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Evidence-Based Beliefs in Many-Valued Modal Logics

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Evidence-Based Beliefs
in Many-Valued Modal Logics

Yuri David Santos

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university of
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Evidence-Based Beliefs in Many-Valued Modal Logics

PhD thesis

to obtain the degree of PhD at the
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on the authority of the
Rector Magnificus Prof. C. Wijmenga
and in accordance with
the decision by the College of Deans.

This thesis will be defended in public on

Thursday 14 January 2021 at 14.30 hours

by

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