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On Linearity, Phasors and Steady-State Response of Electrical Circuits

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Abstract—This note addresses the widespread and often unquestioned assumption that a linear circuit supplied by a sinusoidal input (voltage or current) always results in a sinusoidal output (current or voltage). Likewise, it is generally assumed that only nonlinear circuits can generate harmonic distortion. Based on the linearity assumption, the analysis of power systems properties is usually carried out using Fourier analysis and phasors. Such analyses typically neglect the transients and initial conditions. Two elementary examples are exploited to show that linearity alone is not sufficient to guarantee a sinusoidal steady-state response. It is necessary to invoke some additional assumptions. However, these assumptions contradict the underlying philosophy of phasors or are difficult to accomplish physics-wise.

I. MOTIVATION

In many electrical engineering textbooks, papers, talks, and even guidelines like the IEEE Standard 1459–2010 [8], one often encounters statements along the following lines:

“Most electric power is transmitted and used as alternating current. Specifically, most power is generated as a sinusoidal voltage. Under the assumption that the circuit is *linear*, we have the special case that if the supplied voltage is sinusoidal

$$v(t) = V \cos(\omega t), \quad (1)$$

then the current is also a sinusoid, i.e.,

$$i(t) = I \cos(\omega t - \varphi). \quad (2)$$

Here V and I denote the peak values, φ is the phase difference between the voltage and the current, and ω is the angular frequency.”

This note should be considered as an educational contribution. Its purpose is to enhance the awareness of the electrical engineer that, even in the case of a sinusoidal supply voltage (input), mere linearity of the circuit is not sufficient to guarantee a sinusoidal current (output). Moreover, steady-state analysis and the corresponding phasor-based synthesis of lossless filter networks, as often proposed in power factor optimization problems, leads to surprising results when considered from a dynamical systems point of view. Two *linear* examples are provided to facilitate the discussion.

Notation: Throughout the document, voltages are assumed to be expressed in volt [V], currents in ampere [A], impedances in ohm [Ω], and admittances in siemens [S]. For sake of brevity, these units are omitted in the text.

II. EXAMPLE A – TRIAC WITH FIRING DELAY

Consider a TRIAC circuit depicted in Fig. 1, supplied with a sinusoidal voltage waveform

$$v(t) = V \sin(\omega t), \quad (3)$$

a constant load resistance R , and a firing delay angle δ .

For $\delta \in (0, T]$, the current drawn from the source can be resolved into a Fourier series of the form

$$i(t) = \sum_{n=1}^{\infty} I_n \sin(n\omega t + \phi_n), \quad (4)$$

where $I_n = 0$ for all $n = 2, 4, 6, \dots$, i.e., only odd harmonics are present; see [7] for details.

The presence of harmonics in the current generally leads to the presumption that the circuit must be nonlinear.

A. Time-Varying Conductance

Given the voltage-current waveforms, one might equally well represent the load as a *time-varying* conductance

$$G(t) = \begin{cases} 0 & \text{for } t \in \left[\frac{kT}{2}, \frac{kT}{2} + \delta \right), \quad k = 0, 1, 2, \dots, \\ 1 & \text{otherwise.} \end{cases}$$

Hence, we have that

$$i(t) = G(t)v(t), \quad (5)$$

which enables us to investigate its general properties.

B. Proof of Linearity

To show that the TRIAC circuit is linear, consider (5) as a mapping $g: v \rightarrow i$, i.e.,

$$i(t) = g[v(t)], \quad (6)$$

Fig. 1. Circuit with resistive load and TRIAC.

with $g[v(t)] = G(t)v(t)$, and verify the following two universally accepted properties that are necessary and sufficient for linearity:¹

- *Additivity (superposition)*—suppose we decompose $v(t)$ into two components, say $v(t) = v_1(t) + v_2(t)$, then according to the mapping (6) we have that the respective currents are

$$\begin{aligned} i_1(t) &= g[v_1(t)] = G(t)v_1(t), \\ i_2(t) &= g[v_2(t)] = G(t)v_2(t), \end{aligned}$$

which clearly is the same as

$$i(t) = g[v_1(t) + v_2(t)] = g[v_1(t)] + g[v_2(t)].$$

- *Homogeneity*—let γ be any real constant, then

$$g[\gamma v(t)] = \gamma G(t)v(t) = \gamma g[v(t)].$$

Thus, the voltage-current relationship (5) is *linear* by definition, but since the circuit is varying in time, it should be classified as a *linear and time-varying (LTV)* circuit.

III. PHASORS ANALYSIS

Complex numbers, and phasors in particular, are the ‘bread and butter’ for the electrical engineer. Phasors naturally lead to the important concept of network functions: impedance and admittance. The general idea behind phasor analysis is that when an electric system is operating in steady-state, differential equations are no longer required to describe its behavior since all variables are either constants or, in the AC case, sinusoidal variations in time with constant frequency.

¹These two properties can be found in any standard textbook on signals and systems; see for instance [10].

Before going into an in-depth analysis of the steady-state assumption, let us briefly recall the essential properties of phasors and the associated network functions for linear and time-invariant (LTI) circuits.

A. Mathematical Description

In essence, the phasor concept stems from Euler’s formula

$$e^{jx} = \cos(x) + j \sin(x), \quad (7)$$

where $j^2 = -1$. This formula indicates that sinusoids can be represented mathematically as the sum of two complex-valued functions:

$$A \cos(\omega t + \theta) = \frac{A}{2} [e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)}], \quad (8)$$

or as the real part of one of the functions, i.e.,

$$A \cos(\omega t + \theta) = \operatorname{Re}\{Ae^{j(\omega t + \theta)}\} = \operatorname{Re}\{Ae^{j\theta}e^{j\omega t}\}. \quad (9)$$

Mathematicians denote the term $Ae^{j(\omega t + \theta)}$ as the *analytic representation* of $A \cos(\omega t + \theta)$, while engineers often denote it as a *rotating phasor*. However, the term *phasor* is usually designated to just the static part $Ae^{j\theta}$ as this term contains all the essential information that uniquely characterizes the sinusoid, i.e., amplitude and phase [4].

The use of phasor notation brings a significant mathematical simplification concerning differentiation and integration. Indeed, let

$$\mathbf{F}(t) = \vec{F}e^{j\omega t}, \quad (10)$$

and $\vec{F} = Ae^{j\theta}$, then its time-derivative is simply given by

$$\frac{d}{dt}\mathbf{F}(t) = \vec{F}\frac{d}{dt}e^{j\omega t} = j\omega\vec{F}e^{j\omega t}. \quad (11)$$

TABLE I
TIME-DOMAIN VERSUS PHASOR REPRESENTATION.

time-domain	phasor model	impedance	time-domain	phasor model	admittance
$v_R(t) = Ri_R(t)$	$\vec{V}_R = \vec{Z}_R \vec{I}_R$	$\vec{Z}_R = R$	$i_R(t) = \frac{1}{R}v_R(t)$	$\vec{I}_R = \vec{Y}_R \vec{V}_R$	$\vec{Y}_R = \frac{1}{R}$
$v_L(t) = L \frac{di_L(t)}{dt}$	$\vec{V}_L = \vec{Z}_L \vec{I}_L$	$\vec{Z}_L = j\omega L$	$i_L(t) = \frac{1}{L} \int v_L(t) dt$	$\vec{I}_L = \vec{Y}_L \vec{V}_L$	$\vec{Y}_L = \frac{1}{j\omega L}$
$v_C(t) = \frac{1}{C} \int i_C(t) dt$	$\vec{V}_C = \vec{Z}_C \vec{I}_C$	$\vec{Z}_C = \frac{1}{j\omega C}$	$i_C(t) = C \frac{dv_C(t)}{dt}$	$\vec{I}_C = \vec{Y}_C \vec{V}_C$	$\vec{Y}_C = j\omega C$

Similarly, the time integral of (10) is

$$\int \mathbf{F}(t) dt = \frac{1}{j\omega} \vec{F} e^{j\omega t}. \quad (12)$$

Thus, the derivative of a phasor leads 90 degrees, and multiplied by ω , whereas integration leads to a scaling of $1/\omega$ and a 90 degrees lag. This means that in phasor notation differential and integral operations can be performed by scaling and phase shifting.

B. Impedance and Admittance

Given an LTI one-port network with rotating port phasors

$$\mathbf{V}(t) = \vec{V} e^{j\omega t} \text{ and } \mathbf{I}(t) = \vec{I} e^{j\omega t}, \quad (13)$$

with $\vec{V} = V e^{j\alpha}$ and $\vec{I} = I e^{j\beta}$, then the network impedance or admittance are defined as the ratio of these quantities:

$$\vec{Z} = \frac{\mathbf{V}(t)}{\mathbf{I}(t)} = \frac{\vec{V}}{\vec{I}} = Z e^{j\varphi}, \quad (14)$$

where $\varphi = \alpha - \beta$ is the impedance angle and $Z = |\vec{Z}|$. The real part of (14) is due to the presence of resistive elements, and the imaginary part is due to inductive and capacitive reactances in the system, respectively. Likewise, the admittance function is defined as

$$\vec{Y} = \frac{1}{\vec{Z}}. \quad (15)$$

The impedance and admittance functions for the three individual basic LTI circuit elements are collected in Table I.

IV. EXAMPLE B – LOSSLESS FILTERS

Many classical power quality mitigation strategies involve the use of filter networks to suppress unwanted harmonics [3] or to increase the energy transfer efficiency by improving the power factor under nonsinusoidal conditions [5]. For power factor improvement purposes, such filters are ideally designed using interconnections of lossless LTI inductors and capacitors.

The filter synthesis is often carried out using Fourier analysis and phasors, and therefore assumes that the overall power system operates in a steady-state. Consequently, the analysis neglects the transients and initial conditions. Nevertheless, the

latter play a fundamental role in the behavior of the filter—already for purely sinusoidal supply voltages.

To illustrate this, consider a series connection of an inductor L and a capacitor C as shown in Fig. 2.

Fig. 2. Lossless series LC circuit.

Such lossless filter has an admittance of the form

$$\vec{Y}(j\omega) = \frac{j\omega C}{1 - \omega^2 LC} \quad (16)$$

which is defined for $0 \leq \omega \neq \omega_n$, where

$$\omega_n = \frac{1}{\sqrt{LC}} \quad (17)$$

denotes the natural frequency. Hence, given a supply voltage of the form (3), straightforward application of the phasor analysis of Section III suggests that the associated steady-state current would read

$$i(t) = |\vec{Y}(j\omega)| V \cos(\omega t). \quad (18)$$

Let us compare this result with the steady-state solution of the associated differential equation.

A. The Physical Model

Consider again the LC circuit of Fig. 2. From a physical point of view, the differential equation describing the dynamics of this circuit reads

$$\frac{d^2 i(t)}{dt^2} + \omega_n^2 i(t) = f(t), \quad (19)$$

where the the forcing function equals

$$f(t) = \frac{1}{L} \frac{dv(t)}{dt} = F \cos(\omega t) \quad (20)$$

with $F = \omega V/L$.

B. The Homogeneous and (a) Particular Solution

Recall that (19) belongs to the class of linear nonhomogeneous differential equations [1]. Hence, its solution is given by the superposition of the homogeneous solution $i_h(t)$, i.e., for the case $f(t) = 0$, which is of the form [1]

$$i_h(t) = A \cos(\omega_n t) + B \sin(\omega_n t), \quad (21)$$

and any particular solution $i_p(t)$. The real constants A and B are to be determined by the initial conditions $i_0 = i(t_0)$ and

$$i'_0 = \left. \frac{di(t)}{dt} \right|_{t=t_0}.$$

In this case, a particular solution $i_p(t)$ can be found by the method of *undetermined coefficients* [1], which is based on the assumption that it has the same form as the forcing function $f(t)$. Thus, it is assumed that

$$i_p(t) = I \cos(\omega t). \quad (22)$$

Substituting the latter into (19) and factoring out the cosine terms, yields for its amplitude

$$I = \frac{F}{\omega_n^2 - \omega^2}, \quad (23)$$

provided that $\omega \neq \omega_n$, i.e., as long as the frequency of the supplied voltage does not coincide with the natural frequency of the LC circuit. It is left to the reader to verify that (22), together with the amplitude (23), precisely coincides with the phasor-based solution obtained in (18).

C. The General Solution

Since the system is linear, the total solution to (19) is

$$i(t) = i_h(t) + i_p(t),$$

with $i_h(t)$ and $i_p(t)$ given in (21) and (18), respectively. Thus, the *general solution* of the current reads

$$i(t) = A \cos(\omega_n t) + B \sin(\omega_n t) + \frac{F}{\omega_n^2 - \omega^2} \cos(\omega t) \quad (24)$$

instead of just (18), where it remains to determine A and B .

D. The (Complete) Steady-State Solution

At initial time $t = t_0 = 0$, we have that (24) reduces to

$$i_0 = A + \frac{F}{\omega_n^2 - \omega^2}$$

and $i'_0 = B\omega_n$. Solving the latter for A and B , and substituting the results in (24) yields the *complete steady-state* solution

$$i(t) = \left(i_0 - \frac{F}{\omega_n^2 - \omega^2} \right) \cos(\omega_n t) + \frac{i'_0}{\omega_n} \sin(\omega_n t) + \frac{F}{\omega_n^2 - \omega^2} \cos(\omega t), \quad (25)$$

for all $\omega \neq \omega_n$ and $t \geq 0$.

E. Initial Conditions

Now, comparing (25) to the presumed phasor-based solution (18) reveals that in order to obtain a purely sinusoidal steady-state current from $t_0 = 0$ and onwards, the initial current must be set to

$$i_0 = \frac{\omega C V}{1 - \omega^2 L C},$$

which is in general different from zero, whereas

$$i'_0 = 0.$$

Physically this implies that the inductor must be equipped with an *initial flux*, say $\lambda_0 = Li_0$, which is a rather difficult task, if possible at all.

An additional difficulty is that the slightest mismatch in the required initial conditions results in either $A \neq 0$ or $B \neq 0$, or both. Consequently, an additional current component, with a frequency equal to the natural frequency, appears in the current extracted from the source.

Interestingly, note that the natural frequency (17) of the compensator will generally *not* coincide with an integer multiple of the fundamental frequency in the supplied voltage. Hence, we conclude that even *subharmonic* or *interharmonic* frequencies [12] might appear in an LTI circuit!

F. Numerical Evaluation

Fig. 3 shows the results of simulating the response of the lossless series LC filter for different initial conditions. Notice that when the initial conditions are zero, as the phasor analysis presumes, the response exhibits harmonic frequencies.

Fig. 3. Lossless filter response to a sinusoidal input and different initial conditions for $V = 230\sqrt{2}$, $\omega = 100\pi$, $L = 10^{-3}$, and $C = 10^{-5}$.

V. DISCUSSION

Phasors can simplify the analysis of the behavior of circuits that are driven by a sinusoidal signal after the transients vanish [4]. Indeed, such analysis focuses on the so-called *zero-state response* of the system (also known as the *forced response*), which is related to a particular solution of the differential equation that characterizes the circuit. Since phasor analysis only focuses on the zero-state response, the initial conditions, and

their influence on the overall response, are typically neglected. However, the assumption that the transients will eventually disappear is not valid for certain kinds of LTI circuits—as for instance the lossless LC filter studied in Section IV.

A. Sustained Oscillations

In mechanical engineering, a system described by a differential equation of form (19) corresponds to an undamped system. An intuitive mechanical analogy [9] can be found in the frictionless mass–spring system depicted in Fig. 4. Here the mass plays the role of the inductor ($m \sim L$) and the spring that of the capacitor ($k \sim 1/C$).

Fig. 4. Frictionless mass-spring system.

Even if the externally applied force $f(t)$ is zero, any initial displacement x_0 different from the equilibrium position of the mass will result in an oscillating system. Since there is no friction, this transient behavior will not disappear. Consequently, phasor analysis is *not* suitable to analyze this system.

B. Stability

Stability is a fundamental concept in control systems engineering. A linear system is said to be bounded-input bounded-output (BIBO) stable if, for every bounded input, the output of the system remains bounded. The need for BIBO stability in practice is conspicuous, where unstable systems may lead to disastrous consequences.

Consider the lossless series LC filter example and its phasor analysis provided in Section IV. The phasor-based expression of $i(t)$ given in (18), suggests that the system is BIBO stable. However, this conclusion is not true! Indeed, from (23) it is evident that when the frequency of the supplied voltage coincides with the natural frequency of the filter, the current will tend to infinity.

Fig. 5 shows the simulation results of the lossless LC filter with zero initial conditions. In the particular case $\omega = \omega_n$, the amplitude of the oscillations will continue increasing as time goes by. Hence, the system is not BIBO stable.

C. Final Value Theorem

Another important observation is related to the *Final Value Theorem (FVT)*, which states that if a function $y(t)$ is bounded on the interval $[0, \infty)$ and $\lim_{t \rightarrow \infty} y(t)$ exists, then [6]

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s), \quad (26)$$

where $Y(s)$ is the Laplace transform of $y(t)$.

To verify if the conditions of the FVT are satisfied, it is necessary to analyze the roots of the denominator of $Y(s)$, e.g., the poles of a transfer function. If there is more than one root at the origin or nonzero roots with nonnegative real parts, then the conditions of the theorem are not satisfied as the limit of $y(t)$ is unbounded or does not exist. In the case of a transfer function, the FVT ensures that, eventually, the transients will disappear. Thus, the response of the system will be dominated by a particular solution to the differential equation that characterizes the system's behavior.

D. Validity of Phasor Analysis

For electrical circuits, the discussion of the previous section means that phasor analysis only captures the true steady-state behavior *if, and only if*, it is possible to neglect both the homogeneous solution and the transient part of the forced response. In other words, phasor analysis will be valid *if, and only if*, the FVT is satisfied!

Now, the transfer function associated to (16) is given by

$$Y(s) = \frac{s/L}{s^2 + \omega_n^2}. \quad (27)$$

Not surprisingly, the transfer function (27) does *not* satisfy the conditions of the FVT since its poles are purely imaginary.

On the other hand, a single inductor or capacitor both exhibit one real pole at the origin. Hence, these elements individually satisfy the FVT. However, as soon as they are combined into, e.g., a lossless filter configuration, the application of phasor analysis becomes ambiguous and their responses will become prone to initial conditions.

E. Nonsinusoidal Conditions

The problem of analyzing lossless LC filters using phasors already becomes apparent for a single sinusoidal input. However, as outlined in Section IV, such filters are usually designed to suppress the reactive harmonic components in the supplied current as a result of a distorted voltage. Let

$$v(t) = \sum_{n=1}^{\infty} V_n \sin(n\omega t + \alpha_n), \quad (28)$$

which, after subsequent substitution in (20) and (19), results in a general solution for the current of the form

$$i(t) = A \cos(\omega_n t) + B \sin(\omega_n t) + \sum_{n=1}^{\infty} \frac{F_n}{\omega_n^2 - n^2\omega^2} \cos(n\omega t + \alpha_n), \quad (29)$$

Fig. 5. Lossless LC filter response to a sinusoidal voltage (3) for different values of ω and zero initial conditions.

with $F_n = n\omega V_n/L$. Hence, in order to eliminate the current component associated to ω_n , generally nonzero initial conditions for both i_0 and i'_0 are required. These initial conditions obviously depend on all the parameters present in the system and are therefore highly sensitive to mismatches.

F. Theory Versus Practice

Of course, one could reasonably argue that any practical load or practical filter design always possesses some damping due to the unavoidable presence of dissipation. Consequently, this will guarantee that after some sufficient amount of time, say $t_1 \gg t_0$, the transients will eventually vanish. However, this is generally not how the problems are presented and analyzed in the literature; see e.g., [2], [5], and [8], and the references therein.

Additionally, filters for power factor improvement are ideally designed with a high quality factor as to limit the amount of additional dissipation losses introduced by the filter itself. High quality factors generally means low damping [3]. As such, the time for the transients to vanish may be rather long. In the meanwhile large oscillations at the natural frequency of the filter will be present that might trigger oscillations in other equipment within the same power system. For that, the use of (nearly) lossless filters might not be very practical after all [3]. Such situations might be avoided by using active power filters (APFs) instead [11].

VI. CONCLUSION

In order to guarantee a sinusoidal steady-state in a linear circuit, as well as an unambiguous use of phasor analysis, two additional requirements should be added, namely

- time-invariance (of the circuit components), and
- the Final Value Theorem (FVT) must be satisfied,

whereas the implementation of lossless filter networks generally requires to set non-zero initial conditions. To the best of our knowledge, the validity of phasor analysis in conjunction with the FVT cannot be found in any standard textbook on electrical circuit theory.

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