despite the vast number of papers on this topic, there is a wide discrepancy between what the literature argues to be well designed CoCos and the CoCos that are actually issued by banks. Most papers stress the importance of a conversion to equity (CE) contract based on a market value, see e.g. Sundaresan & Wang (2015) and Pennacchi, Vermaelen & Wolff (2014). Calomiris & Herring (2013) state several conditions CoCos have to adhere to in order to have the desired effect. Two conditions are the previously named market trigger and a sufficiently dilutive conversion to equity. A market trigger is required as accounting values traditionally

¹ Basel Committee on banking supervision (2017)

lag behind the true financial condition of the bank.² Market prices appear to show signs of distress much more timely, increasing the odds of timely conversion. Another advantage of market triggers is that conversion does not bring new information to financial markets, as everyone can observe the decreasing stock price. This prevents investors from panicking due to a sudden realization that a certain bank is in poor shape, a problem considered in Chan & van Wijnbergen (2015). The conversion to equity is necessary to give equity holders an incentive to prevent conversion, as conversion can heavily dilute equity holders. Despite this, 60 percent of the recently issued CoCos is of the principal write down (PWD) type, where instead of being converted to equity, CoCos are written down with a pre defined percentage. In Chan & van Wijnbergen (2015) and Chan & van Wijnbergen (2016) it is argued that this feature is highly undesirable, as it could increase the systemic risk and give equity holders an incentive to increase the risk taken instead of decreasing it. Despite this, the opposite effect can occur as well, as described in Martynova & Perotti (2018), where it is shown that under certain assumptions the risk-taking incentive is higher with CE than with PWD Cocos. The third condition as mentioned in Calomiris & Herring (2013) is that CoCos should form a significant part of the book value of equity of the bank, in order to make sure that conversion is a real concern. Another condition often named in the literature considers the investors in CoCos. When banks buy the CoCos of other banks, conversion can cause a death spiral in which conversion of one bank, could cause other banks to run into trouble themselves. Regulators should therefore discourage banks to buy CoCos from other banks.

In Avdjiev et al. (2015) the first empirical evaluation of CoCos is given along with an overview of the different CoCos that have been issued. The authors find that investors appear to be driven by the search for high yields and do not take the risk of conversion into account. This is problematic, as investors might start to panic when CoCos do convert, leaving banks in a more troublesome situation than they would have been in without CoCos. Academics are still working on understanding the new risks that come along with issuing CoCos. The complexity and hybrid nature of CoCos has led to several pricing methods, which are compared in Wilkens & Bethke (2014). A problem with developing a general pricing method is that there is a large variety in the CoCos issued by different or even individual banks. This feature makes an empirical analysis difficult, as the lack of generality results in small samples to work with.

² See Haldane (2011) for figures.

The multiple equilibrium problem is part of an ongoing debate on the uniqueness of the equilibrium price of the shares for a bank that issues CoCos with a market trigger. Sundaresan & Wang (2015), from now on SW, note that under realistic conditions there can occur multiple equilibria or not exist any equilibrium due to a wealth transfer between equity and CoCo holders. Depending on the belief of the agents whether conversion will take place, several equilibrium share prices are valid. This can lead to jumps in the price or to agents manipulating the market price as described in Maes & Shoutens (2012), where the authors call this phenomenon a death spiral. These death spirals could be induced by CoCo holders that sell the stock of the bank short as a hedge for the CoCo. If, upon conversion, wealth is transferred from the equity holders to the CoCo holders due to heavy dilution, CoCo holders have an incentive to force conversion. Pennacchi et al. (2014) propose to enhance CoCos with a call option given to the original shareholders and let conversion depend on the ratio of the sum of the equity and the CoCo value over the asset value. The original shareholders can, upon conversion, decide to exercise the option to buy the newly issued shares from the former CoCo holders. The authors claim that this results in a unique equilibrium. However, SW note that the proposed trigger makes conversion exogenous as debt is assumed to be always priced at par. This assumption implies that the value of debt is constant and unaffected by conversion. The ratio of the sum of the equity and the CoCo value over the asset value is, therefore, unaffected by conversion and this makes the trigger insensitive to conversion as well, which leads to a unique equilibrium. The result follows from the in practice likely violated assumption that the debt is always priced at par. Furthermore, it requires a liquid CoCo market in order to retain the advantages of the market trigger. Hence, the problem of multiple equilibria is not resolved and requires further research.

The problem of there being no unique equilibrium as addressed in SW, has been refuted by several other authors. Calomiris & Herring (2013) claim that there is no problem when CoCos are properly designed. According to the authors, when conversion is sufficiently dilutive, banks will always issue more equity, eliminating the belief of investors that conversion is a realistic option. SW argue that this only holds for unlevered banks, as levered banks do not always have the option to issue more equity. Hence, the lack of a unique equilibrium in certain circumstances remains valid. Other critiques have come from Glasserman & Nouri (2016) and Pennacchi & Tchisty (2015), who both argue that a unique equilibrium does exist. In Pennacchi & Tchisty (2015) the authors assume that there are no jumps in the asset

value, which is generally considered unrealistic (see e.g. Chernov, Ghysels, Galant & Tauchen 2003). Both papers assume that market prices of both CoCos and shares will adjust continuously. Investors will see conversion coming and adjust the share price continuously and this results in a unique equilibrium. SW, on the other hand, use a framework where the adjustment of market prices is discrete and this causes investors to be caught by surprise by price developments, as they cannot immediately adjust to the development, and this results in the multiple equilibrium problem. The question is, therefore, whether prices adjust in continuous or in discrete time. It is unclear which view of the world is correct. However, given that investors cannot respond to news immediately and will in practice always need time and liquidity to adjust to the new situation, it is arguable that prices adjust in discrete time, although it could be in small intervals. Hence, multiple equilibria could still well be a threat, even though there are scenarios where this is not the case. To resolve the problem, SW consider floating coupons for CoCos when the asset value does not feature jumps. They find that in the absence of jumps, the coupon payments can be set equal to the interest rate to achieve a unique equilibrium. However, they note that in practice this will not work due to jumps being present. An instrument that mitigates the problem completely is, therefore, desirable, as this definitively resolves the threat of multiple equilibria prices.

The multiple equilibria problem is one of the most crucial problems to resolve as it makes CoCos with a market trigger potentially extremely volatile, a highly undesirable feature for an instrument originally designed to bring stability to the financial sector. As a general consensus in the literature is that market triggers should be used, research on how to resolve the multiple equilibria problem for CoCos with market triggers is required.

2.3 Multiple equilibria problem

In this section I will provide an example to illustrate why the multiple equilibria problem occurs.

2.3.1 Illustrative example

In order to illustrate what causes the multiple equilibria problem, I will reiterate the multiple equilibria problem as described in SW and repeat the example (see Sundaresan & Wang 2015, p. 888-891) used to show why multiple equilibria occur. I consider a bank with equity, senior debt, where deposits from savers are part of

the senior debt, and CoCos. In this framework A_t represents the asset value at time t , S_t^i , for $i = u, c$ depending on whether the CoCo has converted, is the stock price at time t of a single equity share, n is the number of shares outstanding, K is the trigger value of equity in the CoCo contract, $\bar{B}$ the face value of the debt of senior debt holders and the default threshold for the firm, $\bar{C}$ is the face value of CoCos, m shares are granted to CoCo holders when the value of equity falls below K , and T is the date of maturity for the CoCos. In the next section I will introduce a perpetual CoCo, but for now I stick to the example in SW.

Following SW, I define a discrete-time model with one period. The time of maturity is given by $T = 1$. In this case, the initial asset value is A_0 and the terminal asset value is the random variable A_1 . The probability distribution function (PDF) and cumulative distribution function (CDF) of the risk-neutral distribution of the asset value are given by $f(\cdot)$ and $F(\cdot)$. In this section I consider all agents to be risk neutral and the interest rate is equal to zero. The initial value of the regular debt, B_0 , and the unconverted CoCo bond, C_0^u , are given by

$$B_0 = \bar{B}(1 - F(\bar{B})) + \int_0^{\bar{B}} A_1 f(A_1) dA_1 \quad (2.1)$$

$$C_0^u = \bar{C}(1 - F(\bar{B} + \bar{C} + K)) + \frac{m}{n + m} \int_{\bar{B}}^{\bar{B} + \bar{C} + K} (A_1 - \bar{B}) f(A_1) dA_1. \quad (2.2)$$

Note that $F(\bar{B})$ is the probability of default and $\int_0^{\bar{B}} A_1 f(A_1) dA_1$ is what regular bond holders get in case of default. CoCo holders get the face value of the CoCo at maturity if the asset value at time $t = 1$ exceeds the sum of the debt, the CoCos and the trigger. In case of conversion at time t , the value of the CoCos, denoted by C_0^c is clearly equal to mS_t^c . The stock price, with and without conversion respectively, equals

$$S_0^c = \frac{(A_0 - B_0)}{n + m} \quad (2.3)$$

$$S_0^u = \frac{(A_0 - B_0 - C_0^u)}{n}. \quad (2.4)$$

Now consider the example in SW where $\bar{B} = 90$, $\bar{C} = 10$, $K = 5$, $m = 2$, and $n = 1$. In this case I have that $m = \frac{n\bar{C}}{K}$. The probability distribution for A_1 is discrete with $P(A_1 = 80) = 0.25$, $P(A_1 = 100) = 0.5$, and $P(A_1 = 120) = 0.25$. The expected value of A_1 equals 100, hence $A_0 = 100$, as investors are rational and I assume risk neutral agents. Using Equations (2.1) to (2.4) I find that without conversion $B_0 = 87.50$, $C_0^u = 5.83$ and $S_0^u = 6.67$ is an equilibrium as all equations

are satisfied and $S_0^u > K$. However, given that conversion does take place, I have $B_0 = 87.50$, $C_0^c = 8.33$ and $S_0^c = 4.17$, which is an equilibrium as well as again all equations are satisfied and $S_0^c < K$. The reason for the multiple equilibria is that in this case, there is a wealth transfer from equity holders to CoCo holders. When conversion takes place, too many shares are given to CoCo holders, causing the expected returns for the equity holders and therefore the share prices to decrease. SW show, furthermore, that there is no equilibrium if there is a transfer of value from the CoCo holder to the equity holders.

There is no simple way around this problem as noted by SW. A unique equilibrium is achieved when, upon conversion, there is no transfer of value between CoCo holders and equity holders. In order to make this intuitively clear, consider a CoCo with coupon payments equal to zero that converts to a number of shares such that the value of the newly obtained equity is equal to the face value of the CoCo upon conversion. Furthermore, I assume the issuing bank has a positive risk of default before maturity. The value of this CoCo, before maturity or conversion, is lower than the face value. Hence, CoCo holders gain from conversion, as then they will get the face value of the CoCo, which is higher than the current value of the CoCo. The equity holders, on the other hand, suffer from conversion. This is because they expected to have to pay less to the CoCo holders, as there was a possibility of not having to pay them at all in case of default. Suppose now that the asset value is such that the equity value is just above the trigger, say at $K + \epsilon$ for some $\epsilon > 0$. Conversion does not occur as the share price is above the trigger level. As upon conversion the equity holders suffer in favour of the CoCo holders, the equity value drops by say 2ϵ upon conversion. Hence, if conversion would occur, the share price drops below the trigger price and, given this lower share price, conversion is justified. Given the underlying asset value, both scenarios are reasonable and there are, therefore, multiple equilibrium prices. This can only be resolved when the number of shares granted to CoCo holders upon conversion is such that the value of the shares given to CoCo holders upon conversion equals the market value of the CoCos. This implies that the number of shares given to the CoCo holders should be equal to the ratio of the market value of the CoCos to the share price after conversion. Hence, in order to have a unique equilibrium, I need

$$m_\tau = \frac{C_\tau^u}{S_\tau^c},$$

where τ is the date of conversion, defined by $\tau = \min\{t \in \Lambda : nS_t < K\}$, with Λ

being the discrete set of all points in time where the equity value is compared to the trigger. The problem with defining an m that always satisfies this equation is that the number of shares granted to CoCo holders should not depend on the future market value of CoCos. Such a contract would introduce a problem as the market value of a CoCo depends on the contract, whereas the contract then depends on the market value of the CoCo. Such a contract cannot be priced properly and is therefore undesirable. This example was simple and assumes discrete time. SW show that in a more complete continuous-time framework, although only discrete trading is allowed, the same problem arises.

2.4 Floating Coupon payments

In this section I will define the model I use to design the new CoCos. Before defining the model, first some basic characteristics for the proposed CoCo need to be determined. I consider CoCos without a date of maturity, with a market trigger, and a conversion to equity (CE) scheme. A perpetual CoCo is most interesting, as in Basel III it is determined that CoCos will only be considered as additional Tier 1 (AT1) capital if there is no date of maturity. Since one of the main reasons CoCos have become so popular is the new regulation as determined in Basel III, CoCos make most sense when they count as AT1 capital. The results will not differ much due to this assumption and the analysis can easily be generalized to include CoCos with a date of maturity. Additionally, Basel III requires banks to allow regulators to trigger the conversion of CoCos. This is a discretionary feature which cannot be modelled and will, therefore, be left out. I consider CE contracts as the problem I address only occurs with CE contracts. Finally I assume no shortselling, as shortselling results in potential arbitrage opportunities for CoCo holders, as they can force a favorable conversion by short selling the shares (see e.g. Hillion & Vermaelen 2004). If shortselling would be allowed, the CoCo I propose should be extended with a call option, that can be exercised on conversion, to the equityholders to buy the shares newly issued to CoCo holders for the face value of the CoCo (as proposed by Pennacchi et al. 2014). This would resolve the shortselling problem as CoCo holders can no longer gain from forcing conversion by selling the shares short, since in this case the equity holders will exercise their option.

We introduce the model in section 2.4.1 for a general CoCo structure. Then, in section 2.4.2 I show how the coupons should be designed in order to offset asset value movements.

2.4.1 Dynamic continuous-time model for the bank's asset value

The notation used in our model is similar to SW. Again I consider a bank with equity, senior debt and CoCos. Define A_t to be the asset value of the bank of interest at time t with some underlying dynamic process. I assume a model along the lines of Merton (1976) as is done in SW:

$$dA_t = \mu A_t dt + \sigma A_t dz_t + A_t(y_t - 1)dq_t,$$

where z_t is a standard Brownian motion, y_t follows an independent log-normal distribution with parameters μ_y and σ_y^2 , while q_t is a poisson processes with arrival rates λ_t . Due to the poisson process the asset value will display jumps, causing the multiple equilibria problem addressed in SW. The occurrence of jumps in the stock prices, and therefore in the asset value, has strong empirical support, see e.g. Bakshi, Chao & Chen (1997) and Chernov et al. (2003). As I deal with a risk-neutral probability measure, I have that $\mu = r - \eta - \lambda E[y - 1]$, where r is the risk-free rate. Here, ηA_s is the outflow of asset value from which the dividends and coupon payments to equity holders and bond holders are paid. Hence, I have $\eta = a(s, A_s)$, where $a(s, A_s)A_s$ is the sum of the dividends to equity holders, $(a(s, A_s)A_s - b(s, A_s)\bar{B} - c(s, A_s)\bar{C})$, and coupon payments to bond holders, $b(s, A_s)\bar{B}$ and $c(s, A_s)\bar{C}$ respectively for senior debt holders and CoCo holders. $c(s, A_s)$ is implicit at this point. I will choose it in the next section in order to satisfy the conditions to prevent a transfer of value between equity holders and CoCo holders.

We assume that the bank defaults when the value of the assets drops below the face value of debt, so when $A_t < \bar{B}$. In case a bank is hit by a sudden shock that causes immediate bankruptcy, CoCos function as a buffer to reduce the social costs of bankruptcy. Several events are important for the value of the bank and the debt, equity, and CoCos. First, I have the date of default, given by δ , which can be defined as

$$\delta = \min\{t \in \Lambda : A_t < \bar{B}\},$$

where Λ is defined in Section 2.3.1. Defaults are costly and I define the costs of default to be equal to ωA_δ , A_δ being the value of assets at the date of bankruptcy, and $\omega \in [0, 1]$. When default occurs, I therefore have that the remaining value of the senior bond holders upon default is given by $(1 - \omega)A_\delta$. Senior debt matures at infinity. As banks will in general issue new bonds once current bonds expire,

this simplification is not unreasonable. If the level of debt changes, the coupon payments can simply be determined for the new situation. Note that default could occur before conversion takes place due to the jumps in the asset value.³

Next, the date of conversion is defined by

$$\tau = \min\{t \in \Lambda : nS_t \leq K\},$$

where conversion will take place when the equity value of the bank, nS_t , drops below the trigger level K , and Λ is defined as before. Similar to SW, if conversion never takes place, so if $nS_t > K$ for all $t \in \Lambda$, I define $\tau = \infty$.

We, furthermore, define a third date that is crucial in order to make sure there is no transfer of value between equity holders and debt holders. Let

$$\gamma = \min\{t \in \Lambda : \bar{B} < A_t < \bar{B} + \bar{C}\},$$

where γ denotes the time at which a shock occurs that is such that all equity value is wiped out but there is some value of the CoCos left. In this case I assume that the CoCo holders will take over the bank and obtain all the shares. This situation only occurs when a jump in the asset value takes place that causes the asset value to fall in the interval $\bar{B} < A_t < \bar{B} + \bar{C}$. The assumption is not uncommon in the literature and is made in, for example, Pennacchi (2010) as well.

Next, I can define the value of the senior debt, the equity and the CoCos. Let the risk free interest rate be given by r_t , resulting in the discount factor

$$R(t, s) = e^{-\int_t^s r_u du}.$$

We consider the period from now until the maturity, T , of the senior debt, where the senior debt includes deposits. The current value of the senior debt is then given by

$$B_t = E_t \left[(1 - \omega) A_\delta R(t, \delta) + \int_t^\delta b(s, A_s) \bar{B} R(t, s) ds \right],$$

where $b(s, A_s)$ is the coupon payment to bondholders and $E_t [\cdot]$ is the expectation operator conditioned on all the information available at time t .

³ We assume default to be exogenous. In our framework there are no costs of bankruptcy or conversion for equity holders due to limited liability and there being no transfer of value between equity and CoCo holders upon conversion (as will be shown in the rest of this section and the appendix). We, furthermore, abstract from the possibility of endogenous bankruptcy as discussed in Diamond & Rajan (2011).

In order to resolve the multiple equilibria problem in SW, the coupon payments to CoCo holders, $c(s, A_s)\bar{C}$, should be such that the market value of the CoCos is equal to $\bar{C}$ at any point in time where conversion can be triggered. Due to the definition of the conversion quantity, this will ensure that upon conversion, the CoCo holders will get $\frac{\bar{C}}{S_\tau} S_\tau = \bar{C}$ worth of shares. Hence, no transfer of value between equity and CoCo holders takes place, resolving the multiple equilibria problem. In order to obtain the required coupon payments I first define the value of a share

$$S_t = E_t \left[\frac{1}{n} I_t \right] + E_t \left[\frac{1}{n+m} \{ J_\tau R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}} \} \right], \quad (2.5)$$

where I_t and J_t are given by

$$I_t = \int_t^{\min\{\tau, \delta, \gamma\}} (a(s, A_s)A_s - b(s, A_s)\bar{B} - c(s, A_s)\bar{C})R(t, s)ds,$$

$$J_t = \int_t^{\min\{\delta, \gamma\}} (a(s, A_s)A_s - b(s, A_s)\bar{B})R(t, s)ds.$$

The first element on the right hand side of Equation (2.5) represents the expected standardized dividend payments before conversion, given by I_t . The second element on the right hand side represents the standardized dividend payments after conversion, given by J_t , when the equity holders are diluted. The total value of the CoCos is given by

$$C_t = E_t \left[\int_t^{\min\{\tau, \delta, \gamma\}} c(s, A_s)\bar{C}R(t, s)ds \right] + E_t \left[\frac{m}{n+m} \{ J_\tau R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}} \} \right] \\ + E_t [(A_\gamma - \bar{B})R(t, \gamma) \mathbf{1}_{\gamma < \min\{\tau, \delta\}}], \quad (2.6)$$

where $\mathbf{1}_{a>b}$ is one if $a > b$ and zero otherwise. The first element in Equation (2.6) represents the expected coupon payments to CoCo holders before conversion. The second element consists of the expected dividend payments to the equity holders that owned CoCos which were converted to equity. The third element gives the expected value CoCo holders receive when the equity holders are wiped out completely and the CoCo holders become the new equity holders.

Note that the main difference with SW is that I assume no date of maturity for the CoCos and the bonds. The value of both the equity and the CoCos is merely the sum of the expected future payments in a risk neutral world discounted by the risk free interest rate.

2.4.2 Floating coupons offsetting movements in the total asset value

In the previous section it was clear that part of the value of the CoCos comes from the coupon payments. If the coupon payments are defined in such a way that the market value of the CoCo is equal to the face value when conversion can occur, then the conversion ratio that ensures that there is no transfer of value can easily be determined.

Note that CoCos are a form of junior debt when default takes place before conversion and that when the equity value is wiped out, in the absence of default, that the CoCo holders will become the sole equity holders. This allows us to rewrite Equation (2.6):

$$C_t = E_t \left[\int_t^{\min\{\tau, \delta, \gamma\}} c(s, A_s) \bar{C}R(t, s) ds \right] + E_t [(A_\gamma - \bar{B})R(t, \gamma) \mathbf{1}_{\gamma < \min\{\tau, \delta\}}] + E_t [\bar{C}R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}}]. \quad (2.7)$$

There are two possible scenarios that influence the value of the CoCos: either the complete equity value is wiped out before time τ and the CoCo holders will only get $\max(A_\gamma - \bar{B}, 0)$, or the CoCo holders will retain the value $\bar{C}$ either in the form of CoCos or, in case of conversion, in shares. In a scenario with an interest rate of zero, the coupon payments only serve the purpose of covering the risk that either default occurs or the equity holders are wiped out before conversion. Note that the CoCo holders face a much lower risk than equity holders, as the value of the CoCos will decrease only after conversion. Before conversion the value of the CoCo is constant, making CoCos debt like, while it becomes equity after conversion. When a large bank is in financial distress, first the equity holders will suffer and the CoCo holders will only be held accountable once the financial situation becomes so bad that the share price breaches the trigger. In the process the bank will be recapitalized and, given a trigger that is set high enough, be relieved from the financial pressure.

Before I determine the desired coupon payment, I first note that as conversion can only take place at certain discrete time intervals, the periods included in Λ , the coupons can be determined at these moments as well. This is the case because the market value of the CoCos only needs to be equal to the face value of the CoCos when conversion can take place. This provides us with a natural transition from the continuous coupon payments in the model to discrete coupon payments that are applicable in practice.

The expected loss upon conversion, default, or when the equity holders are

wiped out, divided by the value of the CoCos is given by

$$\Gamma_t = \left(1 - E_t[R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}}] - \frac{E_t[(A_\gamma - \bar{B})R(t, \gamma) \mathbf{1}_{\gamma < \min\{\tau, \delta\}}]}{\bar{C}} \right).$$

The coupon payments paid to CoCo holders before conversion can be derived by solving the following equation

$$\Gamma_t = E_t \left[\int_t^{\min\{\tau, \delta, \gamma\}} c(s, A_s) R(t, s) ds \right],$$

where I use Equation (2.7) and the fact that $C_t = \bar{C}$. Γ_t can be interpreted as the expected difference after conversion or default of the CoCo at time t between the face value of the CoCo and what the CoCo holders will receive. From now on I will refer to Γ_t as the conversion loss. If the probability of conversion instead of default is high at time t , the conversion loss is small as CoCo holders expect a value close to the face value of the CoCo. If, on the other hand, the probability of instant default is high at time t , the conversion loss will be high. In the extreme case when the probability of conversion is one and the discount rate is zero, the conversion loss will equal zero and the coupons can be set equal to the risk free rate. On the other extreme, when the probability of the CoCo holders being wiped out completely is one, the conversion loss is one and the coupon payment in period t should equal the face value of the CoCo. The coupons should be constructed in such a way that they cover the difference between the expected value after conversion or default and the face value of the CoCo. If this can be achieved, the value of the CoCo will equal the face value of the CoCo independent of the asset value.

We define the coupon payment such that, given the current value of underlying assets, I have that for $s > t$ that $\tilde{c}(s, A_s|t) = c(s, A_s)R(t, s)$. Then I have that the following should be satisfied for a given $c(t, A_t)$,

$$\begin{aligned} \Gamma_t &= \left(1 - E_t[R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}}] - \frac{E_t[(A_\gamma - \bar{B})R(t, \gamma) \mathbf{1}_{\gamma < \min\{\tau, \delta\}}]}{\bar{C}} \right) \\ &= E_t \left[\int_t^{\min\{\tau, \delta, \gamma\}} \tilde{c}(s, A_s|t) ds \right]. \end{aligned} \quad (2.8)$$

Appendix 2.A.1 shows the derivation of the desired coupon payment. I find:

$$\tilde{c}(s, A_s|t) = \Gamma_s - E_s[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - s \geq 0]} \Gamma_{s+1}]. \quad (2.9)$$

By setting the coupon rate as in Equation (2.9), I have that $C_t = \bar{C}$ for all $t < \min\{\tau, \delta, \gamma\}$. The intuition behind the coupon payment is as follows: the CoCo holder knows at time t that in the future coupon payments are such that the expected loss in the next period, $E_t[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - s \geq 0]} \Gamma_{s+1}]$, is compensated. However, the conversion loss is higher as there is a chance that the next period will not be reached and a loss is incurred during this period. The investor needs to be compensated for this additional expected loss in the form of the coupon payment.

It is clear that the coupon payments yield much information about the value of the CoCo without the coupons, or a traditional CoCo with fixed coupons, as well. The coupons are constructed in such a way that the value of the CoCo without a coupon plus the coupon is equal to the face value. Hence, if the coupon payment is high, the value of the CoCo without the coupon, or with a constant coupon, will be lower than for the case where our floating coupon payment is low.

An objection to the proposed design could be that in times of financial distress, when the asset value drops, the increasing coupon payments could increase the probability of default. However, by setting the trigger for conversion well before a bank is in realistic danger of default, conversion will take place before the bank is in any sort of danger and the coupon payments will therefore not form a serious problem regarding the risk of default. When the trigger for conversion is set sufficiently high, default only occurs before conversion when a major shock hits the bank and a jump in the asset value causes the default. In this case the coupon payments will clearly be stopped as the CoCo holders will either become equity holders, and dividend payments can be stopped at any time, or the bank defaults.

It is important to determine a suitable trigger level that is sufficiently high. Regulators provide certain conditions, partially depending on the market price of equity, when they consider the financial situation of a bank to be too dire to pay dividends. When paying dividends poses a risk for the solvency of a bank, coupon payments for CoCos could as well. The trigger could therefore be placed such that conversion takes place before dividends and the coupon payments of CoCos form a danger for the solvency of the bank. As CoCos are an instrument mainly used to satisfy the demand of regulators, a joint effort to determine certain characteristics of the CoCo makes sense.

2.5 Numerical experiment

In this section I compute the numerical value of the floating coupons of the CoCos using a simulation study. I determine the numerical value of the coupon payments in (2.9). To do this in practice, the first challenge is to find a measure for A_t , as the underlying asset value is unknown and unobservable. A common solution in the literature is to define the underlying asset value to be equal to the sum of the market value of equity and all debt, see e.g. Feng & Volkmer (2012). The dependency of the coupon payments on the unobservable asset value is the main drawback of the proposed design. As the asset value cannot reliably be determined, the coupon payments will be inaccurate as well. Despite this, as investors cannot determine the asset value either, the above proposed measure is the best guess I have and although results derived with this measure may be inaccurate, investors do not have a better measure and therefore have no reason to adjust the price of the CoCo.

In the remainder of this section I first describe the simulation design for the most general process considered in this chapter. Next, I compute the coupon payments using a jump diffusion model to model the dynamics of the asset value of a bank. Thirdly, I do a robustness check using the more general distribution for the value of the assets.

2.5.1 Simulation design

In order to simulate the dynamics of the asset value, I will assume that the asset value follows a general stochastic process. In this section I present the most general model, while in the following sections I consider variations of this model. I consider a similar continuous-time dynamic model as in SW, who used the model introduced in Merton (1974) and Black & Cox (1976), and include stochastic volatility as in Heston (1993). This results in the Scott and Bates model developed in Bates (1996) and Scott (2002). I include jumps in the volatility as well and arrive at the following model, first developed by Duffie, Pan & Singleton (2000):

$$dA_t = \mu A_t dt + \sqrt{V_t} A_t dz_t + A_t(y_t - 1)dq_t \quad (2.10)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t} dW_t + x_t dp_t, \quad (2.11)$$

where z_t and W_t are standard Brownian motions, y_t follows an independent log-normal distribution with parameters μ_y and σ_y^2 , x_t follows an independent log-normal distribution with μ_x and σ_x^2 , q_t and p_t are poisson processes with arrival

rates λ_t^q and λ_t^p , and κ and θ are parameters. Note that V_t is the variance of the asset value at time t , while the volatility is given by $\sqrt{V_t}$. μ is again the risk-free rate plus the compensation for the jumps in the asset value and the volatility. The stochastic process is general and is supported by empirical evidence in e.g. Bakshi et al. (1997) and Chernov et al. (2003).

The simulations are performed in C++. I will not estimate the parameter values used in the model from empirical data. Instead I will use the calibration from SW, who based it on empirical papers and empirical data, to compute the floating coupons. As SW only consider a jump diffusion model, I use their calibration for the jump diffusion part of the model and consider a range of parameter values for the stochastic volatility. In the outflow of the asset value I only account explicitly for the coupon payments to the senior bondholders. In practice there will also be dividend payments and the payments to CoCo holders. However, an endogeneity problem occurs when the outflow depends on the coupon payments. To resolve this, the computation becomes more time consuming. To avoid the additional computation time, I assume the outflow of assets is constant, which implies that additional coupon payments to CoCo holders are financed by equity holders. This further enhances the equity feature that equity value is high in good times, while it drops first in bad times.

The obtained CoCo coupon payments give a notion of the characteristics of the CoCos proposed in this chapter. The difference between the coupons for different asset values gives the sensitivity of the price of a CoCo with constant coupons to changes in the asset value. The level of the conversion loss for different levels of the asset value will be given as well, as this is a measure for the loss that CoCo holders are expected to suffer upon conversion or default compared to the face value of the CoCo.

2.5.2 Merton process with jumps

The Merton process with jumps, introduced by Merton (1976), is a common way to model stock prices or the asset value of a bank and forms the basis for more extensive models. The model is given by

$$dA_t = \mu A_t dt + \sigma A_t dz_t + A_t(y_t - 1)dq_t, \quad (2.12)$$

where I recall that z_t is a standard Brownian motion, y_t follows an independent log-normal distribution with parameters μ_y and σ_y^2 , and q_t is a poisson process

with arrival rate λ .

The asset value will be simulated on a monthly basis, so t grows in steps of size $\frac{1}{12}$, whereas the coupon payments will be given on a yearly basis. Following SW, I set the default level equal to the face value of debt, which is given by 87. Hence, when the asset value drops below the debt value, the bank defaults. The drift of the assets is equal to the risk free rate, which is 0.03, minus the coupon payments to bondholders, set to 3.4 percent of the face value of the debt.⁴ The face value of the CoCos is set to 5 and the trigger level, K , is equal to 1. Note that the difference between the asset value and the default level is simply the value of equity plus the value of the CoCos. Hence, a trigger based on asset values can easily be translated to a trigger based on the value of equity. I assume zero default costs. For the remaining parameters in the model I set the baseline values $\sigma = 0.04$, $\sigma_y = 0.03$, $\mu_y = -0.01$, $\lambda = 4$, following SW.

The simulation is done with an algorithm similar to Zhou (2001). First, the conversion loss will be simulated with 1,000,000 iterations for the asset values between the conversion level plus 0.01 and 105 with steps of 0.01. Hence, if the conversion level is 93, the conversion loss will first be computed for an asset value of 93.01, next for a value of 93.02, then for a value of 93.03 and so on. In order to estimate the second term on the right hand side of Equation (2.9), I simulate a one period change in the asset value 100,000,000 times and use the estimates for the conversion loss to compute the expectation. A downside of this approach is that I treat the conversion loss as discrete. This will introduce some bias, although the influence is likely to be limited as the conversion loss is a smooth function of the asset value, as can be seen from, for example, Figure 2.2. Even though the expectations are computed in a rather straightforward manner, the computation is time consuming. In order to reduce computation time, I truncate the asset value in the next period to the 0.1 percent smallest and the 0.1 percent biggest draws. As only a small probability mass is truncated this does not seriously affect the results. Furthermore, Figure 2.2 shows that the value of the conversion loss is constant for high asset values, reducing the impact of the simplification. For practical implications banks could decide to truncate the distribution in a different way.

The results for the baseline model are given in Figures 2.1 and 2.2. The coupon payments decrease in the asset value. The conversion loss first increases in the asset value, then it decreases and it finally stabilizes. When the asset value is close to the trigger the CoCo conversion is more likely to be triggered by the regular diffu-

⁴ This implies that I have $\mu = 0.03 - 0.034 * 87$. Note that the remaining outflow of the assets at time t , η_t , are such that $\eta_t = 0.034 * 87 - \lambda E[y_t - 1]$.

sion instead of a jump than when the asset value is bigger. When the conversion is triggered by the regular diffusion, the CoCo holders receive a value in shares that equals the face value. So when the asset value is low, the conversion loss is relatively low. When the asset value moves away from the conversion threshold, the conversion loss increases at first, while at a certain point, a jump is not likely to be big enough to instantly trigger a loss at conversion and the conversion loss stabilizes. Hence, there is a non monotonic effect in the conversion loss. The coupon payment is monotonic in the asset value though. Close to default the expected number of future coupon payments is low. The current coupon payment has to be high to compensate for this even though the conversion loss is low close to the trigger.

Numerical example coupon payments and conversion loss benchmark case

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 4$, $\sigma_y = 0.03$, $\mu_y = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.1

Figure 2.2

In the Figures 2.3 to 2.18 other scenarios are considered. I vary the most interesting parameters, namely σ^2 , σ_y^2 , the trigger level, and the frequency of jumps in the asset value. All parameters are varied between three different values to get an impression of the impact of changes in these variables. The results are summarized in Table 2.1.

Figures 2.3 to 2.6 give the coupon payments and the conversion loss when the trigger level is adjusted to 92.5 and 94 respectively. Figure 2.3 displays higher

coupon payments with a steeper decrease near the trigger compared to the previous scenario, while Figure 2.5 displays lower coupon payments with a flatter slope. For similar asset values, the coupon payments are similar though. A lower trigger only affects the CoCo when the asset value is in the asset value range that leads to conversion for the higher trigger value but not for the lower trigger value. Figures 2.4 and 2.6 give the values of the conversion loss for different asset values. The shape of the graph is similar to the graph for the case with the higher trigger level. Only the level is different. This makes sense as with a lower trigger to conversion, default instead of conversion will occur more often, implying that the value for CoCo holders will be wiped out more often when trigger levels are lower, resulting in a higher conversion loss.

Figures 2.7 to 2.10 give the coupon payments and levels of the conversion loss both for the case with a higher and for the case with a lower frequency of jumps. The graphs yield few surprises, as the shape of the figures is similar to the benchmark case. The main differences are in levels, as a higher jump frequency result in a higher expected loss upon conversion or default. This leads to increased coupon payments for higher jump frequencies, especially near the trigger.

Table 2.1. Summary of the impact of parameter changes on the coupon payments and conversion loss

	Coupon payments	Conversion Loss
Trigger value	–	–
Jump frequency	+	+
Jump volatility	+	+
Asset volatility	<i>Constant</i>	<i>Constant</i>

The columns of the table represent the sign of the effect of an increase of the parameter in the row on the coupon payments and the conversion loss. A plus implies that when a parameter in the first column and a given row increases, so does the variable in the column. A minus implies the opposite and constant implies that there is no change in the variable for a change in the parameter.

In Figures 2.11 to 2.14 the coupon payments and levels of the conversion loss for different levels of the asset value are given, where the level of σ_y is varied. Figures 2.11 to 2.14 show that variations in σ_y have a similar effect on the coupon payments as variations in the jump frequency, except that the conversion loss is flatter in case σ_y decreases. When σ_y increases the level of the coupon payments, mainly near the

Numerical example coupon payments and conversion loss with low trigger

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 4$, $\sigma_y = 0.03$, $\mu_y = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 92.5.

Figure 2.3

Figure 2.4

trigger, and the conversion loss are higher due to the higher probability of instant default.

In Figures 2.15 to 2.18 the variance of the asset value, σ^2 , is varied. Figures 2.15 and 2.17 show that an increase in volatility has no effect on the coupon payments. Increases in σ^2 make it more likely that when the asset value is near the trigger a conversion without default occurs. This should decrease the coupon payments. Figures 2.16 and 2.18 show that the conversion loss decreases a little when σ^2 increases. However, a higher volatility makes it more likely to have steep drops in the asset value, which should increase the coupon payments as fewer expected payments will be made. From the results it is clear that these effects balance each other. Hence, the coupon payments are robust to changes in the volatility.

Given that the value of the CoCos is sensitive to the underlying asset value, CoCos, as they are structured at the moment, are difficult to price, especially for asset values near the trigger. This implies that for CoCos without floating coupon payments, the price development will be unpredictable. Steep decreases in the price of CoCos of a specific bank as a result of financial setbacks and price deviations between similar CoCos issued by different banks are to be expected. This

Numerical example coupon payments and conversion loss with high trigger

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 4$, $\sigma_y = 0.03$, $\mu_y = -0.01$, $\mu_{\eta} = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 94.

Figure 2.5

Figure 2.6

is undesirable as investors will struggle more with valuing the risks that these instruments bring and this could cause markets to become more volatile. Floating coupon payments that compensate investors for these additional risks can be an attractive alternative to investors.

The coupons are robust to different parameter choices. The level of the coupons changes depending on the risk borne by the CoCo holders, but the shape of the coupon function on the asset value changes little. When a CoCo with a structure similar to what I propose would be implemented in practice, the coupon payments would mostly be stable. The proposed CoCo design will, therefore, in general not differ much from regular CoCos with constant payments. For banks with high asset values, the coupon payments will be lower than the coupons of CoCos that are currently issued, as there is little risk for which the CoCo holders have to be compensated.⁵ There will only be sizeable adjustments in the coupon payments when the asset value drops to values near the trigger, which, in the absence of jumps, will happen gradually. Big adjustments are only necessary when jumps

⁵ See Avdjiev et al. (2015) for an empirical study on CoCos.

Numerical example coupon payments and conversion loss with frequent jumps

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 2$, $\sigma_y = 0.03$, $\mu_y = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.7

Figure 2.8

Numerical example coupon payments and conversion loss with infrequent jumps

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 8$, $\sigma_y = 0.03$, $\mu_y = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.9

Figure 2.10

Numerical example coupon payments and conversion loss with low jump volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 4$, $\sigma_y = 0.01$, $\mu_y = -0.01$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.11

Figure 2.12

Numerical example coupon payments and conversion loss with high jump volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.04$, $\lambda = 4$, $\sigma_y = 0.05$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.13

Figure 2.14

Numerical example coupon payments and conversion loss with low asset volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.02$, $\lambda = 4$, $\sigma_y = 0.03$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.15

Figure 2.16

Numerical example coupon payments and conversion loss with high asset volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equation (2.12). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma = 0.06$, $\lambda = 4$, $\sigma_y = 0.03$, the remaining outflow of the assets is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.17

Figure 2.18

occur, which have serious implications for the value of CoCos, as they are currently designed, as well. Hence, sizable jumps in the asset value are not more of a threat for CoCos with the proposed design than for regular CoCos. In our design the CoCo holders will, furthermore, be more comfortable when markets are volatile due to the stable value of the CoCos.

2.5.3 Merton process with jumps and stochastic volatility with jumps

In this section I compute the coupon payments for a more general stochastic process to test the robustness of the results. In reality the volatility is not constant, as was assumed in the previous section. It is instead stochastic and displays jumps as noted by for example Chernov et al. (2003). Hence, I have the model as given in (2.10) and (2.11). I simulate the asset value using the Euler discretization scheme described in Broadie & Kaya (2006) and assume that jumps in the volatility and the asset value occur simultaneously as well. We, furthermore, set $\kappa = 1$, $\theta = \sqrt{V_0}$, and $\sigma_v = \sqrt{V_0}$, where $\sqrt{V_0}$ is the volatility at $t = 0$. The mean of the jumps in the volatility and the volatility of the volatility will be varied to investigate whether different jump sizes have unexpected consequences. The main purpose of this section is to investigate whether allowing for stochastic volatility and jumps in the volatility will have major consequences for the found coupon payments. The parameters chosen are therefore somewhat arbitrary and the computed coupons should not necessarily be considered to be realistic values for the coupon payments that the CoCo holders will obtain if the proposed CoCos would be issued.

Due to the model with stochastic volatility with jumps in both the asset value and the volatility, from now on SVJD model, the tails of the distribution are fatter, which is considered in the literature to be one of the main goals of introducing the SVJD model (see Gallant, Hsieh & Tauchen 1997 and Eraker, Johannes & Polson 2003). In order to compute the coupon payments and the conversion loss for different asset values, I will use the same approach as in Section 2.5.2 where I replace the previous model with the SVJD model. I will perform experiments with a starting variance of $V_0 = \sigma^2$.

With the regular jump diffusion model changes in the volatility did not affect the coupon payments. Figures 2.19 to 2.26 give the results for different parameter values for the SVJD model. In Figures 2.19 and 2.20 I have $\mu_x = 0.01$ and $\sigma_x = 0.3$. The figures are little different from Figures 2.1 and 2.2. In Figures 2.21 and 2.22 the volatility of the volatility is increased to $\sigma_x = 0.6$, while in the Figures 2.23 and 2.24 the volatility of the volatility is decreased to $\sigma_x = \frac{0.8}{3}$. It is clear from the figures

that there is little effect on the coupon payments and conversion loss. In Figures 2.25 and 2.26 the mean size of the jumps in the volatility is increased to $\mu_x = 0.1$. Again there is no effect on the coupon payments and the conversion loss.

There are no clear difference between the Merton jump diffusion model and the SVJD model. The main driver for the coupon payments and the conversion loss is the probability of default. The model for the volatility appears not to be an important factor for the level or shape of the coupon payments. Instead the process for the jumps in the asset value is most important to determine.

2.6 Policy implications and conclusions

The main contribution of this chapter is the description of a theoretical CoCo that has many desirable properties. The results, furthermore, show the sensitivity of the CoCo's floating coupons to different parameter settings for the underlying asset value. Next to the theoretical interest, the proposed CoCo could serve as a model for a new generation of CoCos that, in a simplified form, work similarly. Even though the proposed CoCo contract solves the multiple equilibria problem of SW, this is of little use when the market is not interested in buying instruments with a similar structure, or when banks are not interested in issuing them. Whereas, currently, CoCo holders receive fixed coupon payments, and the market price of the CoCos fluctuates, for our CoCo design, the value of the CoCos is constant at the moments that coupons are paid due to the coupons being floating. When the payments are frequent, there will be little fluctuation in the value of CoCos in the interval between potential conversion dates. Investors will anticipate that the future coupon payments will adjust to the new situation and the value of CoCos is, therefore, even in between coupon payments, close to the face value. The CoCos are less risky than equity, although more risky than senior debt. Investors that consider regular equity to be too risky, while they are seeking higher returns than senior debt has to offer, can be interested in the proposed CoCos. The strong increase in the magnitude of coupon payments near the trigger illustrates how regular CoCos will lessen in value when approaching conversion.⁶ This could make the instrument, as it is structured now, undesirable for investors. This problem is resolved in our design and could therefore potentially attract more investors.

From the perspective of the bank, CoCos are a new tool to attract additional

⁶ For convertible bonds this does not apply as these bonds usually are converted voluntarily and this is done when the equity value increases instead of decreases.

Numerical example coupon payments and conversion loss for benchmark case stochastic volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equations (2.10) and (2.11). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $\kappa = 1$, $\theta = 0.04$, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma_v = 0.04$, $\lambda^q = \lambda^p = 2.6$, $\sigma_y = 0.03$, $\mu_x = 0.01$, $\sigma_x = 0.3$, the remaining asset outflow is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.19

Figure 2.20

Numerical example coupon payments and conversion loss with high stochastic volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equations (2.10) and (2.11). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $\kappa = 1$, $\theta = 0.06$, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma_v = 0.06$, $\lambda^q = \lambda^p = 2.6$, $\sigma_y = 0.03$, $\mu_x = 0.01$, $\sigma_x = 0.3$, the remaining asset outflow is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.21

Figure 2.22

Numerical example coupon payments and conversion loss with low stochastic volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equations (2.10) and (2.11). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $\kappa = 1$, $\theta = \frac{0.08}{3}$, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma_v = 0.08/3$, $\lambda^q = \lambda^p = 2.6$, $\sigma_y = 0.03$, $\mu_x = 0.01$, $\sigma_x = 0.3$, the remaining asset outflow is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.23

Figure 2.24

Numerical example coupon payments and conversion loss with large jumps in stochastic volatility

The figure displays the coupon payments (left figure) and the conversion loss (right figure) on the vertical axis for a given asset value on the horizontal axis. The coupons and conversion losses are computed using Equations (2.9) and (2.8) respectively and the process for the assets is given in Equations (2.10) and (2.11). The parameter values are given by $\bar{C} = 5$, $\bar{B} = 87$, the bond coupons are $b = 3.4$ percent of the face value of the debt per year, $\kappa = 1$, $\theta = 0.04$, $r_t = 0.03$ for all t , $\mu_t = 0.03 - 0.034 * 87 / A_t$ for all t , where A_t represents the asset value at time t , $\sigma_v = 0.04$, $\lambda^q = \lambda^p = 2.6$, $\sigma_y = 0.03$, $\mu_x = 0.1$, $\sigma_x = 0.3$, the remaining asset outflow is $\eta_t = 0.034 * 87 + \lambda E[y_t - 1]$ for all t and the conversion level is 93.

Figure 2.25

Figure 2.26

capital. Issuing the proposed CoCos instead of new shares does increase the risk for the current share holders, as instead of sharing financial distress with other share holders, they will have to carry the burden themselves. This gives banks more flexibility in increasing the risk profile of equity holders, by issuing the proposed CoCos, without making the bank as a whole more risky. Another advantage for banks to issue CoCos is that in many European countries, the coupon payments on CoCos are tax deductible, which makes the instruments interesting for banks as well. A crucial advantage compared to regular CoCos with a market trigger is that the multiple equilibria problem is resolved, while the recapitalization features that standard CoCos have, are retained. The problem of shifting risk incentives as in Berg & Kaserer (2015) is reduced as upon conversion there is no transfer of value between CoCo holders and equity holders. Furthermore, the trigger is based on the market value of equity and conversion does therefore not yield any new information that could cause unrest in the market. The trigger incorporates the prediction of the market and this has proven to be a more reliable predictor than predictions resulting from either regulators or book values. New CoCos could be inspired by the proposed design, where some of the conditions can be relaxed to make the instruments less complex. Such an intermediate form could on itself be a useful improvement over the current design.

Market manipulations are less of a risk in the proposed design as there is no transfer of value upon conversion and investors in CoCos from a given bank have no longer an incentive to force conversion. This solves a sizeable problem of traditional CoCos with a market trigger. Still banks might dislike conversion based on hitting the trigger once. By only considering conversion on set time intervals, a sudden drop in the market value of equity that is not based on the true financial condition of the bank and is followed by a steep increase, will likely not cause conversion though. Alternatively, Calomiris & Herring (2013) suggest to use a 90-day moving average to smooth fluctuations in the price. Future research should look into the exact conversion conditions though, in order to come up with clearer guidelines that suit the proposed instrument. Despite this, the proposed design has many desirable features and is, even if the multiple equilibrium problem is not a severe threat, an interesting instrument to study as it could potentially add to the stability of the financial sector.

In this chapter I focus on the design of a new hypothetical CoCo that resolves some of the key problems that CoCos with a market trigger face. I investigated the characteristics of a perpetual CoCo that converts to equity based on a mar-

ket trigger. The CoCo has floating coupon payments that are set in such a way that the market value of the CoCos is equal to the face value of the CoCo at any time when conversion can occur. This feature makes determining the conversion ratio of the CoCos to equity simple and resolves the multiple equilibria problem addressed in SW. Numerical experiments were conducted to investigate the behaviour of the coupon payments. The results show that the coupons are highly nonlinear when the asset value is close to the trigger and that the volatility of the underlying stochastic process does not appear to have a major influence on the required coupon payments. However, jumps in the asset value are an important factor for the coupon payments. This has several implications for the proposed design of CoCos compared to the way they are currently structured. The value of CoCos that have a market trigger and constant coupon payments can be expected to decrease steeply when the share price falls and approaches the trigger level. The size of this drop in prices is difficult to determine due to the dependence on the jumps in the asset value and this could cause discomfort among investors. As the asset value can only be measured roughly, proper pricing models for CoCos are difficult to design. The proposed CoCo suffers from the same problem and due to the sensitivity to the characteristics of the underlying process for the asset value, the instrument is for now mostly hypothetical. A design inspired by the proposed hypothetical CoCo could be implemented though, as investors are likely interested in an instrument that better protects them against the possible losses that are associated with CoCos. This design will make CoCos more of a hybrid between equity and debt, compared to existing CoCos, that is less risky than equity but more risky than debt. As market triggered CoCos are widely acknowledged to be more desirable than book value triggered CoCos, there is a high priority to resolve problems faced by market triggered CoCos. Hence, new designs that resolve the current issues of market triggered CoCos are desirable.

The complexity of the proposed CoCo, and CoCos in general, in combination with the sensitivity of the value of CoCos to the underlying process that governs the asset value, is problematic. Investors might not be able to understand the nature of CoCos and this could lead to unrest on financial markets. This uncertainty and lack of understanding cannot be resolved, due to the ambiguous dependence between the unobservable asset value and the value of CoCos. Given that the number of outstanding CoCos is increasing every year and the green light regulators have given banks to issue CoCos in Basel III, research on improving the design of CoCos is necessary. The proposed design gives investors in CoCos more certainty

and the CoCos a clearer place between debt and equity in terms of risk, which is a major improvement. The multiple equilibrium problem is resolved, which will result in more stable prices of CoCos. The increased risk-taking incentive is reduced as well, due to the lack of a transfer of value between equity holders and CoCo holders upon conversion. Future research should focus on developing more accurate methods to use CDS rates to derive the probability of default of banks and look into strategies to measure the asset value more accurately in order to get a better impression of the value of CoCos. The CoCo design proposed here could, moreover, be adjusted to make practical implementations more feasible. An optimal trigger condition should, additionally, be determined.

2.A Appendix

2.A.1 Coupon payments

In this appendix I show that Equation (2.9) gives the desired coupon value. In order to find the desired coupon, first note that coupon payments are only paid out at discrete intervals. Let these intervals be the same intervals as those that are used to evaluate whether the thresholds for τ , γ , or δ are breached. Then, I define that for a given interval the piecewise constant function $g(s, A_s)$ is such that $\int_i^{i+1} g(s, A_s) ds = \int_i^{i+1} \tilde{c}(s, A_s | t) ds$ for all $i = t \dots, \min\{\tau, \delta, \gamma\}$, where $g(\cdot)$ is continuous in s . Next, I recall the following:

$$\Gamma_t = \left(1 - E_t[R(t, \tau) \mathbf{1}_{\tau < \min\{\delta, \gamma\}}] - \frac{E_t[(A_\gamma - \bar{B})R(t, \gamma) \mathbf{1}_{\gamma < \min\{\tau, \delta\}}]}{\bar{C}} \right). \quad (2.A.1)$$

Equation (2.8) can then be rewritten as

$$\begin{aligned} \Gamma_t &= E_t \left[\int_t^{\min\{\tau, \delta, \gamma\}} \tilde{c}(s, A_s | t) ds \right] \\ &= E_t \left[\sum_{i=0}^{\min\{\tau, \delta, \gamma\} - t - 1} \int_{t+i}^{t+i+1} \tilde{c}(s, A_s | t) ds \right] \\ &= E_t \left[\sum_{i=0}^{\min\{\tau, \delta, \gamma\} - t - 1} \int_{t+i}^{t+i+1} g(s, A_s) ds \right] \\ &= E_t \left[\sum_{i=t}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) (i + 1 - i) \right] \\ &= E_t \left[\sum_{i=t}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right], \end{aligned} \quad (2.A.2)$$

where (2.A.2) follows from the mean value theorem, and I write subscript t in the expectation to emphasize that I condition on the current information. I have to show that $\Gamma_t = E_t \left[\sum_{i=t}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right]$ for all $t \in \Lambda$.⁷ For an arbitrary $t \in \Lambda$ I have

$$\Gamma_t = E_t \left[g(\lambda_t, A_t) + \sum_{i=t+1}^{\min\{\tau, \delta, \gamma\} - 1} \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 1 - i \geq 0]} g(\lambda_i, A_i) \right],$$

⁷ In the remainder of this appendix, I write the separate elements of the sum. It can happen that the limit of the sum does not necessarily exceed the starting point of the sum. In these cases I define the value of the sum to be equal to zero. To emphasize this, I include an indicator function for whether this is the case for the elements from time $t + 1$ onward.

which implies that

$$\begin{aligned}
E_t[g(\lambda_t, A_t)] &= \Gamma_t - E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \sum_{i=t+1}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \\
&= \Gamma_t - E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} (g(\lambda_{t+1}, A_{t+1}) \right. \\
&\quad \left. + \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i)) \right] \\
&= \Gamma_t - E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} (\Gamma_{t+1} \right. \\
&\quad \left. - E_{t+1} [\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i)] \right. \\
&\quad \left. + \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i)) \right] \\
&= E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \left(E_{t+1} \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \right. \right. \\
&\quad \left. \left. - \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right) \right] \\
&\quad + \Gamma_t - E_t [\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \Gamma_{t+1}]. \tag{2.A.3}
\end{aligned}$$

We can get rid of the first term in (2.A.3) by observing that

$$\begin{aligned}
&E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \left(E_{t+1} \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \right. \right. \\
&\quad \left. \left. - \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right) \right] \\
&= E_t \left[E_{t+1} \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \right. \\
&\quad \left. - \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \\
&= E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] \\
&\quad - E_t \left[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 3 - t \geq 0]} \sum_{i=t+2}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right] = 0, \tag{2.A.4}
\end{aligned}$$

where I used the tower property to arrive at Equation (2.A.4). Hence, I find that if I set

$$E_t[g(\lambda_t, A_t)] = g(\lambda_t, A_t) = \Gamma_t - E_t[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \Gamma_{t+1}],$$

we have that

$$\Gamma_t = E_t \left[\sum_{i=t}^{\min\{\tau, \delta, \gamma\} - 1} g(\lambda_i, A_i) \right].$$

Finally, as

$$\tilde{c}(t, A_t | t) = g(\lambda_t, A_t)$$

for all t , I conclude that the coupon payments for period s should equal

$$\tilde{c}(s, A_s | t) = \Gamma_s - E_t[\mathbf{1}_{[\min\{\tau, \delta, \gamma\} - 2 - t \geq 0]} \Gamma_{s+1}]$$

for Equation (2.8) to hold.

From theorem 3 in SW I conclude that for this coupon payment and $m = \frac{C_\tau}{S_\tau}$ I have a unique equilibrium.

Chapter 3

Discrete time asset pricing under endogenous changes in the kind of cash-flows

3.1 Introduction

Several assets are subject to an automatic loss (or gain) due to their own market price, in the form of an unanticipated wealth transfer among investors. In presence of heterogeneity in market power across investors, that we define as the individual ability to influence the trading prices, this unanticipated wealth transfer may undermine the fair asset valuation and the market stability. In this paper, we show how to represent the trading prices of assets of this type as a part of a unique dynamic Nash equilibrium in a symmetric information clearing auction market with discontinuous trading. The contribution of this paper, which is purely theoretical, is the proof of the uniqueness of a possible mechanism to reach this equilibrium, under mild conditions on the utilities representative agents for distinct classes of investors.

An unanticipated wealth transfer among investors creates an endogeneity issue that prevents anticipating the kind of cash-flows. This is the case, for example,

This chapter is based on Ronchetti & Vullings (2019).

of equities that may be included in market capitalization-based indexes and are traded in centralized markets. The current valuation of equities of this type must account for the index inclusion premium (see, for example, Chang et al. (2014) for an empirical study). It is also the case of the assets of a firm whose managerial decisions are driven by the market price of the same assets. The current valuation of these assets must account for the managerial decisions based on future prices (see, for example, Bond et al. (2009) for a theoretical analysis). Moreover, distinct forms of contingent capital considered in the financial literature as micro-prudential recapitalization tools for financial firms in distress have similar characteristics.¹ The future kind of cash-flows provided by these contracts may be, for example, either equity- or debt-like depending endogenously on the current contract valuation.²

Asset pricing theory builds upon the idea that a security is worth its expected discounted payoff. Asset pricing models describes how the discounting realizes, by specifying the admissible Stochastic Discount Factors (SDF's) for the market. To price assets characterized by an automatic loss (or gain) due to their own market price, it is necessary to explicitly account for the market power and contributions to the asset demand of the investors who participate to the trades. The investors may indeed differ in terms of their preferences about the kind of cash-flows, and act as to impose them. We combine in the concept of market power the individual ease with which the securities can be traded, the individual trading patience, and any individual exposure to an exogenous shock.

Equilibrium asset pricing models build upon some notions of market equilibrium.³ In particular, models that put emphasis on the welfare aspect of a market equilibrium and explain trading prices as optimizers of the consumption-investment strategy for a market-wise representative agent have a long tradition, which dates back to Negishi (1960) at least. Focusing on just a single representative agent, these models necessarily overlook the heterogeneity of the investors. Intuitively, the market power is cross-sectionally averaged over the investors. As a consequence, these

¹ In the last fifteen years the use of debt converting into equity on the basis of market prices has been widely advocated to recapitalize financially distressed firms (Flannery (2005), Glasserman & Nouri (2016)). In addition, other conversion mechanisms have been discussed, including conversions triggered by accounting-ratios (Squam Lake Working Group (2009) and Glasserman & Nouri (2012)) and optional conversions for the bank's management (Glasserman & Wang (2011)). See, for example, Bolton & Samama (2012) for a discussion on the benefits of contingent capital, and Chen, Glasserman, Nouri & Pelger (2017) for an explanation of its effect on the optimal capital structure, debt overhang, and asset substitution.

² As explained in the introduction of Glasserman & Nouri (2016), these contracts are incomplete because *"there states in which the apportionment of firm value between contingent capital investors and equity holders may not be fully specified"*.

³ See, e.g., Cochrane & Culp (2003) for an introduction to equilibrium asset pricing models.

models are just for assets whose kind of cash-flows is well-defined at the time of their pricing. This is the case, for example, of dividend-paying equities and non-defaultable bonds with no unexpected risk premium. The entity of dividends and coupons may be stochastic, but their identification (what we refer to as "kind") is known at the time of their pricing. Differently, in this paper we propose an alternative equilibrium asset pricing model that explicitly accounts for this heterogeneity and conflicting interests about the kind of cash-flows of an asset. A first relevant implication of our model is the relative pricing of derivatives of assets featuring this change in the kind of their cash-flows. A second relevant implication of our model is the explanation of why uncertainty regarding the likelihood of a change in the kind of cash-flows increases the valuation heterogeneity among investors.

The essential characteristics of our model are as follows. We partition the investors into separate classes, and represent the trading activities in each class as if they resulted from the optimal consumption-investment strategy for an individual class-representative agent. We then describe trading prices as if they resulted from the unique joint optimal consumption-investment strategies for distinct class-representative agents. Because of this feature, our model can then be extended to those assets whose kind of cash-flows are uncertain at the time of their valuation, and that features a wealth transfer across investors at a change of the kind of cash-flows. Distinct class-representative agents may indeed either profit or lose from this wealth transfer, and influence the market in order to make the cash-flows of an asset become of the preferred kind. Consequently, our framework highlights the intrinsic link between endogenous changes in the kind of an asset cash-flows and the heterogeneity in market power across the investors. Intuitively, we show how the market reaches a unique Nash equilibrium, with the class-representative agents accounting for each others' optimal consumption-investment strategy, even when the class-representative agents, before observing quotes and trades, account for different asset demand determinants in their private valuations. We show that, if this is the case, the intertemporal marginal rates of substitution of the distinct class-representative agents converge through a sequential update of the asset demand determinants that they assume.

Previous literature has highlighted that the inability to anticipate the kind of cash-flows of an asset and unanticipated wealth transfers among investors prevent the existence of a unique competitive equilibrium price, in a market represented by means of a unique market-wise representative agent. Sundaresan & Wang (2015) discuss this problem for pricing contingent capital with market-based conversion

triggers. In the same context, Glasserman & Nouri (2016) show a solution under continuous trading, and when the equity holders are penalized by the conversion of the contingent capital.⁴ When applied to this context, our framework is complementary to the one in Sundaresan & Wang (2015), as it focuses on a Nash equilibrium concept, which is more general than the competitive one. It also complements Glasserman & Nouri (2016), as it embraces discontinuous trading and any form of stakeholder penalization at the conversion of the contingent capital. Interestingly, Davis et al. (2014) find by experiments that simplified discrete time markets for contingent capital with market-price triggers systematically realize in a precise equilibrium. Our theoretical predictions match these findings. In another stream of literature, Chang et al. (2014) note that investors do not anticipate the wealth transfer at the swap of the components of market capitalization-based equity indexes (index inclusion effect). When applied to this context, our framework shows how the equity market reaches a unique dynamic Nash equilibrium in both cases of either a gain or a loss due to the index inclusion.

In the literature on credit ratings, Manso (2013) observes the absence of a unique equilibrium due to credit ratings. Rating agencies can choose between different ratings that are all consistent with an equilibrium outcome. Manso (2013) shows that the welfare maximizing equilibrium, where the probability of default is lowest, will realize when firms pay a fee to the rating agency, as this gives the agency an interest in the survival of the firm it is rating. Our framework is similar in that the uniqueness of the dynamic Nash equilibrium is the result of investors preferring a specific equilibrium. However, our setting differs in that there is asymmetric information between the investors and the realized equilibrium is the result of trading.

As already stressed, an essential feature of our model is the explicit inclusion of heterogeneity in the market power of the investors. The limitations of the perfectly competitive market paradigm for several financial applications have been stressed by several authors. Basak (1997) studies the influence of a non-price-taking investor, who consumes and invests accounting for her impact on the price, on a market equilibrium characterized by no arbitrage, market clearing, and rational expectations, under the assumptions of symmetric information and complete and frictionless market.⁵ The concept of market power includes the trading ability at a given price, and then it is strongly linked to the concept of liquidity. The financial

⁴ Tian (2016) shows conditions for a dynamic equilibrium when the CoCo issuer has a call option written on the CoCo. Pennacchi & Tchisty (2018) provide conditions on the variance of the firm's total assets for a unique equilibrium under continuous trading and when the CoCo holders are penalized.

⁵ See also Grinblatt & Ross (1985).

literature has largely recognized the importance of this property for the asset valuations (see, for example, Amihud, Mendelson & Pedersen (2012) for a survey of the relevant literature). Lack of liquidity has been one of the most relevant drivers of the global financial crisis of 2007-2009 (see, for example, Brunnermeier (2009)). In particular, when trading is discontinuous, prices must be studied taking market resiliency and limits to infinite tightness into account (Kyle (1985)). Næs, Skjeltorp & Ødegaard (2011) show that variables related to stock market liquidity can predict real economy variables, such as changes in GDP and investments. He & Milbradt (2014) study endogenous liquidity for a corporate bond. Collin-Dufresne & Fos (2016) study the relationship between the stochastic trading volume and price volatility. The aggregate effect of the market power of distinct investors is also tightly related to the asset demand. Equity demand could result from changes in financial holding regulations or shocks to the individual wealth of the equity investors that are independent of the net present valuation of the firm's total assets (see, for example, Ding, Shawky & Tian (2008) and Brunnermeier & Pedersen (2008) for studies on liquidity shocks to equity investors). Gârleanu, Pedersen & Poteshman (2009) show that accounting for the demand of options can explain observed price patterns. Nonetheless, to the best of our knowledge, existing equilibrium models do not explain prices in markets including assets with a bidirectional causality between their net present valuations and market prices. This sets our framework apart from other theoretical asset pricing studies that explicitly account for liquidity and asset demand.

Example 1: contingent penalization We now exemplify the asset pricing problem we deal with in this paper. We consider an exchange economy for the equity of a firm subject to a government intervention in case of low market value of the firm equity. We model this economy as in Constantinides & Duffie (1996b), which is one of the most influential theoretical papers for the equity premia studies in presence of uninsured idiosyncratic risks. However, we do not make their fundamental assumption for a market equilibrium of independent standard normal random shocks to individual consumption growth.

Investors have homogeneous preferences w.r.t. consumption represented by a time- and state-separable von Neumann-Morgenstern preferences with constant relative risk aversion coefficient $\alpha > 0$ and constant subjective discount rate ρ . At time t , the individual consumption for the k -th investor is $C_{k,t}$, the aggregate consumption is C_t , and the symmetric information for the investors is represented by

the set $\mathcal{F}_t$. A single nondurable consumption good is taken as numéraire. The shock to individual consumption growth for the k -th investor at time t is $\eta_{k,t}$. The shocks are cross-sectionally independent and $\eta_{k,t+1} \sim \mathcal{N}(0, 1/k)$.⁶

Among other assets, the investors trade the equity of a private firm. This equity is the unique source of financing for the firm. We denote its ex-dividend price and the dividend it pays at the trading date t by P_t and D_t , respectively. Because of an outstanding law that cannot be changed, there is a penalization of the private ownership in case of low market capitalization. A value of 5 is indeed nationalized at time $t + 1$ if its market cum-dividend value $P_{t+1} + D_{t+1}$ is 10 or less. By similar steps as those in Constantinides & Duffie (1996b), we find that the k -th investor, who accounts for her own perceived risk factors, reaches at time t this private valuation of the firm equity:

$$\hat{P}_t(k) = \mathbb{E} \left[e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \exp \left[\frac{\alpha(1+\alpha k)}{2} Y_{t+1}^2 \right] (P_{t+1} + D_{t+1}) \middle| \mathcal{F}_t \right], \quad (3.1)$$

for the conditional cross-sectional variance

$$Y_{t+1}^2 := \mathbb{V} \left[\ln \left[\frac{C_{k,t+1}}{C_{t+1}} / \frac{C_{k,t}}{C_t} \right] \middle| \mathcal{F}_{t+1} \right],$$

where we denote the conditional expectation operator and the conditional variance operator given the information set $\mathcal{F}_t$ under the historical probability measure, which is absolutely continuous with respect to the standard Lebesgue integral, by " $\mathbb{E}[\cdot | \mathcal{F}_t]$ " and " $\mathbb{V}[\cdot | \mathcal{F}_t]$ ", respectively.⁷ Without any further assumption, in the private valuation the cum-dividend price $P_{t+1} + D_{t+1}$ is discounted by the factor $e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha}$, which does not depend on the investor's identity, and the factor $\exp \left[-\alpha \left(\eta_{k,t+1} Y_{t+1} - \frac{Y_{t+1}^2}{2} \right) \right]$, which depends on the investor's identity, and gives rise to the term $\exp \left[\frac{\alpha(1+\alpha k)}{2} Y_{t+1}^2 \right]$.

Let us now take a numerical example to highlight a substantially different private valuation set by the k_1 -th and k_2 -th investors. Both investors correctly know that the total market capitalization of the company at future time $t + 1$ is either 9 or

⁶ We could consider any other type of cross-sectional dispersion of the shocks to individual consumption growth that differs from the one with $\eta_{k,t+1} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, 1)$ discussed in Constantinides & Duffie (1996b).

⁷ Equation (3.1) correspond to the equations at pages 227-228 in Constantinides & Duffie (1996b).

11.5 with equal probability. In her own private valuation of the firm equity done as in Equation (3.1), the k_1 -th investor considers the risk-neutral probability of a future penalization at 4%. Differently, this event is 1 percentage point more probable under the risk-neutral probability measure adopted by the k_2 -th investor. We have

$$e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \exp \left[\frac{\alpha(1+\alpha)}{2} Y_{t+1}^2 \right] \mathbb{P} \left[\mathbf{1}_{\{\text{Pen}\}} = 1 \mid \mathcal{F}_t \right] = 4\%,$$

$$e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \exp \left[\frac{\alpha(1+2\alpha)}{2} Y_{t+1}^2 \right] \mathbb{P} \left[\mathbf{1}_{\{\text{Pen}\}} = 1 \mid \mathcal{F}_t \right] = 5\%,$$

where " $\mathbb{P}[\cdot \mid \mathcal{F}_t]$ " denotes the conditional probability operator given the information set $\mathcal{F}_t$ under the historical probability measure, and we denote the indicator function by " $\mathbf{1}_{\{\cdot\}}$ " and the penalization event by "Pen". Consequently, the k_1 -th investor values the position on sale slightly above 10.05, while the k_2 -th investor values it 5.25. This substantial difference in private valuations stems from the fact that because the k_2 -th investor assigns a slightly higher risk-neutral probability to future conversion, the current valuation drops to the value that automatically triggers the partial nationalization of the firm. Therefore this nationalization occurs immediately due to the beliefs of the investor.⁸

Therefore, on the one side, the probability for a particular event that involves the market cum-dividend value of the firm equity at time $t + 1$ induces the loss at time t . On the other side, this loss has a feedback effect on the probability. A similar problem may also arise if the company receives a financial aid from the government when its market value is 10 or less. However, in this case the aid can push the market value above 10, thus resulting in a contradiction. When there is no government aid the market value is below 10, which triggers aid, while when there is government aid the market value is above 10, cancelling the aid. Due to this feature Sundaresan & Wang (2015) and Glasserman & Nouri (2016) conclude that in these circumstances there is no competitive equilibrium.

⁸ We indeed have that

$$\begin{aligned} & \underbrace{0.5}_{\text{probability}} \times \underbrace{9}_{\text{payoff}} + \underbrace{0.5}_{\text{probability}} \times \underbrace{11.5}_{\text{payoff}} = 10.05 > 10, \\ & \underbrace{0.5}_{\text{probability}} \times \underbrace{9}_{\text{payoff}} + \underbrace{0.5}_{\text{probability}} \times \underbrace{11.5}_{\text{payoff}} - \underbrace{0.05}_{\text{probability}} \times \underbrace{5}_{\text{penalization}} = 10 \\ & \rightarrow \underbrace{0.5}_{\text{probability}} \times \underbrace{9}_{\text{payoff}} + \underbrace{0.5}_{\text{probability}} \times \underbrace{11.5}_{\text{payoff}} - \underbrace{1}_{\text{probability}} \times \underbrace{5}_{\text{penalization}} = 5.25. \end{aligned}$$

so that $\hat{P}_t(k_1) = 10.05$ and $\hat{P}_t(k_2) = 5.25$.

Example 2: index inclusion effect In this section we use a simple example to illustrate how our setting is relevant for a market that features an index inclusion effect. Without loss of generality we compare two firms near the cutoff. The first firm, **FIRM_A**, is currently a member of index 1, **INDEX₁**, while the second firm, **FIRM_B** is currently a member of index 2, **INDEX₂**. At any point in time where the composition of the indices can change, the changes are such that that the firm with the highest trading price is included in **INDEX₁** and the other firm in **INDEX₂**. We consider both the case where it is beneficial to be in the high value index and the case where it is beneficial to be in the low value index. We denote the index inclusion effect by $\gamma \in \mathbb{R}$. This value is added to the asset value of the firm in **INDEX₁**. So when $\gamma > 0$ it is beneficial to be in **INDEX₁**, while when $\gamma < 0$ it is beneficial to be in **INDEX₂**. We denote the asset value, ignoring the index inclusion effect, of **FIRM_A** (**FIRM_B**) by $V_A \in \mathbb{R}_{++}$ ($V_B \in \mathbb{R}_{++}$ respectively). Note that when $|V_A - V_B| > |\gamma|$ it is straightforward to determine the composition of the indices. Hence, we assume that $|V_A - V_B| \leq |\gamma|$. Furthermore, we assume that all investors are willing to buy assets for any price lower than or equal to the fundamental value, possibly including the index inclusion effect, and willing to sell for any price greater than or equal to the fundamental value.

Consider the case where $V_A > V_B$ and $\gamma > 0$. In this case the asset holders of both firms want to have a higher value than the other firm. The investors are willing to trade the asset of **FIRM_A** for $V_A + \gamma$ as they know that the trading price for the assets of **FIRM_B** can never exceed this value as $V_A + \gamma > V_B + \gamma$. Hence, **FIRM_A** is included in **INDEX₁** and **FIRM_B** is included in **INDEX₂**. Clearly when $V_A < V_B$ the firms will switch between indices.

Next, consider the case where $V_A > V_B$ and $\gamma < 0$. The asset holders of both firms want to have the lower firm value when the index composition is determined. Suppose the trading price would be lower for **FIRM_A** than for **FIRM_B**. In that case the trading price for **FIRM_B** will be $V_B + \gamma < V_B < V_A$. So in order for this to occur the asset holders of **FIRM_A** need to sell their assets with a greater discount on the fundamental value than $|\gamma|$. This is clearly not optimal, so we have that **FIRM_A** is included in index **INDEX₁** and firm **FIRM_B** is included in index **INDEX₂**. Note that the trading price of **FIRM_B** needs to be lower than V_A . In this example this implies that there is no trading in the assets of **FIRM_B**. In practice this would imply that asset holders can either abstain from trading in the assets of **FIRM_B** or trade for a value below the trading price of **FIRM_A**.

The first example represents the situation for the Russell 2000 and the Russell

3000 (see Chang Hong Liskovitch (2014)), for the case that the index composition only changes at the end of May. Our model predicts that investors can anticipate changes in index compositions. Similarly, the second example represents the situation for the Russell 1000 and the Russell 2000. In this case our prediction is that in May investors cannot fully benefit from the index inclusion effect for firms that are just below the threshold between the indices. In June, when the trading price no longer affects the cash-flows, investors do incorporate the index inclusion effect and this leads to a change in the trading price. The results in Chang Hong and Liskovitch (2014) confirm this prediction. In case we do not assume that investors trade for the fundamental value but optimize their portfolios, the fundamental value no longer identifies in what index a firm near the threshold will be. Indeed, the individual market power becomes relevant as the asset holders of the firms involved will try to influence the market to their advantage. The pricing method introduced in this paper is then required to determine the trading prices of the assets and identify the composition of the indices.

Chang Hong Liskovitch (2014) find an index deletion effect as well. Consider in the above example that we have $\gamma < 0$ and in previous periods we had $V_A > V_B$. Without the index inclusion effect, a drop in V_A does not affect $\mathbf{FIRM}_B$. However, in this case, this drop implies that $\mathbf{FIRM}_B$ will pass the threshold and lose the index inclusion effect. Hence, we would anticipate the index deletion effect as well.

3.2 Market

In this section we show how the trading prices of the financial instrument discussed in Section 3.1 can be represented as if they were set by class-representative agents who have symmetric information and cannot hedge all of their risk exposures.⁹ These class-representative agents may differ in terms of individual investment opportunities, hedging abilities, risk preferences, liquidity needs and constraints. We consider a repeated, open-outcry, order-driven, double auction financial market, with institutional trade-determination and price-setting. The investors trade risky financial instruments (such as for example equities, corporate bonds and options) and default-free discount bonds. We illustrate in Section 3.2.1 the salient features of the class-representative agents and the traded financial instruments. In Section 3.2.2 we introduce the notation to describe the dynamics of the market. We describe

⁹ Any self-financing strategy implemented by an agent does not pointwise replicate any contingent claim.

how the class-representative agents set their own individual private valuations in Section 3.2.3. We describe the behavior of the class-representative agents in Section 3.2.4.

3.2.1 Risky financial instruments and classes of investors

All but one of the N_A traded risky financial instruments are arbitrage-free, and the kind of their cash-flows is well-defined at the time of their valuation. For example, these $N_A - 1$ financial instruments may be equities and defaultable bonds with no unexpected risk premium. Differently, the remaining financial instrument features the characteristics of the financial instrument discussed in Section 3.1, and the kind of cash-flows it offers are uncertain at the time of its valuation. This financial instrument is divided into N_S indistinguishable shares, and we refer to each of them as "the asset". The asset is not a Giffen good. An insufficient demand for it does not make its market price increase, while a drop in its market price does not induce a contemporaneous decrease in its demand. Both these effects are monotonic. At the trading date t , the asset payoff function automatically takes one of several distinct functional forms in response to its contemporaneous market price. This change represents an automatic change in the kind of asset cash-flows, and different kinds of cash-flows may correspond to distinct valuations of the asset. As a result, the investors may not be able to anticipate the kind of the asset cash-flows and the wealth transfer, and the asset may not have a unique competitive equilibrium price. Then, the asset is an incomplete contract, as Glasserman & Nouri (2016) explains in the context of the pricing of CoCos with market-based triggers and equity holder penalization at the CoCo conversion. We focus our attention on how the demand and supply determine the asset trading price. Then, without loss of generality, the supply of each of the arbitrage-free financial instruments is fixed and normalized to one.¹⁰

There is a wealth transfer among the investors any time the kind of cash-flows of the asset changes. We adopt the convention that a wealth transfer in favor of the asset holders has a "positive sign".

Assumption 1. *At any change in the kind of cash-flows of the asset, some wealth is automatically transferred from (or to) the asset holders to (or from, respectively) other (but not all) investors.*

¹⁰ Our discussion does not rely on the assumption of positive and constant supply of the financial instruments.

Under Assumption 1 the wealth transfer does not affect just the investors who have directly invested in the asset. It may also affect some investors who do not hold it. Some of them may indeed either receive or lose wealth because of the change in the kind of cash-flows of the asset. Others may need to rebalance their portfolios because of the same reason, without systematically improving or worsening their wealth. Assumption 1 also implies that each investor who benefits from the wealth transfer cannot invest in the asset. If this would be possible, the same investor may manipulate the market.¹¹

The investors differ in terms of abilities to influence the trading prices. They are indeed individually subject to distinct trading restrictions, and individually exposed to distinct exogenous shocks. Therefore, we divide them into classes on the basis of their market power w.r.t. the asset and contributions to its demand. Each class is characterized by levels for these properties averaged over all the investors in the class. For example, a class may include all the asset holders exposed to a given set of exogenous shocks. Then, the result of the trading activities of all the investors in the same class is modeled as the result of the optimal consumption-investment strategy for a dedicated rational class-representative agent.

Assumption 2. *For each class-representative agent there is a well-defined ranking of the trading prices corresponding to the distinct kinds of cash-flows of the asset, from the one characterized by the largest to the lowest gain.*^{12 13}

For example, the representative agent for a class that includes the holders of the equity of a firm issuing CoCos with equity holder penalization at the CoCo conversion, rank the configuration with no CoCo conversion as first, and then the configuration with CoCo conversion.

For simplicity of exposition, we limit our discussion to class-representative agents who derive utility just from consuming the nondurable consumption good taken as numéraire.¹⁴

Assumption 3. *At each trading date, the individual consumption of a class-representative agent is locally continuous in the asset trading price around a level that, if crossed, induces a change in the kind of cash-flows of the asset.*

Under Assumption 3, when the trading price is at a level that, if crossed, there is a

¹¹ See, for example, Albul, Jaffee & Tchistyi (2010) and Calomiris & Herring (2013).

¹² We consider a loss as a negative gain.

¹³ Bond et al. (2009) introduce the concept of an "agent-preferred equilibrium", which is related to Assumption 2.

¹⁴ Our discussion can be extended to other utilities.

change in the kind of cash-flows of the asset, any of its infinitesimal moves is accompanied at most by just an infinitesimal change in any individual consumption. Hence, at those levels of the trading price, any small change in the asset trading price is not accompanied by any large change in an individual consumption.

Assumption 4. *The lifetime individual utility for a class-representative agent is a function of just her individual consumption of a single nondurable consumption good at each trading date during the period $[t, \infty)$. This function is strictly increasing in each element of the individual consumption process, and it is locally continuous at any level of the asset trading price that, if crossed, makes the kind of cash-flows of the asset change. At each trading date t , the goal of each class-representative agent is to maximize her expected lifetime individual utility conditional on her information.*

Intuitively, Assumption 4 implies that the utility of a class-representative agent for a class of asset holders, that we refer to as “asset holder agent”, moves in the same direction as the asset trading price. Note that this is a weak assumption as the counterfactual is that the utility increases (decreases) when the asset holders face a loss (gain). The composition of locally continuous functions is locally continuous. Therefore, under Assumptions 3 and 4, the utility function is locally continuous in the trading price of the asset around any level that triggers a change in the kind of its cash-flows. We stress that the behavior of an asset holder agent does not necessarily coincide with the behavior of a real investor, and when we refer to an asset holder agent “selling” we mean that at least one real investor in the class she represent “sells”.

We now consider the relationships between distinct classes created on the basis of the market power of the investors w.r.t. the asset and contributions to its demand. We represent these relationships by means of two properties of the class-representative agents that indicate their stakes in the asset. The “individual market power for a first class-representative agent against a second class-representative agent w.r.t. a financial instrument” is the ability of the former to induce the private valuation of the financial instrument set by the latter move as close as possible to her own private valuation. The “individual control for a class-representative agent over the asset demand” is the degree of her active participation to the asset trades.

Assumption 5. *Each class-representative agent uses both her own individual market power against any other class-representative agent w.r.t. the asset and her own individual control over the asset demand to avoid any individual wealth loss due to the change in the kind of cash-flows of the asset as described in Assumption 1, and to exploit any arbitrage opportunity.*

To exemplify the implications of Assumption 5, consider a CoCo with market-based conversion trigger and equity holder penalization at the CoCo conversion. Because of the equity holder penalization, the equity cash-flows under the CoCo conversion are pointwise lower (that is, less profitable for the equity holders) than those under no CoCo conversion. The equity holders then strictly prefer the CoCo to not convert, if the remaining components of her portfolio are not affected by the CoCo conversion. The perspective of the CoCo holders is different. They indeed prefer the CoCo to convert, again considering the case in which the other components of her portfolio are not affected by the CoCo conversion. All in all, conditionally on keeping the remaining part of her portfolio unaltered, on the one side, a class-representative agent prefers to have enough individual market power and individual control over the asset demand to avoid the CoCo conversion, if she represents equity holders. On the other side, she prefers the opposite, if she represents CoCo holders. In addition, under Assumption 1 there are class-representative agents who do not represent actual equity and CoCo investors, and do not have any particular preference.

3.2.2 Market evolution

We represent the uncertainty in the market by a means of a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0, \infty)}, \mathbb{P})$, for the sample space Ω , the σ -algebra $\mathcal{F}$, the sub- σ -algebras $\mathcal{F}_l$ defined for any value of the non-negative integer trading date index l , and the historical probability measure $\mathbb{P}$. The sub- σ -algebra $\mathcal{F}_t$ represents the symmetric information available to all the investors at the trading date t , and the filtration $\{\mathcal{F}_l\}_{l \in [0, t]}$ represents its flow over time from the trading date 0 up to and including the trading date t . It is known whether each event in $\mathcal{F}_0$ has realized. In particular, if an event $A \in \mathcal{F}_0$ is such that $\mathbb{P}[A] = 0$, then all the investors correctly know that the event A is impossible.

The class-representative agents face multiple risks, which directly affect the value of the assets and/or determine the probability of a change in the kind of its cash-flows. Each of these factors are represented by random variables with a well-defined joint historical pdf. At the trading date t , the expected returns of the arbitrage-free financial instruments are completely determined by the N_X -dimensional vector X_t , whose components are random, neither necessarily independent nor identically distributed.

Assumption 6. *There are more financial instruments than sources of risk, that is $N_A > N_X$.*

Under Assumption 6 there exists a self-financing strategy for the $N_A - 1$ arbitrage-free financial instruments that replicates any function of the vector X_t . Let us now focus on the risk factors that directly affect the probability of a change in the kind of cash-flows of the asset. Under Assumption 1 some investors do not trade the asset and/or the other financial instruments, and are not affected by any wealth transfer at the change in the kind of cash-flows of the asset. Therefore, not all the systematic risk factors for the financial instruments affect the change in the kind of cash-flows of the asset, but just those to whom the investors who trade the assets are exposed to. For this reason, we isolate some of the systematic risk factors that directly determine the value of all assets and stack them in the vector $Y_t^{[k]}$, such that we have the partition $X_t = [Y_t^{[k]'} Y_t^{[-k]'}]'$. Note that we can partition the vector X_t in $(2^{N_x} + 1)$ distinct ways, because this is the cardinality of the power set of the set collecting the components of the vector X_t . The process $\{Y_t^{[k]}\}_{t \in [1:t]}$ can realize in any path in the set $\mathcal{Y}_t^{[k]}$, for any $k = 0, \dots, 2^{N_x}$. Among all these vectors $Y_t^{[k]}$, just one includes only all the risk factors that truly matter for the probability of a change in the kind of cash-flows of the asset when the market is at a Nash equilibrium. We now introduce the integer value k_t^* that corresponds to this vector.

Definition 1. *At a Nash equilibrium on the trading date t , there is a unique integer value $k_t^* \in [0 : 2^{N_x}]$ such that the true probability of a change in the kind of cash-flows of the asset is determined by the vector $Y_t^{[k_t^*]}$ and it does not depend on the vector $Y_t^{[-k_t^*]}$. In turn, the asset trading price at the trading date t depends on the vector $Y_t^{[k_t^*]}$.*

The value k_t^* is therefore necessary for the identification of the fundamentals for the entire economy, which may evolve over time.

3.2.3 Security private valuation

We now describe the private valuation of each financial instrument set by a class-representative agent, in the form of a conditional expectation of (discounted) cash-flows. The pricing formula is similar in spirit, but not equal, to the one adopted in classic asset pricing models. We denote the process for the vector X_t over the time interval $[1 : t]$, which can realize in any path included in the set $\mathcal{X}_t$, by $\{X_t\}_{t \in [1:t]}$. The payoffs of these financial instruments are in the linear space of square-integrable functions on $\mathcal{X}_t$ that we denote by " $L_2(\mathcal{X}_t)$ ". We represent a private valuation as the result of the application of an integral operator that operates from the space $L_2(\mathcal{X}_{t+\Delta}) \times \{0 : 2^{N_x}\}$ to the space $L_2(\mathcal{X}_t)$. Such operator, shrinks the space in which it operates and it returns a value dependent on the path

for the vector X_t over the time interval $[t : t + \Delta]$. For each single financial instrument, we take $2^{N_X} + 1$ functions included in the space $L_2(\mathcal{X}_{t+\Delta})$ as reference payoff functions. In particular, each of these functions represents the payoff function for the financial instrument in case the asset assumes a particular payoff function. We then introduce the function $K_{t+\Delta}$, which lies in the subspace $\{0 : 2^{N_X}\}$ and takes on one of the integer values in the interval $[0 : 2^{N_X}]$. The output of the function $K_{t+\Delta}$ indicates which subset of all the systematic risk factors is considered as directly determining the value of all financial instruments. That is, the value k of the function $K_{t+\Delta}$ indicates that the vector $Y_t^{[k]}$ is selected out of $(2^{N_X} + 1)$ admissible ones to perform a **valuation**. The numéraire is the single nondurable consumption good considered in Assumption 4.

The asset Let us start with the **private valuation** of the asset. At the trading date t , if the asset assumes its admissible k -th functional form, each class-representative agent correctly knows that the asset pays a net dividend of $D_{1,t}^{[k]}$ and has ex-dividend market price $P_t^{[k]}$, for any $k = 0, \dots, 2^{N_X}$. Therefore, the sum $P_t^{[k]} + D_{1,t}^{[k]}$ represents the payoff in case the asset cash-flows are of their k -th admissible kind. In that case, the SDF used to set the **private valuation** is the $(\mathcal{F}_t \cup \{K_t\})$ -measurable variable $\mathcal{M}_t^{[K_t]}$. The functions $P_t^{[k]} + D_{1,t}^{[k]}$ are not all pointwise equivalent across k 's.¹⁵ Also the SDF's $\mathcal{M}_t^{[k]}$ differ across the k 's. At the trading date t , the valuation of the asset is represented by the conditional private valuation operator under the historical probability measure

$$\hat{P}_t(k) := E \left[\bar{m}_{t+\Delta}(K_t)' \left(\bar{P}_{t+\Delta} + \bar{D}_{1,t+\Delta} \right) \middle| \mathcal{F}_t \cup \{K_t = k\} \right], \quad (3.2)$$

for any $k = 0, \dots, 2^{N_X}$, and the $(2^{N_X} + 1)$ -dimensional vectors

$$\bar{P}_{t+\Delta} := \left[P_{t+\Delta}^{[0]} \dots P_{t+\Delta}^{[2^{N_X}]} \right]', \quad \bar{D}_{1,t+\Delta} := \left[D_{1,t+\Delta}^{[0]} \dots D_{1,t+\Delta}^{[2^{N_X}]} \right]'$$
 and

$$\bar{m}_{t+\Delta}(K_t) := E \left[\left[\mathbf{1}_{\{K_{t+\Delta}=0\}} \dots \mathbf{1}_{\{K_{t+\Delta}=2^{N_X}\}} \right]' \odot \left[\mathcal{M}_{t+\Delta}^{[0]} \dots \mathcal{M}_{t+\Delta}^{[2^{N_X}]} \right]' \middle| \mathcal{F}_{t+\Delta} \cup K_t \right], \quad (3.3)$$

where the symbol " $\odot$ " denotes the Hadamard product. We interpret the value $\hat{P}_t(k)$ as the private valuation for the asset set by a class-representative agent exploiting the information set $\mathcal{F}_t$ and acting as if any change in the kind of cash-flows of the

¹⁵ For example, a CoCo features just two admissible kinds of cash-flows: equity-like and bond-like.

asset was determined by the risk factors collected in the vector $Y_t^{[k]}$. In particular, the private valuation operator $\hat{P}_t(k)$ takes the functions $\mathbf{1}_{\{K_{t+\Delta}=0\}}, \dots, \mathbf{1}_{\{K_{t+\Delta}=2^{N_X}\}}, \mathcal{M}_{t+\Delta}^{[0]}, \dots, \mathcal{M}_{t+\Delta}^{[2^{N_X}]}$, and those in the vector $\vec{P}_{t+\Delta} + \vec{D}_{1,t+\Delta}$ as inputs. It then returns a function in space $L_2(\mathcal{X}_t)$, as assumed in the classic asset pricing literature (see, for example Hansen & Richard (1987)). We now adopt an assumption on the identification of the risk factors that truly matter for the probability of a change in the kind of cash-flows of the asset based on the private valuation operator $\hat{P}_t(k)$.

Assumption 7. For any $k_1, k_2 = 0, \dots, 2^{N_X}$ such that $k_1 \neq k_2$, we have $\hat{P}_t(k_1) \neq \hat{P}_t(k_2)$.

We assume Assumptions 1 and 7 hold, and think of class-representative agents who know the identity of the vector X_t and how to partition its components into subsets of type $Y_t^{[k]}$. Moreover, the agents know that each of these vectors at the trading date t corresponds to a well-defined functional form of the asset payoff at the same date. Under Assumption 7, the investors describe this interesting partition by means of the conditional private valuation operator $\hat{P}_t(k)$ in Equation (3.2). We denote the sets in this partition by $\mathcal{W}_0, \dots, \mathcal{W}_{2^{N_X}}$, because the asset features at most $2^{N_X} + 1$ distinct kinds of cash-flows.¹⁶ The corresponding payoff functions for the asset are not all identically equal. They are such that $\bigcup_{l=0}^{2^{N_X}} \mathcal{W}_l = \mathbb{R}_+$ and either $\mathcal{W}_l \cap \mathcal{W}_{\tilde{l}} = \emptyset$ or $\mathcal{W}_l \cap \mathcal{W}_{\tilde{l}} = \mathcal{W}_l$ for any $l, \tilde{l} = 0, \dots, 2^{N_X}$ with $l \neq \tilde{l}$. These characteristics ensure that any kind of asset cash-flows is associated to a well defined and unique asset trading price. For example, the partition consists of two sets for the CoCos considered by Sundaresan & Wang (2015) and Glasserman & Nouri (2016), and for an equity possibly affected by the index inclusion effect for market capitalization-based indexes. For any trading date $t \in \mathbb{Z}_0^+$, a change of the functional form of the asset payoff from the one assumed at the previous trading date $\tilde{t}$ is in the form

$$\{ \{ \hat{P}_{\tilde{t}}(k) \in \mathcal{W}_{\tilde{l}} \} \cup \{ \hat{P}_{\tilde{t}}(k) \in \mathcal{W}_l \} \}, \quad (3.4)$$

for trading dates $\tilde{t} < t$ and sets $\mathcal{W}_{\tilde{l}} \neq \mathcal{W}_l$. The k -th class-representative agent considers that, at the change in the kind of asset cash-flows in Expression (3.4), her wealth automatically changes according to Assumption 1.

¹⁶ The discussion can easily be extended to the time-varying partition of the sets, that is to the time-varying partition of the kind $\mathcal{W}_{0,t}, \dots, \mathcal{W}_{2^{N_X},t}$ at the trading date t .

The arbitrage-free securities At the trading date t , if the asset assumes its admissible k -th functional form, each class-representative agent correctly knows that the j -th arbitrage-free financial instrument is risky, has ex-dividend market price $S_{j,t}^{[k]}$ and pays the net dividend $D_{j,t}^{[k]}$, for any $j = 2, \dots, N_A$.

Let us now consider the class-representative agent with the information set $\mathcal{F}_t$ who acts as if the probability of a change in the kind of cash-flows of the asset is determined by the risk factors collected in the vector $Y_t^{[k]}$. She sets her own private valuation of the j -th arbitrage-free financial instrument as follows:

$$\hat{S}_{j,t}(k) := E[m_{t+\Delta}(K_t) (S_{j,t+\Delta} + D_{j,t+\Delta}) | \mathcal{F}_t \cup \{K_t = k\}], \quad (3.5)$$

for any $j = 2, \dots, N_A$, any $\Delta \geq 1$, and the SDF

$$m_{t+\Delta}(K_t) := E \left[\left[\mathbf{1}_{\{K_{t+\Delta}=0\}} \dots \mathbf{1}_{\{K_{t+\Delta}=2^{N_X}\}} \right]' \left[\mathcal{M}_{t+\Delta}^{[0]} \dots \mathcal{M}_{t+\Delta}^{[2^{N_X}]} \right] \middle| \mathcal{F}_{t+\Delta} \cup K_t \right], \quad (3.6)$$

which is measurable w.r.t. $\mathcal{F}_{t+\Delta} \cup K_t$. By the linearity of the expectation operator we have $m_{t+\Delta}(K_t) = \vec{m}_{t+\Delta}(K_t)' \iota$, for the $(2^{N_X} + 1)$ -dimensional vector of ones ι . Therefore, from the standardization provided by the indicator functions, the private valuation of the j -th arbitrage-free financial instrument is based on just the average across the components of vector $\vec{m}_{t+\Delta}(K_t)$, while the private valuation of the asset requires all its components.

There are also T default-free discount bonds with maturities from the following trading date to T trading dates ahead. Without loss of generality, the par value of each bond is one unit of the consumption good. At the trading date t , if the asset assumes its admissible k -th functional form, each class-representative agent correctly knows that the market price of the l -th bond is

$$\hat{B}_{l,t}(k) = E[m_{t+l}(K_t) | \mathcal{F}_t \cup \{K_t = k\}], \quad (3.7)$$

for any $l = 1, \dots, T$ and any $k = 0, \dots, 2^{N_X}$. In the following sections we also use the notation $\hat{S}_t(k) = [\hat{S}_{2,t}(k) \dots \hat{S}_{N_A,t}(k)]'$ and $\hat{B}_t(k) = [\hat{B}_{1,t}(k) \dots \hat{B}_{T,t}(k)]'$.

Equations (3.2), (3.5) and (3.7) highlight the unconventional property of the pricing problem we deal with in this paper. To set the private valuations of the arbitrage-free financial instruments and the default-free bonds, just a single SDF is necessary. Any future functional form of the payoff for each of them is indeed known. In other words, the kind of cash-flows they offer is known at the time

of their valuation. Consequently, just the scalar $m_{t+\Delta}$ defined in Equation (3.6) is necessary to discount future payoffs by time and risk according to the private valuations in Equations (3.5) and (3.7). In particular, this scalar function is obtained by averaging the scalar product between the vector of SDF's $\mathcal{M}_{t+\Delta}^{[0]}, \dots, \mathcal{M}_{t+\Delta}^{[2^{N_x}]}$ and the vector of indicator functions for the sets of risk factors that affect the probability of a change in the kind of the asset cash-flows and the asset payoffs. It is then a SDF that depends on the particular kind of cash-flows provided by the asset. Differently, all the distinct SDF's $\mathcal{M}_t^{[0]}, \dots, \mathcal{M}_t^{[2^{N_x}]}$ are singularly necessary for the private valuation of the asset, since the future functional form of its payoff is not known at the time of its valuation. Therefore, to compute the private valuation in Equation (3.2) we need the vector of SDF's defined in Equation (3.3), which is a conditional mean of the componentwise composition of the vector of SDF's $\mathcal{M}_{t+\Delta}^{[0]}, \dots, \mathcal{M}_{t+\Delta}^{[2^{N_x}]}$ and the vector of indicator functions for the sets of risk factors that affect the probability of a change in the kind of cash-flows of the asset. Finally, note that both the SDF $m_{t+\Delta}$ and the SDF's $\mathcal{M}_t^{[0]}, \dots, \mathcal{M}_t^{[2^{N_x}]}$ vary as functions of K_t .

3.2.4 Strategies of the class-representative agents

We take the initial trading date 0 as a reference date for our discussion. At that date the market is such that the investors can be partitioned into classes so that the trading activities among its investors can be represented as the result of a well-defined optimization. The i -th class-representative agent, on the basis of the symmetric information in set $\mathcal{F}_0$, optimizes her expected lifetime utility from consumption as if any change in the kind of cash-flows of the asset was determined by the risk factors collected in the vector $Y_0^{[i]}$. Therefore, the superscript " i " indicates which variables are taken by the class-representative agent at the trading date 0 as the risk factors for the probability of a change in the kind of the cash-flows of the asset. Then, at any following trading date t , we use the following notation to indicate which variables are considered by an agent as determinant of the probability of a change in the kind of the cash-flows of the asset. The integer value assumed by the variable $k_t^{[i]}$, which lies in the interval $[0 : 2^{N_x}]$, indicates the set of variables considered by the same class-representative agent who considers the vector $Y_0^{[k]}$ at the initial trading date 0. The variation over time of the variable $k_t^{[i]}$ represents the fact that a class-representative agent may revise her assumption on the risk factors over time.

Furthermore, we adopt a notation that highlights the fact that the individual consumption and the portfolio weights are driven by the risk factors for the probability of a change in the kind of the cash-flows of the asset. At the trading date

t , the aggregate labor income and aggregate consumption for the investors in the i -th class are $I_t^{[i]}$ and $c_t^{[i]}(k_t^{[i]})$, respectively. These quantities are the stochastic labor income and individual consumption for the i -th class-representative agent, who holds a portfolio $\theta_t^{[i]}(k_t^{[i]}) := [\theta_{1,t}^{[i]}(k_t^{[i]}) \dots \theta_{N_A,t}^{[i]}(k_t^{[i]})]'$ of shares of securities and a portfolio $\lambda_t^{[i]}(k_t^{[i]}) := [\lambda_{1,t}^{[i]}(k_t^{[i]}) \dots \lambda_{T,t}^{[i]}(k_t^{[i]})]'$ of default-free discount bonds. Note that the variable $\theta_{1,t}^{[i]}(k_t^{[i]})$ denotes the number of shares of the asset. We also use the notation $\theta_{-1,t}^{[i]}(k_t^{[i]}) := [\theta_{2,t}^{[i]}(k_t^{[i]}) \dots \theta_{N_A,t}^{[i]}(k_t^{[i]})]'$, and refer to any set

$$\left\{ \gamma_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)} = \left\{ \left\{ \theta_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ \lambda_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ c_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)} \right\} \quad (3.8)$$

as an individual consumption-investment strategy for the i -th class-representative agent. We consider just bounded strategies. Ex-dividend market prices are functions of the contemporaneous and lagged values of the vector X_t . The symmetric information of the investors at the trading date t , which has been introduced in Section 3.2.2, is then $\mathcal{F}_t = \{X_{\tilde{l}}\}_{\tilde{l} \in [1:t]} \cup \left(\bigcup_{l=1, \dots, N_A} \{D_{l,\tilde{l}}\}_{\tilde{l} \in [1:t]} \right) \cup \left(\bigcup_{i=0, \dots, 2^{N_X}} \{I_{\tilde{l}}^{[i]}\}_{\tilde{l} \in [1:t]} \right)$, which simplifies to $\mathcal{F}_t = \{X_{\tilde{l}}\}_{\tilde{l} \in [1:t]}$ if dividends and labor income at trading date t are deterministic functions of the contemporaneous and lagged values of the vector X_t .

We need an identification assumption on the risk factors that truly affect the probability of a change in the kind of cash-flows of the asset. For this reason, we assume that the strategy adopted by the i -th class-representative agent given in Expression (3.8) varies together with the risk factors assumed as changing the kind of cash-flows of the asset.

Assumption 8. For any $i = 0, \dots, 2^{N_X}$ and any trading dates t and l , for any $k_l^{[i]}, \tilde{k}_l^{[i]} = 0, \dots, 2^{N_X}$ such that $k_l^{[i]} \neq \tilde{k}_l^{[i]}$, we have that $\left\{ \gamma_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)} \neq \left\{ \gamma_l^{[i]}(\tilde{k}_l^{[i]}) \right\}_{l \in [t, \infty)}$.

Assumption 8 ensures the identification of the risk factors that truly affect the probability of a change in the kind of cash-flows of the asset on the basis of pointwise differences between strategies chosen on the basis of distinct sets of systematic risk factors that directly determine the value of all the tradable securities. The absence of arbitrage opportunities for the financial instruments with well-defined cash-flows at the time of their valuation is described in the following assumption.

Assumption 9. For any $k = 0, \dots, 2^{N_X}$, if the non-strict inequality

$$\sum_{l=0}^{\infty} \theta_{-1,l}^{[i]}(k)' (\hat{S}_{l+1}(k) - \hat{S}_l(k)) + \lambda_l^{[i]}(k)' (\hat{B}_{l+1}(k) - \hat{B}_l(k)) \geq 0$$

holds, it holds as an equality.

Under Assumption 9, by the Dalang-Morton-Willinger theorem, there exists a probability measure equivalent to the historical one such that, under that measure, the trading price of the financial instruments with a well-defined cash-flow at the time of their valuation are martingales w.r.t. the filtration $\{\mathcal{F}_l\}_{l \in [0, \infty)}$.

Under Assumption 4, a class-representative agent allocates her individual wealth in the securities and for consumption, in order to maximize her expected lifetime individual utility conditional on her information. Whatever she does, she must respect some budget constraints. In a Nash equilibrium, the i -th class-representative agent obtains at the trading date t the wealth

$$\theta_{1,t-1}^{[i]}(k_t^{[i]}) \hat{P}_t(k_t^{[i]}) + \theta_{-1,t-1}^{[i]}(k_t^{[i]})' \hat{S}_t(k_t^{[i]}) + \lambda_{t-1}^{[i]}(k_t^{[i]})' \hat{B}_t(k_t^{[i]})$$

from her investments in the financial instruments entered at the previous trading date $t - 1$. She then decides, on the basis of the set $\mathcal{F}_t \cup \{K_t = k\}$, how to allocate this wealth and her labor income between consumption and investments in the financial instruments:

$$\begin{aligned} & \theta_{1,t-1}^{[i]}(k_t^{[i]}) \hat{P}_t(k_t^{[i]}) + \theta_{-1,t-1}^{[i]}(k_t^{[i]})' \hat{S}_t(k_t^{[i]}) + \lambda_{t-1}^{[i]}(k_t^{[i]})' \hat{B}_t(k_t^{[i]}) + I_t^{[i]} \\ & = c_t^{[i]}(k_t^{[i]}) + \theta_{1,t}^{[i]}(k_t^{[i]}) \hat{P}_t(k_t^{[i]}) + \theta_{-1,t}^{[i]}(k_t^{[i]})' \hat{S}_t(k_t^{[i]}) + \lambda_t^{[i]}(k_t^{[i]})' \hat{B}_t(k_t^{[i]}). \end{aligned} \quad (3.9)$$

The maximization of the expected utility The i -th class-representative agent maximizes the conditional expectation of her lifetime utility given her conditioning information subject to the market clearing conditions, the consumption-investment constraint in Equation (3.9) and the fact that aggregate labor income and total dividend sum up to the aggregate consumption. She then adopts a particular strategy, which is a set of processes for her individual security and bond portfolio weights and her individual consumption choices. We define now the strategy that realizes this goal, when all the other class-representative agents adopt a given set of reference strategies.

Definition 2. For any $i = 0, \dots, 2^{N_X}$ and any reference group of strategies of all the

class-representative agents but the i -th over the time period $[t, \infty)$, which we denote as

$$\begin{aligned} \left\{ \gamma_l^{[-i]}(k_l^{[-i]}) \right\}_{l \in [t, \infty)} &:= \left\{ \left\{ \theta_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ \lambda_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ c_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \right. \\ &\quad \left. \forall \tilde{i} = 0, \dots, 2^{N_X}, \tilde{i} \neq i \right\} \end{aligned} \quad (3.10)$$

where $k_t^{[-i]} := \left\{ k_t^{[0]}, \dots, k_t^{[i-1]}, k_t^{[i+1]}, \dots, k_t^{[2^{N_X}]} \right\}$, an individual conditionally optimal consumption-investment strategy for the i -th class-representative agent over the same period is defined as

$$\begin{aligned} &\gamma_{[t, \infty)}^{[i \text{ Cond}]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t, \infty)} ; \left\{ \gamma_l^{[-i]} \right\}_{l \in [t, \infty)}, \left\{ k_l^{[-i]} \right\}_{l \in [t, \infty)} \right) \\ &:= \arg \max_{\gamma \in \Gamma_{[t, \infty)}^{[i]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t, \infty)} ; \left\{ \gamma_l^{[-i]} \right\}_{l \in [t, \infty)}, \left\{ k_l^{[-i]} \right\}_{l \in [t, \infty)} \right)} \mathcal{U}_{[t, \infty)}^{[i]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t, \infty)} \right), \end{aligned} \quad (3.11)$$

where the optimization criterion is defined as

$$\mathcal{U}_{[t, \infty)}^{[i]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t, \infty)} \right) := \mathbb{E} \left[U_{[t, \infty)}^{[i]} \left(\{ c_l^{[i]}(K_l^{[i]}) \}_{l \in [t, \infty)} \right) \middle| \mathcal{F}_t \cup \left\{ K_l^{[i]} = k_l^{[i]} \right\}_{l \in [t, \infty)} \right]$$

for the utility function $U_{[t, \infty)}^{[i]}$ from consumption introduced in Assumption 4, and the basic feasible set of the optimization problem is defined as

$$\begin{aligned} &\Gamma_{[t, \infty)}^{[i]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t, \infty)} ; \left\{ \gamma_l^{[-i]} \right\}_{l \in [t, \infty)}, \left\{ k_l^{[-i]} \right\}_{l \in [t, \infty)} \right) \\ &:= \left\{ \left\{ \left\{ \theta_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ \lambda_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)}, \left\{ c_l^{[i]}(k_l^{[i]}) \right\}_{l \in [t, \infty)} \right\} : \forall l \in [t, \infty) \right. \\ &\quad c_l^{[i]}(k_l^{[i]}) = I_l^{[i]} + \left(\theta_{1,l-1}^{[i]}(k_l^{[i]}) - \theta_{1,l}^{[i]}(k_l^{[i]}) \right) \hat{P}_l(k_l^{[i]}) \\ &\quad \quad + \left(\theta_{-1,l-1}^{[i]}(k_l^{[i]}) - \theta_{-1,l}^{[i]}(k_l^{[i]}) \right)' \hat{S}_l(k_l^{[i]}) + \left(\lambda_{l-1}^{[i]}(k_l^{[i]}) - \lambda_l^{[i]}(k_l^{[i]}) \right)' \hat{B}_l(k_l^{[i]}), \\ &\quad c_l^{[i]}(k_l^{[i]}) = \sum_{j=0}^{2^{N_X}} I_l^{[j]} - \sum_{\substack{j=0 \\ j \neq i}}^{2^{N_X}} c_l^{[j]}(k_l^{[j]}) + \sum_{\tilde{l}=1}^{N_A} D_{\tilde{l},t}, \end{aligned}$$

$$\begin{aligned}
\lambda_{i,l}^{[i]}(k_l^{[i]}) &:= - \sum_{\substack{j=0 \\ j \neq i}}^{2^{N_X}} \lambda_{i,l}^{[j]}(k_l^{[j]}), \forall \tilde{l} = 1, \dots, T, \\
\theta_{1,l}^{[i]}(k_l^{[i]}) &= N_S - \sum_{\substack{j=0 \\ j \neq i}}^{2^{N_X}} \theta_{1,l}^{[j]}(k_l^{[j]}), \\
\theta_{i,l}^{[i]}(k_l^{[i]}) &= 1 - \sum_{\substack{j=0 \\ j \neq i}}^{2^{N_X}} \theta_{i,l}^{[j]}(k_l^{[j]}) \quad \forall \tilde{l} = 2, \dots, N_A \}.
\end{aligned}$$

When necessary, we also refer to the distinct elements of the individual conditionally optimal consumption-investment strategy for the i -th class-representative agent in Equation (3.11) and use the following notation:

$$\begin{aligned}
&\gamma_{[t,\infty)}^{[i \text{ Cond}]} \left(\left\{ k_l^{[i]} \right\}_{l \in [t,\infty)}, \left\{ \gamma_l^{[-i]} \right\}_{l \in [t,\infty)}, \left\{ k_l^{[-i]} \right\}_{l \in [t,\infty)} \right) \\
&= \left\{ \left\{ \theta_l^{[i \text{ Cond}]} \left(k_l^{[i]}, \gamma_l^{[-i]}, k_l^{[-i]} \right) \right\}_{l \in [t,\infty)}, \right. \\
&\quad \left. \left\{ \lambda_l^{[i \text{ Cond}]} \left(k_l^{[i]}, \gamma_l^{[-i]}, k_l^{[-i]} \right) \right\}_{l \in [t,\infty)}, \left\{ c_l^{[i \text{ Cond}]} \left(k_l^{[i]}, \gamma_l^{[-i]}, k_l^{[-i]} \right) \right\}_{l \in [t,\infty)} \right\}.
\end{aligned}$$

The bargaining At any trading date t , after observing $j - 1$ subsequent bundles of bid and ask quotes, for a positive integer j , the i -th class-representative agent perceives that the risk factors that affect the probability of a change in the kind of cash-flows of the asset are in the vector $Y_{t+\Delta}^{[k_{t,j}^{[i]}]}$. Consequently, she wants to allocate the wealth $\theta_{1,t}^{[i]}(k_{t,j}^{[i]}) \hat{P}_t(k_{t,j}^{[i]})$ in the asset. However, she also observes a j -th bundle of quotes set by all the other investors, and understands that she can trade the asset at the trading price $\hat{P}_t(\bar{k}) \neq \hat{P}_t(k_{t,j}^{[i]})$. The i -th class-representative agent perceives this valuation mismatch as originating a potential arbitrage opportunity at the trading date t . However, without any further assumption this opportunity is just potential. This opportunity would be concrete only if the i -th class-representative agent was right about the identity of the risk factors for the probability of a change in the kind of the cash-flows of the asset, that is if $k_{t,j}^{[i]} = k_t^*$. Under Assumption 5 each class-representative agent uses her own individual market power against any other in-

vestor to make the other investors update their private valuations toward hers, and make the aforementioned arbitrage opportunity at the trading date t become concrete. However, unless the i -th class-representative agent has so much individual market power against any other investor to maintain her own private valuation $\hat{P}_t(k_{t,j}^{[i]})$, she may need to adjust her portfolio weights accordingly. A trade of the asset at the date t occurs when the weight does not change anymore by bargaining, that is if $\theta_{1,t}^{[i]}(k_{t,j_t^*+1}^{[i]}) = \theta_{1,t}^{[i]}(k_{t,j_t^*}^{[i]})$, for all $i = 0, \dots, 2^{N_X}$, where the positive integer j_t^* is defined as

$$j_t^* := \inf \left\{ j \in \mathbb{Z}^+ : \max_{i, \tilde{i} = 0, \dots, 2^{N_X}} \left| \hat{P}_t(k_{t,j}^{[i]}) - \hat{P}_t(k_{t,\tilde{i}}^{[\tilde{i}]}) \right| = 0 \right\}. \quad (3.12)$$

Therefore, the link between our notation in the previous section and the one used here is $k_{t,j_t^*}^{[i]} \equiv k_t^{[i]}$.

We now introduce a set of criteria to describe the way the two properties of individual market power and individual control over the asset demand for all the representative agents introduced in Section 3.2.1 and discussed in Assumption 5 are exploited. As already pointed out by Sundaresan & Wang (2015), neglecting these properties, there may be alternative sets of market prices for the distinct financial instruments such that the market clears and the investors optimize their individual portfolios. Each of these sets of market prices corresponds to a competitive equilibrium in the market, and it is characterized by a distinct value of the set of criteria we formally introduce in the next Definition 3. However, under Assumption 5 each class-representative agent exploits her individual market power and individual control over the asset demand, and accordingly prefers the configuration in which the corresponding criterion is maximized and any perceived arbitrage opportunity coming from a valuation mismatch is concrete. For sake of generality we do not parameterize these criteria by means of a finite-dimensional parameter. We just constrain them to satisfy minimal features related to measures of the variation of the portfolio weights for the asset during the bargaining process and over subsequent trading dates.

Definition 3. For any $i = 0, \dots, 2^{N_X}$, the individual criterion $\mathcal{Q}_{[t,\infty)}^{[i]}$ for the trading period $[t, \infty)$ depends on the elements of the sequence $\{k_l\}_{l \in [t,\infty)} \equiv \left\{ \left\{ k_l^{[i]} \right\}_{l \in [t,\infty)}, \left\{ k_l^{[-i]} \right\}_{l \in [t,\infty)} \right\}$ and it is

- (i) decreasing in $\left| \theta_{1,l}^{[i \text{ Cond}]} \left(k_{l,j}^{[i]}, \cdot, k^{[-i]} \right) - \theta_{1,l}^{[i \text{ Cond}]} \left(k_{l,j-1}^{[i]}, \cdot, \tilde{k}^{[-i]} \right) \right|$ for any $k^{[-i]}, \tilde{k}^{[-i]} \in [0 : 2^{N_X}]^{2^{N_X}}$ and any $l \in [t, \infty)$;

(ii) increasing in $\left| \theta_{1,l}^{[i \text{ Cond}]} \left(k_{l,j_l^*}^{[j]}; \gamma^{[-j]}, k^{[-j]} \right) - \theta_{1,l-1}^{[i \text{ Cond}]} \left(k_{l-1,j_{l-1}^*}^{[j]}; \tilde{\gamma}^{[-j]}, \tilde{k}^{[-j]} \right) \right|$ for any $k^{[-j]}, \tilde{k}^{[-j]} \in [0 : 2^{N_X}]^{2^{N_X}}$, any pair $\gamma^{[-j]}, \tilde{\gamma}^{[-j]}$ of reference groups of sequences of beliefs about the risk factors of all the class-representative agents but the i -th, and any $l \in [t, \infty)$;

(iii) with a unique global maximum.

Point (i) in Definition 3 represents the fact that the smaller is the change in portfolio weight for the asset during the bargaining process and before trading, the higher is the individual market power of the i -th class-representative agent against the other agents. Intuitively, the smaller is the change in portfolio weight for the asset, the less wealth the agent has lost in previous trades due to the wrong beliefs about the relevant risk factors. Overall, it is then beneficial for the class-representative agent to modify her private valuation the least as possible. Point (ii) in Definition 3 represents the fact that each class-representative agent wants to actively increase the asset liquidity and maximally contribute to the generation of its trading price. Therefore, the greater the portfolio rebalancing for the asset over trading dates, the higher is the individual control over the asset demand. Overall, each class-representative agent has then an incentive to update her beliefs about the risk factors that truly matter for the probability of a change in the kind of cash-flows of the asset, correct her private valuation, and accordingly adopt her optimal consumption-investment strategy. Finally, Point (iii) complements Assumption 2, so that each class-representative agent knows the integer k such that the vector $Y_t^{[k]}$ gives her the highest expected lifetime utility from individual consumption. Essentially, in our setting, this last point allow us to maintain the identification of the integer value k_t^* given in Definition 1 even in a market with distinct class-representative agents.

Definition 4. For any $i = 0, \dots, 2^{N_X}$ and any reference group of sequences of beliefs about the risk factors of all the class-representative agents but the i -th, which we denote as $\left\{ k_l^{[-j]} \right\}_{l \in [t, \infty)}$, an individual conditionally optimal sequence of beliefs about the risk factors for the i -th class-representative agent over the same period is defined as

$$k_{[t, \infty)}^{[i \text{ Cond}]} \left(\left\{ k_l^{[-j]} \right\}_{l \in [t, \infty)} \right) := \arg \max_{\substack{\left\{ k_l^{[j]} \right\}_{l \in [t, \infty)} \\ k_l^{[j]} \in \{0 : 2^{N_X}\}}} \mathcal{Q}_{[t, \infty)}^{[j]} \left(\left\{ \left\{ k_l^{[j]} \right\}_{l \in [t, \infty)}, \left\{ k_l^{[-j]} \right\}_{l \in [t, \infty)} \right\} \right),$$

where the optimization criterion $\mathcal{Q}_{[t,\infty)}^{[i]}$ is introduced in Definition 3.

3.3 Nash equilibrium

In a market Nash equilibrium the class-representative agents account for each other's decisions. Each of them indeed adopts an individual conditionally optimal consumption-investment strategy of the type given in Definition 2, and an individual conditionally optimal sequence of beliefs about the risk factors of the type given in Definition 4.

Definition 5. A market Nash equilibrium is composed by the processes $\{\hat{P}_l(k_l^*)\}_{l \in [t,\infty)}$, $\{\hat{S}_l(k_l^*)\}_{l \in [t,\infty)}$ and $\{\hat{B}_l(k_l^*)\}_{l \in [t,\infty)}$ for the trading prices and the $2^{N_X} + 1$ individual consumption-investment strategies $\left\{ \left\{ \gamma_l^{[0]\star}(k_l^*) \right\}_{l \in [t,\infty)}, \dots, \left\{ \gamma_l^{[2^{N_X}]\star}(k_l^*) \right\}_{l \in [t,\infty)} \right\}$, where (i) the strategies $\left\{ \gamma_l^{[0]\star} \right\}_{l \in [t,\infty)}, \dots, \left\{ \gamma_l^{[2^{N_X}]\star} \right\}_{l \in [t,\infty)}$ solve the system of $2^{N_X} + 1$ functional equations

$$\begin{aligned} \left\{ \gamma_l^{[i]}(k_l^*) \right\}_{l \in [t,\infty)} = \gamma_{[t,\infty)}^{[i \text{ Cond}]} \left(\left\{ k_l^{[i]\star} \right\}_{l \in [t,\infty)} ; \left\{ \gamma_l^{[0]}(k_l^*) \right\}_{l \in [t,\infty)}, \dots, \right. \\ \left. \left\{ \gamma_l^{[i-1]}(k_l^*) \right\}_{l \in [t,\infty)}, \left\{ \gamma_l^{[i+1]}(k_l^*) \right\}_{l \in [t,\infty)}, \dots, \right. \\ \left. \left\{ \gamma_l^{[2^{N_X}]}(k_l^*) \right\}_{l \in [t,\infty)}, \left\{ k_l^{[-i]\star} \right\}_{l \in [t,\infty)} \right), \end{aligned} \quad (3.13)$$

for the individual conditionally optimal consumption-investment strategy $\gamma_{[t,\infty)}^{[i \text{ Cond}]}$ defined in Equation (3.11), for any $i = 0, \dots, 2^{N_X}$, and (ii) the sequence $\{k_l^*\}_{l \in [t,\infty)}$ of variables k_l^* already been introduced in Definition 1, which can be partitioned as $k_l^* = [k_l^{[i]\star} \ k_l^{[-i]\star}]' \in \{0, 2^{N_X}\}^{2^{N_X}+1}$ for any $i = 0, \dots, 2^{N_X}$, solves the system of $2^{N_X} + 1$ functional equations

$$\left\{ k_l^{[i]} \right\}_{l \in [t,\infty)} = k_{[t,\infty)}^{[i \text{ Cond}]} \left(\left\{ k_l^{[-i]} \right\}_{l \in [t,\infty)} \right) \quad (3.14)$$

for any $i = 0, \dots, 2^{N_X}$.

A market Nash equilibrium therefore includes the set of generating functions for the individual conditionally optimal consumption-investment strategies for all the class-representative agents. In turn these strategies are valued for the set of generating functions for the individual conditionally optimal sequence of beliefs about

the risk factors for all the class-representative agents. In a competitive equilibrium, the market pricing functions and market prices $\hat{P}_t(k)$, $\hat{S}_t(k)$ and $\hat{B}_t(k)$ are unique, as otherwise there are arbitrage opportunities. However, the market may have no competitive equilibrium or multiple ones, because of the presence of the asset, which features the characteristics discussed in Section 3.1 (see Sundaresan & Wang (2015)). The next propositions state that the market can reach a unique Nash equilibrium, if each class-representative agent exploits her distinctive market power and contributions to the asset demand.

Proposition 1. *The market Nash equilibrium given in Definition 5 is unique, and the corresponding unique trading prices at the date t are $\hat{P}_t(k_t^*)$, $\hat{S}_{j,t}(k_t^*)$ and $\hat{B}_{l,t}(k_t^*)$ for the pricing formulas in Equations (3.2), (3.5) and (3.7).*

Proof. See Appendix A. ■

Our framework allows to simultaneously explain the trading prices of all the tradable securities in the market. Differently, any existing equilibrium asset pricing model would indicate a misspricing, because it would not explicitly take into account the impact of the change in the asset cash-flows for the prices. We can express the systems composed by Equations (3.13) and (3.14) using a standard notation for systems of functional equations. To do so, we stack the elements of the process $\{k_t^{[i]}\}_{t \in [t, \infty)}$ in the integer-valued infinite-dimensional vector

$$\mathbf{y}^{[1st]} := \left[k_t^{[0]} \ k_{t+1}^{[0]} \ \dots \ k_\infty^{[0]} \ \dots \ k_t^{[2^{N_X}]} \ k_{t+1}^{[2^{N_X}]} \ \dots \ k_\infty^{[2^{N_X}]} \right]',$$

and the elements of the sequence of functions $k_{[t, \infty)}^{[i \text{ Cond}]}$ in the infinite-dimensional vector function

$$\mathbf{F}^{[1st]} := \left[k_t^{[0 \text{ Cond}]} \ k_{[t, t+1]}^{[0 \text{ Cond}]} \ \dots \ k_{[t, \infty)}^{[0 \text{ Cond}]} \ \dots \ k_t^{[2^{N_X} \text{ Cond}]} \ k_{[t, t+1]}^{[2^{N_X} \text{ Cond}]} \ \dots \ k_{[t, \infty)}^{[2^{N_X} \text{ Cond}]} \right]'.$$

Similarly, we stack the elements of the sequence of functions in the LHS of Equation (3.13) in the infinite-dimensional vector function

$$\mathbf{y}^{[2nd]} := \left[\gamma_t^{[0]} \ \gamma_{t+1}^{[0]} \ \dots \ \gamma_\infty^{[0]} \ \dots \ \gamma_t^{[2^{N_X}]} \ \gamma_{t+1}^{[2^{N_X}]} \ \dots \ \gamma_\infty^{[2^{N_X}]} \right]',$$

and the elements of the sequence of functionals in the RHS of Equation (3.13) in the

infinite-dimensional vector function

$$\mathbf{F}^{[2n\partial]} := \left[\gamma_t^{[0 \text{ Cond}]} \gamma_{[t,t+1]}^{[0 \text{ Cond}]} \dots \gamma_{[t,\infty)}^{[0 \text{ Cond}]} \dots \gamma_t^{[2^{N_X} \text{ Cond}]} \gamma_{[t,t+1]}^{[2^{N_X} \text{ Cond}]} \dots \gamma_{[t,\infty)}^{[2^{N_X} \text{ Cond}]} \right]'.$$

We can then write Equations (3.14) and (3.13) as

$$\mathbf{y}^{[1\mathfrak{s}t]} = \mathbf{F}^{[1\mathfrak{s}t]}(\mathbf{y}^{[1\mathfrak{s}t]}),$$

solved by the vector $\mathbf{y}^{[1\mathfrak{s}t]} = \mathbf{y}^{[1\mathfrak{s}t]*}$, and

$$\mathbf{y}^{[2n\partial]}(\mathbf{y}^{[1\mathfrak{s}t]*}) = \mathbf{F}^{[2n\partial]}(\mathbf{y}^{[1\mathfrak{s}t]*}, \mathbf{y}^{[2n\partial]}),$$

respectively.

3.4 CoCo with market-based conversion trigger

In this section we illustrate the absence of a unique competitive equilibrium for the market of a CoCo with market-based conversion trigger, and how the market can reach a unique Nash equilibrium. In Section 3.4.1 we discuss some experimental results in Davis et al. (2014) predicted by our model. In Section 3.4.2 we show how the market considered by Sundaresan & Wang (2015) adopt to illustrate the pricing problem can reach a unique Nash equilibrium.

3.4.1 Experimental results of Davis et al. (2014)

Davis et al. (2014) implement experiments that highlight the informational and allocative inefficiencies for contingent capital with market-based conversion mechanism and the cross-sectional heterogeneity about the likelihood of its conversion across investors. The experiments are conducted in a simplified market with only one security and with investors valuing it differently. We now interpret their results in light of the Nash equilibrium introduced in Section 3.3.

On the basis of the reasoning reported in the proof of Proposition 1, in case of a negative wealth transfer our model predicts the following scenarios. If the investors agree upon the existence of an equilibrium without conversion, only this equilibrium occurs. Differently, if some investors value the security in accordance to a unique conversion equilibrium, they trigger conversion if they have sufficient market power. This is the case, for example, in the presence of inefficiencies in the

asset allocation and/or the price discovery that prevent learning the existence of an equilibrium without conversion. Figure 4 of Davis et al. (2014) confirms these predictions. First, from Panel **c** there is never conversion when all the investors consider a equilibrium without conversion to be feasible. Second, Panels **a** and **b** of the same figure indicate that there are information inefficiencies in the price discovery, which create a range of values where just some investors believe that the unique equilibrium is the one with conversion. From Panel **c** we indeed observe an occasional conversion in this range.

3.4.2 A numerical illustration in Sundaresan & Wang (2015) revisited

Sundaresan & Wang (2015) illustrate the lack of a unique competitive equilibrium price for a CoCo with market-based trigger by means of numerical examples. We now consider one of these examples as just a partial description of the market described in Section 3.2. We complete the description by specifying the asset demand mechanism, and show that in such a case there is a unique Nash equilibrium price.

At the trading date $\tilde{t} < 0$, a bank is entirely capitalized by just one non-dividend paying equity share, and the vector $X_{\tilde{t}}$ explains the systematic variation of the equity returns. At the trading date 0 the bank issues a coupon bond and a zero-coupon CoCo that matures at the trading date $T > 0$. The CoCo contract particulars state that, if at any trading date $t \leq T$ the equity market price is above the fixed value $\mathcal{T} > 0$, the bank's capital structure remains unchanged. Differently, if at that time the equity market price is below or equal to $\mathcal{T}$, the bank's convertible debt is wiped out and the equity holders are penalized. The equity and CoCo holders evenly split the bank's ownership. The equity and the CoCo are then assets characterized by the property illustrated in Assumption 1, with the change in the kind of cash-flows happening at the CoCo conversion.

If the equity already issued at the trading date 0 always ensured the complete bank's ownership, and the CoCo did not convert, the equity and CoCo would offer the payoff functions E_t and C_t at the trading date t , respectively, whose functional form is inarguable. These functions originate the reference payoff functions introduced at the beginning of Section 3.2.3. All the investors believe that the risk factors that affect the probability of conversion are contained in $Y_t^{[k]}$. We denote the competitive equilibrium pricing functions for equity and the CoCo at 0 and $t \leq T$ by the pairs of functions $\hat{P}_{E,0}(k), \hat{P}_{C,0}(k)$ and $\hat{P}_{E,t}(k), \hat{P}_{C,t}(k)$, respectively. A few restrictions constrain them. First, the CoCo conversion is possible, so

that $E \left[\mathbf{1}_{\{\hat{P}_{E,t}(K_0) > \mathcal{T}\}} \middle| \mathcal{F}_0 \cup \{K_0 = k\} \right] \in (0, 1)$ for any k and the indicator function " $\mathbf{1}_{\{\cdot\}}$ ", and where the last set contains real numbers. Second, the bank's ownership remains in the hands of equity and CoCo holders, so that the tradable wealth $V_0 := \hat{P}_{E,0}(k) + \hat{P}_{C,0}(k)$ and $V_t := \hat{P}_{E,t}(k) + \hat{P}_{C,t}(k)$ are such that $V_0 = E[V_t | \mathcal{F}_0 \cup K_0]$. From Equation (3.2) the conditional private valuation of the equity and the CoCo are

$$\begin{cases} \hat{P}_{E,0}(k) = E \left[\vec{m}_t(K_0)' \vec{P}_{E,t} \middle| \mathcal{F}_0 \cup \{K_0 = k\} \right], \\ \hat{P}_{C,0}(k) = E \left[\vec{m}_t(K_0)' \vec{P}_{C,t} \middle| \mathcal{F}_0 \cup \{K_0 = k\} \right], \end{cases} \quad (3.15)$$

respectively, for the inarguable bi-dimensional payoff vector functions $\vec{P}_{E,t} := [E_t \ 0.5V_t]'$ and $\vec{P}_{C,t} := [C_t \ 0.5V_t]'$ and for the SDF vector function $\vec{m}_{t+\Delta}(K_t)$ defined in Equation (3.3) that particularized to this example is bi-dimensional with

$\mathbf{1}_{\{K_t=0\}} = \mathbf{1}_{\{V_t(K_0) > 2\mathcal{T}\}} + \mathbf{1}_{\{V_t(K_0) \in (\mathcal{T}, 2\mathcal{T})\}} \mathbf{1}_{\{\hat{P}_{E,t}(K_0) > \mathcal{T}\}}$ and $\mathbf{1}_{\{K_t=1\}} = \mathbf{1}_{\{V_t(K_0) \leq \mathcal{T}\}} + \mathbf{1}_{\{V_t(K_0) \in (\mathcal{T}, 2\mathcal{T})\}} \mathbf{1}_{\{\hat{P}_{E,t}(K_0) \leq \mathcal{T}\}}$. The private valuation functions $\hat{P}_{E,0}(k)$ and $\hat{P}_{C,0}(k)$ derive from a description of the CoCo conversion triggering mechanism by means of the pricing function $\hat{P}_{E,t}(k)$, because this last function is an input for the indicator functions $\mathbf{1}_{\{K_t=0\}}$ and $\mathbf{1}_{\{K_t=1\}}$. Consequently, System (3.15) does not have a unique solution. For example, any function $\mathcal{V}_0$ such that $\mathcal{V}_0(k) = E[V_t(K_0) | \mathcal{F}_0 \cup \{K_0 = k\}]$ makes both the quadruplet

$$\{\mathcal{V}_0(k)/2, \mathcal{V}_0(k)/2, V_t(k)/2, V_t(k)/2\}$$

and the quadruplet

$$\{\mathcal{V}_0(k)/2 + \delta_0(k), \mathcal{V}_0(k)/2 - \delta_0(k), V_t(k)/2 + \delta_1(k), V_t(k)/2 - \delta_1(k)\}$$

be admissible solutions of type $\{\hat{P}_{E,0}(k), \hat{P}_{C,0}(k), \hat{P}_{E,t}(k), \hat{P}_{C,t}(k)\}$, where

$$\delta_0(k) := E \left[\mathbf{1}_{\{V_t(K_0)(X_t) > K\}} \middle| \mathcal{F}_0 \cup \{K_0 = k\} \right] K/2$$

and $\delta_1(k) := \mathbf{1}_{\{V_t(k) > K\}} \mathcal{T}/2$. For the former solution we have

$$\mathbf{1}_{\{V_t(k) \in (\mathcal{T}, 2\mathcal{T})\}} \mathbf{1}_{\{\hat{P}_{E,t}(k) \leq \mathcal{T}\}} = \mathbf{1}_{\{V_t(k) \in (\mathcal{T}, 2\mathcal{T})\}},$$

while for the latter

$$\mathbf{1}_{\{V_t(k) \in (\mathcal{T}, 2\mathcal{T}]\}} \mathbf{1}_{\{\hat{P}_{E,t}(k) > \mathcal{T}\}} = \mathbf{1}_{\{V_t(k) \in (\mathcal{T}, 2\mathcal{T}]\}}.$$

The same pair of functions $\{V_0, V_t\}$ may then lead to different solutions. The variable δ_1 represents the wealth that is transferred at the CoCo conversion from the equity holders to the CoCo holders (see, for example, Sundaresan & Wang (2015)). The variable δ_0 represents the price discount for the anticipation at the trading date 0 of this wealth transfer. The equity holders, if penalized by the wealth transfer at the CoCo conversion, favor maximized δ_0 and δ_1 . All in all, depending on the choice of the value δ_0 , different solutions are admissible.

We now adapt our discussion to a setting that Sundaresan & Wang (2015) adopt to illustrate the pricing problem. The logarithmic return of the sum of the firm's assets explains the variation of the expected returns. It follows the Cox, Ross & Rubinstein (1979) discrete-time approximation of the Black & Scholes (1973) model. The CoCo matures in 5 years, so that $T = 1260$, counting 252 business days per year. The logarithmic return on total assets is an independent and identically distributed Bernoulli trial. This trial takes either value $\ln[u]$, with probability $p := (\exp[rh] - d) / (u - d)$, or value $\ln[d]$, with parameters $u := \exp[\sigma\sqrt{h}]$ and $d := \exp[-\sigma\sqrt{h}]$, for constant risk-free rate $r = 3\%$, volatility σ annualized to 4% and $h = 5/T \approx 0.004$. The CoCo does not pay any coupon, has USD5 as face value and an equity trading price of USD1 as conversion trigger. At the CoCo conversion, its holder gets 5 new equity shares. As in the Sundaresan & Wang (2015)'s setting, the bank issues non-convertible debt with USD87 and 3.34% as face value and coupon rate, respectively. The resulting firm's capital structure does not alter our discussion in Section 3.4.2. The total market prices of the firm's assets at the initial trading date is USD100, and bankruptcy costs sum up to 10% of the total market value of the firm's assets at default.

We now add a characterization of the equity market price at T . We consider the case $N_X = 2$ and a real scalar asset demand factor $Y_t^{[k_t^*]}$ given in Definition 1 such that under the risk-neutral probability measure we have

$$-\log \left[1 + \xi \left(Y_T^{[k_T^*]}, Y_T^{[-k_T^*]} \right) \right] \Big| Y_T^{[-k_T^*]} \stackrel{i.i.d.}{\sim} \text{Logistic} \left(\mu^{[M]}(Y_T^{[-k_T^*]}), \sigma^{[M]}(Y_T^{[-k_T^*]}) \right),$$

for functions $\mu^{[M]} : \mathbb{R} \rightarrow \mathbb{R}$, $\sigma^{[M]} : \mathbb{R} \rightarrow \mathbb{R}_+$ and $\xi : \mathbb{R}^2 \rightarrow (-1, \infty)$. The event " $\{\text{NU}\}$ " represents the lack of a unique determination of the equity market price at

T. Conditionally on this event, the CoCo converts if

$$-\log \left[1 + \xi \left(Y_T^{[k_T^*]}, Y_T^{[-k_T^*]} \right) \right] \leq T^{[M]}(Y_T^{[-k_T^*]}),$$

for the function $T^{[M]} : \mathbb{R} \rightarrow \mathbb{R}$ that is implicitly defined by

$$\begin{cases} T^{[M]}(Y_T^{[-k_T^*]}) = \mu^{[M]}(Y_T^{[-k_T^*]}) - \sigma^{[M]}(Y_T^{[-k_T^*]}) \log \left[1 + \xi \left(T^{[M]}(Y_T^{[-k_T^*]}), Y_T^{[-k_T^*]} \right) \right] \\ 1 + \xi \left(T^{[M]}(Y_T^{[-k_T^*]}), Y_T^{[-k_T^*]} \right) = 5 \mathbb{E} \left[[1 - 1]' \vec{P}_{E,T} \middle| Y_T^{[-K_T]} \cup \{K_T = k_T^*\} \right]. \end{cases}$$

Let us now consider the condition when the kind of cash-flows of the equity and the CoCo are not defined because of just the uncertainty induced by $Y_T^{[k_T^*]}$. In this condition, the vector of the risk-neutral conditional probabilities for the functional forms of the equity payoff given $Y_T^{[-k_T^*]}$ is

$$\left[1 - \left(1 + \xi \left(T^{[M]}(Y_T^{[-k_T^*]}), Y_T^{[-k_T^*]} \right) \right)^{-1} \quad \left(1 + \xi \left(T^{[M]}(Y_T^{[-k_T^*]}), Y_T^{[-k_T^*]} \right) \right)^{-1} \right]',$$

and the market price for the equity at 0 is

$$\hat{P}_{E,0}(k) = \mathbb{E} \left[\vec{m}_T(K_0)' \vec{P}_{E,T} \middle| Y_0^{[-K_0]} \cup \{K_0 = k\} \right].$$

The initial lowest ask quote $\hat{P}_{E,0}(k_0^{[a,0]})$ and the highest bid quote $\hat{P}_{E,0}(k_0^{[b,0]})$ are sequentially adjusted according to

$$\hat{P}_{E,0}(k_0^{[l,j]}) = \mathbf{1}_{[j-1]} \hat{P}_{E,0}(k_0^*) + \left(1 - \mathbf{1}_{[j-1]} \right) \left(\hat{P}_{E,0}(k_0^{[l,j-1]}) + \epsilon^{[l,j-1]} \right), \quad \mathbf{l} = \mathbf{a}, \mathbf{b},$$

for

$$\mathbf{1}_{[j-1]} := \mathbf{1}_{\left\{ \left| \hat{P}_{E,0}(k_0^{[l,j-1]}) - \hat{P}_{E,0}(k_0^*) \right| \leq 0.01 \right\}}$$

and variables $\epsilon^{[a,j-1]} \stackrel{i.i.d.}{\sim} \mathcal{U}(-0.01, 0)$ and $\epsilon^{[b,j-1]} \stackrel{i.i.d.}{\sim} \mathcal{U}(0, 0.01)$.

We plot in Figures 3.1a-3.1d the competitive equilibrium prices for equity and the CoCo (included by dashed lines), and their Nash equilibrium prices (denoted by solid lines), as functions of either volatility, non-convertible debt par value, CoCo par value or CoCo conversion trigger value, respectively. On the left of each panel, the lowest values are for the CoCo.

Figure 3.1. Competitive equilibrium prices (included between dashed lines) and Nash equilibrium prices (denoted by solid lines) for equity and the CoCo in the numerical experiment of Section 3.4.2. The values are expressed as functions of asset volatility (Panel a), leverage in terms of non-convertible debt (Panel b) and the CoCo (Panel c), and the CoCo conversion trigger level (Panel d). On the left side of each panel, the values for the CoCo are lower than those for the equity.

3.5 Conclusion

We characterize a dynamic Nash equilibrium in double auction markets for assets whose cash-flows are not in a unidirectional causal relationship with their future market price. Assets of this kind are CoCos with market-based conversion trigger, equity in market capitalization-based indexes, and other financial assets that are affected by market-based managerial decisions. Our conditions require that investors either know or learn from quotes and trades the risk factors that impact the asset demand and probability of a change in cash-flows. Our framework can be applied to every asset that is not a Giffen good, with any kind of cash-flows. It is not restric-

ted to any trading frequency, any asset demand mechanism, or any dynamics of the risk factors that directly affect the probability of a change in the kind of cash-flows of the assets. It accommodates any direction of the unanticipated wealth transfer at the change of the kind of cash-flows.

We leave to future research the empirical application of our framework to specific cases. For example, it would be interesting to test some structural implications for market equilibria of equities included in market capitalization-based indexes, to gain insight on the index inclusion effect. We also leave to future research the extension of our framework to markets with discriminatory prices.

3.A Appendix A

We first prove the proposition in the case the class-representative agents know the identity of the risk factors that truly affect the probability of a change in the kind of the cash-flows of the asset. Then, we extend the proof to the case of agents learning this identity from quotes and market prices.

Representative agents knowing the relevant risk factors Let us consider the asset holder agent representing the k -th class of investors. She sets the valuation $\hat{P}_t(k_t^*) \in \mathcal{W}_l$ at the trading date t , and then focuses on her perspective about the valuation $\hat{P}_{\tilde{t}}(k_t^*) \in \mathcal{W}_{\tilde{l}}$ at the trading date $\tilde{t} > t$. At that date, there is a wealth transfer of the type described in Assumption 1 if $\mathcal{W}_l \neq \mathcal{W}_{\tilde{l}}$ and an event of the type in Expression (3.4) realizes.

- Let us first consider the case of a penalization for the asset holder agent at the change of the kind of cash-flows of the asset. Under our convention, this is for a wealth transfer with negative sign.

If the demand for the asset at the price level $\hat{P}_{\tilde{t}}(k_t^*)$ is relatively low and such that the market does not clear, the valuation $\hat{P}_{\tilde{t}}(k_t^*)$ drops. In this way, the valuation adjusts to the demand, and in turn the demand increases. A sufficient drop in the valuation guarantees market clearing. If the only sufficient drop is such that $\mathcal{W}_{\tilde{l}} \neq \mathcal{W}_l$, a unique Nash equilibrium price in the form of Definition 5 exists at the trading date $\tilde{t}$. This equilibrium is characterized by the corresponding change in the kind of the cash-flows of the asset from the trading date t to the trading date $\tilde{t}$. At such a change, the asset holder gains from liquidity more than what she loses from the wealth transfer.

Differently, the demand for the asset at the price level $\hat{P}_{\tilde{t}}(k_t^*)$ may be enough for market clearing and preventing any change in the kind of the cash-flows of the asset. The utility of the asset holder agent decreases in case $\mathcal{W}_l \neq \mathcal{W}_{\tilde{l}}$, because of the penalization at the change in the kind of the cash-flows of the asset. Moreover, under Assumptions 3 and 4 the utility function of each class-representative agent is locally continuous at a level of the asset trading price that, if crossed, induces a change in the kind of cash-flows of the asset. Therefore, under Assumptions 3, 4 and 5, there exists a non-negative real value $\epsilon_{\tilde{t}}(k_t^*)$ that (i) the k -th class-representative agent is willing to pay to avoid a change in the type of the asset cash-flows, and (ii) such that $\hat{P}_{\tilde{t}}(k_t^*) - \epsilon_{\tilde{t}}(k_t^*)$ does still not correspond to a change in the kind of cash-flows of the

asset. We can interpret the amount $\epsilon_{\tilde{t}}(k_i^*)$ as a liquidity premium that the k -th class-representative agent offers to another class-representative agent in order to receive support in avoiding the change in the kind of cash-flows of the asset. Alternatively, this amount can be interpreted as a form of individual market power against any other investor w.r.t. the asset. Alongside this fact, under Assumption 1 some investors are not interested by any wealth transfer at a change in the kind of cash-flows of the asset. Consequentially, under Assumption 2, there must exist a class of investors, created on the basis of their market power w.r.t. the asset and contributions to the asset demand, such that the corresponding class-representative agent is indifferent between the distinct equilibria. This class-representative agent prefers to cash-in the liquidity premium $\epsilon_{\tilde{t}}(k_i^*)$. The asset holder agent representing the k -th class of investors knows this fact, so that she knows that she receives support to avoid the change in the type of the asset cash-flows. Consequently, the market equilibrium characterized by $\mathcal{W}_l \neq \mathcal{W}_{\tilde{l}}$, that is for a change in the type of the asset cash-flows, contrasts the Nash equilibrium in Definition 5. In such a case, the expected lifetime utilities are indeed not maximized. Hence, a unique Nash equilibrium price of the form of Definition 5 exists at the trading date $\tilde{t}$, and it is characterized by no change in the kind of cash-flows of the asset.

- Let us now consider the case of a gain for any asset holder agent at the change of the kind of cash-flows of the asset, that is, under our convention, for a wealth transfer with positive sign.¹⁷ When any asset holder agent commits to not trading the asset, the expected lifetime utilities of all the class-representative agents do not increase. These agents indeed do not have the possibility to reallocate the part of their wealth already allocated in the asset. They cannot optimally rebalance their portfolios. They do not actively contribute to the asset liquidity and the generation of its trading price. Abstaining from trading the asset is particularly costly for the asset holder agent with the lowest individual market power against the other class-representative agents w.r.t. the asset.

We may have a first case where the individual market power w.r.t. the asset of the asset holder agent is lower than the highest individual market power of an agent for a class of potential buyers. Without loss of generality, let us consider that this asset holder agent represents the k -th class of investors.

¹⁷ In this case there exists no competitive equilibrium for the market.

Under Assumptions 3, 4 and 5, there exists a non-negative real value $\epsilon_{\tilde{l}}(k_{\tilde{l}}^*)$ that (i) the asset holders are willing to pay to induce a change in the type of the asset cash-flows, and (ii) such that $\hat{P}_{\tilde{l}}(k_{\tilde{l}}^*) - \epsilon_{\tilde{l}}(k_{\tilde{l}}^*) \in \mathcal{W}_{\tilde{l}}$ with $\mathcal{W}_l \neq \mathcal{W}_{\tilde{l}}$. We can interpret the amount $\epsilon_{\tilde{l}}(k_{\tilde{l}}^*)$ as a liquidity premium that the asset holder agent with the lowest individual market power offers to the other investors in order to induce a change in the kind of cash-flows of the asset. At that change, the asset holder agent cashes-in the related wealth transfer. Alternatively, this amount can be interpreted as a form of individual market power against any other investor w.r.t. the asset. However, both the value $k_{\tilde{l}}^*$ and the utility function for the k -th class-representative agent are known by all the other class-representative agents. Then, from Assumption 1 there must be a class-representative agent willing to buy the asset at a discount. In turn, this fact raises the asset demand. All the class-representative agents agree upon this mechanism and reach the same valuation, which in turn corresponds to the market clearing price.

Alternatively, we may have a second case where the individual market power w.r.t. the asset of the asset holder agent is not lower than the individual market power of any agent for a class of potential buyers. In this case, there is no change in the kind of the cash-flows of the asset and there is possibly no trading.

In addition, for each $\tilde{t} > t$ from Equations (3.5) and (3.7) and Assumptions 6 and 9 the valuation of the financial instruments except for the asset is obtained through the SDF $m_{\tilde{t}}(k_{\tilde{t}}^*)$ defined in Equation (3.6).

Since k_t^* is known, the class-representative agents can determine the probability of a change in the type of the asset cash-flows. As the reasoning applies to any $\tilde{t}$, we have that at the trading date t there is a unique Nash equilibrium for any subsequent period. This implies that the vector $\vec{m}_{\tilde{t}}(K_t)$ and the scalar $m_{\tilde{t}}(K_t)$ are well-defined and consequently that the trading prices $\hat{P}_t(k_t^*)$, $\hat{S}_{j,t}(k_t^*)$, and $\hat{B}_{l,t}(k_t^*)$ are unique.

Representative agents learning the relevant risk factors From the first part of the proof, in the special case in which $k = k^*$, the Nash equilibrium introduced in Definition 5 is unique. A trade only takes place when all the class-representative agents agree on the valuation of the asset, and this is the essential idea behind the definition of the positive integer j_l^* in Equation (3.12). To prove the result, we have to show that the private valuations set by the class-representative agents converge

at each trading date. By Assumption 5 a class-representative agent uses her own individual market power against any other investor to make the other investors update their private valuations toward hers, to make any perceived arbitrage opportunity coming from a mismatch between private valuations concrete. This behavior is represented by the Point (i) of Definition 3. However, the class-representative agents also want to actively contribute to the asset liquidity and the generation of its trading price. From Point (iii) of Definition 3 the global maximization of the criterion $Q_{[t,\infty)}^{[i]}$ is unique for any $i = 0, \dots, 2^{N_X}$.

Let us first consider the case when the maxima of all the criteria $Q_{[t,\infty)}^{[i]}$ for any $i = 0, \dots, 2^{N_X}$ do not correspond to market clearing. In this case, the class-representative agents do not actively contribute to the asset liquidity and the generation of its trading price, and cannot rebalance their portfolios. Since the market does not clear, the asset demand is either relatively low or high. If it is relatively low, there is at least one representative agents for a class of asset holders willing to sell the asset at a discount in order to rebalance her portfolio. Differently, if the asset demand is relatively high, any asset holder has the incentive to raise the asset trading date, by raising the ask quote she sets. Therefore, if the maxima of the criteria $Q_{[t,\infty)}^{[i]}$ for any $i = 0, \dots, 2^{N_X}$ do not correspond to market clearing, they do not jointly maximize the expected utilities of the class-representative agents.

Let us now consider the case when there is an integer value $i \in \{0 : 2^{N_X}\}$ such that the maximum for the criterion $Q_{[t,\infty)}^{[i]}$ is attained with the i -th class-representative agent assuming that the vector $Y_t^{[k_t^{[i]}]}$ includes the risk factors that affect the probability of a change in the type of the asset cash-flows, with $k_t^{[i]} \neq k_t^*$. From the first part of the proof and Assumptions 7 and 8 the i -th class-representative agent does not necessarily maximize her expected lifetime utility from consumption, because she does not necessarily choose her consumption-investment strategy on the basis of the correct optimization problem, which is the problem that she should correctly focus on. In general, she does not surely choose the strategy that maximizes her expected utility from consumption. Hence, the i -th class-representative agent assumes the same risk factors as the other class-representative agents as the determinants of the probability of a change in the type of the asset cash-flows, that is $k_t^{[i]} = k_t^*$ for any $i = 0, \dots, 2^{N_X}$ and at any trading date t . Only under this valuation match each class-representative agent correctly values the asset, chooses her optimal portfolio weights, and consumes optimally.

Chapter 4

Risk incentive compatible bonds

4.1 Introduction

Excessive risk taking is a classic problem in finance. The convex payout structure of equity leads to risk-taking incentives for equity holders. Profits accrue to the equity holders while losses are shared with bond holders, making it attractive for firms to choose riskier projects (see e.g. Jensen & Meckling 1976 and Galai & Masulis 1976). This problem has been one of the causes of the financial crisis (Admati et al. 2013). Banks took large risks, forcing governments to bail these banks out to save them, which has resulted in high costs for society. These bailouts, which were anticipated, along with deposit guarantee schemes, provided a further incentive for banks to engage in excessive risk taking (see respectively Acharya & Yorulmazer 2007, Demirguc-Kunt & Huizinga 2004 and Dam & Koetter 2012). Laeven & Levine (2009) find that banks with stronger control from equity holders take more risk, illustrating that equity holders actively try to increase risk taking in banks. Due to the major consequences for society, there is a call in the literature to solve this problem (see Admati et al. 2013).

The main contribution of this chapter is the introduction of a, to the best of my knowledge, novel debt contract that is able to mitigate risk-taking incentives in a

This chapter is based on Vullings (2020).

multi-period setting. When a firm issues risk incentive compatible (RIC) bonds, the incentives of the equity holders and bond holders are aligned. RIC bonds work like regular bonds complemented with equity or coupon payments in states of the world where the firm has a high cash flow from their investments. These stochastic payments limit the upside of the cash flow for the equity holders of the firm and by extension the risk-taking incentives. Firms can then use this contract to finance investments while credibly signaling that they will not engage in excessive risk taking. This reduces the cost of debt and makes RIC bonds attractive for firms to issue. RIC bonds do not rely on regulatory intervention and are consequently feasible for both regulated and unregulated firms.¹

In this chapter an optimization model is used to analyze what contract, ranging from regular debt and RIC bonds to different types of Contingent Convertible bonds (CoCos) to regular equity, is optimal for the firm to issue in the presence of excessive risk taking and risk-averse investors. In a setting where investors are more risk averse than the original equity holders, I show that RIC bonds can be the optimal financing strategy for the firm. When investors are risk averse, contracts with payouts with a given expected return that have a low variance, are preferable over equity financing. However, when there are bankruptcy costs and the firm can increase the risk of its investments, this preference can be reversed. The expected loss due to bankruptcy costs can, consequently, outweigh the benefits of the low variance in the payments. Hébert (2017) finds that while in the absence of the moral hazard problems considered in this paper firms should be mostly debt financed, a mixture of debt and equity is required in case of severe moral hazard problems. RIC bonds are a hybrid between debt and equity that can fulfill this role as they yield fixed payments to investors complemented with random payments when the cash flow of the firm is high. When the additional payments to investors, in case the firm has a high cash flow, are lower than the additional costs of equity financing over debt financing, RIC bonds are an attractive financing option for firms. I provide a numerical example to illustrate that RIC bonds can be the cheapest financing strategy. This is the case when the firm is exposed to increased risk-taking incentives and there are differences in risk preferences between the original equity holders and investors.

¹One can wonder whether a combination of existing financial instruments can be used to replicate RIC bonds. A firm can with either convertible bonds or call options mimic the payoffs of RIC bonds. However, a crucial element of RIC bonds is that firms contractually commit to provide a specific cash flow to the investors. After the issuance of the contract, firms prefer to deviate. The contract maximizes the value for the equity holders only when the contract is issued. Using convertible bonds or call options will therefore not work, as the firm cannot credibly commit to selling these derivatives in subsequent periods.

The difference in risk preferences represents that if some investors have a bigger appetite for risk than others, the risk-loving investors can create value by selling a claim to a cash flow that has a low variance to the risk-averse investors (see Hébert 2017). Without this mechanism, only issuing equity would be a solution to the model as in this case all investors have the same risk appetite and having an all equity firm mitigates excessive risk-taking incentives.

The model I present considers an optimization problem where the objective is optimized over contract characteristics. I allow the firm to issue regular debt, a variety of CoCos, regular equity and RIC bonds. I show that there are circumstances where RIC bonds are the optimal contract. Given the finding that RIC bonds can be effective at mitigating excessive risk-taking incentives when the original equity holders are less risk averse than investors, future research in these new bonds is warranted.

RIC bonds generalize the idea of Green (1984) to use convertible bonds to reduce risk-taking incentives. Dutordoir et al. (2014) review the literature and note that there is no evidence that firms use convertible bonds to reduce risk shifting. A downside of convertible bonds is that they cannot adjust to changing circumstances. RIC bonds solve this problem and give incentives to the firm to limit risk taking accounting for the conditions in each period.²

An alternative proposal to stabilize the financial system involves Contingent convertible bonds (CoCos) (see Flannery 2005, 2014). CoCos are issued as debt, but convert automatically to equity when the firm is in financial distress. The change in capital structure that results from conversion can be anticipated by equity holders and alters their risk-taking incentives.³

Another proposed solution is to use management compensation to limit risk-taking incentives, e.g. K. John et al. (2000) and Eufinger & Gill (2017). However, this literature typically assumes that management compensation can be perfectly observed by regulators. Eufinger & Gill (2017) find that 50% of banks have some

²Dutordoir et al. (2014) note that there is mixed evidence that firms issue convertible bonds due to uncertainty on the riskiness of the firm, see also Brennan & Kraus (1987) and Brennan & Schwartz (1988). Although I focus on risk shifting, risk uncertainty due to asymmetric information is captured in the model as well. RIC bonds are only attractive to issue when the firm has a specific risk level, making the issuance of these bonds a credible signal for the true risk level.

³RIC bonds share similarities with CoCos in the sense that both depend on a trigger and both involve shares of equity being issued to bond holders. The main difference is that the idea behind CoCos is to reduce the level of debt in times of distress, while RIC bonds force equity holders to take less risk. Hence, RIC bonds and CoCos lower the riskiness of firms using different mechanisms. RIC bonds are a precautionary tool to prevent firms from increasing the risk in times of distress, while CoCos are designed to recapitalize a firm that is in distress. Both can, therefore, coexist and target different problems of the financial sector.

form of discretionary performance based payment. The consequence of this imperfect observation of management compensation on the effectiveness of using management compensation to limit risk-taking incentives, is not studied. Furthermore, Jenter & Kanaan (2015) report that CEOs are fired for things out of their control. When equity holders prefer higher risk taking but reward management with a compensation structure that gives an incentive to limit risk taking, the equity holders have an incentive to either fire management and hire a new management that they incentivize to take more risk or to otherwise influence management to increase risk taking. Furthermore, ownership and management is not always separated. Management compensation schemes are infeasible for these firms as the owner cannot be rewarded with long term debt or payments that are linked to their risk taking.

Currently, debt covenants are widely used to limit risk taking of firms (see Chava & Roberts 2008). Healy & Palepu (1990) find that covenants can be effective, while Anderson, Mansi & Reeb (2004) stress the importance of a neutral board for the reliability of covenants. Nikolaev (2010) argues that a downside of covenants is that lenders set debt covenants too tight to gain control of firms. Leland (1994) uses a theoretical model that shows that, in the presence of bankruptcy costs and with risky debt, the value of debt decreases due to debt protecting covenants. The reason is that these protective covenants increase the probability of default. So, although debt covenants can help to reduce risk taking, the increased probability of default can reduce the value of debt. RIC bonds resolve this problem by making the original equity holders pay in good times instead of bad times. The risk-taking incentives are then mitigated while firms in financial distress are not more likely to default.

Small firms have received little attention, although these firms are effected by excessive risk-taking incentives as well. The equity holders of these firms cannot commit to limit their risk taking and as a result might not get a loan or face high costs to acquire additional capital. A contract that these small firms can use to commit to limit their risk taking can increase the number of positive net present value projects that can be financed. Furthermore, the lower financing costs make it attractive for the equity holders of small firms to issue RIC bonds.

The remainder of the chapter is structured as follows. Section 4.2 provides an overview of the relevant literature. Section 4.3 presents a discrete time model for which I derive the main results of the model and provides a numerical illustration of these results. Section 4.4 discusses policy implications of RIC bonds. Section 4.5 concludes.

4.2 Literature review

An extensive analysis on firm financing under excessive risk taking and differences in risk preferences is done in Hellwig (2009) and Hébert (2017). They find that a mix between debt and equity is optimal when risk shifting is without cost for the firm. Two other major branches of the literature that consider risk-taking incentives by firms are initiated by Green (1984) who proposes to use convertible bonds and T. A. John & John (1993) and K. John et al. (2000) who look at management compensation.

Green (1984) proposes to use convertible bonds to mitigate risk-taking incentives. He shows in a one-period model that equity holders can be given an incentive to limit their risk taking by limiting the upside of their returns. Ozerturk (2008) and Dorion, François, Grass & Jeanneret (2014) show the effectiveness of using convertible bonds to limit risk taking. However, Dutordoir et al. (2014) review the literature on convertible bond financing and find no evidence that firms issue convertible bonds to reduce risk shifting. However, there is mixed evidence that convertible bonds are used to reduce the problem of asymmetric information on firm risk (Brennan & Kraus 1987 and Brennan & Schwartz 1988).

T. A. John & John (1993) propose to use the management compensation structure to reduce risk-taking incentives and find that equity holders have an incentive to incentivize management to reduce risk taking in order to reduce the cost of debt. Several papers call for regulators to limit management bonuses of banks to achieve this.⁴ K. John et al. (2000) and Bolton, Mehran & Shapiro (2015) propose incentive schemes that are aimed to reward the management with debt like compensation to reduce risk-taking incentives of managers.

K. John et al. (2000) recognize the problem of deteriorating financial conditions for the effectiveness of using convertible bonds or management compensation. They consider banks and propose more frequent adjustments of the deposit insurance premium or penalties for changes in the management compensation structure. The first proposal results in the unfortunate problem that in times of financial distress, when the incentives to increase risk are highest, firms have to pay higher coupon payments, thus further increasing the probability of default. Furthermore, investors do not want higher coupons in times of severe financial distress as shown in Leland (1994).

Penalizing changes in the reward of the management is effective as long as there

⁴ See e.g. Thanassoulis (2012), Hakenes & Schnabel (2014), Bebchuk & Spamann (2010) and Freixas & Rochet (2013).

are no other ways for the equity holders to pressure the management to increase the riskiness of the firm. Eufinger & Gill (2017) propose to employ higher capital requirements for banks that have management compensation structures that give higher incentives to increase risk taking. Srivastav & Hagendorff (2016) find evidence that banks that reward management with more debt like instruments do take less risk. Both K. John et al. (2000) and Eufinger & Gill (2017) stress the necessity of transparency in management compensation in order to effectively implement these proposals to limit the risk taking of firms.

Hilscher & Raviv (2014), Berg & Kaserer (2015) and Martynova & Perotti (2018) consider the risk-taking incentives of CoCos and find that although CoCos can be effective in mitigating risk-taking incentives, there is a disagreement on what design is most suitable to achieve this. Currently issued CoCos have mostly book triggers (see Avdjiev et al. 2015), which are known to be flawed in signaling financial distress (see Calomiris & Herring 2013 and Sundaresan & Wang 2015). Although market triggers are good predictors for financial distress, this trigger mechanism can result in a multiple equilibria problem (see Sundaresan & Wang 2015 and Glasserman & Nouri 2016). These shortcomings partly contribute to the lack of consensus among academics and regulators on the desirability of CoCos as micro-prudential tools (see Avdjiev et al. 2015).

The literature has mainly focused on solving the excessive risk-taking problem for major banks. A solution that (i) mitigates risk-taking incentives, (ii) does not depend on the observability of management compensation, (iii) does not increase the probability of default in times of distress and (iv) works for a general firm in a multi-period model, has not been proposed to the best of my knowledge.

4.3 Model

In this section I first discuss the main model and assumptions. Next, I discuss the main propositions of the paper. Lastly, I provide a numerical example to illustrate the main results of the model.

The model takes as starting point a firm that has decided it wants to raise a given level of capital from investors. The original equity holders maximize their expected utility by offering contracts to investors to raise this level of capital. Investors can buy different forms of (hybrid) debt or equity. When the investors buy equity it is in a form that does not give them voting rights. The outcome of the optimization problem is the contract that is optimal for the original equity holders to offer to

investors.

4.3.1 Discrete time model

I use a discrete time model with $n + 1$ periods, with $n \in \mathbb{N}$, ranging from time $t = 0$ to $t = n$. The $n + 1$ dimensional random variable $\mathbf{V}_{n+1}$ is the fundamental and contains all uncertainty in the model, where I use the notation $\mathbf{z}_n = (z_0, \dots, z_{n-1})$. V_t is observed at time t and v_t corresponds to the realization. v_t is equal to the conditional expectation of the final cash flow at time t . The distribution, CDF, of $\mathbf{V}_{n+1}$ is given by $F : \mathbb{R}_+^{n+1} \rightarrow [0, 1]$, with corresponding symmetric probability density function, PDF, $f : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+$ that is characterized by its mean, $\boldsymbol{\mu}_{n+1} \in \mathbb{R}_+^{n+1}$, and variance, $\boldsymbol{\sigma}_{n+1} \in [\boldsymbol{\sigma}_{n+1}^{\min}, \boldsymbol{\sigma}_{n+1}^{\max}]$, where $\boldsymbol{\sigma}_{n+1}^{\min} \in \mathbb{R}_+^{n+1}$, $\boldsymbol{\sigma}_{n+1}^{\max} \in \mathbb{R}_+^{n+1}$ and $\boldsymbol{\sigma}_{n+1}^{\min} \leq \boldsymbol{\sigma}_{n+1}^{\max}$. To be precise: the i -th elements of $\boldsymbol{\mu}_{n+1}$ and $\boldsymbol{\sigma}_{n+1}$ are respectively the mean and variance of V_i . For V_0 the variance is given by $\sigma_0 = 0$ and $\mu_0 = V_0$ is a constant. For V_i , with $i \in \{1, \dots, n\}$, the mean is given by $\mu_i = V_{i-1}$ and its variance is known by the original equity holders at time $t = i - 1$. Consequently at $t = i - 1$, I have that $\mu_i = v_{i-1}$. The information set used to compute the expectation of V_i is expanded every period by the realization of V_j for $j \in \{1, \dots, i - 1\}$. Hence, the conditional expectation of V_i changes every period as the conditioning information changes. With the notation $F(V_t | \mu_t, \sigma_t)$ I denote the marginal distribution of V_t conditional on the mean and variance of V_t . The date of maturity is $t = n$ and V_n is the cash flow at maturity. The final cash flow V_n can be thought of as a liquidation dividend, for example. Furthermore, for $\phi \in [0, \mu_t)$ and $\sigma_t < \tilde{\sigma}_t$ the following inequality applies

$$\int_{-\infty}^{\phi} dF(V_t | \mu_t, \sigma_t) < \int_{-\infty}^{\phi} dF(V_t | \mu_t, \tilde{\sigma}_t). \quad (4.1)$$

The interpretation of Inequality 4.1 is that when the variance increases, the left tail of the distribution gets fatter. A change in the variance is a mean preserving spread in this model (see Rothschild & Stiglitz 1970). Examples of distributions that have these characteristics are the triangular and uniform distribution. I use a distribution with these characteristics to isolate the excessive risk-taking problem. The assumptions on the distribution F are summarized in Assumption 10.

Assumption 10. *The distribution function F satisfies the condition in Inequality 4.1 and its density function is*

1. *symmetric*

2. characterized by its mean, $\mu_{n+1} \in \mathbb{R}_+^{n+1}$, and variance, $\sigma_{n+1} \in [\sigma_{n+1}^{\min}, \sigma_{n+1}^{\max}]$.

Lemma 1 captures several characteristics of distributions that satisfy Assumption 10 that are relevant for the main results of this paper. From now on $\mathbb{E}$ represents the expectation operator.

Lemma 1. *Let the CDFs of the one dimensional random variables X_1 and X_2 be given by the functions F_{X_1} and F_{X_2} , with corresponding PDFs f_{X_1} and f_{X_2} , that satisfy Assumption 10 and have parameters μ, σ_1 and μ, σ_2 respectively, where $\sigma_1 > \sigma_2$. Then (i) $\mathbb{E}[X_1|X_1 < \mu] \leq \mathbb{E}[X_2|X_2 < \mu]$, (ii) $\mathbb{E}[\max\{X_1, \alpha\}] \geq \mathbb{E}[\max\{X_2, \alpha\}]$ and (iii) $\mathbb{E}[\min\{X_1, \alpha\}] \leq \mathbb{E}[\min\{X_2, \alpha\}]$ for $\alpha \in \mathbb{R}_+$.*

Proof. See Appendix A. ■

The main friction in the model is that σ_t for $t \in \{1, \dots, n\}$ is only known to the original equity holders and unobservable for the investors. So the investors cannot directly observe σ_t . However, they can infer the value of σ_t from the contracts issued by the original equity holders. Differently, v_t for $t \in \{1, \dots, n\}$ is observed perfectly by all parties. As a result, it is not possible to write a contract on the variance of the cash flow and all contracts should instead be written on V_t for $t \in \{1, \dots, n\}$.

The model is an optimization problem that represents the choice of the optimal contract from the perspective of the original equity holders of the firm. The original equity holders are in this model the managers as well. The original equity holders need an amount V_0 to finance a project and can themselves only finance $V_0 - B_0$. As a result they need to obtain additional capital from investors with a value of B_0 . B_0 is exogenous and it is outside the scope of this paper to make B_0 endogenous and choose it optimally. The original equity holders aim to choose a contract to offer to the investors that maximizes their initial expected utility from the project. First, I will describe the range of contracts that are considered in the optimization problem informally. Next, several symbols and functions are necessary to define before introducing the equity value and the formal contract features.

Firstly, the firm can issue regular debt, where the coupons are paid at maturity along with the principal. When v_t drops below the principal value of the debt for any $t \in \{1, \dots, n\}$, the firm defaults. Secondly, the firm can issue shares of equity without voting rights to raise B_0 . In this case default cannot occur. I assume there are no dividend payments or other payouts, except for the final cash flows at time n . Thirdly, CoCos can be issued alongside debt and can convert to equity. When the CoCo does not convert, the CoCo holder receives the face value of the CoCo at maturity. When the CoCo does convert, they receive a part of the cash

flow corresponding to their share of the equity. The CoCo can only convert once. Fourthly, the firm can issue RIC bonds. RIC bond works like regular debt, except that the bondholders can obtain shares of equity when the expected value of the future cash flow exceeds a given threshold. The firm can default on RIC bonds when v_t drops below the principal value of the RIC bonds for any $t \in \{1, \dots, n\}$. The contracts considered are limited by the features described in this section. The model in this section can be extended to describe more general contracts, but that is outside the scope of this paper.

Let $B_0 \in \mathbb{R}_{++}$ and $\omega \in [0, 1]$ be given exogenous constants, $\theta_{n+1} : \mathbb{R}_+^{n+1} \rightarrow [0, 1]^{n+1}$ and $\mathbf{V}_{n+1}^* : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+^{n+1}$ are $n+1$ dimensional functions whose definition follows from the optimization problem. Furthermore, $\theta_i : \mathbb{R}_+ \rightarrow [0, 1]$ and $V_i^* : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are the i -th element of θ_{n+1} and $\mathbf{V}_{n+1}^*$ respectively for $i \in \{0, \dots, n\}$. Similarly $K \in \mathbb{R}_+$, $C_1 \in \mathbb{R}_+$, $C_2 \in \mathbb{R}_+$ and $B_n \in \mathbb{R}_+$ are constants whose value is determined by the optimization problem. $U_n^E : \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow \mathbb{R}_+$ and $U_n^B : \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow \mathbb{R}_+$ are C^1 functions representing the utility functions of the original equity holders and investors respectively.

The original equity holders want to raise $B_0 > 0$ and choose the other parameters in a way that achieves this while maximizing their expected utility. K , C_1 , C_2 and B_n only depend on the distribution of V_n at time $t = 0$ and are a threshold, the CoCo face value, the CoCo conversion value and the face value of debt respectively. B_n represents the principal that is to be repaid to the senior debt holders and is equal to the default threshold. When the firm issues a CoCo, $K > B_n$ is the conversion threshold, and will from now on be referred to as the CoCo trigger value. Conversion occurs when $v_t < K$ for any $t \in \{1, \dots, n\}$. C_1 is the size of the cash flow the investors get at maturity when the CoCo does not convert, so when the V_n never drops below K . C_2 determines the equity transferred to investors when the CoCo does convert. CoCos have to convert to equity before the firm defaults, as otherwise it is regular junior debt, which is why $K > B_n$ needs to hold.

The vector function θ_{n+1} is the ratio of the equity transferred to the investors when the benchmark indicated by the vector function $\mathbf{V}_{n+1}^*$ is surpassed. The elements of both vectors are functions of the mean of the distribution of the corresponding elements of $\mathbf{V}_{n+1}$. For example, θ_t , for $t \in \{0, \dots, n\}$ takes μ_t as argument. When $V_0^*(\mu_0) < \mu_0$ and $\theta_0(\mu_0) > 0$, the firm raises the capital they require at $t = 0$ by selling a part $\theta_0(\mu_0)$ of the total equity. When $\theta_0(\mu_0) = 0$, $V_i^*(\mu_i) > B_n$ and $\theta_i(\mu_i) > 0$ for some $i \in \{1, \dots, n\}$ the firm raises the required capital by issuing a RIC bond.

The equity cash flow consists of several elements that I first define separately. Let $DV : \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow \mathbb{R}_+$ be given by

$$\begin{aligned} DV(g, t) = & \sum_{i=t+1}^n \left(\prod_{j=t}^{i-1} \mathbb{1}_{[V_j > g]} \right) \left(\mathbb{1}_{[V_i < g]} (1 - \omega) V_i \right) \\ & + \left(\prod_{i=t}^n \mathbb{1}_{[V_i > g]} \right) g + \mathbb{1}_{[V_t < g]} (1 - \omega) V_t. \end{aligned} \quad (4.2)$$

The function DV gives the cash flow of the debt part of the investment. The first element on the right hand side, from now on RHS, of Equation (4.2) represents the cash flow given to the investors if the firm defaults at any future time. The indicator function $\mathbb{1}_{[V_i < g]}$ represents the default event, where g is the default threshold. The sum makes that all periods are considered and the product indicates that a firm can only default once. The second element represents the cash flow if the firm does not default. The third element represents the cash flow when the firm is currently in default.

Next, let $EV : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow \mathbb{R}_+$ be defined by

$$EV(c, e, g, t) = \left(\prod_{i=t}^n \mathbb{1}_{[V_i > g]} \right) (V_n - g) - \left(\prod_{j=t}^n \mathbb{1}_{[V_j > c]} \right) e. \quad (4.3)$$

This function gives the equity cash flow at maturity. The first product of indicator functions in Equation (4.3) becomes zero when $V_{\tilde{t}}$ drops below g for any $\tilde{t} \in \{t, \dots, n\}$. In this case the equity cash flow will be zero at maturity. The second product of indicators indicates whether the CoCo has converted. If both indicators are equal to one, the cash flow of the equity at maturity is the total cash flow at maturity minus the principal that belongs to the debt holders minus the face value of the CoCo. When the CoCo converts, so the second product of indicators becomes zero, the equity cash flow at maturity is the total cash flow minus the principal that belongs to the investors due to the debt part of the investment.

Thirdly, let $RD : \mathbb{R}_+^{n+1} \times \mathbb{R}_+^{n+1} \times \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow [0, 1]$ be defined by

$$RD(\mathbf{a}_{n+1}, \mathbf{b}_{n+1}, g, t) = \prod_{i=t}^n \left(1 - \mathbb{1}_{[V_i > b_i]} \cdot a_i \frac{(V_i - b_i)}{V_i - DV(g, i)} \right). \quad (4.4)$$

This function gives the dilution due to either issuing equity at $t = 0$ or issuing a

RIC bond. First, note that $V_t - DV(g, t)$, where g is the default threshold, represents the difference between the expected total cash flow and the cash flow of the debt owned by the investors. For convenience, I will from now on refer to this value as the cash flow of the equity holders. The fraction on the RHS of Equation (4.4) is then the percentage of the cash flow to equity holders that exceeds the threshold b_t . For every $\tilde{t} \in \{t, \dots, n\}$ the indicator checks whether $V_{\tilde{t}} > b_{\tilde{t}}$. If this is not true, the function RD takes a value of 1, which corresponds to no dilution. If this is true, a fraction a_t of the fraction of the cash flow of the equity holders that exceeds b_t is given to the investors. One minus this value is then the equity share that the current equity holders retain. By taking the product over every period, the dilution faced by the original equity holders in every period is accounted for.

Fourthly, let $CD : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, \dots, n\} \rightarrow [0, 1]$ be defined by

$$CD(c, f, g, t) = \left(1 - \sum_{i=t}^n \mathbb{1}_{[V_i < c]} \left(\prod_{j=t}^{i-1} \mathbb{1}_{[V_j > c]} \right) \cdot \frac{\min\{f, V_i - DV(g, i)\}}{V_i - DV(g, i)} \right). \quad (4.5)$$

This function gives the dilution due to a CoCo conversion. If there is never CoCo conversion, $\mathbb{1}_{[V_t < c]}$ and the function CD are equal to zero and one respectively for any $t \in \{0, \dots, n\}$. Conversion can only occur once and the first indicator function on the RHS of Equation (4.5) is equal to one at conversion. If conversion has occurred previously, the product of indicators is equal to zero. When conversion is triggered, the investors receive, if possible, the ratio of the CoCo conversion value of the converted CoCo over the cash flow of the equity holders in equity shares. If the CoCo conversion value exceeds the cash flow of equity holders, the investors become the sole equity holders. In order to limit the value of C_2 to sensible values, it holds that $C_2 \leq \max\{0, K - B_n\}$. After conversion there cannot be additional dilution due to the CoCo as the product of indicators in Equation (4.5) will be zero for all subsequent periods.

Using these functions, I define $E : \mathbb{R}_+^{n+1} \times \mathbb{R}_+^{n+1} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \{1, \dots, n\} \rightarrow \mathbb{R}_+$ by

$$E(\mathbf{a}_{n+1}, \mathbf{b}_{n+1}, c, e, f, g, t) = EV(c, e, g, t) \cdot RD(\mathbf{a}_{n+1}, \mathbf{b}_{n+1}, g, t) \cdot CD(c, f, g, t). \quad (4.6)$$

Equation (4.6) gives the equity cash flow of the original equity holders using the time t information. Clearly this is a random variable unless $t = n$. The function gives the cash flow at maturity, incorporating the dilution suffered due to issuing

equity, RIC bonds, or CoCos.

The cash flow of the investors is given by the function $B : \mathbb{R}_+^{n+1} \times \mathbb{R}_+^{n+1} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$

$$B(\mathbf{a}_{n+1}, \mathbf{b}_{n+1}, c, e, f, g) = \left(DV(g, 0) + \left(\prod_{i=1}^n \mathbb{1}_{[V_i > c]} \right) e + EV(c, e, g, 0) \cdot (1 - RD(\mathbf{a}_{n+1}, \mathbf{b}_{n+1}, g, 0) \cdot CD(c, f, g, 0)) \right). \quad (4.7)$$

The elements on the RHS in Equation (4.7) are the cash flow of the debt, the face value of the CoCo and the equity obtained by the investors respectively. Note that the product of the functions RD and CD represent the part of the equity that the original equity holders retain. So one minus this value represents the part of the equity that the investors obtain. For the function B the t index is irrelevant as the investors can only act at $t = 0$.

Before introducing the optimization problem, first I discuss some constraints. The first constraint is given in Assumption 11 and states that the firm defaults when V_i drops below B_n .

Assumption 11. When $v_\delta < B_n$ for any $\delta \in \{1, \dots, n\}$ the equity value is zero for all $t \geq \delta$.

Here δ is the time of default. Next, Assumption 12 gives the second constraint.

Assumption 12. The value of σ_t for $t \in \{1, \dots, n\}$ is given by

$$\sigma_t = \arg \max_x \mathbb{E} \left[U_n^E \left(E(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n, t-1), t-1 \right) \middle| \mu_t, x \right]. \quad (4.8)$$

The first main friction of the model is the excessive risk taking. Assumption 12 states that at each point in time t , the original equity holders will select the parameter σ_t to maximize their expected utility. They do this by optimizing their expected utility at time $t = n$. This means that the original equity holders can at each point in time increase the risk taking of the firm if this is advantageous to them. Only the original equity holders know the value of σ_t , so it is not possible to contract on σ_t . Consequently, excessive risk taking is a potential problem as investors cannot stop the original equity holders to increase σ_t if this suits them. However, the contract issued to the investors determines the optimal choice of σ_t for any t .

Hence, the investors know what level of σ_t is optimal for the original equity holders given the contract they are offered and they demand a cash flow accordingly. It can therefore be optimal for the original equity holders to offer the investors a contract in which the level of σ_t , for each $t \in \{1, \dots, n\}$, is by Assumption 12 equal to $\sigma_t^{\min}$. Due to Assumption 12, only the contract issued to investors can serve as a commitment device to limit risk taking.

Note that by Assumption 12 the original equity holders can switch from a low risk strategy to a high risk strategy. For example the firm can choose $\sigma_l = \sigma_l^{\min}$ at time $t = l$, for some $l \in \{1, \dots, n-1\}$, and $\sigma_{l+1} = \sigma_{l+1}^{\max}$ at time $t = l+1$. This can for example be the case if the firm has raised B_0 partially in CoCos, the CoCos convert at time $t = l$ and the CoCos incentivise the original equity holders to limit risk taking before conversion but not after conversion.

Lastly, Assumption 13 contains the third constraint.

Assumption 13. *The values of $\theta_{n+1}(\cdot)$, $\mathbf{V}_{n+1}^*(\cdot)$, K , C_1 , C_2 and B_n are such that*

$$B_0 = \mathbb{E} \left[M_n^B B(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) \middle| \mu_1, \sigma_1 \right] \quad (4.9)$$

where the SDF M_n^B is given in Appendix A.

This constraint states that the contract offered to the investors should be such that the expected cash flow of the investment, discounted by the SDF of the investors, should be equal to the initial capital that is to be raised by the firm.

Combining the assumptions with the cash flow of the original equity holders gives the following optimization problem.

$$\max_{\theta_{n+1}(\cdot), \mathbf{V}_{n+1}^*(\cdot), K, C_1, C_2, B_n} \mathbb{E} \left[U_n^E \left(E(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n, 0) \right) \middle| \mu_1, \sigma_1 \right], \quad (4.10)$$

subject to

$$\begin{aligned} \sigma_t &= \arg \max_x \mathbb{E} \left[U_n^E \left(E(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n, t-1, 0) \right) \middle| \mu_t, x \right] \\ &\text{for } t \in \{1, \dots, n\}, \\ B_0 &= \mathbb{E} \left[M_n^B B(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) \middle| \mu_1, \sigma_1 \right]. \end{aligned}$$

The objective function in Equation (4.10) is the expected utility of the original equity holders at time $t = 0$. The original equity holders want to maximize this, accounting for the fact that investors know how the variance of the cash flow is chosen, as described in Equation (4.8) of Assumption 12, and demand that the discounted expected cash flow at maturity matches their original investment, as described in equation (4.9) of Assumption 13.

Figure 4.1 depicts the timing of the model. The contract characteristics are all determined at time $t = 0$. At subsequent times $t \in \{1, \dots, n\}$ only the variance of V_t can be chosen. This captures the excessive risk-taking mechanism as the original equity holders can change the risk taking every period. When the contract is negotiated, the investors know that the original equity holders can increase risk taking in subsequent periods. Investors demand contract features that ensure that their discounted expected cash flow matches the investment as a competitive market is assumed for the credit market.

Figure 4.1. Investment and contract choices with RIC bonds in an n-period model

This figure gives a schematic overview of the model. The investment decisions and contract characteristics that need to be determined for a given time t are given in the column below the solid line that separates the value of t with the variables that need to be determined. B_0 represents the value to be borrowed, B_n the face value of the debt, μ_i is the expected value of the cash flow at maturity conditional on all information at $t = i$, $\mathbf{V}_{n+1}^*(\mu_{n+1})$ is the threshold above which debt holders receive a fraction $\theta_{n+1}(\mu_{n+1})$ of equity as additional cash flow, K is the conversion threshold for the CoCo, C_1 the face value of the CoCo, C_2 the CoCo conversion value and σ_t is the volatility of the cash flow of the investment chosen by management at time t .

$t = 0$	$t = i$	$t = n$
Investment opportunity arises	μ_{i+1} is observed	v_n is observed
Firm attracts B_0 in capital	The original equity holders choose σ_{i+1}	Claims are settled
Stakeholders choose $\theta_{n+1}(\cdot)$,		
$\mathbf{V}_{n+1}^*(\cdot), K, C_1, C_2, B_n$		
The original equity holders choose σ_1		

4.3.2 Main results

To show RIC bonds can be optimal, I introduce several propositions that are used in the main proposition of this paper. The aim of the paper is not to solve the optimization problem in Equation (4.10) as this requires more assumptions on the

parametric form of F . Instead I first focus on showing how different contracts affect the main frictions of the model. The incorporation of the first friction, namely excessive risk taking, is clear from Assumption 12. The second friction, differences in risk preferences between the original equity holders and investors, is frequently considered in the literature to represent that entrepreneurs are likely less risk averse than investors.⁵ By choosing these two frictions I follow Hébert (2017). Differences in the functional forms of U_n^B and U_n^E capture the differences in risk preferences. Next, I will show that RIC bonds can be the optimal contract when both these frictions are present.

For regular debt and an all equity firm we have the following standard results regarding excessive risk taking (see Hébert (2017)). Here $\Pr$ represents the probability operator.

Proposition 2. *When $\theta_{n+1}(\mu_{n+1}) = 0_{n+1}$, $C_1 = C_2 = 0$ and σ_n^{max} is such that $\Pr(V_n < B_n) > 0$ then there exists a concave U_n^E such that $\sigma_n = \sigma_n^{max}$.*

Proof. See Appendix A. ■

Proposition 2 states that when the original equity holders issue only regular debt and it is possible for the firm to default, there exists a concave utility function U_n^E such that they will in all periods choose the highest possible risk.

Proposition 3. *When $\theta_0(\mu_0) > 0$, $\theta_i(\mu_i) = 0$ for $i \in \{1, \dots, n\}$, $V_0^*(\mu_0) < \mu_0$, $C_1 = C_2 = 0$ and U_n^E is strictly concave then $\sigma_n = \sigma_n^{min}$.*

Proof. See Appendix A. ■

Proposition 3 states that when the original equity holders only issue additional equity and they are risk averse, the firm will always choose the lowest risk level. The strict concavity of U_n^E is necessary to guarantee that the original equity holders are not indifferent between all values of σ_n .

Proposition 4. *Let U_n^E be concave. Then for any distribution F of $\mathbf{V}_{n+1}$ that satisfies Assumption 10 there exist functions θ_{n+1} and $\mathbf{V}_{n+1}^*$ with $\theta_0(\mu_0) = C_1 = C_2 = 0$ and $B_n > 0$ such that $\sigma_{n+1} = \sigma_{n+1}^{min}$.*

Proof. See Appendix A. ■

⁵ The second friction is more a conflict of interest than a friction. However, for convenience I will refer to it as a friction throughout the chapter.

Proposition 4 states that a RIC bond can be designed to mitigate the risk-taking incentives when no CoCos or additional equity are issued. So RIC bonds can be used to mitigate the moral hazard of excessive risk taking. CoCos cannot generally be used to mitigate risk-taking incentives as stated in the following proposition.

Proposition 5. *Let U_n^E be linear with slope 1. Then there exists a distribution F for $\mathbf{V}_{n+1}$ that satisfies Assumption 10, such that for given $\sigma_{n+1}^{\min}, \sigma_{n+1}^{\max}, B_n$ and $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$ there does not exist $K > B_n$ with $C_1 > 0 \cup C_2 > 0$ such that by Assumption 12 $\sigma_{n+1} \neq \sigma_{n+1}^{\min}$ with probability one where $\sigma_i^{\min} \neq \sigma_i^{\max} \forall i \in \{1, \dots, n\}$.*

Proof. See Appendix A. ■

The proposition states that CoCos are not effective at mitigating risk-taking incentives for a general distribution. CoCos are especially ineffective when the probability of conversion is high regardless of the risk choice. Whereas RIC bond features change to adjust for changes in the probability of default, CoCos do not and this makes them not effective in general. See Hilscher & Raviv (2014), Berg & Kaserer (2015) and Martynova & Perotti (2018) for more elaborate studies on the risk-taking incentives of CoCos. The conflicting results in the papers illustrate the sensitivity of the results to modeling assumptions.⁶

Proposition 6. *If $\sigma_{n+1}^{\min} = \sigma_{n+1}^{\max}$, U_n^B is strictly concave and U_n^E is linear with slope 1, $\theta_n(\mu_{n+1}) = \mathbf{0}_{n+1}$, $C_1 = C_2 = 0$ and $B_n > 0$ is a solution to the Optimization problem (4.10).*

Proof. See Appendix A. ■

Proposition 6 states that regular debt is the solution to Optimization problem (4.10) when the excessive risk-taking friction is not there while the original equity holders are risk neutral and the investors are risk averse. Hence, in the absence of excessive risk taking, debt can be used as a benchmark contract. If it can be shown that other contracts can achieve this benchmark even when, due to the possibility of excessive risk taking, debt cannot, those contracts are optimal.

Proposition 7. *If U_n^B is linear, $\forall x \in \mathbb{R}_+ U_n^B(x) = U_n^E(x)$ and $\omega > 0$. Then a solution to the Optimization problem (4.10) satisfies $\theta_0(\mu_0) > 0$ and $V_0^*(\mu_0) < \mu_0$ and $\theta_i(\mu_i) = 0 \cup V_i^*(\mu_i) > \sup \{V_i\} \forall i \in \{1, \dots, n\}$, $C_1 = C_2 = 0$ and $B_n = 0$.*

⁶ In Martynova & Perotti (2018) an important difference is that the authors focus more on risk-taking incentives after conversion. When C_1 is positive, there is more equity after conversion and by Proposition 3 the risk incentives are lower when there is more equity.

Proof. See Appendix A. ■

Propositions 6 and 7 are in line with Hébert (2017). They state that regular debt (Proposition 6) and regular equity (Proposition 7) are optimal in the absence of excessive risk taking and discrepancies in risk preferences between original equity holders and investors respectively. When both frictions are present the authors argue that a combination of equity and debt is optimal. RIC bonds are a specific combination of debt and equity that are designed to minimize the impact of the two frictions.

Proposition 8. *Let U_n^B be strictly concave and U_n^E be linear with slope 1. Then $\exists \sigma_{n+1}^{\min}, \sigma_{n+1}^{\max}$ and a distribution F of $\mathbf{V}_{n+1}$ that satisfies Assumption 10 such that a necessary condition for an optimal solution to Optimization problem (4.10) is that $C_1 = C_2 = 0$, $\theta_i(x) > 0$ for $i \in \{1, \dots, n\}$ and some $x \in \mathbb{R}_{++}$, $\theta_0(\mu_0) = 0 \cup V_0^*(\mu_0) > \mu_0$ and $\mathbf{V}_i^*(\mu_i) < \sup\{\mathbf{V}_i\}$ for some $i \in \{1, \dots, n\}$.*

Proof. See Appendix A. ■

Proposition 8 states that there exist conditions where RIC bonds are the optimal contract and where issuing regular debt, equity or CoCos are not optimal solutions. So within the model presented here there are circumstances where the original equity holders will want to issue RIC bonds as opposed to the other options. The proposition does not say anything about how realistic these circumstances are. The purpose of the paper is to show that RIC bonds are effective in a model with just two frictions, namely excessive risk taking and differences in risk preferences between the original equity holders and investors. This is a first step to investigate whether RIC bonds can be feasible in practice. However, models to test whether RIC bonds perform well with other frictions are needed to draw stronger conclusions about the effectiveness of RIC bonds in practice.

In Propositions 5, 6 and 8 it was assumed that U_n^E is linear with slope 1. For Proposition 6 the proof can straightforwardly be extended to the case where the original equity holders are risk averse as well, but less so than the investors. This means that Proposition 8 applies for risk-averse original equity holders, though less risk averse than investors, as well if the CoCos are ignored. Proposition 5 and by extension 8 are less straightforward to extend to the case of risk-averse original equity holders for a general utility function. The proofs still apply if the original equity holders are not too risk averse, but it is hard to set a bound on the allowed level of risk aversion.

4.3.3 Numerical Illustration

In this section I give a simple numerical example that illustrates the settings in which RIC bonds are desirable. I use the model introduced in section 4.3.1 extended with several further simplifying assumptions to focus on the core mechanism. The optimal design of a CoCo is hard to determine, which is why in this section I focus only on regular debt, equity and RIC bonds. Compared to regular bonds, RIC bonds have an advantage due to the lower risk-taking incentives, at the cost of having more variance in the payout. Compared to an equity only firm, RIC bonds typically have a lower variance in the payout while they do induce the possibility of a costly default.

In this section I use a three period model and set $\sigma_1 = 0$, implying that at time $t = 1$ no new information on the value of the investment is revealed. The value of the assets of the firm at time $t = 2$ is normally distributed and has an expected value of $\mu_2 = 100$. If the value of the assets of the firm is below the default threshold at time $t = 2$, the firm will default. To present the results I will show the regions where regular debt, equity and RIC bonds are optimal. To do The horizontal axis takes values of $\sigma^{\max} - \sigma^{\min}$ and the vertical axis takes values of the parameter that indicates the difference in risk preference. Hence, the axes represent the main frictions of the model. I consider different settings in terms of the ω , $\sigma^{\min}$ and B_0 . To capture the difference in risk-taking preferences I assume that the original equity holders are risk neutral. The investors are risk averse and exhibit CRRA preferences. Their utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined by

$$u(x) = \frac{x^{1-\rho} - 1}{1-\rho},$$

where $\rho \in \mathbb{R}_+$ represents the risk aversion of the investors. When ρ decreases, the investors become more risk neutral. The aim of the example is to further develop an intuition for when RIC bonds can be useful. When a firm considers to issue a RIC bond, it will have to investigate whether the utility function of their investors and the distribution of their cash flow yield the same implications as the simplified example considered in this section.

To mitigate the risk-taking incentives I will set

$$V_2^*(\mu_2) = 2\mu_2 - B_n$$

and $\theta_2(\mu_2) = 1$. As the normal distribution is symmetric this will make the value of

the equity invariant to σ_2 and thus mitigate the risk-taking incentives for the firm (see the proof of Proposition 4).

For the computation of the face value of the debt and equity value at time $t = 0$ see Appendix B. I numerically compute the expected utility of the original equity holders of a firm that issues, next to the existing equity: regular debt; RIC bonds; or additional equity. I vary the parameters to illustrate when RIC bonds are desirable to issue. The results are summarized in Figures 4.2 to 4.6.

Figure 4.2. **Investment and contract choices with RIC bonds in a 3-period model**

The figure gives the region where regular debt, additional equity and RIC bonds are optimal to issue. As the results are based on numerical estimates, there are discrete changes on the axes. Specifically, the value of the horizontal axis is divided in ten intervals and the value of the vertical axis in six intervals. The area with the parameters for which issuing additional equity is optimal is depicted in dark grey. The areas for which regular debt and RIC bonds are optimal are depicted in light grey and white respectively. The parameter values are $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 20$.

Figure 4.2 takes the parameter values $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 20$. Intuitively RIC bonds are desirable when there is the possibility to increase risk taking by a large magnitude and there is a large differences in risk preferences between the original equity holders and investors. This is confirmed by the figure. When there is little opportunity to increase risk taking, debt is preferable unless there is no difference in risk preferences, in which case equity is optimal.

In Figure 4.3 the parameter values are $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 25$. The increase in $\sigma_2^{\min}$ makes the possibility to increase risk taking relatively lower. In this case the downside of regular debt is relatively less severe. As a result, there are more cases where regular debt is the optimal contract, as is clearly illustrated by the figure. There are two regions where equity is optimal; equity

Figure 4.3. Investment and contract choices with RIC bonds in a 3-period model

The figure gives the region where regular debt, additional equity and RIC bonds are optimal to issue. As the results are based on numerical estimates, there are discrete changes on the axes. Specifically, the value of the horizontal axis is divided in ten intervals and the value of the vertical axis in six intervals. The area with the parameters for which issuing additional equity is optimal is depicted in dark grey. The areas for which regular debt and RIC bonds are optimal are depicted in light grey and white respectively. The parameter values are $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 25$.

is optimal for both a low and for a high level of risk aversion. For high levels of risk aversion the cash flow at maturity for RIC bonds and regular debt are so high that the investors get almost the complete cash flow of the firm. In this case there is little difference in cash flows between regular debt, equity and RIC bonds. As the firm cannot default when it only issued equity, it is optimal for the original equity holders to issue only equity. In Figure 4.4 the parameters are the same as in Figure 4.3 except that $\omega = 0$. In this case issuing only equity is not optimal for high levels of risk aversion as one would expect. Furthermore, even for low levels of risk aversion issuing only equity is no longer better than issuing debt.

In Figure 4.5 the parameter values are $B_0 = 65$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 20$. In this scenario a relatively small part of the investment is financed by the original equity holders. This makes the probability of default relatively high. The expected outcome is therefore that both equity and RIC bonds are the optimal solution in a large part of the cases. The figure confirms the hypothesis. When the possibility to increase risk taking is small, equity is overtaken in desirability by debt for increased differences in risk preferences, while when the possibility to increase risk taking is large, RIC bonds overtake equity as the most desirable contract.

Finally, Figure 4.6 considers the parameter values $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.4$,

Figure 4.4. Investment and contract choices with RIC bonds in a 3-period model

The figure gives the region where regular debt, additional equity and RIC bonds are optimal to issue. As the results are based on numerical estimates, there are discrete changes on the axes. Specifically, the value of the horizontal axis is divided in ten intervals and the value of the vertical axis in six intervals. The area with the parameters for which issuing additional equity is optimal is depicted in dark grey. The areas for which regular debt and RIC bonds are optimal are depicted in light grey and white respectively. The parameter values are $B_0 = 50$, $\mu_2 = 100$, $\omega = 0$, and $\sigma_2^{\min} = 25$.

Figure 4.5. Investment and contract choices with RIC bonds in a 3-period model

The figure gives the region where regular debt, additional equity and RIC bonds are optimal to issue. As the results are based on numerical estimates, there are discrete changes on the axes. Specifically, the value of the horizontal axis is divided in ten intervals and the value of the vertical axis in six intervals. The area with the parameters for which issuing additional equity is optimal is depicted in dark grey. The areas for which regular debt and RIC bonds are optimal are depicted in light grey and white respectively. The parameter values are $B_0 = 65$, $\mu_2 = 100$, $\omega = 0.2$, and $\sigma_2^{\min} = 20$.

Figure 4.6. Investment and contract choices with RIC bonds in a 3-period model

The figure gives the region where regular debt, additional equity and RIC bonds are optimal to issue. As the results are based on numerical estimates, there are discrete changes on the axes. Specifically, the value of the horizontal axis is divided in ten intervals and the value of the vertical axis in six intervals. The area with the parameters for which issuing additional equity is optimal is depicted in dark grey. The areas for which regular debt and RIC bonds are optimal are depicted in light grey and white respectively. The basic parameter values are $B_0 = 50$, $\mu_2 = 100$, $\omega = 0.4$, and $\sigma_2^{\min} = 20$.

and $\sigma_2^{\min} = 20$. When the default costs are increased, the desirability of debt suffers most. Depending on $\sigma_2^{\min}$ RIC bonds will also decrease in desirability. The figure reveals that for the chosen parameters, some cases where in Figure 4.2 debt was the optimal contract now have RIC bonds as the optimal contract.

The goal of this section was to support the result in Proposition 8 that stated that there are cases where RIC bonds are the optimal contract. The results in this section support that idea for a normal distribution.⁷ This illustrates that RIC bonds are worthwhile to further look into to find whether the results hold as well for practical settings.

4.4 Policy implications

In this section some policy implications of RIC bonds are discussed. First, I review the policy relevance of RIC bonds. Second, I discuss the problem of lax effort. Third, I address potential pricing problems. Last, I discuss whether hedging can mitigate

⁷ Note that strictly speaking a normal distribution does not satisfy Assumption 10. However, given the parameter choices and that I use a numerical estimation of the required expectations, the normal distribution does satisfy Assumption 10 in practice in this example.

the effectiveness of RIC bonds.

The policy relevance of RIC bonds is due to its potentially wide application. These bonds deviate from most of the other proposals in the literature in that they can be issued by both small and large firms, as other proposals in the literature are not feasible for small firms. Furthermore, RIC bonds have the advantage for large firms these these bonds do not depend on the intervention of regulators. Frictions due to differences in risk-taking preferences are reduced as the incentives of investors and original equity holders are aligned when a firm issues RIC bonds. The need for a third party to regulate the firm is therefore reduced, as the firm does not have an incentive to deviate from the agreement.

When a firm does not have a separation between the original equity holders and management, management compensation structures cannot be used to limit risk-taking incentives as the manager is the equity holder and cannot be rewarded with long term debt. Differently, RIC bonds are feasible for these firms.

The lax effort problem arises due to the fact that the original equity holders of a firm can decide not to put in enough effort to maximize the value of the firm. Traditionally this problem is resolved by making management compensation performance dependent. When a firm issues RIC bonds and rewards management with equity, management has an incentive to exert effort in a similar way to traditional performance dependent management compensations. However, the model excludes a few scenarios where it is not possible to reward the original equity holders, which are the managers in the model, in such a way that it is optimal for the original equity holders to exert effort. When the original equity holders have more frequent updates on the expected cash flow of the firm, it could be that the trigger on the cash flow for which the RIC bond holders get additional payments is not adjusted, while the original equity holders do observe that the expected cash flow of the firm is high. As the original equity holders receive a smaller part of the additional profit than in the case of regular debt, it could be optimal not to exert additional effort in this scenario. For this to occur, the private benefits from the lack of effort should be high compared to the effect of putting in effort on the value of the cash flow of the firm. The original equity holders should, furthermore, have more frequent updates on the firm value than the other stakeholders. This problem mostly occurs in those states of the world that are relatively unlikely when the firm does not increase risk taking, as RIC bonds primarily make additional payments to bond holders in states of the world that mainly occur when the firm has chosen a risky strategy. Note that the problem could still be less severe for firms with RIC

bonds than in the case where the firm has a management that is rewarded with long term debt. In the latter case the management compensation is still less dependent the expected cash flow when the cash flow is high than in the case of RIC bonds. Hence, firms with RIC bonds does not necessarily suffer from this problem more than firms with regular debt that reward management with long term debt.

When a firm is close to default, additional payments will be made to the RIC bond holders even for relatively small increases in the firm value. This reduces the value of the equity and for lower equity values it could become infeasible to incentivize the original equity holders to exert effort. However, when the firm issues debt, the investors will demand a higher debt repayment as the original equity holders will increase the riskiness of the firm. This implies that in states of the world where a firm with RIC bonds cannot incentivize management to exert effort, firms with regular debt could already be in default. Nevertheless being limited in the ability to reward the original equity holders in times of distress is a potential downside of RIC bonds.

The trigger mechanism depends on the asset value of the firm. The asset value of the firm needs to be determined correctly and be insensitive to fraud. This has to be done by either market or book values. In the CoCo literature, there is mostly consensus that market values are more reliable than book values to judge the value of a major bank (see e.g. Calomiris & Herring 2013 and Sundaresan & Wang 2015). When the trigger is based on market values, it is necessary for these market values to be the result of sufficiently liquid derivatives. For major banks and other major firms this is not a problem. For small firms this is not possible though, making RIC bonds with market triggers infeasible for these firms. However, as small firms are less complex than major banks, it is easier for small firms than for major banks to get a clear picture of the firm value using book values.

In the CoCo literature the problem of multiple and no equilibrium prices is often raised. This problem occurs when the equity value does not strictly increase in the asset value. To prevent this from happening for RIC bonds, the share of equity given to investors when the asset value passes the RIC bonds benchmark should be strictly less than one. In the proof of Proposition 4 this was not the case. However, by continuity arguments, the result in Propositions 4 and 8 still apply when this is the case.

The robustness of RIC bonds to hedging is important for their effectiveness to mitigate risk-taking incentives. Equity holders could look for ways to buy a claim on the high cash flow of their own firm from third parties. If this claim gives the

equity holders an incentive for excessive risk taking, RIC bonds are not effective. However, note that the investors are protected against this. Although the investors suffer due to the higher expected default costs when the firm increases risk taking, they gain due to their claim to part of the cash flow on top of the regular debt in case the firm has a high cash flow. By issuing RIC bonds, the firm sells the claim on a part of the cash flow that passes a specific benchmark. The traditional advantage of increased risk taking for equity holders, namely the higher probability of a high cash flow, is no longer present as the investors will receive part of this cash flow. The investors have no incentive to sell this claim back to the original equity holders as they know this will induce the original equity holders to increase risk taking, which reduces the value of the remaining claim of the investors. Hence, if the original equity holders were to buy a claim on the high cash flow, this would have to be from a third party. However, this third party knows that original equity holders will increase risk taking to increase the value of this claim. The price the original equity holders would have to pay for this claim takes this into account. The advantage taken of the third party is, as a result, incorporated in the price of the claim. The original equity holders can therefore not gain by hedging the effects of RIC bonds away. As increased risk taking does not shift value from investors to the original equity holders, the original equity holders have no incentive to hedge the effects of RIC bonds away.

4.5 Conclusion

In this chapter I introduce a new hybrid debt contract that mitigates risk-taking incentives of the original equity holders of a firm. By issuing this hybrid debt type, firms commit credibly to a strategy where the risk of the firm will not be increased. This is attractive for the firm as it lowers the cost of debt and there is a lower probability of default.

In a discrete time model where I optimize the expected utility of the original equity holders of a firm over different contract characteristics, I show that RIC bonds are effective in mitigating risk-taking incentives and can be the optimal financing strategy for firms. In the presence of excessive risk-taking and a difference in risk aversion between the original equity holders and investors, issuing RIC bonds can strike a better balance than issuing equity, regular debt or CoCos.

Whereas other proposals to reduce risk-taking incentives focused mostly on banks, RIC bonds can be issued by firms of all sizes and by firms where the own-

ership and management are not separated. When the value of the firm can be observed, RIC bonds have a wider application than proposals that require regulators or good observability of the management compensation.

Future research where the capital structure of firms is modeled more elaborately is necessary to further compare the welfare effects of RIC bonds. Furthermore, the effect of RIC bonds on moral hazard on the effort side should be further studied.

4.A Appendix

This appendix consists of Appendix A and Appendix B. Appendix A provides the proofs for Sections 4.3.1 and 4.3.2 while Appendix B provides the expressions used in Section 4.3.3 to numerically compare the performance of different contracts.

4.A.1 Appendix A

This section contains the proofs for the results in Section 4.3.1.

Proof of Lemma 1. Note that due to the symmetry

$$\mathbb{E}[X_i | X_i < \mu] = 2\mathbb{E}[\min \{X_i, \mu\}] - \mu \text{ for } i = 1, 2.$$

As X_1 has the same mean as X_2 but a higher variance, it follows by Jensen's inequality that

$$\begin{aligned} 2\mathbb{E}[\min \{X_1, \mu\}] - \mu \\ \leq 2\mathbb{E}[\min \{X_2, \mu\}] - \mu. \end{aligned}$$

This gives

$$\begin{aligned} \mathbb{E}[X_1 | X_1 < \mu] &= 2\mathbb{E}[\min \{X_1, \mu\}] - \mu \\ &\leq 2\mathbb{E}[\min \{X_2, \mu\}] - \mu \\ &= \mathbb{E}[X_2 | X_2 < \mu]. \end{aligned}$$

Using the same argument it immediately follows that

$$\mathbb{E}[\min \{X_1, \alpha\}] \leq \mathbb{E}[\min \{X_2, \alpha\}]$$

and by using that the maximum operator is convex

$$\mathbb{E}[\max \{X_1, \alpha\}] \geq \mathbb{E}[\max \{X_2, \alpha\}].$$

■

Derivation of the SDF in Assumption 13. The SDF M_n^B is derived by maximizing the utility of the investors, where their decision variables are the number of risky assets, s , to buy from the firm and the number of risk free bonds, m , to buy from a third

party. This gives the investor optimization problem

$$\max_{s,m} \mathbb{E} \left[U_n^B(W_n) \middle| \mu_1, \sigma_1 \right] \quad (4.A.1)$$

with

$$W_n = sB(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) + m(1 + r_f),$$

where r_f is the risk free rate, and budget constraint

$$W_0 = sB_0 + m.$$

To solve this problem, the Lagrangian

$$\begin{aligned} \max_{s,m} L = \mathbb{E} \left[U_n^B \left(sB(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) + m(1 + r_f) \right) \middle| \mu_1, \sigma_1 \right] \\ + \lambda(W_0 - sB_0 - m) \end{aligned}$$

with FOC's

$$\begin{aligned} \mathbb{E} \left[U_n^{B'}(W_n) B(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) \middle| \mu_1, \sigma_1 \right] - \lambda B_0 &= 0 \\ \mathbb{E} \left[U_n^{B'}(W_n) \middle| \mu_1, \sigma_1 \right] (1 + r_f) - \lambda &= 0 \end{aligned}$$

is used. Solving these conditions gives

$$B_0 = \frac{\mathbb{E} \left[U_n^{B'}(W_n) B(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) \middle| \mu_1, \sigma_1 \right]}{\mathbb{E} \left[U_n^{B'}(W_n) \middle| \mu_1, \sigma_1 \right] (1 + r_f)},$$

which in SDF representation can be written as

$$B_0 = \mathbb{E} \left[M_n^B B(\theta_{n+1}(\mu_{n+1}), \mathbf{V}_{n+1}^*(\mu_{n+1}), K, C_1, C_2, B_n) \middle| \mu_1, \sigma_1 \right],$$

with SDF

$$M_n^B = \frac{U_n^{B'}(W_n)}{\mathbb{E} \left[U_n^{B'}(W_n) \middle| \mu_1, \sigma_1 \right] (1 + r_f)}.$$

I further normalize the supply of s to one, the risk-free asset is in zero net supply, and I assume the market clears. ■

Proof of Proposition 2. Equation (4.6) evaluated at time t , for any $t \in \{0, \dots, n-1\}$ becomes

$$\left(\prod_{i=t+1}^n \mathbb{1}_{[V_i > B_n]} \right) (V_n - B_n). \quad (4.A.2)$$

Note that this is a maximum operator if $\Pr(V_n < B_n) > 0$. Then, by Lemma 1, an increase in σ_t monotonously increases the expected value of (4.A.2). As a linear function is concave, there exists a concave utility function U_n^E for which this effect is maintained. Hence, by Assumption 12 there exists a concave U_n^E such that $\sigma_n = \sigma_n^{\max}$ when $\theta_{n+1}(\mu_{n+1}) = 0_{n+1}$, $C_1 = C_2 = 0$ and $\sigma_n^{\max}$ is such that $\Pr(V_n < B_n) > 0$. ■

Proof of Proposition 3. Equation (4.6) evaluated at time t , for any $t \in \{0, \dots, n-1\}$, $\theta_0(\mu_0) > 0$, $\theta_i(\mu_i) = 0$ for $i \in \{1, \dots, n\}$, $V_0^*(\mu_0) > B_n$ and $C_1 = C_2 = 0$ becomes a multiple of V_n . As M_n^E is strictly concave by Assumption 12 it holds that $\sigma_n = \sigma_n^{\min}$. ■

Proof of Proposition 4. Consider $t = n-1$ and let $\theta_n(\mu_n) = 1$ and $V_n^*(\mu_n) = 2\mu_n - B_n$. Plugging the arguments in Equations (4.3), (4.4) and (4.5) gives

$$\begin{aligned} EV(0, 0, B_n, n-1) &= \mathbb{1}_{[B_n < V_n]} (V_n - B_n), \\ RD((\theta_n(\cdot), 1), (V_n^*(\cdot), 2\mu_n - B_n), B_n, n-1) &= \left(1 - \mathbb{1}_{[2\mu_n - B_n < V_n]} \right. \\ &\quad \left. \cdot \frac{V_n - 2\mu_n + B_n}{V_n - B_n} \right), \\ CD(0, 0, B_n, n-1) &= 1. \end{aligned}$$

Using these arguments, Equation (4.6) evaluated at time t is piece wise linear in V_n and the distance of the mean of V_n , denoted by μ_n , to the two break points is equal. Let $\zeta \in \mathbb{R}_+$ represents the previous dilution. Then Equation (4.6) becomes

$$\begin{aligned} &\mathbb{1}_{[V_n > B_n]} (V_n - B_n) \left(1 - \mathbb{1}_{[2\mu_n - B_n < V_n]} \frac{V_n - 2\mu_n + B_n}{V_n - B_n} \right) \cdot \zeta \\ &= \left(\mathbb{1}_{[B_n < V_n < 2\mu_n - B_n]} (V_n - B_n) + \mathbb{1}_{[2\mu_n - B_n \leq V_n]} 2(\mu_n - B_n) \right) \cdot \zeta. \end{aligned} \quad (4.A.3)$$

The RHS in Equation (4.A.3) is a random variable that only depends on V_n and a symmetric truncation is applied. So if the distribution of V_n is symmetric, so is the distribution of the RHS in equation (4.A.3). Then, by Assumption 10 on the distribution of V_n , Inequality 4.1 and the mean preserving spread in Assumption 10, an increase in σ_n does not affect the expectation of Equation (4.A.3) for some functions $\theta_{n+1}(\cdot)$ and $\mathbf{V}_{n+1}^*(\cdot)$ with $\theta_0(\mu_0) = C_1 = C_2 = 0$ and $B_n > 0$. Clearly, when the value of the $V_n^*(\mu_n)$ argument of Equation (4.6) is smaller than $2\mu_n - B_n$, an increase in σ_n decreases the expectation in Equation (4.A.3).

An increase in σ_n does not increase the optimization objective (4.10) as U_n^E is a concave function. Hence, by Assumption 12 we have that $\sigma_n = \sigma_n^{\min}$.

Next, let $t = n - 2$. The reasoning above does not apply as in this case Equation (4.6) is no longer linear. The optimization objective in (4.10) for functions $\theta_{n-1}(\mu_{n-1}) = 1$ and $\mathbf{V}_{n-1}^*(\mu_{n-1}) = \mu_{n-1}$ with $\theta_0(\mu_0) = C_1 = C_2 = 0$ becomes

$$\frac{1}{2}\mathbb{E}[\max\{V_{n-1} - B_n, 0\} | V_{n-1} < \mu_{n-1}, \mu_{n-1}, \sigma_{n-1}] + \frac{1}{2}\mu_{n-1}.$$

This value decreases in σ_{n-1} due to Inequality (4.1) and as changes in σ_{n-1} do not affect μ_{n-1} . Hence, by Assumption 12 there exists a function V_{n-1}^* such that $\sigma_{n-1} = \sigma_{n-1}^{\min}$.

For all $t = i$ with $i \in \{1, \dots, n - 3\}$ a similar argument can be made. The concavity of U_n^E does not diminish this effect. Hence, with concave U_n^E there exists, for any distribution F of $\mathbf{V}_{n+1}$ that satisfies Assumption 10, functions $\theta_{n+1}(\cdot)$ and $\mathbf{V}_{n+1}^*(\cdot)$ with $\theta_0(\mu_0) = C_1 = C_2 = 0$ and $B_n > 0$ such that $\sigma_{n+1} = \sigma_{n+1}^{\min}$. ■

Proof of Proposition 5. I have to show that, when U_n^E is linear with slope 1, there exists a distribution F for $\mathbf{V}_{n+1}$ such that for $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$ there exist $K > B_n$ with $C_1 > 0 \cup C_2 > 0$ such that by Assumption 12 the event $\sigma_{n+1} \neq \sigma_{n+1}^{\min}$ occurs with a positive probability where $\sigma_i^{\min} \neq \sigma_i^{\max} \forall i \in \{1, \dots, n\}$. The result is proven when for $t = n - 1$ there is a $\sigma_n^{\max}$ such that for some V_{n-1} and by Assumption 12 the event $\sigma_n = \sigma_n^{\max}$ occurs with a positive probability.

Suppose $v_i > K \forall i \in \{1, \dots, n - 1\}$. This is possible as F satisfies Assumption 10. Let U_n^E be linear with slope 1 and consider $t = n - 1$ and denote μ_n by μ . By Assumption 12 and as U_n^E is linear with slope 1, σ_n , for this proof for notational simplicity denoted as σ , is chosen to maximize

$$\mathbb{E} \left[\mathbb{1}_{[B_n < V_n]} \left(V_n - B_n - \mathbb{1}_{[K < V_n]} C_1 \right) \left(1 - \mathbb{1}_{[V_n < K]} \frac{\min\{C_2, V_n - B_n\}}{V_n - B_n} \right) \middle| \mu, \sigma \right]$$

$$\begin{aligned}
&= \int_{B_n}^{\infty} (x - B_n) dF(x|\mu, \sigma) \\
&\quad - \int_K^{\infty} C_1 dF(x|\mu, \sigma) - \int_{B_n}^K (x - B_n) \frac{\min\{C_2, x - B_n\}}{(x - B_n)} dF(x|\mu, \sigma) \\
&= \int_{B_n}^{\infty} (x - B_n) dF(x|\mu, \sigma) - \int_K^{\infty} C_1 dF(x|\mu, \sigma) \\
&\quad - \int_{B_n}^K \min\{C_2, x - B_n\} dF(x|\mu, \sigma), \tag{4.A.4}
\end{aligned}$$

where I make use of the fact that $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$. Let F be the triangular distribution whose PDF is characterized by

$$f(x|\mu, \sigma) = \begin{cases} 0 & x < \mu - \sigma \\ \frac{\sigma - \mu + x}{\sigma^2} & \mu - \sigma < x < \mu \\ \frac{1}{\sigma} & x = \mu \\ \frac{\sigma + \mu - x}{\sigma^2} & \mu < x < \mu + \sigma \\ 0 & \mu + \sigma < x. \end{cases} \tag{4.A.5}$$

As C_1 and C_2 enter Equation (4.A.4) additively, their effect on σ can be analyzed separately. So first I analyze the case where $C_1 > 0$ and $C_2 = 0$ and second the case where $C_1 = 0$ and $C_2 > 0$. The effect of a change in σ on the optimization objective for given C_1 and C_2 can be categorized in three scenarios. In the first scenario $\mu - \sigma > K > B_n$, in the second scenario $K > \mu - \sigma > B_n + C_2$ and in the third scenario $K > B_n + C_2 > \mu - \sigma$.

For Scenario 1 and $C_2 = 0$ Equation (4.A.4) boils down to

$$\begin{aligned}
\int_{\mu - \sigma}^{\mu + \sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{\mu - \sigma}^{\mu + \sigma} C_1 dF(x|\mu, \sigma) &= \int_{\mu - \sigma}^{\mu + \sigma} x dF(x|\mu, \sigma) - B_n - C_1 \\
&= \mu - B_n - C_1. \tag{4.A.6}
\end{aligned}$$

For Scenario 2 with $C_2 = 0$ Equation (4.A.4) becomes

$$\begin{aligned}
&\int_{\mu - \sigma}^{\mu + \sigma} (x - B_n) dF(x|\mu, \sigma) - \int_K^{\mu + \sigma} C_1 dF(x|\mu, \sigma) \\
&= \mu - B_n - C_1 \left(1 - \left[\frac{(\sigma - \mu)x + \frac{1}{2}x^2}{\sigma^2} \right]_{\mu - \sigma}^K \right) \\
&= \mu - (B_n + C_1) + C_1 \frac{(\sigma - \mu)K + \frac{1}{2}K^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\mu^2 - \mu\sigma}{\sigma^2}. \tag{4.A.7}
\end{aligned}$$

Note that Equation (4.A.6) is smaller than Equation (4.A.7). Let $h_1 : \mathbb{R}_+ \rightarrow [0, 1]$ be defined by

$$h_1(a) = \frac{(a - \mu)K + \frac{1}{2}K^2 + \frac{1}{2}a^2 + \frac{1}{2}\mu^2 - \mu a}{a^2}. \quad (4.A.8)$$

This function represents the last term in Equation (4.A.7). From Equation (4.A.8) it follows that

$$\frac{\partial h_1(a)}{\partial a} = (\mu - K)a^{-2} - (\mu - K)^2a^{-3}. \quad (4.A.9)$$

From Equation (4.A.9) it is clear that

$$\frac{\partial h_1(a)}{\partial a} > 0 \text{ for } a > \mu - K.$$

As in Scenario 2 $\sigma > \mu - K$ the RHS in Equation (4.A.7) is increasing in σ . For Scenario 3 with $C_2 = 0$ Equation (4.A.4) can be written as

$$\begin{aligned} & \int_{B_n}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_K^{\mu+\sigma} C_1 dF(x|\mu, \sigma) \\ &= \int_{B_n}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - C_1 \left(1 - \left[\frac{(\sigma - \mu)x + \frac{1}{2}x^2}{\sigma^2} \right]_{\mu-\sigma}^K \right) \\ &= \frac{1}{2}\mu + \frac{\sigma}{6} + \frac{\frac{1}{2}(\sigma - \mu)\mu^2 + \frac{1}{3}\mu^2 - \frac{1}{2}(\sigma - \mu)B_n^2 - \frac{1}{3}B_n^3}{\sigma^2} \\ &\quad - B_n \frac{(\sigma - \mu)\mu + \frac{1}{2}\mu^2 - (\sigma - \mu)B_n - \frac{1}{2}B_n^2}{\sigma^2} \\ &\quad - C_1 + C_1 \frac{\sigma K - \mu K + \frac{1}{2}K^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\mu^2 - \mu\sigma}{\sigma^2}. \end{aligned} \quad (4.A.10)$$

Both Equation (4.A.7) and Equation (4.A.10) are increasing in σ . Furthermore, the value in Equation (4.A.10) is greater than the values in Equations (4.A.6) and (4.A.7). Hence, for $C_2 = 0$, when U_n^E is linear with slope 1, there exists a distribution F for $\mathbf{V}_{n+1}$ such that for $\theta_n(\mu_{n+1}) = \mathbf{0}$ there do exist $K > B_n$ with $C_1 > 0$ such that by Assumption 12 $\sigma_{n+1} \neq \sigma_{n+1}^{\min}$ with positive probability where $\sigma_i^{\min} \neq \sigma_i^{\max} \forall i \in \{1, \dots, n\}$.

Next consider $C_1 = 0$ and $C_2 > 0$. Scenario 1 becomes

$$\int_{\mu-\sigma}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) = \mu - B_n. \quad (4.A.11)$$

For Scenario 2 the expression becomes

$$\begin{aligned} & \int_{\mu-\sigma}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{\mu-\sigma}^K \min\{C_2, x - B_n\} dF(x|\mu, \sigma) \\ &= \int_{\mu-\sigma}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{\mu-\sigma}^K C_2 dF(x|\mu, \sigma), \end{aligned} \quad (4.A.12)$$

where the equality follows because in Scenario 2 it holds that $\mu - \sigma > B_n + C_2$. Note that Equation (4.A.11) is greater than Equation (4.A.12).

In Scenario 3 Equation (4.A.4) reduces to

$$\begin{aligned} & \int_{\mu-\sigma}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{\mu-\sigma}^K \min\{C_2, x - B_n\} dF(x|\mu, \sigma) \\ &= \int_{\mu-\sigma}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{B_n+C_2}^K C_2 dF(x|\mu, \sigma) - \int_{\mu-\sigma}^{B_n+C_2} (x - B_n) dF(x|\mu, \sigma) \\ &= \int_{B_n+C_2}^{\mu+\sigma} (x - B_n) dF(x|\mu, \sigma) - \int_{B_n+C_2}^K C_2 dF(x|\mu, \sigma). \end{aligned} \quad (4.A.13)$$

To prove the result, it needs to be shown that there exists a distribution F and $\sigma_n^{\max}$ such that the value in Equation (4.A.11) is smaller than the value in 4.A.13 for all $K > B_n$ and $0 < C_2 < K - B_n$ for some realizations of μ where $\sigma < \mu - K$ and $\sigma = \sigma_n^{\max}$ for Equations (4.A.11) and (4.A.13) respectively. Some tedious computations show that for $\mu < \frac{3}{2}B_n$ this is true for all K and C_2 .⁸ There exists a distribution F for $\mathbf{V}_{n+1}$ such that this can occur with a positive probability. Hence, for $C_1 = 0$, when U_n^E is linear with slope 1, there exists a distribution F for $\mathbf{V}_{n+1}$ such that for $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$ there exist $K > B_n$ with $C_2 > 0$ such that by Assumption 12 $\sigma_{n+1} \neq \sigma_{n+1}^{\min}$ with a positive probability where $\sigma_i^{\min} \neq \sigma_i^{\max} \forall i \in \{1, \dots, n\}$. As the parallel result was previously established for $C_2 = 0$ and $C_1 > 0$, this concludes the proof. ■

Proof of Proposition 6. Let F be the distribution function of $\mathbf{V}_{n+1}$ with PDF f . Next, let $g_1 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be equal to Equation (4.7) where the first argument is $\mathbf{0}_{n+1}$ the second and third arguments are arbitrary, the fourth and fifth are zero and the last argument equals $B_n^1 \in \mathbb{R}_{++}$. Clearly g_1 is a function of V_n . Similarly, let $g_2 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be equal to Equation (4.7) where either the first or the fourth or fifth arguments differ from the arguments used to construct g_1 , while $\mathbb{E}[g_1(V_n)|\mu_1, \sigma_1] = \mathbb{E}[g_2(V_n)|\mu_1, \sigma_1]$. This implies that the seventh argument of Equation (4.7) used to obtain g_2 , from now on, $B_n^2 \in \mathbb{R}_+$, is such that $B_n^2 \leq B_n^1$, as the increase of

⁸ Wolfram Mathematica was used to do these computations due to the number of unknown parameters and to guarantee precision.

g_2 due to changes in the other arguments needs to be compensated to maintain $\mathbb{E}[g_1(V_n)|\mu_1, \sigma_1] = \mathbb{E}[g_2(V_n)|\mu_1, \sigma_1]$. Note that by Equations (4.2) to (4.7) it holds that $g_1(V_n) \leq V_n$ and $g_2(V_n) \leq V_n$.

As $\sigma_{n+1} = \sigma_{n+1}^{\min} = \sigma_{n+1}^{\max}$, the arguments used to generate g_1 and g_2 do not affect the distribution of V_n .

Let $X_1 = g_1(V_n)$ and $X_2 = g_2(V_n)$. By Assumption 13 and the construction of the functions g_1 and g_2 , X_2 can be created by applying a mean preserving spread to X_1 . So we can write $X_2 = X_1 + H_1$ where H_1 has mean zero, nonnegative variance and $\mathbb{E}[H|X_1] = 0$ (see Rothschild & Stiglitz 1970). Since U_n^E is linear with slope one, the original equity holders are indifferent between issuing a contract with the features of function g_1 and a contract with the features of function g_2 . I revisit the optimization problem in the derivation of the SDF in Assumption 13. The investor optimization problem in Equation 4.A.1 becomes

$$\max_{s,m,h} \mathbb{E} \left[U_n^B(W_n) \mid \mu_1, \sigma_1 \right]$$

with

$$W_n = sX_1 + hH_1 + m(1 + r_f)$$

and budget constraint

$$W_0 = sB_0 + hH_0 + m$$

where H_0 represents the price to get cash flow H_1 . As the model does not allow for the investors to sell assets to the firm, it has to hold that $h \geq 0$. As U_n^B is strictly concave

$$\begin{aligned} \mathbb{E}[U_n^B(g_1(V_n)) \mid \mu_1, \sigma_1] &= \mathbb{E}[U_n^B(X_1) \mid \mu_1, \sigma_1] \geq \\ \mathbb{E}[U_n^B(X_1 + H_1) \mid \mu_1, \sigma_1] &= \mathbb{E}[U_n^B(X_2) \mid \mu_1, \sigma_1] = \mathbb{E}[U_n^B(g_2(V_n)) \mid \mu_1, \sigma_1]. \end{aligned}$$

Similarly,

$$\mathbb{E}[U_n^B(X_1) \mid \mu_1, \sigma_1] \geq \mathbb{E}[U_n^B(X_1 + hH_1) \mid \mu_1, \sigma_1]$$

for all $h > 0$. Hence, if $H_0 \geq 0$ the investors will set $h = 0$. As the original equity holders are indifferent between issuing X_1 and both X_1 and H_1 , they will set

$H_0 \geq 0$.

Let the arguments of g_1 be such that Assumption 13 is satisfied. Note that g_2 was defined as any deviation from this contract and for $\mathbb{E}[g_1(V_n)|\mu_1, \sigma_1] = \mathbb{E}[g_2(V_n)|\mu_1, \sigma_1]$ investors do not have a lower willingness to pay for the contract represented by g_1 . The original equity holders seek to raise a value B_0 . To achieve this with a contract, say $g_3 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, different from the one represented by g_1 the expected cash flow of this contract has to be such that $\mathbb{E}[g_1(V_n)|\mu_1, \sigma_1] \leq \mathbb{E}[g_3(V_n)|\mu_1, \sigma_1]$.

As U_n^E is linear with slope one, the optimization objective in (4.10) for the functions g_1 and g_3 becomes $\mathbb{E}[V_n - g_1(V_n)|\mu_1, \sigma_1]$ and $\mathbb{E}[V_n - g_3(V_n)|\mu_1, \sigma_1] \leq \mathbb{E}[V_n - g_1(V_n)|\mu_1, \sigma_1]$ respectively. Note that equality only holds when the last argument of Equation (4.7) used to construct g_1 and g_3 are the same, meaning that the cash flows resulting from the contracts are the same. As g_3 was defined to deviate from g_1 , it holds that $\mathbb{E}[V_n - g_3(V_n)|\mu_1, \sigma_1] < \mathbb{E}[V_n - g_1(V_n)|\mu_1, \sigma_1]$. So no deviation from the solution where the first argument equals $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$, the second and third, $\mathbf{V}_{n+1}^*$ and K , are arbitrary, the fourth and fifth, C_1 and C_2 , are zero, and the last argument equals $B_n^1 \in \mathbb{R}_{++}$ increases the optimization objective. Hence, when U_n^E is linear with slope one and U_n^B strictly concave, $\theta_n(\mu_{n+1}) = \mathbf{0}_{n+1}$, $C_1 = C_2 = 0$ and $B_n > 0$ is a solution to Optimization problem (4.10). ■

Proof of Proposition 7. U_n^B is linear, $\forall x \in \mathbb{R}_+ U_n^B(x) = U_n^E(x)$ and by Assumption 13 the solution to the optimization problem (4.10) maximizes

$$\begin{aligned} & \mathbb{E} \left[M_n^B \left(DV(g, 0) + \left(\prod_{i=1}^n \mathbb{1}_{[V_i > c]} \right) e \right. \right. \\ & \quad \left. \left. + EV(c, e, g, 0) \cdot (1 - RD(\mathbf{a}_n, \mathbf{b}_n, g, 0) \cdot CD(c, f, g, 0)) \right) \right. \\ & \quad \left. + U_n^E(EV(c, e, g, t) \cdot RD(\mathbf{a}_n, \mathbf{b}_n, g, t) \cdot CD(c, f, g, t)) | \mu_1, \sigma_1 \right] \\ & = \mathbb{E} \left[V_n - \sum_{i=1}^n (\mathbb{1}_{[V_i < B_n]} ((1 - \omega)V_i - V_n) | \mu_1, \sigma_1 \right]. \end{aligned}$$

This value only depends on the value of ω and $\sum_{i=1}^n \Pr(V_i < B_n)$. Setting the latter term equal to zero by choosing $B_n = 0$ clearly maximizes the optimization problem. Hence, a solution to Optimization problem (4.10) satisfies $\theta_0(\mu_0) > 0$ and $V_0(\mu_0) < \mu_0$ and $\theta_i(\mu_i) = 0 \cup V_i^*(\mu_i) > \sup \{V_i\} \forall i \in \{1, \dots, n\}$, $C_1 = C_2 = 0$ and $B_n = 0$. ■

Proof of Proposition 8. Let F , the distribution of $\mathbf{V}_{n+1}$, be the same triangular distribution as given in the proof of Proposition 5. Let μ_0 and $\sigma_{n+1}^{\min}$ be such that for $\sigma_{n+1} = \sigma_{n+1}^{\min}$ and $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$, $K = C_1 = C_2 = 0$, and where B_n is chosen such that Assumption 13 is satisfied, it holds that $\text{Prob}(V_i < B_n) = 0 \forall i \in \{1, \dots, n\}$.

By Proposition 6 if $\sigma_n^{\min} = \sigma_n^{\max}$ then the above contract is optimal. However, by Proposition 2 $\exists \sigma_{n+1}^{\max}$ such that $\sigma_{n+1} = \sigma_{n+1}^{\max}$. By Proposition 4 there exist functions θ_{n+1} and $\mathbf{V}_{n+1}^*$ with $\theta_0(\mu_0) = C_1 = C_2 = 0$ and $B_n > 0$ such that $\sigma_{n+1} = \sigma_{n+1}^{\min}$. As $\sigma_{n+1} = \sigma_{n+1}^{\min}$ implies that $\text{Prob}(V_i < B_n) = 0 \forall i \in \{1, \dots, n\}$ Equation (4.6) is equal for the solutions considered in Propositions 4 and 6. Hence, there exist functions θ_{n+1} with $\theta_0(\mu_0) = 0$ and $\theta_i(x) > 0$ for $i \in \{1, \dots, n\}$ and some $x \in \mathbb{R}_{++}$, and $\mathbf{V}_{n+1}^*(\mu_{n+1}) < \sup\{\mathbf{V}_{n+1}\}$ that solve Optimization problem (4.10).

It remains to be shown that for an optimal solution of (4.10) it is a necessary condition that $C_1 = C_2 = 0$ and $\theta_0(\mu_0) = 0 \cup V_0^*(\mu_0) > \mu_0$, $\theta_i(x) > 0$ for $i \in \{1, \dots, n\}$ and some $x \in \mathbb{R}_{++}$, and $\mathbf{V}_{n+1}^*(\mu_{n+1}) < \sup\{\mathbf{V}_{n+1}\}$. The proof of Proposition 2 combined with the strict concavity of U_n^B reveals that $\exists \sigma_{n+1}^{\max}$ where $\theta_{n+1}(\mu_{n+1}) = \mathbf{0}_{n+1}$, $C_1 = C_2 = 0$ and where B_n is chosen such that Assumption 13 is satisfied, is not optimal.

As U_n^E is linear while U_n^B is strictly concave, the argument of the proof of Proposition 7 does not apply. The proof of Proposition 6 reveals that $\theta_0(\mu_0) > 0$ and $V_0^*(\mu_0) < \mu_0$ is not optimal.

Due to the strict concavity of U_n^B and the proof of Proposition 6 it follows that for the given μ_0 and $\sigma_{n+1}^{\min}$ an optimal solution has $\text{Pr}(V_n < K) = 0$. Furthermore, from the strict concavity of U_n^B and the proof of Proposition 5 it follows that an optimal solution has $C_1 = 0$. Consider a model where $\text{Pr}(\mu_n < \frac{3}{2}B_n) > 0$. It follows from the proof of Proposition 5 that there is no combination of K and C_2 such that $\text{Pr}(\sigma_{n+1} = \sigma_{n+1}^{\min}) = 1$.

Note that by previous arguments there exist for this parameter setting functions θ_{n+1} with $\theta_0(\mu_0) = 0$ and $\theta_i(x) > 0$ for $i \in \{1, \dots, n\}$ and some $x \in \mathbb{R}_{++}$, and $\mathbf{V}_{n+1}^*(\mu_{n+1}) < \sup\{\mathbf{V}_{n+1}\}$ that solve Optimization problem 4.10. For Assumption 13 and $\sigma_{n+1} = \sigma_{n+1}^{\min}$ both to hold when $\theta_i(x) = 0$ for $i \in \{1, \dots, n\}$ and some $x \in \mathbb{R}_{++}$, either $\theta_0(\mu_0) > 0$ and $V_0^*(\mu_0) < \mu_0$ or $K > B_n$ with $C_1 > 0 \cup C_2 > 0$ need to hold. However, it has been shown that neither $\theta_0(\mu_0) > 0$ and $V_0^*(\mu_0) < \mu_0$ nor $K > B_n$ with $C_1 > 0 \cup C_2 > 0$ with $\theta_i(\cdot) = 0$ for $i \in \{1, \dots, n\}$ is an optimal solution. Hence, when U_n^B is strictly concave and U_n^E is linear with slope 1 there exist functions $\sigma_{n+1}^{\min}, \sigma_{n+1}^{\max}$, a constant μ_0 and a distribution F of $\mathbf{V}_{n+1}$ that

satisfies Assumption 10 such that a necessary condition for an optimal solution of Optimization problem (4.10) is that $\theta_0(\mu_0) = 0$ or $V_0^*(\mu_0) > \mu_0$, $C_1 = C_2 = 0$, $\theta_i(x) > 0 \forall i \in \{1, \dots, n\}$ for some $x \in \mathbb{R}_{++}$ and $\mathbf{V}_i^*(\mu_i) < \sup\{\mathbf{V}_i\}$ for some $i \in \{1, \dots, n\}$. ■

4.A.2 Appendix B

In this Appendix the expressions for the expected utility of the original equity holders of a firm for different financing strategies, as described in section 4.3.3, are given. Note that in the setting of this section, μ_2 and σ_2 are known using the assumption that $\sigma_1 = 0$ and Propositions 2, 3 and 4.

The expected utility of the original equity holders when the firm issues regular debt is given by

$$\begin{aligned} & \mathbb{E} \left[U_2^E(E(\mathbf{0}_3, \mathbf{0}_3, 0, 0, 0, B_n, 0)) | \mu_2, \sigma_2^{\max} \right] \\ &= \int_{B_n}^{\infty} V_2 - B_n dF(V_2 | \mu_2, \sigma_2^{\max}), \end{aligned}$$

where B_n is found by solving

$$B_0 = \frac{\int_0^{B_n} ((1 - \omega)V_2)^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\max}) + \int_{B_n}^{\infty} B_n^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\max})}{\int_0^{B_n} ((1 - \omega)V_2)^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\max}) + \int_{B_n}^{\infty} B_n^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\max})}.$$

When the firm issues additional equity let $\xi \in \mathbb{R}_+$ denote the fraction of equity issued to the investors. The expected utility of the original equity holders is

$$\begin{aligned} & \mathbb{E} \left[U_2^E(E((1, 0, 0), ((1 - \xi)V_0, 0, 0), 0, 0, 0, 0, 0)) | \mu_2, \sigma_2^{\min} \right] \\ &= \int_0^{\infty} (1 - \xi)V_2 dF(V_2 | \mu_2, \sigma_2^{\min}), \end{aligned}$$

where ξ is such that

$$B_0 = \frac{\int_0^{\infty} (\xi V_2)^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\min})}{\int_0^{\infty} (\xi V_2)^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\min})}.$$

Finally, when RIC bonds are issued, the expected utility of the original equity holders is given by

$$\mathbb{E} \left[U_2^E(E((0, 0, 1), (0, 0, 2\mu_2 - B_2), 0, 0, 0, B_n, 0)) | \mu_2, \sigma_2^{\min} \right]$$

$$= \int_{B_n}^{2\mu_2 - B_n} V_2 - B_n dF(V_2 | \mu_2, \sigma_2^{\min}),$$

where B_n is such that

$$\begin{aligned} B_0 = & \left(\int_0^{B_n} ((1 - \omega)V_2)^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) + \int_{B_n}^{2\mu_2 - B_n} B_n^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) \right. \\ & \left. + \int_{2\mu_2 - B_n}^{\infty} (B_n + V_2 - (2\mu_2 - B_n))^{1-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) \right) \\ & \left(\int_0^{B_n} ((1 - \omega)V_2)^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) + \int_{B_n}^{2\mu_2 - B_n} B_n^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) \right. \\ & \left. + \int_{2\mu_2 - B_n}^{\infty} (B_n + V_2 - (2\mu_2 - B_n))^{-\rho} dF(V_2 | \mu_2, \sigma_2^{\min}) \right)^{-1}. \end{aligned}$$

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English summary

In this thesis I investigate the design and pricing of hybrid debt instruments that are used to resolve the problems of over leveraged firms and excessive risk taking. As these frictions were an important factor in the financial crisis, both regulators and scientists have focused on finding ways to overcome these problems. One of the causes of these problems is the capital structure of firms that gives decision makers incentives to deviate from socially optimal decisions.

Chapter 2 considers a new design for CoCos with a market trigger that circumvents the multiple or no competitive equilibrium prices problem. The lack of a unique price is often cited as the reason why this type of CoCo has not been implemented in practice. When there is a discrepancy between the value of CoCos before and after conversion, there can be multiple or no competitive equilibrium prices. The discrepancy makes that there is a transfer of value upon conversion, which affects the value of equity. As conversion is triggered based on the market price of equity, this leads to an endogeneity problem. I show that a change in the coupon payment type, namely from fixed to floating, can resolve this problem. The floating coupons are designed to mitigate the transfer of value and thus resolve the endogeneity problem.

One of the main drawbacks of CoCos with a market trigger can be resolved when coupons are floating. The proposed CoCo design is a first step towards a new type of CoCo that is more suitable for policy purposes. The asset value of a firm, which is required to derive the floating coupons, is unobservable, which limits the applicability of these CoCos. However, even when inaccurate proxies for the asset values are used, the design reduces the severity of the pricing problem. Furthermore, incentives of either CoCo holders or equity holders to manipulate the market are reduced as the gains for either party are lower due to the floating coupons.

Chapter 3 proposes a Bayesian Nash equilibrium pricing model instead of the

established competitive equilibrium model for assets with endogenous changes in cash flows. Instead of a single representative investor that is a price taker, several representative investors, that are heterogeneous in their market power, are considered. The difference in market power is due to differences in portfolio liquidity, individual patience and preferences. Investors want to use their market power to trigger a beneficial or avoid a costly transfer of value. Whereas previous literature found multiple equilibrium prices using competitive equilibrium models, this model yields a unique Bayesian Nash equilibrium price.

By using the proposed pricing methodology, the pricing problem for CoCos with a market trigger is mitigated. Experimental evidence matches the predictions of the model. This alleviates concerns of academics and policy makers that CoCos are not suitable to stabilize the financial sector due to pricing problems. Furthermore, it provides a new framework to price other assets with endogenous changes in cash flows such as equities included in a market capitalization based index and equities of firms where management bases its decision on the market capitalization of the firm.

The proposed pricing method is general and can be applied to a broad class of markets, assets and investor utility functions. Future research can use the framework to consider specific assets. This chapter is a starting point for a new branch of literature that accounts for market power when pricing assets.

Chapter 4 proposes a new debt contract named risk incentive compatible bond (RIC bond). RIC bonds pay fixed coupons comparable to the coupons of regular bonds and complement these coupons with additional floating coupons in times of high cash flows for the firm. As high cash flows are more likely when the firm has risky assets, the incentive of firms to engage in excessive risk taking is limited. Issuing RIC bonds can therefore be used as a credible signal to investors that the firm will not raise risk taking above a specific level. The cost of debt is reduced as the firm can commit to limit its risk taking. When potential investors are risk averse and there are default costs, issuing RIC bonds instead of regular debt, additional equity or CoCos can increase the equity value of the original owners. RIC bonds do not suffer from several issues of other frequently proposed solutions to excessive risk taking. For example, convertible bonds are not suitable due to a lack of adaptability, while long term debt as management compensation only aligns the incentives of management and bondholders and is therefore vulnerable to exploits from equity holders to get around these management incentives.

The assumption that the mean cash flows of the firm are observable is required

for RIC bonds to be effective. In practice this parameter is uncertain. Future research can investigate the impact of uncertainty about this parameter on the effectiveness of RIC bonds in terms of mitigating risk taking incentives and their potential to increase the firm value. The problem of lax effort should be further considered as well. In the model considered in Chapter 4 the possibility that owners do not exert effort in a welfare maximizing way is ignored. Future research can investigate what the consequence of this assumption is on the effectiveness of RIC bonds.

This dissertation investigates the effectiveness and pricing of several hybrid debt contracts that are designed to recapitalize firms in distress and limit risk-taking incentives. The contribution to the scientific literature is the proposal of two new hybrid debt instruments and a new pricing framework for a general class of assets. Future research can build on these results to further understand the benefits and limitations of the proposed contracts and framework and enable policy makers to apply the ideas in practice.

Nederlandse samenvatting

In dit proefschrift worden hybride schuldinstrumenten bestudeerd. Hybride schuldinstrumenten combineren kenmerken van aandelen en obligaties. Doordat excessief risico nemen en te lage kapitaalbuffers worden aangewezen als belangrijke oorzaken voor de financiële crisis van 2008, hebben toezichthouders en wetenschappers in het afgelopen decennium veel aandacht besteed aan hybride schuldinstrumenten die deze problemen kunnen verminderen. Schuld aantrekken en aandelen uitgeven zijn traditioneel twee veelgebruikte opties voor bedrijven om nieuwe projecten te financieren. Eén van de voordelen van aandelen uitgeven is dat, vergeleken met het aantrekken van schuld, het de kans op faillissement vermindert. Een nadeel is dat aandelen uitgeven gevoelig is voor asymmetrische informatie tussen het bedrijf en investeerders. Deze asymmetrische informatie kan tot een lagere aandelenprijs leiden, wat kostbaar is voor het bedrijf. Schuld aantrekken kan dit probleem van asymmetrische informatie verminderen, maar vergroot ook de kans op een duur faillissement en kan tot excessief risicogedrag leiden. Een hogere kans op een faillissement kan bijvoorbeeld onwenselijk zijn omdat een faillissement van een maatschappelijk belangrijk bedrijf, of ingrijpen van de overheid om het bedrijf te redden, tot grote maatschappelijke kosten kan leiden. Hybride schuldinstrumenten combineren karakteristieken van schuld en aandelen met als doel het beste van twee werelden te creëren. Deze instrumenten richten zich in de regel of op het vergroten van kapitaalbuffers of op het verminderen van risicogedrag.

In Hoofdstuk 2 wordt een nieuw ontwerp voor een hybride schuldcontract, dat bekend staat als een contingent convertible bond (CoCo), voorgesteld. CoCos lijken bij uitgifte op schuld, met het belangrijkste verschil dat de CoCos, als het bedrijf in financieel zwaar weer zit, omgezet worden in aandelen. Doordat de CoCos converteren naar aandelen, kan het bedrijf niet failliet gaan op deze vorm van schuld. De conditie waarvan de conversie van schuld naar aandelen afhangt, varieert. De recente wetenschappelijke literatuur ziet CoCos die converteren op basis van de

aandelenprijs van het bedrijf dat de CoCo uitgeeft als een veelbelovende optie. Sundaresan & Wang (2015) laten echter zien dat deze CoCos geen unieke competitieve evenwichtsprijs hebben. Dit kan leiden tot een hoge prijsvolatiliteit met onrust op financiële markten als gevolg. Het gebrek aan een uniek evenwicht wordt veroorzaakt door een mogelijke waardeoverdracht van de aandeelhouders naar de CoCo-houders bij conversie van de CoCo. Als de markt verwacht dat er conversie gaat plaatsvinden, dan wordt de waarde van de aandelen naar beneden bijgesteld. Dit kan ervoor zorgen dat conversie in gang wordt gezet. Als de markt echter geen rekening houdt met conversie, dan hoeft de prijs niet bijgesteld te worden en is er geen conversie. De verwachting dat conversie gaat plaatsvinden is dus een zichzelf vervullende voorspelling. In Hoofdstuk 2 wordt aangetoond dat door de CoCo-houders een rente te betalen die afhangt van de verwachte CoCo waarde bij een conversie, de marktwaarde van zowel de CoCos als de aandelen van het bedrijf onafhankelijk worden van conversie. Dit zorgt ervoor dat er niet langer een zichzelf vervullende voorspelling voor conversie is. Dit type CoCo heeft een unieke evenwichtsprijs en lost daarmee het grootste probleem van CoCos, met een conversie op basis van aandelenprijzen, op.

In Hoofdstuk 3 wordt een Bayesiaans Nash evenwichtsmodel voorgesteld om activa met endogene veranderingen in de kasstroom te beprijsen. Endogene veranderingen in de kasstroom zijn veranderingen in de kasstroom die het gevolg zijn van de hoogte van de kasstroom zelf. Voorbeelden van dit type activa zijn: CoCos met een conversie gebaseerd op de marktwaarde van de aandelen van het bedrijf, aandelen die opgenomen zijn in een index op basis van marktkapitalisatie en aandelen van bedrijven waarbij het management beslissingen neemt op basis van de marktkapitalisatie van het bedrijf. Anders dan standaard in de literatuur, waar één type investeerder wordt bekeken, zitten er in het voorgestelde model investeerders die verschillen in marktmacht. Het verschil in marktmacht wordt gedreven door verschillen in de liquiditeit van het portfolio van de investeerders en individuele karakteristieken zoals geduld en risico-afkerigheid. Investeerders gebruiken hun marktmacht om een gunstige verandering in de kasstroom van activa te forceren of een nadelige verandering te vermijden. Door dit mechanisme hebben de activa in het model een unieke evenwichtsprijs.

CoCos beprijsen zoals voorgesteld in Hoofdstuk 3 lost het probleem van meerdere evenwichtsprijzen op. De voorspellingen van het model komen overeen met experimenteel bewijs uit de literatuur. Dit resultaat kan een zorg van academici en beleidsmakers met betrekking tot CoCos weghalen. Verder geeft het hoofdstuk

een nieuw raamwerk om aandelen in een index op basis van marktkapitalisatie en aandelen van bedrijven waarbij marktkapitalisatie het beleid van het management beïnvloedt te beprijzen.

In Hoofdstuk 4 wordt een nieuw schuldcontract voorgesteld. De risk incentive compatible bond (RIC-bond) betaalt een vaste rente met daar bovenop, als het bedrijf dat de RIC-bond uitgeeft hoge kasstromen heeft, een variërende rente. RIC-bonds uitgeven is een geloofwaardig signaal dat het bedrijf het risico van de activa niet gaat verhogen. Dit komt omdat hoge kasstromen, die tot extra rentebetalingen aan de schuldhouders leiden, waarschijnlijker zijn wanneer het bedrijf risicovolle activa bezit. Het is daarom niet winstgevend om het risico van de activa van het bedrijf te verhogen als het bedrijf RIC-bonds heeft uitgegeven. Het uitgeven van RIC-bonds signaleert daarmee dat het risicogedrag beperkt zal blijven, wat leidt tot lagere kosten voor het aantrekken van nieuwe schuld. Als investeerders risico-afkerig zijn en er zijn faillissementskosten, dan kan het uitgeven van RIC-bonds de waarde van het bedrijf verhogen. Deze eigenschappen maken RIC-bonds een geschikt hybride schuldcontract om excessief risicogedrag van bedrijven te beperken.

Dit proefschrift bestudeert de effectiviteit en het beprijzen van verschillende hybride schuldcontracten die zijn ontworpen om bedrijven in financiële problemen te herkapitaliseren en om de prikkels om risico te nemen te beperken. De toevoeging aan de wetenschappelijke literatuur bestaat uit het ontwerp van twee nieuwe hybride schuldinstrumenten en een nieuw raamwerk om een algemene klasse van activa te beprijzen. Toekomstig onderzoek kan op deze resultaten voortbouwen en kijken wat ervoor nodig is om de ideeën in de praktijk toe te passen.