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Probing the nature of dark matter with the power spectrum of small-scale mass structure in massive elliptical lens galaxies

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Probing the nature of dark matter with the power spectrum of small-scale mass structure in massive elliptical lens galaxies

PhD thesis

to obtain the degree of PhD at the
University of Groningen
on the authority of the
Rector Magnificus Prof. C. Wijmenga
and in accordance with the
decision by the College of Deans.

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*For all those who have the courage to follow
their passion, and the Capricorn inside me*

Cover: designed by the author based on the Australian Aboriginal dot painting *Antara Dreaming Time* by Tuppy Ngintja Goodwin.

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Chapter 1

Introduction

"In a spiral galaxy, the ratio of dark-to-light matter is about a factor of ten. That's probably a good number for the ratio of our ignorance-to-knowledge. We're out of kindergarten, but only in about third grade."

Vera Rubin

1.1 The dark-matter problem

One of the major unsolved puzzles in cosmology and fundamental physics is to find a conclusive evidence for the existence of dark matter, and to unveil its nature. The idea of this hypothetical *dark* (i.e. non-luminous) form of matter emerged as a speculation already more than a hundred years ago, in an attempt to explain the high orbital velocities observed in the Milky Way (e.g. Oort, 1928). Since then, its presence has been postulated in numerous studies of various astrophysical systems in order to reconcile the unexpectedly high velocities exhibited by their constituents (such as stellar velocities in individual galaxies or galaxy velocities inside galaxy clusters) with predictions from the standard Newtonian theory of gravity (e.g. Zwicky, 1937; Bosma, 1978; Roberts & Whitehurst, 1975; Rubin et al., 1980). Nowadays, the existence of dark matter is a widely accepted assumption lying at the core of the concordance *dark-energy-plus-cold-dark-matter* (Λ CDM) cosmological model.

According to the Λ CDM model, dark matter dominates the total mass content of the Universe, making up approximately 84 per cent of all matter (Planck Collaboration et al., 2018). However – due to the apparent lack of its interactions (except from gravity) with the ordinary baryonic matter and the observable electromagnetic radiation – the properties of dark matter are still unknown. The quest to understand its fundamental nature is an ongoing endeavour at the intersection of astrophysics, particle physics and cosmology (e.g. de Swart et al., 2017; Bertone & Tait, 2018; Buckley & Peter, 2018). The research carried out in this thesis is intended to contribute to the diverse efforts of the scientific community aiming at solving this *dark-matter problem*.

To set the stage for the main chapters, in this introduction we briefly summarise the current status of the dark-matter problem and explain how the properties of dark matter could potentially be constrained by means of the phenomenon of galaxy-galaxy strong gravitational lensing. This question will be explored further in the main body of the thesis. While this introductory chapter highlights aspects particularly relevant for this work, the interested reader is referred to the recent reviews, such as e.g. Treu (2010), Bullock & Boylan-Kolchin (2017), Bertone & Tait (2018), Buckley & Peter (2018) or Zavala & Frenk (2019), for a much more thorough consideration of the discussed topics.

1.1.1 Evidence for the existence of dark matter

Over the last century, the astronomical community has accumulated a compelling body of evidence supporting the dark-matter hypothesis. This evidence comes from a variety of independent astrophysical observations on many different scales, such as the rotational velocities of stars and gas orbiting the centres of spiral galaxies (e.g. Roberts & Whitehurst, 1975; Bosma, 1978; Rubin et al., 1980), the velocity dispersion of galaxies within galaxy clusters (e.g. Zwicky, 1937), the gravitational lensing effect by individual galaxies and by galaxy clusters (e.g. Taylor et al., 1998; Treu, 2010), the centre-of-mass estimates in merging galaxy clusters (e.g. Clowe et al., 2004; Markevitch et al., 2004), the type-Ia-supernova distance measurements (e.g. Perlmutter et al., 1999), the weak gravitational lensing by the large-scale structure of the Universe (e.g. Refregier, 2003) or the temperature anisotropies in the cosmic microwave background (CMB, Planck Collaboration et al., 2018). All these observations reveal anomalous

gravitational effects that are in strong disagreement with our current understanding of gravity (i.e. the theory of General Relativity), unless the total mass of the studied astrophysical systems is considerably larger than what can be estimated from the observations of their luminous constituents, such as stars, gas and dust. Assuming the validity of General Relativity, the only coherent way to resolve these discrepancies is by postulating the presence of a supplemental non-luminous matter component providing the missing mass – the *dark matter*.

The most prominent and widely known among these anomalous gravitational effects are the *flat rotation curves* measured in spiral galaxies (more precisely a combination of stellar rotation curves measured in the optical bands and the 21-cm-line mapping of the atomic hydrogen, e.g. Roberts & Whitehurst, 1975; Bosma, 1978; Rubin et al., 1980). According to these measurements, the actual rotational velocities of stars and gas in the outer regions of spiral galaxies are much higher than what is expected from the Newtonian theory of gravity when applied to the observed distribution of the luminous matter. Instead of the predicted decline, as is the case in the Solar System, the rotational velocities measured in spiral galaxies remain nearly constant within the entire radius range accessible to observations. The commonly accepted explanation for this discrepancy is that the visible galaxies are embedded in significantly larger *dark-matter haloes*, whose gravitational potential prevents the stars and gas in the outer regions from escaping the host galaxy.

Another important line of evidence is the key role dark matter is believed to play as the main driver of the *cosmological structure-formation process*. According to the Λ CDM model, the growth of the cosmological structure starts in the early Universe, when tiny quantum (density) fluctuations in the nearly uniform primordial matter distribution grow in a hierarchical manner to eventually form massive dark-matter haloes. The potential wells of these haloes enable the baryonic matter to cool, condense and, finally, form galaxies (White & Rees, 1978; White & Frenk, 1991). These, in turn, are the building blocks of a large web-like structure made of filaments, sheets and voids. The observed appearance of this *large-scale structure* of the Universe turns out to be difficult to explain without the supplemental gravitational pull of dark matter.

Finally, a very strong indication for the existence of dark matter comes from the observations of the CMB (Planck Collaboration et al., 2018). This relic radiation originates from the epoch of recombination in the early

Universe, when protons and electrons of the previously ionised hydrogen plasma were able to combine for the very first time to form neutral atoms. Starting from this point in time, photons have propagated through the Universe without being continuously scattered by charged particles. All these photons combined are the source of a faint nearly isotropic black-body radiation, whose emissivity in the present-day Universe peaks in the microwave region of the electromagnetic spectrum. A crucial feature of the CMB is the characteristic pattern of tiny anisotropies, which is commonly analysed in terms of the angular power spectrum. The measured power spectrum of the CMB (i.e. the spacing and amplitudes of its peaks) turns out to match the predictions from the Λ CDM cosmological model with extremely high precision (Planck Collaboration et al., 2018), thus providing another powerful piece of evidence in support of the dark-matter hypothesis.

Modified gravity as an alternative solution

For completeness, we note that there exists a class of theories known as *modified gravity*, including the *modified Newtonian dynamics* (MOND, Milgrom, 1983), the *tensor-vector-scalar gravity* (TeVeS, Bekenstein, 2004) and the *entropic gravity* (Verlinde, 2011). These question the very existence of dark matter and instead attempt to account for the anomalous observations attributed to dark matter by introducing modifications to the current theory of gravity (i.e. general relativity and its Newtonian limit). The key assumption of modified gravity is that general relativity is valid only for large accelerations, such as those observed in the Solar System. However, it does not longer hold for objects experiencing much lower accelerations, such as e.g. the stars in the outer regions of spiral galaxies. This idea is encapsulated in a more general acceleration-dependent gravitational-force law, which is consistent with the Newtonian limit for large accelerations, but leads to substantially different predictions in the limit of extremely small accelerations.

Modified gravity succeeded in predicting numerous phenomena on galactic scales that have indeed been observed, and are difficult to explain by the presence of dark matter. However, accounting for the observed properties of galaxy clusters, in particular merging galaxy clusters (e.g. Clowe et al., 2004; Markevitch et al., 2004), turned out to be challenging. Finally, as yet, there has been no satisfactory cosmological model built upon this hypothesis.

1.1.2 Dark-matter candidates

Despite the strong empirical evidence and the wide acceptance by the scientific community, the constituents of dark matter are yet to be discovered. Over the last forty years, numerous dark-matter candidates have been considered, ranging from the *ultralight bosons* to the massive *primordial black holes* and spanning approximately 90 orders of magnitude in mass (Bertone & Tait, 2018), see Fig. 1.1 for a graphical overview. Essentially, all these candidates can be divided into two distinct categories. The first category includes the *massive compact halo objects (MACHOs)* – a variety of massive non-luminous astronomical bodies, such as black holes, faint stars or stellar remnants. While most of these have been ruled out as the main constituents of dark matter (for example based on the absence of the gravitational microlensing effect in the Milky Way), the hypothetical primordial black holes (formed in the early Universe through the direct gravitational collapse of highly overdense regions) still remain a viable dark-matter candidate (e.g. Bird et al., 2016; Sasaki et al., 2018). The second category of candidates encompasses *particle dark matter* – as-yet unknown non-baryonic elementary particles, not included in the standard model of particle physics.

The most popular among the particle candidates are the *natural* candidates – particles whose existence would simultaneously solve another critical problem of fundamental physics. Among these, the best-studied class is that of the *weakly-interacting massive particles (WIMPs)* (Steigman & Turner, 1985) – thermal relics from the Big Bang with masses ranging from ~ 10 GeV to ~ 1 TeV, originally postulated in the framework of the supersymmetric extensions to the standard model of particle physics in an attempt to alleviate the *hierarchy problem*. Other popular natural dark-matter-particle candidates include the *sterile neutrinos* and the *axions*. The sterile neutrino is a more massive (with a mass of the order of 1 keV) slower form of the standard-model neutrino, that does not interact electroweakly, unlike ordinary neutrinos. Axions, with a mass of the order of 1-10 meV, have been originally invoked to explain the strong CP problem in quantum chromodynamics.

The reason for the enormous popularity of WIMPs as the primary dark-matter candidate is the *WIMP miracle*. As thermal relics, WIMPs are postulated to have been produced thermally in the early Universe, along with the standard-model particles. Initially, as long as the temperature

Figure 1.1: Possible solutions to the dark-matter problem. Credits: Bertone & Tait (2018).

of the Universe is sufficiently high (i.e. larger than the energy equivalent of the WIMP particle mass), the annihilation of the particle-antiparticle pairs is counterbalanced by the creation of new particles, and WIMPs are in thermal equilibrium with the photon-baryon plasma. However, as the Universe expands and cools down, the annihilation starts to dominate over the creation, and the abundance of WIMPs decreases exponentially. Over the course of time, the suppression of the interaction rate between WIMPs and the standard model particles becomes so strong that WIMPs decouple, or *freeze out*, at a co-moving number density which remains effectively constant till the present day. Surprisingly, the existence of WIMPs with the mass ~ 100 GeV (similar to the mass of a silver atom) would not only alleviate the aforementioned hierarchy problem in particle physics, but at the same time also lead to just the same relic number density (in the co-moving sense) as the one inferred from cosmological observations for the present-day density of dark matter.

Numerous particle-physics experiments have been dedicated to the search for WIMPs, axions and sterile neutrinos; however no dark-matter particles have been convincingly detected so far (e.g. Bertone & Tait, 2018).

1.1.3 Phenomenological dark-matter models

In parallel with the aforementioned efforts of the particle-physics community to detect the actual dark-matter particle and characterise its micro-physical properties, more general phenomenological dark-matter models have been studied in cosmology. These aim at constraining the nature and the macroscopic properties of dark matter, based on astronomical observations. In this framework, the dark-matter-particle candidates are divided into three broader categories – the *hot*, *cold* and *warm* dark matter – depending on the mean velocity of their random motion in the early Universe and the resulting effect on the cosmological structure-formation process (Bond & Szalay, 1983; Davis et al., 1985). This underlying thermal velocity determines the *free-streaming length* (FSL, e.g. Schneider et al., 2013) of the dark-matter particles, defined as the average distance covered by the particles beyond which they clump together and form cosmological structures. The FSL constitutes the limit on the smallest scales on which cosmological structure can form, as discussed below. All density fluctuations smaller than this length are smoothed out by free streaming, whereas larger fluctuations remain intact.

Hot dark matter

Historically, the first plausible dark-matter model has been the hot dark matter (HDM), defined as being composed of abundant low-mass (of the order of 1 eV) particles, such as e.g. the standard-model neutrinos (e.g. Primack, 2001), that are born highly-relativistic at early times and remain relativistic for a relatively long time in the history of the Universe. This relativistic velocity results in a FSL that is so large that all density fluctuations corresponding to the present-day galaxies and their substructure would have been washed out by free streaming, which is clearly contradicted by the existence of galaxies in the present-day Universe. The only way to nevertheless bring the HDM paradigm into accordance with observations would be to assume the *top-down structure-formation scenario*, according to which the largest cosmological structures formed first and fragmented into smaller mass clumps (such as individual galaxies) only at a later time in the evolution of the Universe. However, this scenario is excluded based on observations of the high-redshift Universe. These show that in reality the first objects to form are individual galaxies, which subsequently merge in a hierarchical manner to finally form galaxy clusters and superclusters. All in all, HDM can at most account for only a tiny fraction of the total matter in the Universe and is ruled out as a sole solution to the dark-matter problem (e.g. Primack, 2001).

Cold dark matter

The currently favoured dark-matter model is the cold-dark-matter (CDM) paradigm, which assumes that dark matter is made of classical, collisionless elementary particles with negligible thermal velocities at early times, and is associated with the hierarchical *bottom-up structure-formation scenario*. Due to the relatively large particle mass (~ 100 GeV) and the resulting small FSL, CDM retains practically all density fluctuations seeded in the primordial matter distribution during the epoch of inflation and does not suppress structure formation except on very small scales ($\sim 10^{-6} M_{\odot}$). The CDM paradigm is in an excellent agreement with a wide variety of observations on the scales of galaxies and larger, such as the power spectrum of the temperature anisotropies in the CMB or the large-scale (larger than ~ 1 Mpc) distribution of galaxies in the Universe (e.g. Hoekstra et al., 2004; Vogelsberger et al., 2014; Schaye et al., 2015; Guo et al., 2016; Planck Collaboration et al., 2018).

Yet, the theory faces a number of challenges in the non-linear sub-galactic regime, i.e. on length scales smaller than ~ 1 Mpc and mass scales smaller than $\sim 10^{11} M_{\odot}$ (for a recent review on the small-scale challenges to the CDM paradigm see Bullock & Boylan-Kolchin, 2017). The best-documented and discussed small-scale tensions of the CDM model have been the *core-cusp problem* (i.e. the observed cores of many dark-matter dominated galaxies are less dense and less cuspy than predicted for CDM), the *too-big-to-fail problem* and the *missing satellites problem* (MSP).

In particular, the MSP refers to the fact that the number of dwarf satellite galaxies observed in the Local Group (~ 100 , e.g. McConnachie, 2012; Drlica-Wagner et al., 2015) is significantly lower than the predicted abundance of sub-haloes populating galactic haloes in CDM-based numerical simulations of the cosmological structure-formation process (e.g. Klypin et al., 1999; Moore et al., 1999; Diemand et al., 2007; Nierenberg et al., 2016; Dooley et al., 2017). One possible way to resolve the MSP is by assuming that the missing sub-haloes do exist, but are either devoid of baryons or extremely inefficient at forming stars (due to a variety of baryonic processes, such as feedback from massive stars and active galactic nuclei, tidal stripping or photo-ionization squelching, e.g. Thoul & Weinberg, 1996; Bullock et al., 2000; Somerville, 2002; Sawala et al., 2014; Kim et al., 2017; Despali et al., 2018), and thus remain invisible for conventional imaging surveys. Alternatively, the MSP might point towards alternative dark-matter models which suppress the structure formation on sub-galactic scales by assuming higher thermal velocities of dark-matter particles in the early Universe, as discussed below.

Warm dark matter

One of the best-studied alternative dark-matter models, postulated in an attempt to alleviate the challenges faced by the CDM model on the sub-galactic scales, is the warm dark-matter (WDM) hypothesis (Bode et al., 2001; Menci et al., 2012; Nierenberg et al., 2013; Viel et al., 2013; Lovell et al., 2014; Vegetti et al., 2018a). Compared to CDM, the WDM particles have a lower mass (of the order of 1 keV), higher thermal velocities and, thus, a larger FSL in the early Universe. As a result, WDM is able to stream out of primordial density fluctuations corresponding to the present sub-galactic scales and, thus, suppress the formation of mass structure in galactic haloes. The smaller the mass of the WDM particle, the stronger

Figure 1.2: Predicted dark-matter distribution in a cold-dark-matter (*left panel*) vs. warm-dark-matter (*right panel*) galactic halo. Credits: Lovell et al. (2014).

the resulting suppression. Consequently, the WDM hypothesis leads to similar predictions as CDM on large spatial scales (above ~ 10 Mpc), but results in less abundant mass structure on the smaller galactic and sub-galactic scales, as is illustrated in Fig. 1.2. One of the most popular WDM candidates is the sterile neutrino, introduced in Section 1.1.2.

1.1.4 Sub-galactic mass structure as a key probe of the dark-matter physics

As concluded in the previous section, the alternative dark-matter models predict significantly different levels of mass structure on the sub-galactic scales. Within the standard CDM paradigm, dark matter at sub-galactic scales is expected to be distributed among a numerous population of sub-haloes. In particular, a fundamental prediction of the CDM paradigm is the existence of a large number of very faint or even purely dark *low-mass* (less massive than $\sim 10^8 M_\odot$) sub-halos (Sawala et al., 2016). On the other hand, theories in which dark matter has a significant FSL, such as WDM, predict a dearth of such low-mass dark matter sub-haloes. Therefore, the actual mass distribution on galactic and sub-galactic scales (i.e. kpc and

sub-kpc scales) is very likely to hold important clues about the nature and properties of dark matter.

A common way to characterise the population of such sub-galactic structures is the *sub-halo mass function* $dN/d\ln(m_{sub})$, which quantifies the abundance of dark-matter sub-haloes per unit (logarithmic) sub-halo mass m_{sub} . Fig. 1.3 presents the sub-halo mass function for CDM and WDM with different dark-matter particle mass (Hsueh et al., 2020). As is apparent from Fig. 1.3, the different phenomenological dark-matter models are practically indistinguishable with respect to the predicted characteristics of the high-mass sub-haloes (with masses above $\sim 10^8 - 10^9 M_\odot$). The critical difference lies in the abundance and statistical properties of low-mass sub-galactic structure.

For this reason, constraining the abundance and properties of low-mass sub-galactic structure in real galaxies constitutes a unique avenue to probe the nature of dark matter and distinguish between the alternative phenomenological dark-matter models. The outcome of such a test might either verify the CDM paradigm or require its substantial revision. A powerful tool to detect and quantify (dark) sub-galactic mass structure is strong gravitational lensing, discussed in the next section.

1.2 Probing sub-galactic mass structure with galaxy-galaxy strong gravitational lensing

As discussed in the previous section, the disparate predictions from the alternative dark-matter models can be tested by investigating the clumpiness of the mass distribution inside real galaxies and their haloes. However, dark matter does not appear to interact with electromagnetic radiation and can, thus, only be studied via its gravitational effects on the visible baryonic matter. For this reason, strong gravitational lensing by galaxies, being equally sensitive to both baryonic and dark matter, has emerged as a unique tool to map out the spatial mass distribution in galaxies and probe the sub-galactic mass structure (Mao & Schneider, 1998).

In this section, we elaborate on the fundamentals of galaxy-galaxy strong gravitational lensing, the knowledge of which is vital for the understanding of the main chapters. After that, we focus on *substructure lensing*, i.e. the application of strong gravitational lensing to the study of

Figure 1.3: The sub-halo mass function in the CDM and alternative WDM models with different thermal-relic particle mass m_{WDM} . WDM models assume a smaller particle mass and, thus, a larger free-streaming length than the CDM model, which leads to a suppression of the low-mass sub-halo abundance. Credits: Hsueh et al. (2020).

the sub-galactic mass structure, which is the main technique used in this thesis.

1.2.1 The fundamentals of galaxy-galaxy strong gravitational lensing

In this section, we summarise the fundamentals of galaxy-galaxy strong gravitational lensing and discuss the basic equations quantifying this effect, in the framework of gravitational lens theory. We focus on the topics most relevant for the research presented in this thesis. The reader is referred to Meylan et al. (2006) for a more detailed discussion.

The phenomenon of *galaxy-galaxy strong gravitational lensing* occurs when, from the perspective of an observer on the Earth, two (or more) galaxies happen to lie along the same line of sight. As a consequence of this serendipitous alignment, the light rays propagating from the background *source galaxy* towards the observer are deflected by the curvature of space-time caused by the intervening mass of the foreground *lens galaxy*, in accordance with the theory of General Relativity. As is illustrated in Fig. 1.4, this *gravitational light deflection* leads to the emergence of extended lensed images, either in the form of *gravitational arcs* or, in the case of a perfect alignment and nearly spherical lens- and source galaxies, even a complete *Einstein ring*. The so far largest sample of such galaxy-galaxy strong gravitational lens systems is the Sloan Lens ACS Survey (SLACS, Bolton et al., 2008), which comprises more than 100 spectroscopically-confirmed systems. Fig. 1.5 presents 60 of these SLACS lenses, observed with the *Hubble Space Telescope* (HST).

As can be seen in Fig. 1.4, the light rays from the source galaxy are deflected smoothly as they approach the lens galaxy. However, since the radial extent of a typical lens galaxy is much smaller than the distances between the observer, the lens- and the source galaxy, the lens galaxy can be considered as *geometrically thin* and modelled as if the entire mass lay in one plane (and thus at one single redshift). Considering further that the deflection angles are typically very small and the gravitational field of the lens galaxy is weak (as compared for example to a black hole), the *thin-lens approximation* allows one to describe a galaxy-galaxy strong gravitational lens system within a much simpler framework of the geometrical optics. The trajectory of the deflected light ray can be in this case well approximated by two straight lines with a kink near the lens galaxy.

Figure 1.4: Visualisation of a galaxy-galaxy strong gravitational-lens system with a nearly complete Einstein ring. Credits: ALMA (ESO/NRAO/NAOJ), L. Calçada (ESO), Y. Hezaveh et al.

Fig. 1.6 depicts a sketch of a typical galaxy-galaxy strong gravitational lens system in the thin-lens approximation. The spatial configuration of the lens system is parameterised by the angular-diameter distances from the observer to the lens galaxy D_d , from the observer to the source galaxy D_s and from the lens to the source galaxy D_{ds} . The *lens plane* is defined as a plane perpendicular to the optical axis (dashed line connecting the observer and the lens galaxy), located at the distance of the lens galaxy. Similarly, the *source plane* is defined as a plane perpendicular to the optical axis at the distance of the source galaxy.

The three-dimensional mass distribution (including the possible line-of-sight structure) of the lens galaxy $\rho(\boldsymbol{\xi}, z)$, expressed as a function of the two-dimensional angular coordinate $\boldsymbol{\xi} = D_d \boldsymbol{\theta}$ in the lens plane (Fig. 1.6) and the location along the optical axis z , is then projected onto the lens plane to obtain the two-dimensional *surface-mass density*:

$$\Sigma(\boldsymbol{\xi}) = \int dz \rho(\boldsymbol{\xi}, z). \quad (1.1)$$

The commonly used (dimensionless) *convergence*:

$$\kappa(\boldsymbol{\xi}) = \frac{\Sigma(\boldsymbol{\xi})}{\Sigma_{cr}} \quad (1.2)$$

Figure 1.5: HST-imaging (colour-enhanced) of 60 strong gravitational-lens systems discovered by the SLACS Survey. In each image, the massive foreground lens galaxy is seen in yellow to red, whereas the lensed features of the more distant background galaxy are seen in blue. The images are arranged in order of increasing distance of the foreground lens galaxy from the Earth. Credits: A. Bolton (UH/IfA) for SLACS and NASA/ESA.

Figure 1.6: Sketch of a galaxy-galaxy strong gravitational lens system in the thin-lens approximation. Credits: Meylan et al. (2006).

is defined as the surface-mass density of the lens galaxy $\Sigma(\xi)$ in units of the *critical surface-mass density* for strong gravitational lensing:

$$\Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}}, \quad (1.3)$$

necessary for the emergence of multiple lensed images.

A quantity commonly used to characterise the lensing strength of a (spherical) lens galaxy is the *Einstein radius*, defined as the radius of a circular region surrounding the centre of the lens galaxy within which the average $\kappa = 1$. In the case of a perfect alignment of two nearly spherical galaxies, the background source galaxy is lensed into an *Einstein ring* with the radius equal to the Einstein radius.

The curvature of space-time caused by the lens galaxy is conveniently quantified in terms of the two-dimensional *lensing potential* ψ , which is defined as the solution of the Poisson's equation:

$$\nabla^2 \psi = 2\kappa. \quad (1.4)$$

A given convergence $\kappa(\theta)$ can be related to the corresponding lensing potential $\psi(\theta)$ by solving the following integral:

$$\psi(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \kappa(\theta') \ln |\theta - \theta'|. \quad (1.5)$$

In this picture, galaxy-galaxy strong gravitational lensing can be seen as a transformation between the direction on the sky from which a light ray would reach the observer in the absence of the deflecting lens galaxy (angle β) and the direction of the actually observed deflected light ray (angle θ). The lensing potential and the resulting *deflection-angle field* $\hat{\alpha}(D_d \theta)$ experienced by the light rays are linked via the gradient operation:

$$\alpha = \nabla \psi, \quad (1.6)$$

where α is the *scaled deflection angle* defined as:

$$\alpha(\theta) = \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \theta). \quad (1.7)$$

The deflection-angle field determines a mapping between the corresponding angular coordinates $\xi = D_d \theta$ and $\eta = D_s \beta$ in the lens- and the source

plane, respectively (Fig. 1.6). This mapping is encapsulated in the *lens equation*:

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}). \quad (1.8)$$

The lens equation together with the principle of surface-brightness conservation in strong gravitational lensing allows one to simultaneously reconstruct both the mass distribution in the foreground lens galaxy and the intrinsic unlensed surface-brightness distribution of the background source galaxy. Such reconstructions, obtained in the process of the *lens modelling*, are at the core of the research performed in this thesis.

1.2.2 Smooth lens modelling

An observed image of a galaxy-galaxy strong gravitational lens system constitutes a superposition of light coming from the foreground lens galaxy, the lensed images of the background source galaxy and possible additional (foreground or background) sources present in the covered field of view. The overall goal of the lens modelling is to reconstruct the intrinsic properties of the observed lens system – the mass distribution of the foreground lens galaxy (projected along the line of sight) and the intrinsic (unlensed) surface-brightness distribution of the background source galaxy. Since most of the known lens galaxies are massive ellipticals (e.g. Treu, 2010), which are empirically known to have a very smooth featureless morphology, the most common approach to modelling galaxy-scale lens systems is the *smooth lens modelling*. The underlying assumption of smooth lens modelling is that the mass in the lens galaxy is distributed smoothly, i.e. there are no substructures. While this assumption has proven to be sufficient to determine the overall properties of strong lens galaxies (such as the total projected mass inside the Einstein radius or the overall ellipticity, orientation and the radial density profile of the mass distribution), for the investigation performed in this thesis, the smooth lens modelling constitutes merely the initial step preceding the actual search for the imprints of possible subgalactic mass structure in the lens galaxy on the lensed images. As will be discussed in the next section, mass structure in the lens galaxies is commonly modelled in terms of deviations from the best-fitting smooth-lens model. The latter is in this case treated as a benchmark fiducial model used for a comparison with models containing substructure.

The smooth component of the lens galaxy is usually modelled by inferring the parameter values of a chosen parametric mass-density distribution. A well known dynamical model, found to provide a good fit to the kinematics (i.e. the motion of stars and gas) of galaxies in general, is the *singular isothermal sphere* (SIS). The SIS model is characterised by the following three-dimensional density profile (e.g. Meylan et al., 2006; Treu, 2010), fully determined by the stellar velocity dispersion σ_{SIS} in the modelled galaxy:

$$\rho(r) = \frac{\sigma_{SIS}^2}{2\pi G r^2}, \quad (1.9)$$

with the Einstein radius θ_E given by:

$$\theta_E = 4\pi \left(\frac{\sigma_{SIS}}{c} \right)^2 \frac{D_{ds}}{D_s}. \quad (1.10)$$

However, in reality massive elliptical galaxies are found to be ellipsoidal in projection. For this reason, lens galaxies are typically modelled with the more realistic elliptical generalisation of the SIS profile – the *singular isothermal ellipsoid* (SIE, Kormann et al., 1994), which has been found to provide a remarkably good description of the lensed images. In addition, the smooth-lens model usually includes an *external-shear field* that quantifies the perturbative effect of the local environment (e.g. from the large-scale structure or nearby galaxies) on the lensing potential.

1.2.3 Signatures of sub-galactic mass structure in the lensed images

The main question to be answered at this point is what kind of signatures in gravitationally-lensed images one can search for in order to detect or constrain possible mass structure in the lens galaxy. With this question in mind, in this section we elaborate on the perturbations that arise in the lensed images if the mass distribution of the lens galaxy is not smooth, as assumed in the previous section, but contains (dark) mass structure on sub-galactic scales, as predicted by the CDM paradigm.

Two main observables have been widely used to probe sub-galactic mass structure with strong gravitational lensing: the *flux-ratio anomalies* in observations of distant gravitationally-lensed quasars and the *surface-brightness perturbations* in extended lensed images of background galaxies,

caused either by individual massive subhaloes appearing close in projection to the lensed images or by the collective lensing effect of the entire population of small-scale mass structure, quantified in terms of the substructure power spectrum (e.g. Mao & Schneider, 1998; Dalal & Kochanek, 2002; Koopmans, 2005; Hezaveh et al., 2016). Below, we briefly discuss these different approaches.

Flux-ratio anomalies in multiply imaged quasars

The most prominent lensing effect of sub-galactic mass structure are the *flux-ratio anomalies* in observations of gravitationally-lensed distant quasars (see e.g. Mao & Schneider, 1998; Dalal & Kochanek, 2002; Treu, 2010). Even though the multiple lensed images in such lens systems originate from the same background source, their fluxes are not identical. Instead, the flux of each individual lensed image depends on the magnification factor, which is determined by the projected mass density of the lens galaxy at the specific position of the image in the lens plane. In the specific case of quadruply imaged quasars (arranged either in the *cusp* or the *fold* configuration), a globally smooth lens model predicts the flux values of the lensed images to obey the *magnification relations* (i.e. the sum of the magnification factors for the closest lensed images is equal zero), which provides a corresponding prediction for the flux ratios between the lensed images. However, for most of the known quadruply-imaged lensed quasars, the observed flux ratios are significantly different from this prediction, even though smooth lens models are sufficient to reproduce the observed positions of the lensed images with an excellent accuracy. These flux-ratio anomalies are most likely caused by the presence of mass structure in the lens galaxy, which locally perturbs the magnification pattern of the lensed images. Over the last two decades, the study of such flux-ratio anomalies in multiply lensed distant quasars has remained one of the main methods to probe sub-galactic mass structure beyond the Local Universe and has provided some of the most stringent constraints on the particle mass of dark matter (see e.g. Birrer et al., 2017; Gilman et al., 2019; Hsueh et al., 2020).

Gravitational imaging of single massive sub-haloes in lens galaxies

An alternative approach to search for sub-galactic structure in (massive elliptical) lens galaxies, which builds the basis for the methodology

developed in this thesis, is the *gravitational-imaging technique* (Koopmans, 2005; Vegetti & Koopmans, 2009a). This allows one to constrain the projected position and mass of individual sub-haloes in the lens galaxy via their imprints on the extended lensed images (such as Einstein rings and gravitational arcs) of the background galaxy (see Fig. 1.7 for an example of a gravitational arc perturbed by a massive luminous substructure). Sub-galactic mass structure, if present in the lens galaxy (or along its line-of-sight), induces perturbations to the otherwise smoothly-distributed lensing potential, which leads to deviations between the observed surface-brightness distribution of the lensed images and the predictions from a smooth-lens model. The gravitational-imaging technique makes it possible to model the resulting observable *surface-brightness anomalies* in the lensed images and trace them back to the underlying substructure in the lens galaxy.

Applications of the gravitational-imaging technique to deep HST-observations of galaxy-scale lens systems have so far lead to a detection of two dark-matter sub-structures with masses $3.5 \times 10^9 M_\odot$ and $1.9 \times 10^8 M_\odot$ at the redshifts $z = 0.2$ and $z = 0.881$, respectively, with the latter one being the smallest and most distant galactic substructure discovered up to now beyond the Local Universe (Vegetti et al., 2010c, 2012). These detections demonstrate that galactic sub-haloes can indeed be successfully identified as localised corrections to the smooth gravitational potential. However, despite these encouraging results, the current mass-detection threshold of the gravitational-imaging technique ($\sim 10^8 M_\odot$ for HST-observations of SLACS lenses) still lies above the mass regime in which the alternative dark-matter models could be distinguished, given the current sample sizes and data sensitivity, see Fig. 1.3.

Collective effect of low-mass sub-galactic structure

In response to the current observational limitations of the gravitational-imaging technique, an alternative statistical approach has emerged in the literature over the recent years (Bus, 2012; Hezaveh et al., 2016; Diaz Rivero et al., 2018a; Chatterjee & Koopmans, 2018; Díaz Rivero et al., 2018c; Cyr-Racine et al., 2019). Instead of single localised potential corrections representing individual massive sub-haloes in the original gravitational-imaging technique (Vegetti & Koopmans, 2009a), the alternative statistical approach models the entire population of low-mass sub-galactic structure in the lens galaxy in a statistical manner – in terms of the projected

Figure 1.7: An example of a gravitational arc perturbed by a massive substructure in the lens galaxy SDSS J120602.09+514229.5. The long gravitational arc splits and bends around a luminous satellite galaxy with the mass of $2.75 \times 10^{10} M_{\odot}$. The gravitational-imaging technique makes it possible to model such surface-brightness perturbations in the lensed images, and trace them back to the underlying substructure in the halo of the lens galaxy, even if the substructure is a few orders of magnitude less massive and purely dark. Credits: Vegetti et al. (2010b).

matter power spectrum. Within this framework, the critical low-mass sub-galactic structure can potentially be constrained based on the collectively-induced surface-brightness perturbations in the extended lensed images of the background-source galaxy. Despite the considerable number of recent theoretical studies (Bus, 2012; Hezaveh et al., 2016; Díaz Rivero et al., 2018a; Chatterjee & Koopmans, 2018; Díaz Rivero et al., 2018c; Cyr-Racine et al., 2019), there has been not a single study carried out so far with the aim of actually measuring or inferring observational constraints on the projected matter power spectrum from galaxy-galaxy strong gravitational lensing. This thesis aims to close this gap.

1.3 Thesis aims and objectives

The research presented in this thesis aims at constraining the properties of low-mass structure in the total (dark and baryonic) mass distribution of massive early-type lens galaxies (and, if present, along the line-of-sight), based on a statistical analysis of surface-brightness anomalies revealed in deep HST observations of selected galaxy-galaxy strong gravitational lens systems from the SLACS Survey. In this approach, we treat the hypothetical sub-galactic mass structure in a statistical sense – as an ensemble of small-scale density variations (i.e. deviations from a smooth elliptical power-law density model with external shear) arising from a combined effect of low-mass dark-matter sub-haloes, line-of-sight haloes and small-scale density inhomogeneities in the baryonic mass distribution, such as e.g. globular clusters or tidal streams. The associated population of surface-brightness anomalies arising in the extended lensed images of the background galaxies (i.e. Einstein rings or gravitational arcs) due to the collective perturbative lensing effect of such small-scale density variations resembling a random field is quantified in terms of the power spectrum, as first proposed by Bus (2012) and later by Hezaveh et al. (2016) and Chatterjee & Koopmans (2018). With this goal in mind, we introduce and test a new methodology to reliably extract this power spectrum from deep HST-imaging of galaxy-scale strong gravitational lens systems and trace it back to the underlying collective effect of small-scale sub-galactic mass structure in (massive elliptical) lens galaxies.

This approach has its roots in the gravitational-imaging technique (Koopmans, 2005; Vegetti & Koopmans, 2009a), however it applies two remedies in order to circumvent the current limitations of the original

technique. First, we implement the statistical framework by Chatterjee & Koopmans (2018), which relates the collective effect of low-mass sub-galactic structure to the power spectrum of the resulting surface-brightness anomalies in the lensed images. Second, in order to increase the sensitivity towards low-mass structure, our analysis is performed based on the ultra-violet imaging data of lens systems with highly structured star-forming lensed galaxies. To this end, we analyse the first deep HST/WFC3/F390W-imaging data of a SLACS sub-sample observed in the ultra-violet. The thesis is structured as follows.

Thesis outline

In **Chapter 2**, we discuss in detail all steps of our new methodology, allowing us to reliably extract the power spectrum of surface-brightness anomalies in extended lensed images from deep HST-imaging of galaxy-scale strong gravitational lens systems. We specifically focus on the observational aspects of the power-spectrum measurement, such as the most suitable observational strategy and sample selection. Moreover, we investigate the degeneracies in our measurement, i.e. other effects that can mimic the surface-brightness anomalies induced by mass structure in the lens galaxy, and the modelling challenges inherent in our approach. Finally, we test the feasibility of our methodology by applying it to a sub-sample of galaxy-galaxy strong gravitational lens systems from the SLACS Survey with the most extended, bright, high-signal-to-noise-ratio lensed images, observed with HST/WFC3/F390W in the ultra-violet.

In **Chapter 3**, we relate the measured power spectrum of surface-brightness anomalies to the underlying statistical properties of sub-galactic mass structure in the lens galaxy. Following the formalism introduced by Chatterjee & Koopmans (2018), we model this mass structure in terms of Gaussian-Random-Field (GRF) potential perturbations superposed on the best-fitting smoothly-varying lensing potential. We assume these potential perturbations (on the scales between the FWHM of the PSF and the extent of the lensed images) to be characterised by a power-law power spectrum $P_{\delta\psi}(\sigma_{\delta\psi}^2, \beta, k)$ with two free parameters: the total variance of the GRF-fluctuations $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β . The core of our approach is a systematic study of mock surface-brightness anomalies induced in the lensed images by a priori known low-mass structure with varying statistical properties. The corresponding mock power spectra of

surface-brightness anomalies can finally be compared to a real measurement in order to infer constraints on the statistical properties of low-mass structure in (massive elliptical) lens galaxies. As a proof of concept, we apply the methodology to deep high-resolution HST/WFC3/F390W observations of the strong gravitational lens galaxy SDSS J0252+0039 from the SLACS Survey (Bolton et al., 2008). This analysis leads to the first (conservative) upper-limit constraints on the level of small-scale mass structure in the total (dark and baryonic) mass distribution of a (massive elliptical) lens galaxy.

The goal of **Chapter 4** is to narrow down these constraints by improving the modelling technique and including bias corrections in the lens modelling, thereby paving the way for a future detection of small-scale mass structure in massive elliptical lens galaxies. A key problem in the current methodology is the degeneracy between the investigated surface-brightness anomalies due to mass structure in the lens galaxy and the possible intrinsic structure of the background-source galaxy itself, which might lead to a bias in our inference. An unbiased detection of small-scale mass structure is possible only if this degeneracy can be understood in detail and mitigated. With this in mind, we introduce and test a modified methodology that applies a new implementation of a Bayesian grid-based smooth-lens modelling code, developed by Vernardos et al. (in prep.), allowing us to model the mock perturbed lens systems in exactly the same manner as the real observations. By doing so, we intend to thoroughly account for the potential biases inherent in the lens-modelling procedure and make a proper comparison with the results obtained from real data. This analysis is based on the deep high-signal-to-noise-ratio HST/ACS/F814W-observations of the strong gravitational lens system SDSS J0946+1006, commonly known as the Double Einstein Ring (Gavazzi et al., 2008), with a substantially higher signal-to-noise ratio and a less-structured source galaxy than the HST/WFC3/F390W-observations of SDSS J0252+0039 investigated in Chapter 3.

Finally, **Chapter 5** provides a summary of the main results obtained in this thesis and discusses their implications for future work.

Chapter 2

Power-spectrum analysis of surface-brightness anomalies in deep ultra-violet HST-observations of SLACS lens systems

— Based on

**Power spectrum of surface-brightness anomalies in deep
ultra-violet HST-observations of SLACS lens systems as a
probe of small-scale sub-galactic mass structure**

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Abstract

While a direct detection of the dark-matter particle remains very challenging, the nature of dark matter can be constrained indirectly – by comparing the statistical properties of low-mass sub-galactic structure with predictions from phenomenological dark-matter models, such as the cold, warm or hot dark matter. Galaxy-galaxy strong gravitational lensing provides a unique opportunity to search for gravitational signatures of such low-mass structure in (massive elliptical) lens galaxies beyond the Local Group. In this chapter, we test a novel methodology to measure the power spectrum of surface-brightness anomalies imprinted on the extended lensed images (i.e. Einstein rings or gravitational arcs) due to the presence of small-scale density fluctuations in the lens galaxy. This methodology will be used in the next chapter to infer the first observational constraints on the statistical properties of small-scale sub-galactic structure in a massive elliptical lens galaxy. Here we focus on the observational aspect of the power-spectrum measurement and the various modelling challenges. In particular, we discuss and mitigate the presence of noise correlations in the drizzled *Hubble Space Telescope* (HST) images. Finally, we demonstrate the feasibility of our approach by applying it to a sub-sample of galaxy-galaxy strong gravitational lens systems from the Sloan Lens ACS Survey with the most extended, bright, high-signal-to-noise-ratio lensed images, from new data obtained with HST/WFC3/F390W in the rest-frame ultra-violet.

2.1 Introduction

Over the last forty years, studying the mass distribution within galaxies has provided valuable insights into the complex processes of galaxy formation and evolution. Most importantly, the mass-density profiles inferred from dynamical studies of spiral galaxies provided evidence for the existence of a dominant non-baryonic matter component (Bosma, 1978; Rubin et al., 1978), referred to as dark matter, which nowadays constitutes a crucial pillar of the concordance dark-energy-plus-cold-dark-matter (Λ CDM) cosmological model. According to this model and the associated hierarchical structure-formation scenario, the initial gradual collapse of dark matter into haloes was necessary to create potential wells for the eventual collapse of the baryonic matter and the formation of the galaxies. Despite the essential role of dark matter in this galaxy-formation process, its nature and properties remain unknown. The standard cold-dark-matter (CDM) paradigm is still challenged by various alternative models, such as for example warm dark matter (WDM, see e.g. Bode et al., 2001; Lovell et al., 2014) or self-interacting dark matter (SIDM, see e.g. Spergel & Steinhardt, 2000; Tulin & Yu, 2018). These were proposed to explain discrepancies between the predictions from Λ CDM-based simulations and the observed number of dwarf satellite galaxies in the Local Group (i.e. the Missing Satellites Problem, see e.g. Klypin et al., 1999; Moore et al., 1999; Diemand et al., 2007; McConnachie, 2012; Drlica-Wagner et al., 2015; Nierenberg et al., 2016; Dooley et al., 2017).

While a direct detection of the dark-matter particle remains challenging (see e.g. Bertone & Tait, 2018), it might be possible to constrain the properties of dark matter indirectly by comparing the observed abundance of low-mass sub-galactic structure with predictions from the alternative phenomenological dark-matter models. These crucially depend on the free-streaming length of the dark-matter particles in the early Universe, which in turn is determined by the assumed properties of the dark-matter particle, such as the particle mass or the strength of interactions between the individual particles. The effect of varying the free-streaming length on the resulting level of sub-galactic mass structure (quantified in terms of the subhalo mass function) can be investigated by means of numerical simulations of structure formation. Recent studies have shown that the predictions from the alternative dark-matter models vary considerably with respect to the abundance of sub-galactic structure with mass lower

than $\sim 10^8 M_\odot$ (see, for example, Bullock & Boylan-Kolchin, 2017), thus making the low-mass regime critical for dark-matter studies. For example, simulations based on the Λ CDM model, which assumes a greater dark-matter particle mass and, thus, a lower free-streaming length, lead to a substantially higher abundance of sub-galactic structure with mass below $\sim 10^8 M_\odot$ than the alternative WDM models with a lower dark-matter particle mass and, consequently, a higher free-streaming length.

These predictions could in principle be used to distinguish between the CDM and WDM models. However, in addition to the particle properties of dark matter, the level of mass structure in galactic haloes is crucially altered by processes related to the baryonic matter component (e.g. star formation, supernova feedback or AGN feedback), which are not yet sufficiently understood. The recent hydrodynamical simulations of the cosmological structure-formation process, such as Illustris or EAGLE (Vogelsberger et al., 2014; Schaye et al., 2015), attempt to incorporate the impact of such baryonic processes by considering several distinct galaxy-formation scenarios within a particular dark-matter model.

In order to distinguish between the alternative dark-matter models and galaxy-formation scenarios, the outcomes of the simulations need to be compared with observational constraints inferred from a statistically representative sample of real galaxies. However, investigating the low-mass structure beyond the Local Group is a demanding undertaking. Even if luminous, sub-galactic mass structure at cosmological distances might be too faint to be observed directly with the currently available instruments. In addition, the mass structure might be dark-matter dominated or even purely dark (i.e. completely devoid of stars) and, thus, intrinsically invisible, which makes these studies even more challenging. For this reason, constraints on the properties of mass structure in galaxies at cosmological distances have so far been mainly inferred from observations of galaxy-scale strong gravitational lens systems. The phenomenon of galaxy-galaxy strong gravitational lensing makes it possible to study mass structure in galaxies that, fortuitously, happen to lie in the same line-of-sight and act as a strong gravitational lens on another galaxy (or a quasar) located at a larger distance. The so far largest sample of such optically selected galaxy-galaxy strong gravitational lens systems is the Sloan Lens ACS Survey (SLACS, Bolton et al., 2008), comprising more than 100 lens systems.

The surface-brightness distribution of the lensed images in such galaxy-galaxy strong gravitational lens systems critically depends on the total

(baryonic and dark) projected mass distribution in the lens galaxy and along its line of sight. Thus, it can be used as an observational probe of the hypothetical small-scale (relative to the Einstein radius of the lens system) mass structure in the lens galaxy and the line-of-sight effects. Even if the mass structure in the lens galaxy is completely dark, its gravitational signature might be observable in the form of tiny surface-brightness anomalies in the extended lensed images of the background-source galaxy, measured with respect to the reference best-fitting smooth-lens model (Blandford et al., 2001; Koopmans, 2005; Rau et al., 2013). The presence of mass structure results in perturbations to the otherwise smooth lensing potential, which, in turn, causes perturbations in the deflection angles of the light rays crossing the lens plane in the proximity to the mass structure. The resulting surface-brightness anomalies in the lensed images can be modelled and traced back to the underlying mass structure in the lens galaxy.

One of the most successful methods in this respect has been the *gravitational-imaging technique*, developed by Koopmans (2005) and Vegetti & Koopmans (2009a) with the aim of searching for individual substructure in (massive elliptical) lens galaxies. Thanks to this approach, two dark-matter sub-haloes with masses $3.5 \times 10^9 M_\odot$ and $1.9 \times 10^8 M_\odot$ at the redshifts $z = 0.2$ and $z = 0.881$, respectively, have been detected in deep *Hubble Space Telescope* (HST) observations of galaxy-scale lens systems (Vegetti et al., 2010c, 2012), with the latter one being the smallest and most distant galactic substructure discovered up to now beyond the local Universe. However, despite these encouraging results, the current mass-detection threshold of the gravitational-imaging technique ($\sim 10^8 M_\odot$ for HST-observations of SLACS lenses; Vegetti et al., 2014, 2018a) still lies above the mass regime in which the alternative dark-matter models could be distinguished.

The approach discussed in this work has its roots in the original gravitational-imaging technique (Koopmans, 2005; Vegetti & Koopmans, 2009a). However, it applies two remedies in order to circumvent the current limitations. First, in order to increase the sensitivity towards the low-mass structure, our analysis is performed based on the ultra-violet imaging data of lens systems with highly structured star-forming lensed galaxies. To this end, we obtain and analyse the first deep HST-imaging data (HST/WFC3/F390W) of a SLACS sub-sample in the ultra-violet (Program 12898, Koopmans, 2012). Second, we implement the statistical framework

recently introduced by Chatterjee & Koopmans (2018). This relates the collective effect of small-scale sub-galactic structure (including the low-mass dark-matter sub-haloes and line-of-sight haloes, as well as small-scale density variations in the baryonic mass distribution, for example from tidal streams or globular clusters) to the power spectrum of the resulting surface-brightness anomalies in the lensed images.

Combining these two ideas, in this chapter, we carry out the first power-spectrum analysis of surface-brightness anomalies measured in observed extended lensed images of galaxy-galaxy strong gravitational lens systems. The principal goal of our research is to use this measurement to infer observational constraints on the statistical properties of the underlying low-mass structure in our SLACS sub-sample. This, in turn, might finally allow us to distinguish between the alternative dark-matter models and galaxy-formation scenarios. While the complete approach is presented in detail in the next chapter, here we focus on the observational aspect of our methodology. In particular, we elaborate on the most suitable observational strategy, sample selection, all the steps necessary to reliably extract the power spectrum of surface-brightness anomalies from HST-imaging of galaxy-scale strong gravitational lens systems, and the degeneracies in our models. One of the biggest challenges in our analysis is to distinguish between the real anomalies induced by mass structure and the spurious surface-brightness fluctuations present in the data due to the presence of observational noise.

The present chapter is structured as follows. In Section 2.2, we formalise the concept of surface-brightness anomalies in the lensed images and present a brief overview of the methodology to quantify these in terms of the power spectrum. Section 2.3 moves on to discuss the observational strategy, sample selection, the observations used for our analysis and the final data-reduction process. Sections 2.4, 2.5 and 2.6 describe the lens-galaxy subtraction, smooth lens modelling and the novel power-spectrum analysis of the residual surface-brightness fluctuations, respectively. In Section 2.7, we quantify the noise properties of our observational data in terms of the noise-power spectrum and perform a noise correction of the residual surface-brightness fluctuations. Section 2.8 demonstrates the feasibility of our approach in recovering known mock surface-brightness anomalies from perturbed lensed images mimicking real observations. Section 2.9 provides conclusions and implications for further analysis.

For a consistent comparison of the lens models with earlier studies by Vegetti et al. (2014), throughout this chapter we assume the following cosmology: $H_0 = 73 \text{ km s}^{-1}\text{Mpc}^{-1}$, $\Omega_M = 0.25$ and $\Omega_\Lambda = 0.75$. Given this cosmology, 1 arcsec corresponds to $\sim 4 \text{ kpc}$ at the redshift of the studied lens galaxies ($z_L \sim 0.2 - 0.4$).

2.2 Theoretical background and analysis overview

The research presented in this thesis aims at constraining statistical properties of low-mass structure in the total (dark and baryonic) mass distribution of massive early-type lens galaxies. We treat the hypothetical low-mass sub-galactic mass structure in a statistical sense – as an ensemble of small-scale density variations (i.e. deviations from a smooth elliptical power-law density model with external shear). These can arise from a combined effect of low-mass dark-matter sub-haloes, line-of-sight haloes and small-scale density inhomogeneities in the baryonic mass distribution, such as for example globular clusters or tidal features. The associated population of surface-brightness anomalies arising in the extended lensed images (i.e. Einstein rings or gravitational arcs) of the background galaxies due to the collective perturbative lensing effect of such small-scale density variations, resembling a random field, can be quantified in terms of the power spectrum, as recently proposed by Bus (2012) and Chatterjee & Koopmans (2018). Our novel methodology is based on a power-spectrum analysis of such surface-brightness anomalies revealed in HST-observations of galaxy-galaxy strong gravitational lens systems from the SLACS Survey.

In this section, we first discuss the notion of surface-brightness anomalies in extended lensed images (see Section 2.2.1) and elaborate on the observational challenges that need to be circumvented in order to measure them accurately. Subsequently, we present a concise overview of the analysis carried out in this chapter (see Section 2.2.2).

2.2.1 Surface-brightness anomalies in extended lensed images

Let us consider a galaxy-galaxy strong gravitational lens system with extended lensed images that are described by the surface-brightness distribution $I(\mathbf{x})$ measured as a function of the position $\mathbf{x}$ in the lens plane. The spatial configuration of the lens system is parameterized by

the angular diameter distances from the observer to the lens galaxy D_d , from the observer to the source galaxy D_s and from the lens to the source galaxy D_{ds} . Following the convention of strong gravitational lensing, we express the projected mass density $\Sigma(\mathbf{x})$ of the foreground lens galaxy (and the possible line-of-sight effects) in units of the critical surface-mass density:

$$\Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}} \quad (2.1)$$

to obtain the commonly used (dimensionless) convergence:

$$\kappa(\mathbf{x}) = \Sigma(\mathbf{x}) / \Sigma_{\text{cr}}. \quad (2.2)$$

Furthermore, we define the lensing potential $\psi(\mathbf{x})$:

$$\psi(\mathbf{x}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d\mathbf{x}' \kappa(\mathbf{x}') \ln |\mathbf{x} - \mathbf{x}'| \quad (2.3)$$

as a solution of the Poisson's equation:

$$\nabla^2 \psi(\mathbf{x}) = 2\kappa(\mathbf{x}). \quad (2.4)$$

The associated (scaled) deflection-angle field:

$$\alpha(\mathbf{x}) = \nabla \psi(\mathbf{x}) \quad (2.5)$$

determines a mapping between the corresponding positions $\mathbf{x}$ and $\mathbf{y}$ in the lens- and the source plane, respectively, which is encapsulated in the lens equation:

$$\mathbf{y}(\mathbf{x}) = \mathbf{x} - \alpha(\mathbf{x}). \quad (2.6)$$

This mapping together with the principle of the surface-brightness conservation in strong gravitational lensing:

$$I(\mathbf{x}) = S(\mathbf{y}(\mathbf{x})) \quad (2.7)$$

builds the foundation for numerical grid-based smooth-lens-modelling codes (e.g. the adaptive grid-based Bayesian lens-modelling code by Vegetti & Koopmans, 2009a, used in this work). By means of such codes it is possible to simultaneously reconstruct both the (parametrized) lensing potential

$\psi_0(\mathbf{x})$ and the unlensed surface-brightness distribution of the background source $S(\mathbf{y})$, under the assumption of a smooth mass distribution in the lens galaxy.

A comparison of the observed surface-brightness distribution of the lensed background galaxy $I(\mathbf{x})$ with the best-fitting smooth-lens model $I_0(\mathbf{x})$ allows one to search for the imprints of mass structure in the lens galaxy. As can be seen from equation 2.6, a possible deviation between the true and the best-fitting smooth lensing potential would lead to a slight modification in the mapping between the lens- and the source plane. This, in turn, would result in a surface-brightness change $\delta I(\mathbf{x})$ between the observed surface brightness of the lensed images and the prediction from the best-fitting smooth-lens model, such that:

$$\delta I(\mathbf{x}) = I(\mathbf{x}) - I_0(\mathbf{x}) = S(\mathbf{x} - \nabla\psi(\mathbf{x})) - S(\mathbf{x} - \nabla\psi_0(\mathbf{x})) \quad (2.8)$$

(Blandford et al., 2001; Koopmans, 2005). In what follows, we refer to such surface-brightness residuals $\delta I(\mathbf{x})$ caused by a deviation of the true lensing potential $\psi(\mathbf{x})$ from the best-fitting smooth lensing potential $\psi_0(\mathbf{x})$ as *surface-brightness anomalies*.

In reality, the extraction of such surface-brightness anomalies from the imaging of real lens systems is complicated by three independent phenomena.

- First, observational effects make it impossible to measure the true surface-brightness distribution of the lensed images $I(\mathbf{x})$. Instead, the true lensed images are blurred by the convolution with the point-spread function (PSF) and the pixellation of the imaging. Moreover, they are affected by the presence of the observational noise and subject to data processing, for example drizzling of the raw data to obtain the final science image (Gonzaga et al., 2012).
- Second, the reconstructed best-fitting unlensed surface-brightness distribution of the background source cannot be assumed to perfectly represent the reality. This is due to a degeneracy between the gravitational imprints of small-scale density fluctuations in the lens galaxy and the intrinsic surface-brightness fluctuations in the source galaxy. To mitigate this problem, the adaptive grid-based Bayesian smooth-lens-modelling code by Vegetti & Koopmans (2009a), applied in this work, incorporates a regularization of the source reconstruction by penalizing solutions with overly strong surface-brightness

fluctuations in the source galaxy, as discussed by Warren & Dye (2003) and Koopmans (2005). However, the imposed level of regularization itself is fitted as a free parameter in the lens-modelling procedure, which might lead to either suppressed or enhanced surface-brightness fluctuations in the source as a result of an over- or underregularised source reconstruction, respectively. In other words, the reconstructed smooth-lens model might potentially "absorb" the effect of mass structure in the lens galaxy into spurious source structure or vice versa. The surface-brightness anomalies induced by the mass structure might be artificially enhanced if the background-source reconstruction is overregularised (i.e. too smooth).

- Third, the effect of strong gravitational lensing is sensitive to the total mass present in the line-of-sight along which the gravitational lens is observed. Thus, the measured surface-brightness anomalies might arise not only from the investigated density fluctuations in the lens galaxy, but also from possible mass structure along the line-of-sight (Despali et al., 2018).

Thus, one of the main challenges in our approach is to reliably extract the true surface-brightness anomalies $\delta I(\mathbf{x})$ (as defined in equation 2.8) from the smooth lens-model residual, taking into consideration all the effects discussed above. The intended procedure to achieve this goal is outlined in the next section.

2.2.2 Analysis synopsis

In order to uncover possible surface-brightness anomalies in the extended lensed images of galaxy-galaxy strong gravitational lens systems and measure their power spectrum, we intend to carry out the following data-analysis steps:

1. HST-observations and data reduction by means of the DRIZZLEPAC package (Gonzaga et al., 2012), see Section 2.3;
2. modelling and subtraction of the surface-brightness contribution from the foreground lens galaxy using GALFIT (Peng et al., 2002) or, alternatively, the B-SPLINE algorithm (Bolton et al., 2006), see Section 2.4;

3. smooth lens modelling of the lensed images using the adaptive and grid-based Bayesian lens-modelling technique by Vegetti & Koopmans (2009a), see Section 2.5;
4. statistical quantification of the residual surface-brightness fluctuations in the lensed images in terms of the one-dimensional azimuthally-averaged power spectrum, see Section 2.6;
5. estimation of the noise power spectrum based on blank-sky fields from the HST-observations, modified to account for the Poisson noise of the specific observation, see Sections 2.7.

In the following sections, we discuss the individual steps in more detail and illustrate them with examples from our analysis of the selected SLACS sub-sample.

2.3 Observational strategy and data

In this section, we first motivate the choice of the ultra-violet band for our analysis and discuss our sub-sample selection criteria for the first HST/WFC3/F390W-observations of SLACS lens systems, performed for our study. Subsequently, we elaborate on the undersampling problem, the dithering strategy and the data reduction.

2.3.1 Selection of the observational filter

As pointed out by Blandford et al. (2001) and Koopmans (2005), the level of surface-brightness anomalies arising in extended lensed images of a background-source galaxy due to the presence of mass structure in the foreground lens galaxy depends not only on the mass of this substructure, but also on the surface-brightness gradient in the lensed galaxy itself. More specifically, if the population of mass structure in the plane of the lens galaxy is represented by a potential-perturbation field $\delta\psi(\mathbf{x})$ and the intrinsic surface-brightness distribution of the background galaxy in the source plane is described by $S(\mathbf{y})$, then the level of the resulting surface-brightness anomalies in the lensed images $\delta I(\mathbf{x})$ can be calculated as the inner product of the respective gradient fields:

$$\delta I(\mathbf{x}) = -\nabla S(\mathbf{y}(\mathbf{x})) \cdot \nabla \delta\psi(\mathbf{x}). \quad (2.9)$$

As can be seen from Eq. 2.9, the surface-brightness anomalies $\delta I(\mathbf{x})$ induced in the lensed images by a given potential-perturbation field $\delta\psi(\mathbf{x})$ can be enhanced by increasing the gradient (i.e. level of variations) in the intrinsic surface-brightness distribution of the background galaxy $\nabla S(\mathbf{y}(\mathbf{x}))$.

This directly leads to the choice of ultra-violet observations of gravitational lens systems with active star-forming lensed galaxies for the first application of our methodology. The young stellar population in such galaxies has its maximal emissivity in the ultra-violet and is distributed among numerous bright compact regions. While the observations in the optical and infrared bands, used in earlier gravitational-imaging studies (e.g. Vegetti et al., 2014), mostly capture the smooth old stellar populations, the compact star-forming regions prominent in the ultra-violet band are expected to substantially enhance the surface-brightness gradients in the lensed galaxies, allowing us to improve the sensitivity of our approach to low-mass structure in the lens galaxies, see also Ritondale et al. (2019). We perform our observations using the Wide Field Camera 3 (WFC3) onboard the HST, which offers the highest resolution and signal-to-noise ratio currently available in the rest-frame ultra-violet band for our sample.

2.3.2 Sample selection

We follow the idea of observing galaxy-galaxy strong gravitational lens systems with highly-structured star-forming lensed galaxies in the rest-frame ultra-violet (see Section 2.3.1) and select a sub-sample of 10 lens systems from the SLACS Survey (Bolton et al., 2006) for the HST/WFC3/F390W-observations (Program 12898, Koopmans, 2012). The SLACS lenses constitute the largest homogeneous sample of more than 100 galaxy-galaxy strong gravitational lens systems (Bolton et al., 2008). Given the already existing multi-colour HST-imaging data and the extensive lens-modelling and kinematic studies (e.g. Auger et al., 2009; Vegetti et al., 2014), the SLACS lens systems are an excellent choice for the application of our new statistical approach. An additional advantage is the availability of the spectroscopic redshifts for both the lens and the lensed galaxies, thanks to the initial selection of the lens candidates from the spectroscopic database of the Sloan Digital Sky Survey (SDSS, York et al., 2000). Most of the SLACS lens systems consist of a massive early-type lens galaxy aligned with a blue star-forming lensed galaxy, which allows us to easily implement our observational strategy discussed in Section 2.3.1.

We limit our choice to lens systems with the most extended, bright, high signal-to-noise-ratio lensed images and intentionally exclude all systems with late-type lens galaxies, in order to avoid possible degeneracies between the star formation or dust extinction in the lens galaxy and the lensed images. For all remaining systems, we obtain the SDSS spectra and determine the [OII]-flux of the lensed galaxies, which acts as a standard proxy for their star-formation rate. Furthermore, for each system we calculate the average F814W-magnitude within the most compact area of the lensed images containing half of the total lensed flux. We find a good correlation between the measured [OII]-flux and the average F814W-magnitude of the lensed images. This empirical relation indicates that by selecting SLACS lens systems with the brightest lensed images in F814W, we simultaneously choose systems with a high star-formation rate in the lensed galaxies, in compliance with our observational strategy.

In Table 2.1, we present our final target list of 10 selected SLACS lens systems, together with their basic astrometric and spectroscopic properties: the location on the sky, the spectroscopic redshifts for the lens and the lensed galaxies as well as the velocity dispersion of the lens galaxy (Auger et al., 2009). To improve the quality of the imaging data in comparison to the already existing observations in the F814W-filter, we require an average signal-to-noise ratio of 10 in the pixels covering the lensed images and calculate the observing time for each single lens system accordingly. The resulting time devoted to the observations of each selected lens system, varying between one and six HST-orbits, can be found in the last column of Table 2.1. Our HST/WFC3/F390W-observations were performed between January 26 and September 16, 2013.

2.3.3 Point-spread function

When observed with a telescope, the true image is blurred by the telescope optics. This blurring is commonly modelled as a convolution of the true image with the point-spread function (PSF) of the telescope optics. The convolution with the PSF has important consequences for our study. First of all, the full-width half-maximum (FWHM) of the PSF (equal to 0.07 or ~ 2 pixels for the HST/WFC3/F390W-optics) sets a limit on the smallest length scale that can be considered in our analysis. Besides this, a precise knowledge of the PSF is essential for the accuracy of both the lens-galaxy fitting procedure (discussed in Section 2.4) and the smooth lens modelling

Table 2.1: Astrometric and spectroscopic properties of the first SLACS sub-sample observed with the Hubble Space Telescope in the ultra-violet (HST/WFC3/F390W): the target names, location on the sky, spectroscopic redshifts of the lens z_l and the source galaxies z_s , the velocity dispersion of the lens galaxy σ_l (from Auger et al., 2009), and the number of HST-orbits devoted to the observations of each lens system. For the purpose of this study, we perform the smooth lens modelling and the power-spectrum analysis of the surface-brightness anomalies in the lensed images for only five most suitable lens systems from this sample – SDSS J0252+0039, SDSS J0737+3216, SDSS J1430+4105, SDSS J1627-0053 and SDSS J0903+4116 – having relatively simple geometry and no substantial galaxy-core residuals in the galaxy-subtracted images.

Target lens galaxy	Right Ascension	Declination	z_l	z_s	$\sigma_l[km/s]$	Orbits
SDSS J0252+0039	02 52 45.21	+00 39 58.40	0.280	0.982	164 ± 12	2
SDSS J0737+3216	07 37 28.45	+32 16 18.60	0.322	0.581	338 ± 16	2
SDSS J0903+4116	09 03 15.19	+41 16 09.10	0.430	1.065	223 ± 27	6
SDSS J0912+0029	09 12 05.31	+00 29 01.20	0.164	0.324	326 ± 12	5
SDSS J0956+5100	09 56 29.78	+51 00 06.60	0.241	0.470	334 ± 15	2
SDSS J0959+0410	09 59 44.07	+04 10 17.00	0.126	0.535	197 ± 13	1
SDSS J1430+4105	14 30 04.10	+41 05 57.10	0.285	0.575	322 ± 32	2
SDSS J1627- 0053	16 27 46.45	- 00 53 57.60	0.208	0.524	290 ± 14	3
SDSS J1630+4520	16 30 28.16	+45 20 36.30	0.248	0.793	276 ± 16	5
SDSS J2341+0000	23 41 11.57	+00 00 18.70	0.186	0.807	207 ± 13	2

of the lensed images (discussed in Section 2.5). The inferred best-fitting smooth-lens models need to be first convolved with the PSF before they can be compared to the observational data. Finally, for a proper interpretation of the recovered surface-brightness anomalies, the convolution with the PSF needs to be incorporated in our simulations of mock perturbed lens systems described in the next chapter.

We use the PSF-modelling software TINYTIM¹ (Krist et al., 2010) to obtain a PSF-model of the HST/WFC3/F390W-optics for the brightest central pixel of each individual lens system from our SLACS sub-sample. For the sake of simplicity, we assume that the PSF in all the other pixels does not differ significantly from the central pixel. We plan to incorporate individual PSF-models for each pixel and lens system in future work. The obtained PSF-models take into account the telescope aperture, camera and filter as well as the location of the image cutouts on the CCD and the spectral properties of each specific lens system. In our analysis, we approximate the spectral type for each of the investigated lens systems individually based on the magnitudes in F555W, F614W, F814W and F160W inferred by Auger et al. (2009). Fig. 2.1 depicts an example of such a PSF-model generated using TINYTIM for the central pixel of the lens galaxy SDSS J0737+3216.

2.3.4 Dithering strategy and data reduction

The raw imaging data obtained with the HST are generally known to be undersampled, which means that the blurred point sources are not covered by enough pixels to precisely sample the PSF and fully benefit from the superb resolution of the HST-optics. The pixel width of the WFC3 (0.04 arcsec on a side) is relatively large in comparison to the FWHM of the HST/WFC3/F390W-optics (0.07 arcsec), whereas a well-sampled image is supposed to have at least 2-2.5 pixels across the FWHM of the PSF.

In order to alleviate this undersampling problem and be able to capture the tiny surface-brightness anomalies possibly present in the observed lensed images, we apply the standard dithering strategy. This generates multiple *dithered* exposures (Fruchter & Hook, 2002) of each selected lens system by slightly shifting the telescope pointing every time the next raw image is taken. Besides the improvement in the PSF-sampling, this technique makes it possible to compensate for cosmic rays, possible dead

¹<http://www.stsci.edu/hst/observatory/focus/TinyTim>

Figure 2.1: An example of a point-spread-function model, obtained from the TINYTIM modelling software for the brightest central pixel in our HST/WFC3/F390W-observations of the lens galaxy SDSS J0737+3216 assuming the SED inferred by Auger et al. (2009). The pixel scale corresponds to 0.04 arcsec.

pixels or columns and flat-field effects. However, as will be discussed in Sections 2.3.4 and 2.7.2, combining such dithered exposures introduces correlations between adjacent pixels in the final science image, which needs to be taken into account when analyzing surface-brightness anomalies in the lensed images.

We retrieve the HST/WFC3/F390W-imaging data from the MAST archive² in the form of pipeline-preprocessed flat-field calibrated FITS files (flt.fits files) and make use of the Variable-Pixel Linear Reconstruction algorithm (Fruchter & Hook, 2002), informally known as *Drizzle*, to combine the dithered exposures into a set of final science images. We perform the drizzling in an automatic way using the ASTRODRIZZLE task from the DRIZZLEPAC package (Gonzaga et al., 2012) in the default configuration. In Section 2.7, we investigate the impact of other settings on our final results in comparison to this reference configuration.

After obtaining the drizzled images, we generate cutouts, centred on the brightest pixel of the lens galaxies, with the linear size equal to approximately two Einstein radii for each individual lens system. The image cutouts are presented in Fig. 2.2. For the purpose of illustration, Fig. 2.3 additionally depicts the HST/WFC3/F390W-imaging of an almost complete Einstein ring in SDSS J1430+4105, compared to the existing multi-band observations in F606W, F814W and F160W. The final colour-composite image, obtained using the STIFF³ software, combines our new F390W-imaging with the archival F814W and F160W imaging for this system.

2.4 Lens-galaxy subtraction

For a typical SLACS lens system, the Einstein radius (equal to ~ 1 arcsec) is comparable to half the effective (half-light) radius of the lens galaxy (Auger et al., 2009). This means that the lensed images of the background galaxies in our sample are typically projected on the very inner region (i.e. $\sim 5 - 10$ kpc from the centre) of the respective lens galaxies. As a result, we can expect the surface brightness of the lensed images, which we intend to model, to be contaminated with a non-negligible contribution from the lens galaxies. Thus, before proceeding with the lens modelling, we need to

²<http://archive.stsci.edu/hst/search.php>

³<https://www.astromatic.net/software/stiff>

Figure 2.2: HST/WFPC3/F390W-imaging of our SLACS sub-sample observed in the rest-frame ultra-violet. Each of these images depicts a foreground massive elliptical galaxy acting as a strong gravitational lens on a background star-forming galaxy. The lensed galaxies are mapped onto high-signal-to-noise-ratio gravitational arcs or (in some cases) an almost complete Einstein ring.

Figure 2.3: A high-resolution multi-band view of the Einstein ring in the strong gravitational-lens system SDSS J1430+4105. *From left to right:* the archival HST-imaging in F606W, F814W and F160W, the new HST-observations in F390W and the colour-composite image combining the multi-band photometry, obtained using the STIFF software. All images are oriented with north pointing up and east pointing left.

estimate and correct for the surface-brightness contribution from the lens galaxies in the regions covered by the respective lensed images.

Massive elliptical galaxies are empirically known to be characterised by a smoothly-varying light distribution and a very regular isophotal structure, i.e. concentric ellipses with the same orientation. Thus, a common practice for the lens-galaxy correction is to fit the surface-brightness distribution of the lens galaxy with a smooth model, which can be carried out using standard galaxy-fitting algorithms. In this work, we apply two techniques that have been successfully used in earlier studies of SLACS lenses to model and subtract the smooth lens-galaxy contribution at the location of the lensed images – the more general B-SPLINE algorithm (Bolton et al., 2006) and the empirical galaxy-fitting procedure GALFIT (Peng et al., 2002). In this section, we first briefly introduce these two methods and subsequently compare their performance with respect to the lens-galaxy subtraction in our SLACS sub-sample.

2.4.1 The radial b-spline technique

The radial b-spline technique (Bolton et al., 2006) allows one to find the best-fitting coefficients b_{mk} and c_{mk} of the smooth two-dimensional surface-brightness distribution $I(R, \phi)$, which is parameterized as follows:

$$I(R, \theta) = \sum_{m,k} (b_{mk} \cos(m\phi) + c_{mk} \sin(m\phi)) f_k(R), \quad (2.10)$$

where the radial dependence (R) is modelled with a piecewise polynomial function $f_k(R)$ defined on a set of radial intervals k and the angular dependence (ϕ) is fitted with an arbitrary number of multipole orders m .

The optimal choice for the order of the piecewise polynomial function (linear, quadratic or cubic), the spacing of the interval breakpoints and the most appropriate number of multipole orders depends on the structural properties of each individual modelled galaxy and is usually determined in a trial-and-error process. For a given choice of these settings, the best-fitting model is obtained by inferring the least-squares values of the coefficients b_{mk} and c_{mk} in each interval k independently, while the final solution needs to satisfy the derivative-continuity condition at the breakpoints between the intervals. The goodness of the fit is assessed by means of the reduced χ^2 -statistic.

2.4.2 Galaxy-fitting algorithm GALFIT

GALFIT (Peng et al., 2002) is an empirical technique for surface-brightness fitting and structural decomposition of galaxies, originally developed to model spatially well-resolved nearby galaxies observed with the HST. The two-dimensional surface-brightness distribution of a galaxy is modelled with an arbitrary combination of standard parametric functions, such as e.g. the de Vaucouleurs profile or its generalized version - the Sérsic profile, the Nuker law, the exponential disk or the Gaussian profile. These are empirically known to provide a good fit to the usual structural components of observed galaxies. As pointed out by Peng et al. (2002), in general at least three of such structural components are necessary to model the light distribution of any galaxy to a high precision. The most relevant parametric function for the modelling of the massive elliptical (lens) galaxies in this work is the Sérsic profile (Sérsic, 1963).

During the fitting procedure, GALFIT iteratively generates a model image of the analysed galaxy by summing the user-selected parametric functions with the current values of the parameters to be optimized. This model image is then convolved with the PSF of the telescope optics and compared to the observed real galaxy image in terms of the χ^2 -statistic (least-squares method). The final values of the model parameters are determined when the χ^2 -statistic has converged to its putative global minimum.

2.4.3 Mask generation

When modelling the smooth surface brightness of galaxies by means of either b-spline or GALFIT, it is necessary to mask out all pixels that, for various reasons, should not be taken into account during the galaxy-fitting procedure. Usually, a mask is used to flag bad pixels and detector artifacts in the CCD. Besides that, it is common to mask out astronomical features that should not influence the smooth model of the analysed galaxy, such as e.g. stellar streams, nearby satellite galaxies or other objects not associated with but close in projection to the modelled galaxy. In our case, when modelling lens galaxies, the mask is primarily used to reject from the lens-galaxy fit all pixels overlapping with the extended lensed images as well as galaxy cores that are often poorly fit due to the presence of a cusp and/or PSF effects.

In order to generate a mask for a particular lens system, we first manually outline the lensed features (Einstein ring or gravitational arcs) with a polygon using the DS9 software⁴. Subsequently, we process the list of vertices in this polygon by means of the DS9POLY and the FILLPOLY software (freely available on the GALFIT webpage⁵) to compute a list of pixels contained inside the polygon. Finally, we use the PYRAF-task BADPIXIMAGE to create a FITS-image representing the mask. In the course of the galaxy-fitting procedure, this initial mask might be adjusted in order to account for possible additional faint features revealed in the residual image. Examples of masks created in this way are depicted in Figs. 2.4 to 2.6.

2.4.4 Surface-brightness models of the lens galaxies

We model the two-dimensional smooth surface-brightness distribution of the lens galaxies from our SLACS sub-sample by applying both the B-SPLINE and the GALFIT algorithms.

The B-SPLINE modelling is carried out by means of the implementation developed by Bolton et al. (2006). We fit the radial dependence of the surface-brightness distributions with piecewise-defined cubic polynomials and set the breakpoints between the radial intervals every 0.2 arcsec. To account for the angular dependence, we fit multiple orders m , as defined in Eq. 2.10. We begin the modelling of each investigated lens system in the default configuration including the $m = 0$ (monopole), $m = 1$ (dipole) and $m = 2$ (quadrupole) modes. If substantial angular structure is still found in the residual (data-model) image, we iteratively add further multiple orders i.e. $m = 4$ (octopole) and higher even terms until the reduced χ^2 -statistic is maximized.

For the GALFIT modelling (Peng et al., 2002), we begin by fitting the lens galaxies with one Sérsic component (Sérsic, 1963) and add more components if justified by the residual pattern or a high value of the reduced χ^2 -statistic. Moreover, in some cases, we try to improve the model by additionally fitting the diskiness/boxiness of the isophotes or by adjusting the mask to account for faint features revealed in the residual image.

The resulting best-fitting smooth models of the surface-brightness distribution in the lens galaxies are finally interpolated over the masked

⁴<http://ds9.si.edu/site/Home.html>

⁵<http://users.obs.carnegiescience.edu/peng/work/galfit/galfit.html>

regions and subtracted from the original HST-images. In Figs. 2.4 to 2.6, we illustrate the procedure of the lens-galaxy modelling and subtraction for three lens systems from the sample, SDSS J0252+0039, SDSS J0737+3216 and SDSS 1627+0053, and compare the outcome of GALFIT and B-SPLINE. More specifically, for each lens system we present the original HST-image in the F390W-band, the mask used to reject the lensed images from the fit, the best-fitting smooth GALFIT and B-SPLINE models of the surface-brightness distribution in the lens galaxy and the respective galaxy-subtracted residual images.

We find that for most of the lens systems in our sample B-SPLINE provides a very good fit to the central region of the lens galaxy, however, the inferred global surface-brightness model has an irregular shape, deviating from our empirical expectations for massive elliptical galaxies. GALFIT, on the other hand, provides more realistic models, but it very often yields significant galaxy-core residuals, even when fitting multiple Sérsic components. Thus, both techniques need to be applied with caution. The flexibility of B-SPLINE allows to fit features that are not well described by standard parametric functions, but it might also lead to unrealistic models. To the contrary, fitting empirically-based parametric functions yields a reasonable solution in most cases, but might result in a poor fit if the galaxy deviates from the assumed typical morphology.

Since our objective is to estimate the surface-brightness contribution of the lens galaxy in the region of the lensed images and not necessarily to obtain the best estimate of the parameters characterising the overall surface-brightness distribution, we choose GALFIT as a better method for the purpose of our analysis. Fig. 2.7 presents the galaxy-subtracted images obtained using GALFIT for all the lens systems in our SLACS sub-sample. These residual images contain only the lensed emission of the background-source galaxy – an Einstein ring or gravitational arcs – and can as such be used to constrain the smooth-lens models, see Section 2.5.

The significant galaxy-core residuals present in two of the GALFIT galaxy-subtracted images shown in Fig. 2.7 might point towards central star formation in the lens galaxies, which is in a good agreement with the fraction of star-forming ellipticals inferred from local NUV/FUV and stellar population studies (about 15-20 per cent, see e.g. Kaviraj et al., 2011). A possible way of dealing with this issue might be either to exclude the central region from the lens-galaxy fit or to initially model the central region separately and only then search for the best-fitting global model.

However, for the purpose of this study, we exclude these problematic lens systems from further analysis. We instead proceed with five most suitable systems – SDSS J0252+0039, SDSS J0737+3216, SDSS J1430+4105, SDSS J1627–0053 and SDSS J0903+4116 – having relatively simple geometry and no substantial galaxy-core residuals in the galaxy-subtracted images.

2.5 Smooth lens modelling

After correcting for the light contribution from the foreground lens galaxy, we model the lensed images of the lens systems under the assumption that the mass of the lens galaxy is smoothly distributed. This smooth lens modelling aims at finding the best-fitting smooth-lens model – a smoothly-varying lensing potential of the foreground lens galaxy and the intrinsic (unlensed) surface-brightness distribution of the background galaxy which, combined, most accurately reproduce the observed lensed images.

2.5.1 Modelling technique and the best-fitting smooth-lens models

We use the adaptive and grid-based Bayesian smooth lens-modelling technique developed by Vegetti & Koopmans (2009a) to model five lens systems from our SLACS sub-sample: SDSS J0252+0039, SDSS J0737+3216, SDSS J1430+4105, SDSS J1627–0053 and SDSS J0903+4116 (see Section 2.4). The lens galaxies are assumed to be well described by the smooth power-law elliptical mass-distribution model (PEMD, Barkana, 1998), with the surface-mass density of the lens galaxy parameterized in terms of the convergence κ :

$$\kappa(x, y) = \frac{b \left(2 - \frac{\gamma}{2}\right) q^{\gamma-3/2}}{2(x^2 q^2 + y^2)^{(\gamma-1)/2}}, \quad (2.11)$$

in the convention used by Vegetti & Koopmans (2009a). The additional lensing effect of possible companion objects in the vicinity to the main lens galaxy is accounted for in the form of an external shear field. The lens-modelling code allows us to reconstruct the (pixellated) surface-brightness distribution of the background-source galaxy on an adaptive grid in the source plane and simultaneously find the best-fitting values for the parameters characterizing the lensing potential of the foreground lens galaxy. These are the lens strength b , the position angle of the major axis

Figure 2.4: Lens-galaxy subtraction for SDSS J0252+0039 performed using alternatively GALFIT or the B-SPLINE algorithm. *Top row:* the science image in the F390W-band (*left panel*) and the applied mask covering the lensed images (*right panel*). *Middle row:* the best-fitting GALFIT model of the surface-brightness distribution in the lens galaxy (*left panel*) and the lens-galaxy subtracted residual image (*right panel*). *Bottom row:* the best-fitting B-SPLINE model of the surface-brightness distribution in the lens galaxy (*left panel*) and the corresponding lens-galaxy subtracted residual image (*right panel*).

Figure 2.5: Idem as in Fig. 2.4 for the lens system SDSS J0737+3216.

Figure 2.6: Idem as in Fig. 2.4 for the lens system SDSS J1627+0053.

Figure 2.7: The lens-galaxy-subtracted images for all lens systems from our SLACS sub-sample, obtained using GALFIT. These residual images showing the lensed emission of the background-source galaxy – an Einstein ring or gravitational arcs – are used to constrain the smooth-lens models.

θ (with respect to the original telescope rotation), the (minor to major) axis ratio q , the (three-dimensional) mass-density slope γ ($\gamma = 2$ in the isothermal case), the centre coordinates of the lensing-mass distribution in the lens plane x_0 and y_0 as well as the external-shear strength Γ and its position angle Γ_θ .

Table 2.2 presents the obtained best-fitting parameter values of the PEMD model in comparison to the values from earlier F814W and F555W reconstructions carried out by Vegetti et al. (2014) (when available), whereas Figs. 2.8 to 2.12 show the corresponding best-fitting smooth-lens models (having the highest marginalized Bayesian evidence) of the source galaxy and the lensed images. More specifically, the top row in each of these figures shows the input galaxy-subtracted HST-imaging in F390W and the inferred best-fitting smooth-lens model of the lensed images. The bottom row depicts the reconstructed pixellated surface-brightness distribution of the background-source galaxy on an adaptive grid in the source plane (more specifically a Delaunay triangulation regridded to pixels for plotting purposes) and, finally, the residual (data-model) image, which represents the deviation of the best-fitting smooth-lens model from the data. The key idea of our approach is that these residual surface-brightness fluctuations might have possibly been induced by the hypothetical low-mass structure in the lens galaxy.

2.5.2 The problem of over-fitting

The smooth-lens-modelling technique by Vegetti & Koopmans (2009a) offers several options to control the mapping of the pixels and flux values from the image (lens) plane onto the source plane. The user can choose between the adaptive or non-adaptive, variance, gradient or curvature source-plane regularization (see e.g. Suyu et al., 2006b). Moreover, it is possible to adjust the resolution of the source grid by varying the fraction of pixels that are cast back from the image plane to the source plane in order to create a grid for the source reconstruction. The source-grid resolution is controlled by the parameter n , which sets the linear size of a square in the image plane out of which only the central pixel is cast back to the source plane. For example, if $n = 3$ only one pixel out of each contiguous 3×3 -pixel area is used to create the reconstruction grid in the source plane, while $n = 1$ corresponds to casting back every single pixel.

Table 2.2: Parameter values of the best-fitting smooth lensing potential for five SLACS lens systems, inferred in our U-band (F390W) analysis based on GALFIT or B-SPLINE lens-galaxy-subtracted lensed images, in comparison to earlier F814W and F555W reconstructions performed by Vegetti et al. (2014) (if available). The mass distribution of the lensing galaxy is modelled with the following set of free parameters: the lens strength b , the position angle θ (with respect to the original telescope rotation), the axis ratio q , the (three-dimensional) mass-density slope γ , the external shear strength Γ and its position angle Γ_θ . Additionally, we quote the source-grid resolution n_{src} and the type of regularization (Reg.) that lead to models with the highest marginalized Bayesian evidence. G indicates the gradient regularization, C the curvature regularization and ada an adaptive source-plane regularization. The typical statistical errors of the reconstructed parameter values are of the order 10^{-1} for the angles and 10^{-3} for the remaining parameters.

Lens system	Filter	Subtraction	b [arcsec]	θ [deg.]	q	γ	Γ	Γ_θ [deg.]	n_{src}	Reg.
J0252+0039	F390W	Gaflt	0.996	150.1	0.978	2.066	-0.015	81.4	1	G_{adp}
	F390W	b-spline	0.996	149.2	0.978	2.066	-0.015	81.4	1	G_{adp}
	F814W	b-spline	1.022	26.2	0.943	2.047	0.009	101.8	1	G_{adp}
J0737+3216	F390W	Gaflt	0.926	66.1	0.862	2.110	0.066	72.8	1	G_{adp}
	F390W	b-spline	0.933	66.9	0.869	2.102	0.062	72.2	1	G_{adp}
	F814W	b-spline	0.951	78.3	0.705	2.066	0.050	100.8	1	C
J1430+4105	F555W	b-spline	0.951	77.2	0.709	2.073	0.052	102.5	2	G_{adp}
	F390W	b-spline	1.527	85.4	0.647	2.073	0.029	139.2	1	G_{adp}
	F814W	b-spline	1.484	61.5	0.710	2.048	0.051	128.6	1	C
J1627-0053	F390W	b-spline	1.217	16.9	0.856	2.006	0.008	101.3	1	G_{adp}
	F390W	Gaflt	1.218	17.0	0.855	2.007	0.008	101.5	1	G_{adp}
	F814W	b-spline	1.229	14.3	0.912	1.998	0.004	80.0	2	G_{adp}
	F555W	b-spline	1.212	14.2	0.869	2.058	0.014	87.8	2	C_{adp}

Figure 2.8: Best-fitting smooth-lens model of SDSS J0252+0039 in HST/WFC3/F390W, obtained by means of the adaptive and grid-based Bayesian lens-modelling technique of Vegetti & Koopmans (2009a). *Top row*: the lens-galaxy subtracted image overlaid with a mask, used as input for the smooth lens modelling (*left panel*) and the reconstructed smooth-lens model of the lensed images (*right panel*). *Bottom row*: the unlensed surface-brightness distribution of the background galaxy reconstructed with the source-grid resolution $n = 1$, i.e. casting back every pixel from the image plane to the source plane (*left panel*), and the residual image showing the remaining surface-brightness fluctuations which are not explained by the smooth-lens model (*right panel*). The reconstructed parameter values of the best-fitting smooth lensing potential as well as the chosen modelling options can be found in Table 2.2.

Figure 2.9: Idem as in Fig. 2.8 for the lens system SDSS J0737+3216.

Figure 2.10: Idem as in Fig. 2.8 for the lens system SDSS J1430+4105.

Figure 2.11: Idem as in Fig. 2.8 for the lens system SDSS J1627-0053.

Figure 2.12: Idem as in Fig. 2.8 for the lens system SDSS J0903+4116 and $n = 2$, i.e. casting back one pixel out of each contiguous 2×2 -pixel area in the image plane.

The optimal choice for n and the form of the source regularization depends on both the level of structure in the data and the signal-to-noise ratio, and is usually found by testing and comparing the possible options in terms of the marginalized Bayesian evidence. With respect to the criterion of the highest Bayesian evidence, we obtain the best-fitting smooth-lens models for all five analysed lens systems when choosing $n = 1$ and the adaptive gradient regularization of the pixellated source reconstruction, as shown in Table 2.2. However, as will be discussed in Section 2.6.3, this choice leads to power spectra of the residual surface-brightness fluctuations lying at or below the estimated noise-power spectrum level. This means that the hypothetical low-mass structure in the lens galaxy, and even a substantial fraction of the background noise, have been interpreted as spurious structure in the surface-brightness distribution of the background-source galaxy. This problem is known as overfitting (i.e. modelling the inherent noise present in the data) and generally arises when a model includes too many parameters in comparison to the number of the imposed constraints. In our case, it might also arise from an incorrect or incomplete macro model, deviating from the assumed PEMD model.

In order to mitigate this overfitting problem, we perform tests with a lower resolution of the source grid i.e. a larger value of n . However, we find that for almost all considered lens systems even the choice of $n = 2$ still leads to the residual surface-brightness fluctuations lying below the noise level. The choice of $n = 3$, on the other hand, does not contain enough information for the lens-modelling code to converge to a physical solution and very often leads to a nonphysical solution. A practical solution to this problem, which we apply, is to reconstruct the background source with a lower source resolution (e.g. $n = 3$) while fixing the best-fitting parameter values obtained in the highest-resolution model corresponding to $n = 1$.

In Chapter 4, we additionally find that the overfitting problem is partly alleviated by the choice of a larger mask covering not only the lensed images, but also additional pixels located close to the edge of the cutout. These pixels carry additional information about the noise level in the observed image and provide more constraints for the lens-modelling procedure, which improves the quality of the resulting smooth-lens models.

2.6 Residual surface-brightness fluctuations in the lensed images

As demonstrated in Figs. 2.8 to 2.12, the residual images of the studied lens systems reveal surface-brightness fluctuations that can not be explained by the assumed smooth elliptical power-law plus external shear macro model of the mass distribution. In this section, we quantify such residual surface-brightness fluctuations in terms of their power spectrum, following the statistical approach recently proposed by Bus (2012) and Chatterjee & Koopmans (2018). Moreover, we investigate how the different choices made in the process of the lens-galaxy subtraction (see Section 2.4) and the smooth lens modelling (see Section 2.5) affect the resulting power spectrum of the residual surface-brightness fluctuations.

2.6.1 Power-spectrum analysis of the residual surface brightness fluctuations

The power-spectrum analysis allows us to decompose the residual surface-brightness fluctuations $\delta I(x)$ in terms of their length scale λ or the corresponding wavenumber k . In this analysis, we follow the convention in which the wavenumber k is equal to the reciprocal length scale λ :

$$k \equiv \lambda^{-1} \quad (2.12)$$

and is measured in arcsec^{-1} .

We calculate the power spectrum of the residual surface-brightness fluctuations for each lens system individually, within the respective mask covering the lensed images. For this, we set the flux values of all pixels located outside this mask to zero and apply the Python package `NUMPY.FFT`⁶ to compute the corresponding two-dimensional discrete Fourier transform (DFT) $\hat{\delta I}(k)$. The squared magnitude of the (complex-valued) Fourier coefficients, assigned to each pixel of the Fourier-transformed image, yields the power spectrum of the residual surface-brightness fluctuations $P_{\delta I}(k)$.

Assuming that $\langle \delta I \rangle = 0$, this power spectrum can be used to calculate the total variance of the residual surface-brightness fluctuations $\sigma_{\delta I}^2$ in each

⁶<https://docs.scipy.org/doc/numpy/reference/routines.fft.html>

image. According to Parseval's theorem for the DFT:

$$\frac{1}{N} \sum_{k=0}^{N-1} P_{\delta I}(k) \equiv \frac{1}{N} \sum_{k=0}^{N-1} |\hat{\delta I}(k)|^2 = \sum_{n=0}^{N-1} (\delta I)^2(n), \quad (2.13)$$

with N denoting the number of pixels in the image. Thus, $\sigma_{\delta I}^2$ can be obtained by summing the power spectrum $P_{\delta I}(k)$ over the two-dimensional Fourier-transformed image:

$$\sigma_{\delta I}^2 \equiv \langle (\delta I - \langle \delta I \rangle)^2 \rangle = \frac{1}{N} \sum_{n=0}^{N-1} \delta I^2(n) = \frac{1}{N^2} \sum_{k=0}^{N-1} P_{\delta I}(k). \quad (2.14)$$

An important consequence following from equation 2.14 is that the summation of the power spectrum $P_{\delta I}(k)$ over a specific range of modes k yields the contribution to the total variance $\sigma_{\delta I}^2$ in the analysed image from the corresponding range of length scales λ .

We further assume the residual surface-brightness fluctuations to be isotropic and average the two-dimensional power-spectrum along a set of ten equidistant concentric annuli covering the full Fourier-transformed image. The resulting one-dimensional azimuthally-averaged power spectrum constitutes the final statistic allowing us to quantify the surface-brightness fluctuations in a statistical sense, and compare the populations of such fluctuations resulting from different models. As an example, in Fig. 2.13 we present the one-dimensional azimuthally-averaged power spectrum of the residual surface-brightness fluctuations in the lens system SDSS J0252+0039.

2.6.2 Effect of the lens-galaxy subtraction

In order to assess whether the choice of the galaxy-subtraction technique might be a source of a systematic bias, we calculate the one-dimensional azimuthally-averaged power spectrum of the galaxy-subtracted images within the mask covering the lensed images for both GALFIT and B-SPLINE. The results for three of the selected lens systems are presented in Fig. 2.14. As expected, by removing a smooth large-scale surface-brightness component, the lens-galaxy subtraction reduces the measured power spectrum (i.e. variance) only on the largest spatial scales (smallest k -values). Moreover, in the case of SDSS J0252+0039, SDSS J0737+3216 and SDSS 1627+0053, GALFIT and B-SPLINE lead to almost identical results.

Figure 2.13: Power spectrum of the residual surface-brightness fluctuations remaining in the lensed images of SDSS J0252+0039 after the GALFIT lens-galaxy subtraction and the smooth lens modelling.

Figure 2.14: Effect of the lens-galaxy subtraction, performed using alternatively GALFIT and B-SPLINE, on the power spectrum of the surface-brightness fluctuations within the region of the lensed images for SDSS J0252+0039.

Thus, in the case of the ultra-violet band, where the surface-brightness distribution of elliptical galaxies is concentrated in the centre and decreases very quickly towards the outskirts, the choice of the galaxy-subtraction technique does not seem to influence our final results. However, if the analysis is performed in a different band, especially in the infra-red where the lens galaxy dominates the surface-brightness in the region of the lensed images, the choice of the lens-galaxy model might significantly alter the analysis outcome.

Figure 2.15: Idem as in Fig. 2.14 for the lens system SDSS J0737+3216.

Figure 2.16: Idem as in Fig. 2.14 for the lens system SDSS 1627+0053.

2.6.3 Effect of the smooth lens modelling

As was pointed out in Section 2.5, the inferred smooth-lens models and the level of the resulting residual surface-brightness fluctuations in the lensed images might depend on the applied source-plane regularization (adaptive or non-adaptive, variance, gradient or curvature regularization) and the chosen resolution of the source reconstruction. As an example, in Fig. 2.17 we present the impact of these different options on the measured power spectrum of the residual surface-brightness fluctuations in the lens system SDSS J0737+3216. From Fig. 2.17, we conclude that the choice of $n = 1$ might not always yield the best model, due to over-fitting (discussed in more detail in Section 2.5.2). In the case of SDSS J0737+3216, the computed power spectrum of the residual surface-brightness fluctuations lies below the noise level (see Section 2.7.4 for the estimation of the noise power spectrum) for all models with $n = 1$ (and even $n = 2$), irrespective of the chosen type of regularization. On the other hand, higher n -values might deteriorate the sampling of the lensed images and, consequently, lower the accuracy of the source reconstruction. Thus, a balance needs to be found between the possible over- and under-fitting, taking into account the specific characteristics of each individual lens system.

2.7 Noise correction

The residual surface-brightness fluctuations in the lensed images might point towards the presence of mass structure (or more general – deviations from the assumed smooth mass distribution) in the investigated lens galaxies. However, a variety of different observational effects might mimic the surface-brightness anomalies caused by mass structure in the lens galaxy, most prominently the observational noise. For this reason, in this section we quantify and correct for the noise properties in our HST/WFC3/F390W-observations.

2.7.1 The noise-sigma maps

The two most prominent contributions to the observational noise in HST-images are the sky-background variations and the (Poisson-distributed) photon-shot noise of the lens system itself. Formally, the observational noise can be thought of as a random variable assigned to each individual

Figure 2.17: Power spectrum of the residual surface-brightness fluctuations remaining in the lensed images of SDSS J0737+3216 after the lens-galaxy subtraction and the smooth lens modelling for varying levels of the source-grid resolution and different types of the source-grid regularization. In the legend, *G* indicates the gradient regularization, *C* the curvature regularization, *ada* an adaptive and *nada* a non-adaptive source-plane regularization. The symbols *n1* and *n2* stand for the source-grid resolution corresponding to $n = 1$ and $n = 2$, respectively.

pixel (i.e. a *random field*), with the expectation value equal to zero for all pixels and the variance σ_n^2 varying from pixel to pixel. While the variance of the sky background σ_{sky}^2 can be well approximated based on the flux values in close-by regions of blank sky, the variance σ_P^2 of the Poisson-distributed photon-shot noise (equal to the expected flux value in a specific pixel) is a priori not known. However, the large number of counts in our imaging allows us to approximate the photon-shot noise by a Gaussian noise $\mathcal{N}(0, \sigma_P^2)$, with the variance σ_P^2 equal to the number of photons measured in a specific pixel (as a proxy for the expected flux value in this pixel) after subtracting the sky background.

The corresponding standard-deviation values of the total noise σ_n in each pixel are commonly presented in the form of a noise-sigma map. We construct such maps for our HST-images according to the following formula:

$$\sigma_n = \sqrt{\sigma_P^2 + \sigma_{sky}^2} = \sqrt{N/W + \sigma_{sky}^2}, \quad (2.15)$$

where N is the number of photo-electrons per second detected in a particular pixel (after the sky-background subtraction) and W is the inverse-variance weight of this pixel taken from the weight map automatically created in the drizzling procedure. The variation of the sky background σ_{sky} is estimated based on the standard deviation of the flux values in a sample of close-by blank-sky cutouts with the same size as the particular science image. Fig. 2.18 illustrates the generation of such a noise-sigma map for the drizzled HST/WFC3/F390W-image of the lens galaxy SDSS J0737+3216 with $\sigma_{sky} = 0.002 \text{ e}^- \text{ sec}^{-1}$.

The generated noise-sigma maps provide a full description of the noise properties in the analysed science images, provided that the level of the random surface-brightness fluctuations due to noise is statistically independent between different pixels. If this assumption holds, a realistic noise realisation can be simply generated by drawing random numbers from a Gaussian distribution with the expected value equal to zero and the standard-deviation assigned to each individual pixel by the noise-sigma map. However, as will be discussed in the next Section 2.7.2, the drizzled HST/WFC3/F390W-images show noise correlations that need to be included in a proper description of noise properties.

Figure 2.18: Generation of the noise-sigma map for the HST/WFC3/F390W-image of the lens system SDSS J0737+3216 (according to equation 3.15): the drizzled science image of the lens system N (*left panel*), the inverse-variance weight map W from the drizzling procedure (*middle panel*) and the final noise-sigma map σ_n (*right panel*). The variation of the sky background σ_{sky} is estimated to be equal $0.002 \text{ e}^- \text{ sec}^{-1}$, based on the standard deviation of the flux values in a sample of close-by blank-sky cutouts with the same size as the science image.

2.7.2 Noise correlations due to drizzling

As pointed out in Section 2.3, making use of the *Drizzle* algorithm (Fruchter & Hook, 2002) to combine multiple dithered exposures allows one to improve the sampling of the resulting science image, however it is also known to introduce noise correlations between adjacent pixels, since the flux of a given input pixel is divided between several neighbouring output pixels. During the process of drizzling, pixels from the dithered input frames are aligned and mapped onto the final output-image grid. The location of each input pixel on the output grid is found based on the shift and rotation (called *dither*) of the respective input image, recorded as the World-Coordinate System (WCS) pointing information in the headers of each single dithered exposure. The flux of each input pixel is redistributed over the output pixels according to its fractional overlap with the output pixels. Consequently, the noise in the individual pixels of our imaging data cannot be considered as statistically independent (see Casertano et al. (2000) for a more detailed discussion on the noise correlations in the drizzled images).

In order to investigate the noise-correlation pattern in our data, we create a sample of drizzled blank-sky image cutouts located in the proximity to each studied lens system. For consistency reasons, we match the size of these blank-sky regions to the respective size of the analysed science image. As an example, Fig. 2.19 compares one of the drizzled blank-sky fields in proximity to the lens system SDSS J0252+0039 with a realisation of an uncorrelated Gaussian noise, representing the true, statistically independent, fluctuations of the sky background. Both images have the same total variance of the flux values, however, the drizzled image shows a distinct blotchy correlation pattern imposed on the data by the drizzling procedure.

We treat the surface-brightness fluctuations in the blank-sky images as a statistical population and quantify their properties in terms of the one-dimensional azimuthally-averaged power spectrum. As an example, Fig. 2.20 presents the mean power spectrum obtained in a sample of twenty selected blank-sky regions located in proximity to SDSS J0252+0039 in comparison to a sample of twenty mock realisations of uncorrelated Gaussian noise with the same total variance. Whereas the power spectrum measured in the realisations of the uncorrelated Gaussian noise is flat (as expected), the power spectrum of the drizzled HST/WFC3/F390W blank-sky cutouts is scale-dependent and carries the signature of the correlation pattern imposed on the data by the drizzling procedure. As apparent from Fig. 2.20, the noise variance in the drizzled images is larger than in the uncorrelated mock noise realisations on large spatial scales, but smaller on small spatial scales (below 0.1 arcsec or 2.5 pixels).

According to Fruchter & Hook (2002), the crucial feature of the *Drizzle* algorithm is the possibility to reduce the correlation of adjacent pixels by simultaneously decreasing the pixel size of the output grid and shrinking the input pixels before they are mapped (or informally *drizzled*) onto the final output-image grid. The choice of a finer output-grid size compared to the original detector pixel is controlled by the *final pixscale* parameter and allows one to improve the spatial resolution of the final drizzled image. The size of the shrunken input pixels, called *drops*, can be varied by means of another important parameter – the *final pixfrac* – describing the proportional relationship between the linear sizes of the drop and the original input pixel. The flux of the input pixel is then transferred onto an output pixel with a weight proportional to the overlap between its shrunken version (i.e. the drop) and the output pixel. After shrinking, the flux of

Figure 2.19: Noise correlation in the drizzled HST/WFC3/F390W images: a realisation of an uncorrelated Gaussian noise simulating statistically independent fluctuations of the sky-background, (*left panel*) vs. a blank-sky cutout from the drizzled HST/WFC3/F390W-image in proximity to SDSS J0252+0039 (*right panel*). Both images have the same total variance of the flux values, however, the drizzled image shows a distinct blotchy correlation pattern imposed on the data by the drizzling procedure.

Figure 2.20: Noise correlation in the drizzled HST/WFC3/F390W-images: the mean one-dimensional azimuthally-averaged power spectrum measured in a sample of twenty selected blank-sky regions located in proximity to SDSS J0252+0039 vs. the flat power spectrum measured in a sample of twenty mock realisations of uncorrelated Gaussian noise with the same variance as in the blank-sky cutouts (see Fig. 2.19 for an example).

any given input pixel is redistributed among fewer output pixels, which is supposed to reduce the noise correlations between adjacent pixels.

To test this possibility, we select one particular blank-sky region located in proximity to SDSS J0252+0039 and perform the drizzling procedure with different parameter values for the *final pixscale* and *final pixfrac*. The value of the *final pixfrac* can in principle be varied between 0 (equivalent to sampling with a delta function) and 1 (drop size equal to the original pixel size), however the recommended practice is to choose the size of the drop such that it is slightly larger than the final output-pixel size. Fig. 2.21 depicts the resulting power spectra of the surface-brightness fluctuations in the blank-sky cutout in comparison to the default configuration with both the output-pixel size and the drop size equal to the original pixel size 0.04 arcsec (corresponding to *final pixfrac* = 1). As can be seen in Fig. 2.21, lowering the output-pixel size (i.e. the value of the *final scale*) and shrinking the size of the drops (i.e. lowering the value of *final pixfrac*) when combining the raw dithered images does not significantly affect the measured power spectrum on the most relevant scales, corresponding to the lowest- k bins on which the residual surface-brightness fluctuations have the highest levels. Taking into account that the choice of a lower final output-pixel scale while maintaining the same field of view would substantially increase the number of pixels and, thus, the computational effort of our study, we conclude that using the default configuration (i.e. both the output-pixel size and the drop size equal to the original pixel size 0.04 arcsec) in the drizzling procedure is a good choice for the further analysis.

2.7.3 Charge-transfer inefficiency

Due to a gradual degradation processes of the HST/WFC3/UVIS-CCDs, the flux values in the images are additionally affected by the charge-transfer inefficiency (CTI) (see e.g. Baggett et al., 2015). The radiation damage in space leads to defects in the silicon lattice of the CCDs. During the readout process, some of the photoelectrons get trapped within these defects and are released only with a short delay, which results in the formation of spurious trails in the image (Massey et al., 2014).

In order to test whether the CTI has a substantial effect on the noise properties, we apply the drizzling procedure to CTI-corrected flat-field calibrated exposures (flc.fits files) of a selected blank-sky cutout (located in vicinity to SDSS J0252+0039) and compare the resulting CTI-corrected

Figure 2.21: The effect of the drizzling parameters *pixscale* (final scale of the output pixels) and *final pixfrac* (size of the *drops*) on the one-dimensional azimuthally-averaged power spectrum of surface-brightness fluctuations in a selected blank-sky cutout close to SDSS J0252+0039. The default configuration, used in this work, corresponds to *pixscale*=0.04 arcsec and *final pixfrac* = 1.

image with the original one. A careful visual inspection of these two images, presented in Fig. 2.22, leads to the conclusion that the CTI-correction reduces the level of surface-brightness structure present in the original image on the largest spatial scales. This effect becomes more apparent in the Fourier space. Fig. 2.23 compares the power spectrum measured in the blank-sky cutout before and after the CTI-correction. As is apparent from Fig. 2.23, the CTI-correction does not affect the measured power spectrum on small spatial scales (high k -values), but it leads to a reduction in the noise variance on the largest considered spatial scale - the power for $k_{\min} = 0.88$ corresponding to the spatial length scale $\lambda = 1.13$ arcsec (~ 25 pixels) is roughly 40 per cent lower for the CTI-corrected drizzled science image.

Thus, we recommend to use the CTI-corrected flat-field calibrated exposures (flc.fits files) in the future analyses of lens systems observed with the HST, due to a lower variance on the largest considered spatial scales. However, the current analysis is still valid since the science images and blank-sky cutouts were created in a consistent way (based on the original flt.fits files). Nevertheless, after inferring constraints on the power spectrum of the density fluctuations in the studied lens galaxies, we perform the same analysis while excluding the largest scales and show that it does not significantly alter the results (see Chapter 3 for the case of the lens system SDSS J0252+0039).

2.7.4 Noise power spectrum

In this section, we discuss our approach to quantify the combined effect of the drizzling procedure (Section 2.7.2), the CTI of the HST/WFC3/UVIS-CCDs (Section 2.7.3) as well as the flux-dependent photon-shot (Poisson) noise, in terms of the noise power spectrum. As a first step, we generate a sample of blank-sky regions with the size of the science image, located in the proximity to the specific lens system. We assume that the flux values measured in the blank-sky regions (after subtracting the mean value) are realizations of a Gaussian Random Field with the expected value equal to zero and a constant level of the standard deviation over the entire field-of-view. We approximate this by the standard deviation of the flux values measured in the blank-sky regions. Dividing the flux values measured in each blank-sky region by this value allows us to standardize the random field (i.e. transform the data into a realization of the normal distribution with the mean equal to zero and the standard deviation equal to one).

Figure 2.22: The effect of the charge-transfer-inefficiency (CTI) correction: a blank-sky cutout from the drizzled HST/WFC3/F390W-image in proximity to SDSS J0252+0039 before (*left panel*) vs. after the CTI-correction (*right panel*). The CTI-correction reduces the level of surface-brightness structure on the largest spatial scales, which becomes more apparent in the Fourier space, see Fig. 2.23.

Figure 2.23: The effect of the charge-transfer-inefficiency (CTI) correction on the measured power spectrum of surface-brightness fluctuations in drizzled HST/WFC3/F390W-images: the one-dimensional azimuthally-averaged power spectrum of a blank-sky field in proximity to SDSS J0252+0039 before vs. after CTI-correction (corresponding to Fig. 2.22).

To incorporate the flux-dependent photon-shot (Poisson) noise in the observed image of the lens system, we subsequently multiply the standardized blank-sky regions by the noise-sigma map of the science image. These modified sky-background realizations account for both the realistic noise-correlation pattern of the drizzled HST/WFC3/F390W-images and the specific spatially-varying flux-dependent photon-shot noise in the observed image of the lens system. The power spectrum measured in this sample constitutes our best estimate of the total noise power spectrum in the observed image of a lens system. To measure the power spectrum of the generated modified sky-background realizations, we proceed in exactly the same way as with the residual images from the smooth lens modelling in Section 2.6. In particular, before calculating the Fourier transform, the sky-background realisations are overlaid with the mask outlining the lensed images, which was used to calculate the power spectrum of the residual surface-brightness fluctuations in Section 2.6. As an example, in Fig. 2.24, we plot both the power spectrum of the sky background, based on a sample of twenty original blank-sky regions located in the proximity to the lens system SDSS J0252+0039, and the total noise-power spectrum, based on the modified sky-background realizations as outlined above.

2.7.5 Power spectrum of surface-brightness anomalies in the lensed images

Finally, we correct for the noise contribution to the power spectrum of the residual surface-brightness fluctuations measured in the lensed images in Section 2.6. Assuming that the observational noise and the density fluctuations in the lens galaxy are statistically independent, we consider the corresponding power spectra to be additive, which allows us to subtract the estimated noise power spectrum from the power spectrum of the residual surface-brightness fluctuations, as presented in Fig. 2.24 for the case of SDSS J0252+0039. The difference of these two power spectra (illustrated by the red line in Fig. 2.24) constitutes our best estimate for the noise-corrected power spectrum of the residual surface-brightness fluctuations in the lensed images.

Figure 2.24: Upper-limit constraints on the power spectrum of surface-brightness anomalies in the lensed images of SDSS J0252+0039: the power spectrum of the residual surface-brightness fluctuations after the smooth lens modelling (*blue line*), the power spectrum of the sky background estimated based on a sample of twenty blank-sky regions located in the proximity to the lens (*black line*), the total noise power spectrum (*green line*), and the noise-corrected power spectrum of the residual surface-brightness fluctuations, constituting our upper limit on the power spectrum of surface-brightness anomalies (*red line*).

2.8 Feasibility test with a mock perturbed lens system

In this section, we test the performance of our approach in recovering known mock surface-brightness anomalies from perturbed lensed images, mimicking real HST/WFC3/F390W-observations of the lens system SDSS J0252+0039 from our SLACS sub-sample.

The mock image is created using the best-fitting smooth-lens model inferred from the HST/WFC3/F390W-data in Section 2.5. To achieve this, we repeat the lensing operation of the reconstructed intrinsic surface-brightness distribution of the background galaxy in SDSS J0252+0039, by applying the best-fitting smooth lensing potential perturbed by Gaussian potential fluctuations $\delta\psi_{\text{GRF}}(\mathbf{x})$. The realization of the GRF potential perturbations is generated from a power-law power spectrum:

$$P_{\delta\psi}(k) = A \times k^{-\beta} \quad (2.16)$$

with the following normalization condition:

$$A(\sigma_{\delta\psi}^2, \beta, L) = \frac{L^2 \sigma_{\delta\psi}^2}{\sum_{k_x} \sum_{k_y} \left(\sqrt{k_x^2 + k_y^2} \right)^{-\beta}}, \quad (2.17)$$

where L indicates the length of the image, measured in arcsec. In the above equation, the wavenumbers k_x and k_y correspond to the reciprocal wavelength λ of the associated harmonic waves $e^{-2\pi i \mathbf{k} \cdot \mathbf{x}}$ (propagating in the x and y direction, respectively) in the Fourier representation of the GRF-field and are measured in arcsec^{-1} . We set the variance $\sigma_{\delta\psi}^2 = 4 \times 10^{-4}$ and the power-law slope $\beta = 4$, such that the power spectrum of the resulting surface-brightness anomalies in the lensed images resembles the residuals revealed in the real system. In order to account for observational effects, we convolve the perturbed lensed images with the Tiny-Tim PSF of the HST/WFC3/F390W optics and add a realistic mock noise realisation. For simplicity, this overlaid noise realisation is generated based solely on the noise-sigma map, described in Section 2.7.1, and does not reflect the effect of noise correlations found in the cutouts of blank sky from the observed HST/WFC3/F390W full-sky image (see Section 2.7.2). This simplifying assumption is justified by the low level of noise compared to the measured residual surface-brightness fluctuations.

For such a mock lens system with a known potential-perturbation field $\delta\psi_{\text{GRF}}(\mathbf{x})$, full information is given about the power spectrum of the surface-brightness anomalies caused by $\delta\psi_{\text{GRF}}(\mathbf{x})$ and the superposed noise realisation (Fig. 2.25). This knowledge can be used to test the performance of the smooth-lens-modelling code in recovering the known true power spectrum of surface-brightness anomalies induced by the perturbing potential fluctuations $\delta\psi_{\text{GRF}}(\mathbf{x})$ and fine-tune the choice of parameter values controlling the numerical algorithm. Here, we present the relevant results allowing us to determine the optimal number of pixels n cast back from the lens plane to the source plane when using the previously chosen mask (depicted in Fig. 2.4) and an adaptive source regularisation.

With this aim, we use the methodology applied in Sections 2.5 and 2.6 to the observed data and perform the smooth lens modelling of the mock lensed images for different values of $n = 1, 2, 3, 4$ to determine the resulting power spectra of the residual surface-brightness fluctuations. The smooth lens modelling is carried out without re-optimising for the parameter values describing the smooth lensing potential. Hence, in this test we assume that the smooth lensing potential can be recovered accurately and focus on investigating the degeneracy between the perturbing mass structure in the lens and the small-scale surface-brightness fluctuations in the source galaxy. From Fig. 2.25 it can be seen that the power spectrum of the residual image from the smooth lens modelling lies significantly below the noise level when each pixel is cast back from the lens plane to the source plane ($n = 1$; the highest resolution). As in the analysis of the real system, this overfitting can be explained by a partial absorption of the surface-brightness anomalies and the observational noise in the source structure.

While the underlying degeneracy between the potential perturbations in the lens and the intrinsic structure in the source is less problematic when trying to identify individual sub-haloes with masses above the detection limit, as in Vegetti et al. (2014), this issue requires a more careful consideration in our power-spectrum analysis. As is apparent from Fig. 2.25, this degeneracy can be alleviated by lowering the source-grid resolution (choosing higher n -values), which leads to a better agreement between the reconstruction and the true power spectrum of the induced mock surface-brightness anomalies. A comparison of the residual power spectra corresponding to $n = 3$ and $n = 4$ (Fig. 2.25) suggests that convergence is reached for $n = 3$ and lowering the source resolution even further does not allow us to suppress this effect thoroughly (but would

lead to a considerably less accurate source model). The absorption of the potential fluctuations in the source structure persists on the smallest considered k -scales.

All in all, based on the results of the above test with a realistic mock lens, we conclude that our approach allows us to recover the power spectrum of surface-brightness anomalies induced by the superposed potential perturbations in the lens, when choosing the $n = 3$ resolution mode as the most suitable for the given data quality and choice of mask. However, since the test is based on the previously reconstructed unlensed surface-brightness distribution of the source galaxy, which by construction can be easily modelled using the same smooth-lens-modelling code, in what follows we still conservatively treat the recovered power spectrum as an upper limit to the effect caused solely by the potential fluctuations in the lens galaxy.

2.9 Conclusions and outlook

In this chapter, we have introduced a methodology allowing us to estimate the power spectrum of surface-brightness anomalies in extended lensed images of galaxy-scale strong gravitational lens systems, possibly induced by small-scale density fluctuations in the lens galaxy. Furthermore, we have illustrated our methodology by applying it to the first SLACS sub-sample observed with HST/WFC3 in the ultra-violet, and discussed the modelling challenges. Finally, we have proven the feasibility of our approach in recovering known mock surface-brightness anomalies from perturbed lensed images mimicking real HST/WFC3/F390W-observations of the lens system SDSS J0252+0039.

Next steps

Our main objective is to extend this methodology such that the residual surface-brightness fluctuations can be modelled and traced back to the underlying low-mass structure in the lens galaxy, treated in a statistical sense. With this goal in mind, the focus of the following chapters is a systematic study of mock surface-brightness anomalies caused by a priori known low-mass structure with varying statistical properties, which can be finally compared to the real measurement. We model the mock low-mass structure in the lens galaxy in terms of Gaussian-Random-Field (GRF)

Figure 2.25: Performance test of our methodology to recover surface-brightness anomalies caused by mass structure in the lens galaxy, based on a mock lens system mimicking SDSS J0252+0039. We perturb the best-fitting smooth lensing potential of SDSS J0252+0039 with GRF potential fluctuations characterised by the power-law power spectrum $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$ with the variance $\sigma_{\delta\psi}^2 = 4 \times 10^{-4}$ and the slope $\beta = 4$. The plot depicts the azimuthally-averaged power spectrum of the true surface-brightness anomalies induced in the lensed images (*blue line*), the power spectrum of the overlaid white-noise realisation (*black line*), the power spectrum of these two effects combined (i.e. the power spectrum of the true residual surface-brightness fluctuations to be recovered, *red line*), and the power spectra of the residual surface-brightness fluctuations actually recovered from the perturbed lensed images when performing the smooth lens modelling with fixed parameter values of the lensing potential, for different source-grid resolutions ($n = 1, 2, 3, 4$; *green lines from bottom to top*).

potential perturbations superposed on the best-fitting smoothly-varying lensing potential. Following the formalism introduced by Chatterjee & Koopmans (2018), we assume these potential perturbations to be uniquely characterized by a power-law power spectrum $P_{\delta\psi}(\sigma_{\delta\psi}^2, \beta, k)$ with two free parameters: the total variance of the GRF fluctuations $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β . Thus, our study is performed on a grid in the parameter space containing different combinations of values for $\sigma_{\delta\psi}^2$ and β – referred to as *matter-power-spectrum models*. For each point of this grid, we generate realizations of the corresponding mock Gaussian potential fluctuations, superpose them on the best-fitting lensing potential, compute the resulting perturbed lensed images and, finally, the induced mock surface-brightness anomalies. The power spectra of such mock surface-brightness anomalies are directly comparable to the power spectrum of the real surface-brightness fluctuations, measured in this chapter.

We follow two different ways to infer such mock power spectra of surface-brightness anomalies and compare them to the real measurement. In Chapter 3, we use the best-fitting smooth-lens model obtained for the real data to generate the perturbed mocks and assume that the smooth-lens-modelling technique would simply recover the input parameter values of the smooth lensing potential and the input image of the source galaxy as the best-fitting smooth-lens model for all the generated perturbed lens systems. Under this assumption, no lens modelling is necessary to compute the power spectra of the surface-brightness anomalies that would be measured if the mock lens systems were actually observed. Instead, the best-fitting smooth model obtained for the real data can be directly subtracted from all the mock perturbed lensed images. For a pilot application of this approach, discussed in the next chapter, we choose one of the lens systems from our SLACS sub-sample, SDSS J0252+0039, due to its simple geometry and the high signal-to-noise ratio of the extended lensed images.

To the contrary, in Chapter 4, we more realistically assume that in order to properly account for all numerical effects present in the smooth-lens-modelling procedure, the mock perturbed lens systems need to be modelled in exactly the same way as the real observations. Thus, to thoroughly mimic the smooth-lens-modelling procedure of the observational data, we let the smooth-lens-modelling code find the best-fitting set of parameter values for the smooth lensing potential and the surface-brightness distribution of the background source for each mock individually. This approach allows us to compare the observed and the mock perturbed lens systems in a

fully consistent way. However, it is significantly more time-consuming and introduces another layer of complexity and modelling choices to be made. Our pilot study applies this more accurate methodology to deep high-signal-to-noise-ratio HST/ACS/F814W-observations of the strong gravitational lens system SDSS J0946+1006, commonly known as the Double Einstein Ring.

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Chapter 3

Observational constraints on the sub-galactic matter power spectrum from galaxy-galaxy strong gravitational lensing

— Based on

**Observational constraints on the sub-galactic matter power
spectrum from galaxy-galaxy strong gravitational lensing**

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Abstract

Constraining the sub-galactic matter power spectrum on 1-10 kpc scales would make it possible to distinguish between the concordance Λ CDM and various alternative dark-matter models, due to the significantly different levels of mass structure predicted by these models on these critical sub-galactic scales. Galaxy-galaxy strong gravitational lensing provides a unique opportunity to search for gravitational signatures of sub-galactic mass structure in lens galaxies beyond the Local Group. Here, we demonstrate a novel approach to observationally constrain the population of the overall (dark and baryonic) low-mass density fluctuations in the inner regions of massive elliptical lens galaxies, based on the power spectrum of the associated surface-brightness anomalies measured in highly magnified galaxy-scale Einstein rings and gravitational arcs. The application of our method to the SLACS lens system SDSS J0252+0039 results in the following limits (at the 99 per cent confidence level) on the dimensionless convergence power spectrum (and the associated standard deviation in the aperture mass): $\Delta_{\delta\kappa}^2 < 1$ ($\sigma_{AM} < 0.8 \times 10^8 M_\odot$) on 0.5-kpc scale, $\Delta_{\delta\kappa}^2 < 0.1$ ($\sigma_{AM} < 1 \times 10^8 M_\odot$) on 1-kpc scale and $\Delta_{\delta\kappa}^2 < 0.01$ ($\sigma_{AM} < 3 \times 10^8 M_\odot$) on 3-kpc scale. The estimated effect of CDM sub-haloes lies considerably below these first observational upper-limit constraints on the level of density fluctuations in the projected total mass distribution of galactic haloes. Future analysis based on a larger sample of galaxy-galaxy strong gravitational lens systems will narrow down these constraints, enabling us to rule out all cosmological models that predict a significantly larger level of clumpiness on these critical sub-galactic scales.

3.1 Introduction

The dark-energy-plus-cold-dark-matter (Λ CDM) concordance cosmological model successfully reproduces the observed large-scale (larger than ~ 1 Mpc) distribution of matter in the Universe (e.g. Vogelsberger et al., 2014; Schaye et al., 2015; Guo et al., 2016; Planck Collaboration et al., 2018). However, on smaller galactic and sub-galactic scales, theory and observations appear to diverge (see Bullock & Boylan-Kolchin, 2017, for a recent review on the small-scale challenges to the Λ CDM paradigm). One of the main discrepancies, known as the Missing Satellites Problem (MSP), lies in the fact that the number of dwarf satellite galaxies observed in the Local Group (~ 100 , e.g. McConnachie, 2012; Drlica-Wagner et al., 2015) is much lower than the numerous abundance of sub-haloes populating galactic haloes in Λ CDM-based numerical simulations of the cosmological structure-formation process (e.g. Klypin et al., 1999; Moore et al., 1999; Diemand et al., 2007; Nierenberg et al., 2016; Dooley et al., 2017; Bullock & Boylan-Kolchin, 2017; Tanaka et al., 2018; Fielder et al., 2019; Navarro, 2019).

The currently favoured interpretation of the MSP is that the missing sub-haloes do exist but are extremely inefficient at forming stars, due to a variety of baryonic processes, such as feedback from massive stars and active galactic nuclei, tidal stripping or photo-ionization squelching (e.g. Thoul & Weinberg, 1996; Bullock et al., 2000; Somerville, 2002; Sawala et al., 2014; Kim et al., 2017; Despali et al., 2018), and thus remain undetectable for conventional imaging surveys. Alternatively, the MSP might point towards alternative dark-matter models, referred to as warm dark matter (WDM, Bode et al., 2001), with higher thermal velocities in the early Universe leading to a suppression of structure formation on sub-galactic scales (see e.g. Menci et al., 2012; Nierenberg et al., 2013; Viel et al., 2013; Lovell et al., 2014; Vegetti et al., 2018a; Zavala & Frenk, 2019; Hsueh et al., 2020, for some recent studies). Lastly, the Local Group might just be a biased environment with less abundant substructure and, thus, not representative of the entire Universe (e.g. Muller et al., 2018). Therefore, in order to fully clarify these ambiguities and test the mentioned solutions to the small-scale tensions of the Λ CDM paradigm, it is crucial to constrain the properties of mass structure in galaxies beyond the Local Universe and on mass scales below that of luminous satellites.

The key techniques used to search for the faint, or even truly dark, sub-haloes in galaxies at cosmological distances are based on an indirect study of their gravitational imprints on the lensed images of galaxy-scale strong gravitational lens systems. Mass structure, if present in the lens galaxy (or along its line-of-sight), introduces perturbations to the otherwise smooth lensing potential, which slightly modifies the observed surface brightness of the lensed images with respect to the smooth-lens model. The resulting anomalies, e.g. in the form of the flux-ratio anomalies in multiply imaged gravitationally lensed quasars (e.g. Mao & Schneider, 1998; Metcalf & Madau, 2001; Dalal & Kochanek, 2002; Nierenberg et al., 2014; Birrer et al., 2017; Gilman et al., 2017a, 2019; Hsueh et al., 2020) or the surface-brightness anomalies in the lensed emission of extended background galaxies (e.g. Blandford et al., 2001; Koopmans, 2005; Vegetti & Koopmans, 2009a; Vegetti et al., 2010a,d; Rau et al., 2013; Vegetti et al., 2014, 2018b; Ritondale et al., 2019), can be modelled and traced back to the underlying mass structure in the lens galaxy.

In particular, the *gravitational-imaging technique* developed by Koopmans (2005) and Vegetti & Koopmans (2009a) aims at the detection of individual massive dark-matter sub-haloes in (massive elliptical) lens galaxies at cosmological distances, based on surface-brightness anomalies in deep high-resolution imaging of galaxy-galaxy strong gravitational lens systems. The resulting detection of two dark-matter sub-haloes with masses $3.5 \times 10^9 M_\odot$ and $1.9 \times 10^8 M_\odot$ in lens galaxies at the redshift $z = 0.2$ and $z = 0.881$, respectively, reported in Vegetti et al. (2010e, 2012), demonstrates that galactic sub-haloes can be successfully identified as localised corrections to a smooth gravitational potential, with the detection threshold depending on the angular resolution of the available data. Recently, Despali et al. (2018) showed that this technique can also be used to detect individual line-of-sight haloes and Vegetti et al. (2018a) derived the corresponding constraints on the properties of dark matter.

In order to investigate smaller sub-galactic mass structures, numerous populating galactic haloes in Λ CDM-based cosmological simulations, Bus (2012), Hezaveh et al. (2016) and Chatterjee & Koopmans (2018) proposed a complementary statistical approach. Instead of single localised potential corrections, representing individual massive sub-haloes in the original gravitational-imaging technique by Vegetti & Koopmans (2009a), this alternative approach models the entire population of low-mass density fluctuations in the lens galaxy in a statistical manner – in terms of the pro-

jected matter power spectrum – based on the collectively-induced surface-brightness anomalies in the extended lensed images of the background source galaxies.

Contrary to several recent theoretical studies (e.g. Bus, 2012; Hezaveh et al., 2016; Diaz Rivero et al., 2018a; Chatterjee & Koopmans, 2018; Díaz Rivero et al., 2018c; Cyr-Racine et al., 2019), little research has been so far devoted to the question how to actually measure or infer observational constraints on the projected matter power spectrum from galaxy-galaxy strong gravitational lensing. The main challenge in this respect is that in reality the surface-brightness anomalies may arise not only due to the presence of mass structure in the lens galaxy (or along its line of sight), but also from a variety of other inhomogeneities in the total (dark and baryonic) projected mass density of the lens galaxy, such as e.g. small-scale density fluctuations in the distribution of dark matter, globular clusters, tidal streams, dynamical distortions in the baryonic mass distribution or edge-on discs (Vegetti et al., 2014; Gilman et al., 2017b; Hsueh et al., 2016, 2017a,b), which are not explicitly included in the smooth-lens model. Thus, instead of focusing on CDM substructure only, a more feasible way of approaching this problem is to investigate the overall departures of the mass distribution in the lens galaxy from the best-fitting smooth-lens model. The inferred statistical constraints could be then compared to predictions from hydrodynamical simulations (such as Illustris or EAGLE, Vogelsberger et al., 2014; Schaye et al., 2015) which incorporate both dark and baryonic matter.

Further developing this idea, Chatterjee & Koopmans (2018) suggested that the population of such overall density inhomogeneities in the total projected mass distribution of a lens galaxy can be treated as a statistical ensemble and modelled in terms of Gaussian-Random-Field (GRF) potential perturbations superposed on the best-fitting smoothly-varying lensing potential. In the framework of their theoretical formalism, both the potential perturbations and the resulting observable surface-brightness anomalies in the lensed images are quantified in terms of the power spectrum, and related to each other. Successful tests of this new approach on mock perturbed lens systems, presented by Chatterjee & Koopmans (2018) and Chatterjee (2019), demonstrate that it might be possible to reconstruct the variance of such potential fluctuations and their power-law power-spectrum slope from the power spectrum of the surface-brightness anomalies imprinted on the lensed images.

The goal of this study is to develop and test a methodology to apply this statistical framework to real observational data and infer constraints on the statistical properties of small-scale density fluctuations in the projected total (baryonic and dark) mass distribution of (massive elliptical) lens galaxies on sub-galactic 1-10 kpc scales, based on the collectively-induced surface-brightness anomalies in extended lensed images of observed galaxy-galaxy strong gravitational lens systems. As a proof of concept, we apply the developed methodology to high-resolution Hubble Space Telescope (HST/WFC3/F390W) observations of the strong gravitational lens galaxy SDSS J0252+0039 (Koopmans, 2012) from the Sloan Lens ACS Survey (SLACS, Bolton et al., 2008). This analysis leads to the first observational constraints on the power spectrum of GRF potential perturbations, and the corresponding mass density fluctuations, in a (massive elliptical) lens galaxy. A comparison of these constraints with predictions from hydrodynamical simulations for the Λ CDM and alternative dark-matter models will be the subject of our future research.

The present chapter is divided into four parts. Section 3.2 provides a concise description of the problem under consideration, the GRF-formalism applied to model the mass-density fluctuations in galactic haloes and our procedure adopted to uncover, quantify and interpret the resulting surface-brightness anomalies in the lensed images. In Section 3.3, we elaborate on the observations of our pilot lens system SDSS J0252+0039. Sections 3.4, 3.5 and 3.6 describe the lens-galaxy subtraction, smooth-lens modelling and the power-spectrum analysis of the residual surface-brightness fluctuations, respectively. In Section 3.7, we present the generated sample of mock perturbed lensed images to be compared with the real observations. The resulting observational upper-limit constraints on the projected sub-galactic matter power spectrum are discussed in Section 3.8. The final Section 3.9 provides conclusions and implications of this work for further research.

For a consistent comparison of the lens models with earlier studies by Vegetti et al. (2014), throughout this work we assume the following cosmology: $H_0 = 73 \text{ km s}^{-1}\text{Mpc}^{-1}$, $\Omega_M = 0.25$ and $\Omega_\Lambda = 0.75$. Given this cosmology, 1 arcsec corresponds to 4.11 kpc at the redshift of the lens galaxy ($z_L = 0.280$) and 7.88 kpc at the redshift of the source galaxy ($z_S = 0.982$).

3.2 Methodology

In Chapter 2, we presented a procedure to uncover surface-brightness anomalies in the lensed images and quantify them by means of the power spectrum. In this chapter, we extend this methodology and present a procedure allowing us to relate the revealed surface-brightness anomalies to the underlying small-scale density fluctuations in the lens galaxy. These are modelled in terms of Gaussian-Random-Field (GRF) potential perturbations superposed on the best-fitting smoothly-varying lensing potential, as first suggested by Bus (2012) and Hezaveh et al. (2016).

In Section 3.2.1, after summarising the concept of such GRF potential perturbations, we relate them to the underlying perturbations in the projected mass distribution of the lens galaxy and the associated perturbations in the deflection-angle field. Section 3.2.2 outlines the complete procedure to uncover the surface-brightness anomalies in the lensed images of observed galaxy-scale lens systems and use them to infer constraints on the power spectrum of the underlying potential perturbations in the lens galaxy.

3.2.1 GRF potential perturbations in the lens galaxy

Low-mass perturbations in lens galaxies are commonly modelled in terms of corrections to the best-fitting smoothly-varying parametric model of the lensing potential $\psi_0(\mathbf{x})$ (e.g. Koopmans, 2005; Vegetti & Koopmans, 2009a). While previous studies have mainly focused on the detection of individual massive dark matter sub-haloes (e.g. Vegetti et al., 2012, 2014) and line-of-sight haloes (Despali et al., 2018), here we investigate the overall departures from a smooth-lens model, i.e. density fluctuations in the total mass arising not only from sub-haloes in the lens galaxy or haloes along its line-of-sight, but from the complex (dark and baryonic) matter distribution of real lens galaxies in general.

We treat such density fluctuations as a statistical ensemble and model the associated potential corrections in terms of a homogeneous and isotropic Gaussian potential-perturbation field $\delta\psi_{\text{GRF}}(\mathbf{x})$ (with $\langle\delta\psi_{\text{GRF}}(\mathbf{x})\rangle = 0$) superposed on a smoothly-varying lensing potential $\psi_0(\mathbf{x})$ (Bus, 2012; Hezaveh et al., 2016; Chatterjee & Koopmans, 2018). More specifically, we assume that the lensing potential $\psi(\mathbf{x})$ of a lens galaxy can to the first order be approximated by the sum of a smoothly-varying parametric

component $\psi_0(\mathbf{x})$ and a Gaussian potential-perturbation field $\delta\psi_{\text{GRF}}(\mathbf{x})$:

$$\psi(\mathbf{x}) \approx \psi_0(\mathbf{x}) + \delta\psi_{\text{GRF}}(\mathbf{x}), \quad (3.1)$$

with no covariance between $\delta\psi_{\text{GRF}}(\mathbf{x})$ and $\psi_0(\mathbf{x})$.

In this approximation, the deflection angle $\alpha(\mathbf{x})$ can be separated into the deflection caused by the smooth component of the lensing potential $\alpha_0(\mathbf{x})$ and a perturbative deflection-angle field $\delta\alpha_{\text{GRF}}(\mathbf{x})$ due to the differential lensing effect of the potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$:

$$\alpha(\mathbf{x}) = \nabla\psi(\mathbf{x}) \approx \nabla\psi_0(\mathbf{x}) + \nabla\delta\psi_{\text{GRF}}(\mathbf{x}) = \alpha_0(\mathbf{x}) + \delta\alpha_{\text{GRF}}(\mathbf{x}). \quad (3.2)$$

The resulting differential deflection-angle field $\delta\alpha_{\text{GRF}}(\mathbf{x})$ can be in this case directly linked to the underlying potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ via:

$$\delta\alpha_{\text{GRF}}(\mathbf{x}) = \nabla\delta\psi_{\text{GRF}}(\mathbf{x}), \quad (3.3)$$

independently of the smooth lensing component $\psi_0(\mathbf{x})$. Similarly, the associated convergence-perturbation field $\delta\kappa_{\text{GRF}}(\mathbf{x})$ (i.e. surface-mass density perturbations in units of the critical surface-mass density for lensing) is related to the potential-perturbation field $\delta\psi_{\text{GRF}}(\mathbf{x})$ via the Poisson equation:

$$\nabla^2\delta\psi_{\text{GRF}}(\mathbf{x}) = 2 \times \delta\kappa_{\text{GRF}}(\mathbf{x}). \quad (3.4)$$

A crucial feature of a Gaussian Random Field is that its properties are entirely characterised by the second-order statistics. Hence, the statistical behaviour of the Gaussian potential perturbations $\delta\psi(\mathbf{x})$ is fully described by the 2-point correlation function or, alternatively, its Fourier transform – the power spectrum. Following Chatterjee & Koopmans (2018), we assume the power spectrum of $\delta\psi(\mathbf{x})$ to be isotropic and to follow a power law:

$$P_{\delta\psi}(k) = A \times k^{-\beta}. \quad (3.5)$$

The amplitude A is related to the total variance of the potential perturbations $\sigma_{\delta\psi}^2$, while the power-law slope β describes the distribution of this variance over the different length scales (i.e. k -modes) and, thus, determines the scale dependence of the hypothetical small-scale structure in the lens galaxy. In what follows, we refer to a specific combination of $\sigma_{\delta\psi}^2$ and β as a *matter-power-spectrum model*. We choose the convention in which

the wavenumber k , measured in arcsec^{-1} , corresponds to the reciprocal wavelength λ :

$$k \equiv \lambda^{-1} \quad (3.6)$$

of the associated harmonic wave $e^{-2\pi i k \cdot x}$ in the Fourier representation of the GRF field.

For a science image with a side length L (measured in arcsec), we determine A by specifying the overall variance of the potential perturbations $\sigma_{\delta\psi}^2 \equiv \langle (\delta\psi - \langle \delta\psi \rangle)^2 \rangle = \langle \delta\psi^2 \rangle$ in the considered field-of-view:

$$\int_{k_x} \int_{k_y} P_{\delta\psi}(A, \beta, \sqrt{k_x^2 + k_y^2}) dk_x dk_y = \sigma_{\delta\psi}^2, \quad (3.7)$$

where the integration is performed over the corresponding Fourier grid and the wavenumbers k_x and k_y are calculated according to equation (3.6). Substituting $P_{\delta\psi}(k)$ in equation (3.7) with equation (3.5) and replacing the integrals by a summation over discrete pixels with the size $dk_x = dk_y = L^{-1}$ in k -space finally leads to the following normalisation condition:

$$A(\sigma_{\delta\psi}^2, \beta, L) = \frac{L^2 \sigma_{\delta\psi}^2}{\sum_{k_x} \sum_{k_y} \left(\sqrt{k_x^2 + k_y^2} \right)^{-\beta}}. \quad (3.8)$$

As can be seen from equation (3.8), this normalisation depends not only on the specific combination of β and $\sigma_{\delta\psi}^2$, but also on the field-of-view and the pixel scale of the analysed image. Using the methodology developed in this work, we seek to derive observational constraints on the variance $\sigma_{\delta\psi}^2$ and the slope β of the power-law power spectrum $P_{\delta\psi}(k)$, as defined in equations (3.5–3.8).

The perturbative effect of such Gaussian potential fluctuations $\delta\psi_{\text{GRF}}(\mathbf{x})$ in the lens galaxy leads to observable *surface-brightness anomalies* in the lensed images $\delta I_{\text{GRF}}(\mathbf{x})$, with respect to the smooth-lens model $I_0(\mathbf{x})$:

$$\begin{aligned} \delta I_{\text{GRF}}(\mathbf{x}) &= I_{\text{GRF}}(\mathbf{x}) - I_0(\mathbf{x}) = \\ &= S(\mathbf{x} - \nabla\psi_0(\mathbf{x}) - \nabla\delta\psi_{\text{GRF}}(\mathbf{x})) - S(\mathbf{x} - \nabla\psi_0(\mathbf{x})) = \\ &= S(\mathbf{x} - \alpha_0(\mathbf{x}) - \delta\alpha_{\text{GRF}}(\mathbf{x})) - S(\mathbf{x} - \alpha_0(\mathbf{x})). \end{aligned} \quad (3.9)$$

To infer constraints on $\sigma_{\delta\psi}^2$ and β , we perform a systematic study of such surface-brightness anomalies caused by Gaussian potential perturbations

$\delta\psi_{\text{GRF}}(\mathbf{x})$ with different values of $\sigma_{\delta\psi}^2$ and β , and compare the results to the observed residual surface-brightness fluctuations. For a future comparison with the Λ CDM predictions, the constraints on the potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ can be translated into constraints on the associated convergence perturbations $\delta\kappa_{\text{GRF}}(\mathbf{x})$ or, alternatively, the perturbations in the deflection angle $\delta\alpha_{\text{GRF}}(\mathbf{x})$, by expressing equations (3.2) and (3.4) in Fourier space. Accordingly, the power spectrum of potential perturbations $P_{\delta\psi}(k)$ can be related to the corresponding power spectrum of convergence perturbations $P_{\delta\kappa}(k)$:

$$P_{\delta\kappa}(k) = 4\pi^4 k^4 P_{\delta\psi}(k) \quad (3.10)$$

and to the power spectrum of the resulting perturbations in the deflection angle $P_{\delta\alpha}(k)$:

$$P_{\delta\alpha}(k) = 4\pi^2 k^2 P_{\delta\psi}(k). \quad (3.11)$$

Finally, the inferred constraints can be expressed in terms of the dimensionless convergence power spectrum:

$$\Delta_{\delta\kappa}^2(k) \equiv 2\pi k^2 P_{\delta\kappa}(k), \quad (3.12)$$

which quantifies the contribution of a particular length scale $\lambda = k^{-1}$ to the total variance of the convergence perturbations. For completeness, the dimensionless deflection power spectrum is defined as:

$$\Delta_{\delta\alpha}^2(k) \equiv 2\pi k^2 P_{\delta\alpha}(k). \quad (3.13)$$

3.2.2 Analysis overview

Our approach to constrain the level of small-scale density fluctuations in massive elliptical lens galaxies consists of three main components:

- First, we extract the surface-brightness anomalies in the lensed images of an observed lens system and quantify their statistical properties in terms of the power spectrum $P_{\delta I}(k)$. To be conservative in coping with the degeneracies discussed in Chapter 2, we only correct this power spectrum for the effect of the observational noise and treat any residual surface-brightness fluctuations remaining in the lensed images after the lens-galaxy subtraction, the smooth-lens modelling and the noise correction as an upper limit to the effect caused by the density fluctuations in the lens galaxy. With this in mind, we perform the following steps:

1. HST-observations and data reduction by means of the DRIZ-ZLEPAC package (Gonzaga et al., 2012), see Section 3.3;
 2. modelling and subtraction of the surface-brightness contribution from the foreground lens galaxy using GALFIT (Peng et al., 2002), see Section 3.4;
 3. simultaneous reconstruction of the smooth lensing potential $\psi_0(\mathbf{x})$ and the intrinsic surface-brightness distribution of the background galaxy $S(\mathbf{y})$, using the adaptive and grid-based Bayesian lens-modelling technique by Vegetti & Koopmans (2009a), see Section 3.5;
 4. statistical quantification of the residual surface-brightness fluctuations in the lensed images in terms of the azimuthally-averaged power spectrum $P_{\delta I}(k)$, see Section 3.6.1;
 5. estimation of the noise power spectrum and correction for the noise contribution (including both the sky background and the flux-dependent photon shot noise) to the power spectrum of the residual surface-brightness fluctuations, see Section 3.6.2.
- Second, we generate mock realisations of lensed images perturbed by GRF potential fluctuations $\delta\psi_{GRF}(\mathbf{x})$ with known statistical properties (i.e. the total variance $\sigma_{\delta\psi}^2$ and the power-law slope β of the GRF-power spectrum $P_{\delta\psi}(k)$, as defined in equations 3.5-3.8). For a systematic comparison with the observed data, we create a grid in the parameter space $\sigma_{\delta\psi}^2 - \beta$, containing different combinations of values for the variance and the power-law slope, and generate mock perturbed lensed images for each point of this grid. This procedure allows us to statistically assess the impact of the potential perturbations on the power spectrum of the resulting surface-brightness anomalies $P_{\delta I}(k)$; see Section 3.7.
 - Third, we compare the surface-brightness anomalies in the mock perturbed lensed images with observational data and infer the exclusion probability for each considered combination of $\sigma_{\delta\psi}^2$ and β given the observational data. These upper-limit constraints are additionally expressed in terms of the corresponding power spectrum of perturbations in the deflection angle $P_{\delta\alpha}(k)$ and the convergence (i.e. projected mass density) fluctuations $P_{\delta\kappa}(k)$; see Section 3.8.

We emphasize that we intend to apply our method solely to lens systems for which single dominant sub-haloes and satellite galaxies have already been modelled in previous studies. Moreover, in the future, we plan to additionally use hydrodynamical simulations to thoroughly test the validity of the Gaussianity assumption in our description of the hypothetical departures of the total mass distribution in massive elliptical (lens) galaxies from a smooth model.

3.3 Observations and data reduction

The mass of the smallest structures that can be recovered in a galaxy-scale halo using the method of gravitational imaging (Koopmans, 2005; Vegetti & Koopmans, 2009a) is limited by the sensitivity of the observational setup to tiny surface-brightness perturbations in the lensed images. As argued by Blandford et al. (2001) and Koopmans (2005), and explicitly demonstrated by Rau et al. (2013), the surface-brightness anomalies induced in the lensed images by a given density fluctuation in the lens galaxy are enhanced when the lensed background source is either compact or highly structured. Taking into account that blue star-forming regions are considerably more structured than the redder old stellar populations, the choice of a lens system with a star-forming source galaxy combined with the selection of an ultra-violet observational filter might substantially improve the overall sensitivity of our approach.

Following this idea, we apply our methodology to HST/WFC3/F390W-imaging data (with the central wavelength of 390 nm, referred to simply as *U-band* in the remainder of the paper) of the galaxy-galaxy strong gravitational lens system SDSS J0252+0039 from the SLACS Survey. This lens system consists of a massive elliptical lens galaxy at redshift $z_l = 0.280$ and a blue star-forming source galaxy at redshift $z_s = 0.982$ (Bolton et al., 2008). The choice of SDSS J0252+0039 is motivated by the large mass of its lens galaxy (total mass of $10^{11.25} M_\odot$ inside the Einstein radius), the high surface-brightness gradient of the star-forming background galaxy and the relatively simple lens geometry that reduces the uncertainty in the lens modelling (Auger et al., 2009).

As one of the SLACS gravitational lens systems, SDSS J0252+0039 has already been modelled, based on HST-observations at near-infrared (in F160W: Program 11202, PI Koopmans) and optical wavelengths (in F814W: Program 10866, PI Bolton and in F606W: Program 11202, PI

Koopmans), see Bolton et al. (2008) and Auger et al. (2009, 2010). In particular, Vegetti et al. (2014) applied the technique of gravitational imaging (Koopmans, 2005; Vegetti & Koopmans, 2009a) to the HST/ACS data in the V- and I-bands to search for signatures of individual massive sub-haloes in the lens galaxy. No gravitational signatures of localised substructure have been identified above the mass-detection threshold of $\sim 10^8 M_\odot$. Here we make use of the enhanced source gradient in the U-band observations and search for collectively-induced gravitational imprints of much smaller density fluctuations.

The HST/WFC3/F390W-observations of SDSS J0252+0039 were carried out on August 25, 2013 (Program 12898, PI Koopmans, Koopmans, 2012). We retrieve the acquired eight dithered flat-field calibrated images from the MAST archive¹ and apply the drizzle method (Gonzaga et al., 2012) to combine them into the final science image. For this, we use the ASTRODRIZZLE task from the DRIZZLEPAC package in its default configuration, with the original HST/WFC3 rotation, the output-pixel size of 0.0396 arcsec and the drop size equal to the original pixel size (*final pixfrac* = 1). Our analysis is based on an image cutout of 121 by 121 pixels (corresponding to an area of 4.48 arcsec on a side) centered on the brightest pixel of the lens galaxy, which we present in Fig. 3.1.

A known effect of the drizzling procedure are the correlations between adjacent pixels in the output science image (see Casertano et al., 2000). This correlations are additionally enhanced by the effect of the charge-transfer inefficiency (CTI) in the HST/WFC3/UVIS-CCDs (see e.g. Baggett et al., 2015). To investigate and possibly alleviate these instrumental effects, we repeat the drizzling procedure with different settings (i.e. different values of the *final pixfrac* parameter and the output pixel scale) and compare the results to those obtained for the default configuration. Moreover, we perform a comparison between the power spectra obtained by drizzling either the original or the CTI-corrected flat-field calibrated exposures in a sample of blank-sky cutouts (located in vicinity to the lens system). Based on this analysis, presented in detail in Appendix 3.A, we decide to proceed with our analysis by drizzling the original dithered exposures in the default configuration of the ASTRODRIZZLE task.

¹<http://archive.stsci.edu/hst/search.php>

Figure 3.1: HST/WFC3/F390W-imaging of the strong gravitational lens system SDSS J0252+0039: the drizzled science image with the side length of 4.48 arcsec.

To account for further observational effects present in the science image, we obtain the Point-Spread-Function (PSF) of the HST/WFC3/UVIS optics using the PSF-modelling software TINYTIM² (Krist et al., 2010), with the approximation of the G8V spectral type for the lens galaxy (based on its known magnitudes in the V-, I- and H-bands from Auger et al., 2009). Even though the TinyTim-PSF might not be a perfect representation of the real telescope optics, any minor deviations from the true PSF and the assumed spectral-energy distribution of the lens system affect the measured power spectra of the residual surface-brightness fluctuations only on scales below the full width at half maximum (FWHM) of the PSF, corresponding to wavenumbers $k \gtrsim \text{FWHM}^{-1} = (0.07 \text{ arcsec})^{-1} \approx 14 \text{ arcsec}^{-1}$, which is beyond the regime considered in this work (see Section 3.6.2).

Finally, for visual purposes and in order to assess the possible presence of dust in the studied lens system, we use the STIFF³ package to create a colour-composite image of SDSS J0252+0039. The resulting Fig. 3.2 shows a smooth early-type lens galaxy and uniformly blue lensed images of the background galaxy. A visual inspection of Fig. 3.2 exhibits no indication for dust extinction either in the lens or the source galaxy. This conclusion is confirmed by a more quantitative dust analysis, discussed in Appendix 3.B.

3.4 Lens-galaxy subtraction

The Einstein radius of SDSS J0252+0039 ($\sim 1 \text{ arcsec}$) is comparable to the effective radius of its massive elliptical lens galaxy. Consequently, the lensed images overlap with the inner region of the lens galaxy, as can be seen in Fig. 3.1. To correct for the light contribution from the lens galaxy, we model its surface-brightness distribution with a combination of several Sérsic components (Sérsic, 1963) – a parametric class of smooth light profiles widely used to model early-type galaxies.

The exact lens-galaxy-subtraction procedure is illustrated in Fig 3.3. We perform the modelling by means of the two-dimensional fitting algorithm GALFIT (Peng et al., 2002) after masking out all pixels covering the lensed images, as shown in the top row. The best-fitting GALFIT model, shown in the bottom left panel of Fig 3.3, consists of two Sérsic components

²<http://www.stsci.edu/hst/observatory/focus/TinyTim>

³<https://www.astromatic.net/software/stiff>

Figure 3.2: Colour-composite image of the strong gravitational lens system SDSS J0252+0039 combining the new HST/WFC3/F390W-photometry (*blue*) with the already existing HST-observations in the visual (F814W; *green*) and infrared (F160W; *red*) bands, obtained using the STIFF software.

with Sérsic indices $n = 4.44$ and $n = 0.09$ and effective radii 1.2 and 0.96 arcsec (corresponding to ~ 4.8 and ~ 3.8 kpc at the redshift of the lens), respectively. The small value of the second Sérsic index suggests that the light distribution of the lens galaxy can be well described by the standard de Vaucouleurs profile (i.e. Sérsic profile with $n = 4$) and a diffuse stellar halo. As an alternative, we additionally carry out the lens-galaxy subtraction using the B-SPLINE algorithm (Bolton et al., 2006), but choose to continue our analysis based on the GALFIT model due to the slightly higher Bayesian evidence of the resulting best-fitting smooth-lens model, see Section 3.5. The final lens-galaxy-subtracted image is presented in the bottom right panel of Fig 3.3. This residual image constitutes our best estimate of the lensed emission from the background galaxy only and is as such used for the smooth-lens modelling.

3.5 Smooth lens modelling

In search of possible surface-brightness anomalies in the lensed images, we first model the studied lens system assuming that the global mass distribution in the lens galaxy is smooth and well described by the power-law elliptical mass-distribution model (PEMD, Barkana, 1998) in an external shear field. This smooth lens modelling is performed by means of the adaptive and grid-based Bayesian lens-modelling technique of Vegetti & Koopmans (2009a) that allows us to find the best-fitting parameter values of the PEMD model for the lens galaxy and, simultaneously, reconstruct the unlensed surface-brightness distribution of the background galaxy on an adaptive grid in the source plane. We follow the parametrization used by Vegetti & Koopmans (2009a) and model the projected mass density of the lens galaxy in terms of the convergence κ :

$$\kappa(x, y) = \frac{b \left(2 - \frac{\gamma}{2}\right) q^{\gamma-3/2}}{2(x^2 q^2 + y^2)^{(\gamma-1)/2}}. \quad (3.14)$$

This is a function of the position in the lens plane (x, y) and has the following parameters – the lens strength b , the position angle of the major axis θ (with respect to the original telescope rotation), the (minor to major) axis ratio q , the (three-dimensional) mass-density slope γ (with $\gamma = 2$ for the isothermal case), the centre coordinates of the mass distribution in the lens plane x_0 and y_0 , the external shear strength Γ and its position angle Γ_θ .

Figure 3.3: Lens-galaxy subtraction for SDSS J0252+0039 using GALFIT. *Top row*: the drizzled science image in F390W (*left panel*) and the applied mask, covering the lensed source emission (*right panel*). *Bottom row*: the best-fitting GALFIT model of the surface-brightness distribution in the lens galaxy (*left panel*) and the lens-galaxy-subtracted residual image (*right panel*).

Several options for the inversion of the lensing operation are available to choose from when using the smooth lens modelling code by Vegetti & Koopmans (2009a). To prevent overfitting, the reconstruction of the background galaxy can be regularized by applying an adaptive or non-adaptive, variance, gradient or curvature source-plane regularization. Furthermore, one can choose the source-grid resolution, which is characterised by the fraction and spacing of pixels that are cast back from the image plane to the source plane when generating a grid for the source reconstruction. More specifically, the source-grid resolution is controlled by the parameter n that sets the linear size of a square in the image plane out of which only one pixel is cast back to the source plane. For example, if $n = 3$ only one pixel out of each contiguous 3×3 -pixel area is used to create the Delauney-tessellation grid in the source plane. We note that, even if $n > 1$, *all* pixels are still used to calculate the Bayesian evidence and compare the model to the data. As pointed out by Suyu et al. (2006a) and Vegetti et al. (2014), the optimal choice for the regularization and the source-grid resolution depends crucially on the smoothness of the modelled lensed images and may be different for each specific lens system. Thus, the common practice is to perform the smooth lens modelling for different combinations of the above options, and find the optimal settings based on the highest marginalized Bayesian evidence. Our tests in this respect show that the chosen source-grid properties hardly affect the best-fitting parameter values of the lensing potential. We find, however, that they have a significant effect on the reconstructed unlensed surface-brightness distribution of the background galaxy and, consequently, on the level of the residual surface-brightness fluctuations remaining in the lensed images after the best-fitting smooth-lens model has been subtracted.

We achieve the highest value of the marginalized Bayesian evidence when the smooth lens modelling is performed using the adaptive gradient source-grid regularization and casting back each pixel from the lens plane to the source plane (referred to as $n = 1$), see Fig. 2.8. The best-fitting parameter values of the lensing potential are presented in Table 3.1. These are in a good agreement with the parameter values inferred by Vegetti et al. (2014) in an earlier analysis of the HST/ACS/F814W-data (I-band), except for the apparent discrepancy in the position angle of the major axis θ . This discrepancy can be explained by the rotational invariance owing to the nearly spherical symmetry of the modelled lens galaxy (axis ratio q very close to 1) and the negligible external shear Γ , hardly altering the lensing

potential. All in all, the best-fitting parameter values inferred in both bands indicate that SDSS J0252+0039 has a nearly spherical isothermal mass-density profile ($\gamma \approx 2$) with the Einstein radius of $\Theta_E \approx b \approx 1$ arcsec.

However, despite a remarkably good agreement between the model and data, the obtained source reconstruction turns out to be under-regularised, i.e. most of the surface-brightness fluctuations in the lensed images, and partially even the observational noise, are "absorbed" in the source structure. The power spectrum of the residual surface-brightness fluctuations (Section 3.6), remaining in the lensed images after the subtraction of the best-fitting smooth-lens model with $n = 1$, lies below the noise power spectrum for both adaptive and non-adaptive source-grid regularisation (see Fig. 3.6). From this over-fitting, we conclude that if the smooth-lens modelling of SDSS J0252+0039 in the U-band is carried out with the highest resolution ($n = 1$) and a relatively tight mask, the inversion problem to be solved is under-constrained and degenerate.

While a thorough analysis of the degeneracy between the lensing potential and the source is the subject of a future work (Chatterjee et al. in prep), we continue our study based on the (more conservative) source reconstruction obtained using a lower grid resolution ($n = 3$), while keeping fixed the parameter values inferred from the highest-resolution model ($n = 1$), see Fig. 3.4 and Table 3.1. The particular choice of $n = 3$ is motivated by the results of our tests carried out for a mock lens system mimicking SDSS J0252+0039 (see Chapter 2). These tests suggest that the degeneracy between the density fluctuations in the lens galaxy and the intrinsic source structure is significantly reduced when the modelling is performed with a lower source-grid resolution (i.e. higher values of n).

The model of the lensed images (top right panel of Fig. 3.4), which would be observed if the lensing potential was indeed smooth, is subsequently subtracted from the observed data to uncover residual surface-brightness fluctuations. These indicate a possible deviation of the true lensing potential from the assumed smooth (PEMD) model. As can be seen from the residual image in the bottom right panel of Fig. 3.4 and the corresponding signal-to-noise ratio image presented in Fig. 3.5, the revealed residual surface-brightness fluctuations significantly exceed the noise level.

Figure 3.4: Smooth lens modelling of SDSS J0252+0039 in the U-band (HST/WFC3/F390W) by means of the adaptive and grid-based Bayesian lens-modelling technique of Vegetti & Koopmans (2009a), performed with the source-grid resolution $n = 3$, i.e. casting back only one pixel out of each contiguous 3×3 -pixel area from the image plane to the source plane. *Top row*: the lens-galaxy-subtracted image overlaid with a mask, used as input for the smooth lens modelling (*left panel*) and the best-fitting smooth-lens model of the lensed images (*right panel*). *Bottom row*: the reconstructed intrinsic surface-brightness distribution of the background galaxy (*left panel*) and the residual image showing the remaining surface-brightness fluctuations in the lensed images, possibly caused by small-scale mass structure in the lens galaxy (*right panel*).

Figure 3.5: The signal-to-noise ratio of the residual surface-brightness fluctuations remaining in the lensed images of SDSS J0252+0039 after subtraction of the smooth-lens model obtained with the source-grid resolution of $n = 3$.

Table 3.1: Parameter values of the best-fitting PEMD model for the lens galaxy SDSS J0252+0039 inferred based on the U-band (F390W) imaging, in comparison to the I-band (F814W) reconstruction by Vegetti et al. (2014). The optimized parameters are the lens strength b , the position angle θ (with respect to the original telescope rotation), the axis ratio q , the (three-dimensional) mass-density slope γ , the external shear strength Γ and its position angle Γ_θ . In both bands, the reconstruction was carried out using the adaptive gradient source regularization and casting back each pixel ($n = 1$). The typical statistical errors of our U-band reconstruction are of the order 10^{-1} for the angles and 10^{-3} for the remaining parameters.

Filter	$b[arcsec]$	θ [deg.]	q	γ	Γ	Γ_θ [deg.]
F390W	0.996	150.1	0.978	2.066	-0.015	81.4
F814W	1.022	26.2	0.943	2.047	0.009	101.8

3.6 Power-spectrum analysis of the surface-brightness anomalies in the lensed images

In this section, we estimate the power spectrum of surface-brightness anomalies in SDSS J0252+0039, defined as residual surface-brightness fluctuations in the lensed images caused by the presence of density fluctuations in the lens galaxy (i.e. deviations of the real mass distribution from the best-fitting PEMD model). To start with, in Section 3.6.1, we take into account all the residual surface-brightness fluctuations revealed in the lensed images after the subtraction of the lens-galaxy light and the best-fitting smooth-lens model. We note, however, that such residual fluctuations might originate not only from the possible mass structure in the lens galaxy, but also from other phenomena, for example the observational noise, uncertainties in the PSF-model or intrinsic structure in the lensed source that has not been recovered in the lens-modelling procedure due to the choice of a relatively low source-grid resolution ($n = 3$). Hence, in order to constrain the surface-brightness anomalies resulting solely from the density fluctuations in the mass distribution of the lens galaxy, it is crucial to quantify the other effects, in particular the observational noise (see Section 3.6.2).

3.6.1 Power spectrum of residual surface-brightness fluctuations

To quantify the overall residual surface-brightness fluctuations, we compute the azimuthally-averaged power spectrum of the residual image within the mask covering the lensed images (shown in Fig. 3.3). To achieve this, we set the flux values of the pixels located outside the mask to zero and calculate the two-dimensional discrete Fourier transform (DFT) of the masked residual image, using the Python package `NUMPY.FFT`⁴. The squared magnitude of the Fourier coefficient assigned to each pixel yields the two-dimensional power spectrum. Assuming isotropy of the modelled potential perturbations $\delta\psi(\mathbf{x})$, we average the power-spectrum values along a set of ten equidistant concentric annuli spanning from $k_{\min} = 0.88$ to $k_{\max} = 16.79 \text{ arcsec}^{-1}$ (corresponding to the spatial scales between $\lambda_{\min} = 0.22$ and $\lambda_{\max} = 4.65 \text{ kpc}$ at the redshift of the lens galaxy $z_1 = 0.280$). The resulting azimuthally-averaged power spectrum of the residual surface-brightness fluctuations $P_{\delta I}(k)$ is presented as the blue dashed line in Fig. 3.6.

As apparent from Fig. 3.6, a good alternative when dealing with the over-fitting in the source reconstruction is to increase the size of the mask encompassing all the pixels taken into account in the smooth lens modelling procedure, in combination with applying a high non-adaptive source-grid regularisation. In this case, the outcome of the modelling cannot be compared in terms of the Bayesian evidence, since models with different masks need to be considered as based on different data sets. However, these two options lead to very similar power spectra of the residual surface-brightness fluctuations (see Fig. 3.6). Our tests show that the small difference between these power spectra in the lowest analysed k -bin does not affect our final results (i.e. exclusion probabilities for the matter-power spectrum models, Section 3.8.1) significantly.

3.6.2 Noise correction

In order to characterise the noise properties in the analysed HST-image, we create a sample of 20 selected blank-sky regions with the same size (121 by 121 pixels), located in proximity to the lens system. The first rough estimate of the noise level is given by the standard deviation of the flux

⁴<https://docs.scipy.org/doc/numpy/reference/routines.fft.html>

Figure 3.6: Power spectrum of the residual surface-brightness fluctuations in the lensed images of SDSS J0252+0039 after the smooth lens modelling. The lens modelling is performed with the highest source-grid resolution ($n = 1$) using either an adaptive (*magenta line*) or a non-adaptive (*yellow line*) source-grid regularisation, or, alternatively, with a lower source-grid resolution ($n = 3$; *blue line*), chosen for this study. The *red line* shows the effect of using a larger mask combined with $n = 1$ and a non-adaptive source-grid regularisation. The sky-background power spectrum (*black line*) is estimated based on a sample of twenty selected blank-sky regions located in the proximity to the lens. The estimated total noise power spectrum in the science image (*green line*) accounts for the additional flux-dependent (Poisson) photon-shot noise.

values in this blank-sky sample: $\sigma_{sky} = 0.002 \text{ e}^- \text{ sec}^{-1}$. However, this estimate does not take into account the photon-shot (Poisson-distributed) noise, which depends on the number of detected photons and, consequently, varies from pixel to pixel.

A more precise description of noise properties, including both the sky-background and the photon-shot noise, is provided by the noise-sigma map, which quantifies the standard deviation of noise for each pixel separately. Considering that the Poisson variance of the photon counts is similar to the measured number of photons and the raw HST images of the lens are drizzle-combined using an inverse-variance map weighting, we construct the noise-sigma map for our drizzled HST science image according to the following formula:

$$\sigma_n = \sqrt{N/W + \sigma_{sky}^2}, \quad (3.15)$$

where N is the number of photo-electrons per second detected in a particular pixel (after the sky-background subtraction) and W is the weight of this pixel taken from the weight map of our image provided by the drizzling pipeline. This noise-sigma map is also used in the Bayesian smooth-lens modelling procedure, presented in Section 3.5. Since the Poisson noise approaches Gaussian noise for large number counts, as is the case for the studied image, in the remaining part of our analysis we approximate the photon-shot noise by an additive Gaussian noise $\mathcal{N}(0, \sigma_n^2)$ with a variance σ_n^2 adapted to the flux value in a particular pixel.

However, due to noise correlations in the drizzled images and the charge-transfer inefficiency (CTI), discussed in more detail in Appendix 3.A, the noise correction of the residual surface-brightness fluctuations requires a more extended approach. To mimic the true noise properties in the science image, we use the selected blank-sky regions to generate a sample of 20 scaled sky-background realisations, which account for both the realistic noise-correlation pattern and the spatially-varying flux-dependent photon-shot noise. This is implemented by first dividing the flux values of the blank-sky regions by their standard deviation and subsequently multiplying them by the noise-sigma map of the science image (equation 3.15). For consistency reasons, these scaled sky-background realisations are finally overlaid with the same mask as the one used in the analysis of the real data (Section 3.6). The average power spectrum measured in this sample is used as our best estimate for the total noise power spectrum of the science image (see Fig. 3.6).

A comparison between the power spectrum of the residual surface-brightness fluctuations revealed in the lensed images of SDSS J0252+0039 with the estimated noise power spectrum, presented in Fig. 3.6, shows that for the highest considered k -values (corresponding to scales below three pixels) the residual fluctuations reach the noise level. This indicates that no surface-brightness anomalies are detected on these scales (see Fig. 3.6). For this reason, in our further analysis we consider only the perturbation wave-number range between $k_{\min} = 0.88$ and $k_{\max} = 7.95 \text{ arcsec}^{-1}$, which corresponds to the spatial scales between $\lambda_{\min} = 0.52$ and $\lambda_{\max} = 4.65 \text{ kpc}$ at the redshift of the lens galaxy. Performing the analysis on scales above three pixels together with our choice of $n = 3$ in the smooth-lens-modelling procedure allows us to neglect the effects of small errors in the PSF (with FWHM = 0.07 arcsec), the correlations between adjacent pixels due to drizzling and the possible residual errors in the source-light modelling.

Finally, we use this estimated total noise power spectrum to perform the noise correction of the residual surface-brightness fluctuations. We assume that the observational noise and the potential fluctuations $\delta\psi$, perturbing the smooth lensing potential, are independent stochastic processes and consider the corresponding power spectra to be additive. This allows us to subtract the estimated noise power spectrum from the power spectrum of the total residual surface-brightness fluctuations (see Fig. 3.7). The difference of these two power spectra constitutes our best estimate of the residual surface-brightness fluctuations and is treated in our further analysis as an upper limit to the power spectrum of surface-brightness anomalies $P_{\delta I}(k)$ caused by the hypothetical small-scale mass structure in the lens galaxy.

3.7 Mock perturbed lensed images

In this section, we create a set of mock perturbed lensed images, mimicking SDSS J0252+0039, which will be compared with the observed data.

The starting point is the unperturbed smooth-lens model of the lensed images, shown in the top left panel of Fig. 3.8, corresponding to the best-fitting smooth-lens model inferred in Section 3.5. In order to simulate mock surface-brightness anomalies, we generate pixelated realisations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ from the power-law power spectrum $P_{\delta\psi}(k; \sigma_{\delta\psi}^2, \beta)$ (equations 3.5 and 3.8) and superimpose them on the best-fitting smooth lensing potential $\psi_0(\mathbf{x})$. Repeating the lensing operation of the pixelated source

Figure 3.7: Noise correction of the residual power spectrum. The difference between the power spectrum of the residual surface-brightness fluctuations in the lensed images after the smooth lens modelling with $n = 3$ (*blue line*) and the estimated total noise power spectrum (*green line*) constitutes the upper limit on the power spectrum of surface-brightness anomalies due to mass structure in the lens galaxy SDSS J0252+0039 (*red line*).

galaxy (reconstructed in the smooth-lens-modelling procedure) with the perturbed lensing potential $\psi_0(\mathbf{x}) + \delta\psi_{\text{GRF}}(\mathbf{x})$ results in the corresponding mock perturbed lensed images.

To investigate the impact of the potential perturbations on the power spectrum of the surface-brightness anomalies in the lensed images in a systematic way, we consider a grid of 100×100 different combinations of β and $\sigma_{\delta\psi}^2$. The interval for β consists of 100 equidistant values within the interval $[3, 8]$, whereas $\sigma_{\delta\psi}^2$ (within the studied field-of-view) is varied by obtaining 100 points evenly spaced in the logarithmic range $[10^{-6}, 10^{-1}]$. Fig. 3.8 shows three examples of such mock perturbed lensed images generated for different values of $\sigma_{\delta\psi}^2$ and β , together with the underlying realisations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ and $\delta\kappa_{\text{GRF}}(\mathbf{x})$.

Following the methodology applied to the observational data, we subtract the unperturbed smooth-lens model from these mock perturbed lensed images and compute the power spectrum of the residual surface-brightness fluctuations within the same mask and using the same set of ten bins for the azimuthal averaging as for the observed lensed images. To reduce the sample variance, we generate ten different realisations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ for any considered combination of $\sigma_{\delta\psi}^2$ and β , and average the power spectra over this sample, see Fig. 3.9 for some examples of the resulting mock residual power spectra. A quantitative comparison between these and the real measurement, illustrated in Fig. 3.9, allows us to determine exclusion probabilities for the considered combinations of $\sigma_{\delta\psi}^2$ and β , see Section 3.8.1.

3.8 Constraints on the sub-galactic matter-power spectrum

In this section, we first discuss our methodology used to assess whether a particular matter-power-spectrum model (i.e. combination of $\sigma_{\delta\psi}^2$ and β) leads to surface-brightness anomalies that are significantly different from the measured residual surface-brightness fluctuations (Section 3.8.1). Subsequently, we present and discuss the resulting constraints on the statistical properties of the potential perturbations in the lens galaxy SDSS J0252+0039 (Section 3.8.2). Finally, we express these constraints in terms of the variance of the resulting deflection-angle perturbations (Section 3.8.3), the dimensionless convergence power spectrum $\Delta_{\delta\kappa}^2(k)$ and

Figure 3.8: Mock surface-brightness anomalies induced in the gravitational arcs of SDSS J0252+0039 by Gaussian potential perturbations $\delta\psi_{GRF}(\mathbf{x})$ in the lens galaxy, with the power spectrum $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$. A realistic noise realisation is overlaid for visualisation purposes. *Left column*: unperturbed mock lensed images (*upper panel*) and three examples of perturbed mock lensed images (with $\sigma_{\delta\psi}^2 = 2.154 \times 10^{-5}$ and $\beta = 5.5$, $\sigma_{\delta\psi}^2 = 2.783 \times 10^{-4}$ and $\beta = 4.25$, $\sigma_{\delta\psi}^2 = 10^{-3}$ and $\beta = 3$; *from top to bottom*). *Middle column*: the underlying realisations of $\delta\psi_{GRF}(\mathbf{x})$. *Right column*: the corresponding realisations of the convergence perturbations $\delta\kappa_{GRF}(\mathbf{x}) = \frac{1}{2}\nabla^2\delta\psi_{GRF}(\mathbf{x})$.

Figure 3.9: Power spectrum of residual surface-brightness fluctuations remaining in the lensed images of SDSS J0252+0039 after the lens-galaxy subtraction, the smooth lens modelling and the noise correction (*red line*), in comparison to the power spectra of mock surface-brightness anomalies induced in the lensed images by Gaussian potential perturbations with different values of the integrated variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β (*dashed lines*). For clarity of presentation, only 9 out of 10^4 mock power spectra in our catalogue are presented.

the aperture mass on three specific spatial scales: 0.5, 1 and 3 kpc (Section 3.8.4).

3.8.1 Exclusion probability of matter-power-spectrum models

A comparison between the power spectra of the mock surface-brightness anomalies and the power spectrum of the observed residual surface-brightness fluctuations enables us to determine exclusion probabilities for different combinations of $\sigma_{\delta\psi}^2$ and β . As discussed in Section 3.2.2, we conservatively treat the estimated residual surface-brightness fluctuations as an upper limit to the anomalies caused solely by potential perturbations in the lens galaxy. Consequently, we rule out a particular matter power-spectrum model only if the power spectrum of the resulting surface-brightness anomalies exceeds the measured power spectrum on all considered scales, i.e. in all five analysed k -bins between $k_{\min} = 0.88$ and $k_{\max} = 7.95 \text{ arcsec}^{-1}$.

In order to compute the exclusion probability for a given matter-power-spectrum model, we treat the power-spectrum value measured in each k -bin as the expected value of a Gaussian random variable with the standard deviation (corresponding to the error bars in Fig. 3.9) estimated based on the variance in our sample of mock noise realisations discussed in Section 3.6.2. Under this assumption, the probability that the power spectrum of the mock surface-brightness anomalies exceeds the power spectrum measured in the data in an individual k -bin can be calculated in the following way:

$$P_{EX}(\sigma_{\delta\psi}^2, \beta, k) = \text{Prob}\left(P_{\delta I}^M(k) > P_{\delta I}^D(k) \mid \sigma_{\delta\psi}^2, \beta, P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}\right) \quad (3.16)$$

with $P_{\delta I}^D(k)$ denoting the observed power spectrum and $P_{\delta I}^M(k)$ representing the mock power spectrum for a particular choice of $\sigma_{\delta\psi}^2$ and β .

Assuming that our exclusion-probability estimates for different perturbation wavenumbers k are statistically independent, we compute the final exclusion probability for a particular matter-power-spectrum model as the product of the exclusion probabilities in all considered k -bins:

$$P_{EX}(\sigma_{\delta\psi}^2, \beta) = \prod_k P_{EX}(\sigma_{\delta\psi}^2, \beta, k). \quad (3.17)$$

The inferred exclusion probabilities are presented in Fig. 3.10. The chosen grid contains 50 different values for $\sigma_{\delta\psi}^2$, evenly spaced in the logarithmic range $[10^{-5.0}, 10^{-2.5}]$, and 100 values for β , equidistant in the interval $[3, 8]$. Colour-coded is the inferred exclusion probability of the corresponding matter-power-spectrum model $P_{\text{EX}}(\sigma_{\delta\psi}^2, \beta)$, defined as the probability that the resulting surface-brightness anomalies exceed the observed ones on all considered spatial scales. The superposed (black) isoproability line indicates models with exclusion probability larger than 99 per cent.

3.8.2 Upper limits on the potential perturbations in SDSS J0252+0039

Based on the exclusion plot presented in Fig. 3.10, we rule out matter-power-spectrum models with $\sigma_{\delta\psi}^2$ exceeding $\sim 10^{-2.5}$ (on the spatial scales between L^{-1} and pixel scale $^{-1}$ and within the range of the power-law power-spectrum slope $3 \leq \beta \leq 8$) at the 99 per cent confidence level. This corresponds to the upper bound on the standard deviation of such potential fluctuations $\sigma_{\delta\psi} \leq 0.06$. The excluded models would lead to power spectra of surface-brightness anomalies that significantly exceed the observed upper limits on all considered scales.

Moreover, we find that for a specific value of the integrated variance $\sigma_{\delta\psi}^2$ the exclusion probability depends on the power-spectrum slope β and, thus, on the exact distribution of the integrated variance between the different spatial scales. Shallower slopes, assigning a larger fraction of $\sigma_{\delta\psi}^2$ to small scales, i.e. high k -modes, are found more likely to be excluded. This leads to the conclusion that potential fluctuations on smaller spatial scales have a stronger perturbative effect on the lensed images than potential fluctuations on larger scales.

3.8.3 Upper limits on the deflection-angle perturbations in SDSS J0252+0039

To explain the asymptotic shape of the exclusion plot, presented in Fig. 3.10, we make use of equation (3.11) and convert our original constraints on the power spectrum of potential perturbations $P_{\delta\psi}(k, \sigma_{\delta\psi}^2, \beta)$ into constraints on the power spectrum of the corresponding perturbations

Figure 3.10: Exclusion probabilities for a set of sub-galactic matter-power-spectrum models $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$, inferred from our analysis of SDSS J0252+0039. Each point of the exclusion plot corresponds to a particular combination of the integrated variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β , uniquely characterizing the hypothetical potential perturbations in the lens galaxy. The superposed *black isoprobability line* indicates models with exclusion probability larger than 99 per cent. The *white contour lines* connect matter-power-spectrum models with the same variance of the deflection-angle perturbations $\sigma_{\delta\alpha}^2$ in the analysed field-of-view.

in the deflection angle $P_{\delta\alpha}(k, \sigma_{\delta\psi}^2, \beta)$. According to equation (3.11), the slope of $P_{\delta\alpha}(k, \sigma_{\delta\psi}^2, \beta)$ decreases by 2.0 (i.e. becomes shallower) with respect to the slope of $P_{\delta\psi}(k, \sigma_{\delta\psi}^2, \beta)$. The total variance in the differential deflection angle $\sigma_{\delta\alpha}^2$ over the analysed field-of-view is obtained by integrating $P_{\delta\alpha}(k)$ over all pixels of the two-dimensional Fourier grid:

$$\sigma_{\delta\alpha}^2(\sigma_{\delta\psi}^2, \beta) = \int_{k_x} \int_{k_y} P_{\delta\alpha}(\sqrt{k_x^2 + k_y^2}, \sigma_{\delta\psi}^2, \beta) dk_x dk_y. \quad (3.18)$$

The result is presented in Fig. 3.10, where the overlaid (white) isolines correspond to the same values of $\sigma_{\delta\alpha}^2$. These isolines almost perfectly follow the overall shape of the exclusion plot, which indicates a strong correlation between the exclusion probability of a matter-power-spectrum model and the total variance of the associated deflection-angle perturbations. This suggests that $\sigma_{\delta\alpha}^2$ is the fundamental quantity that determines the level of the resulting surface-brightness anomalies in the lensed images. Consequently, the exclusion probability is almost insensitive to the slope β of the $P_{\delta\alpha}(k)$, i.e. the distribution of the total variance in the deflection angle over the different length scales. Based on Fig. 3.10, we exclude (with 99 per cent probability) a matter-power-spectrum model if the corresponding total variance in the differential deflection field is larger than 6×10^{-3} , independently of the slope β . This insight is valuable for a future analysis of additional lens systems, which instead of considering potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ might more efficiently focus on the corresponding deflection-angle perturbations $\delta\alpha_{\text{GRF}}(\mathbf{x})$. The inferred threshold value of $\sigma_{\delta\alpha}^2$, however, might vary between different lens systems and depend on the chosen field-of-view, the PSF and the signal-to-noise ratio of the analysed image.

For completeness, we derive the corresponding upper-limit constraints on the dimensionless deflection-power spectrum $\Delta_{\delta\alpha}^2(k)$, see equation (3.13), for three different spatial scales 0.5, 1 and 3 kpc, and present them in Fig. 3.11. The results exclude $\Delta_{\delta\alpha}^2(k)$ larger than 0.001 on all considered spatial scales at the 99 per cent confidence level.

3.8.4 Upper limits on the surface-mass density fluctuations in SDSS J0252+0039

We additionally express and interpret our constraints with respect to the dimensionless convergence power spectrum $\Delta_{\delta\kappa}^2(k)$, defined in equation (3.12). With this aim, we determine $\Delta_{\delta\kappa}^2(k)$ corresponding to each combination of $\sigma_{\delta\psi}^2$ and β for three different spatial scales: 0.5, 1 and 3 kpc. The smallest considered scale of 0.5 kpc corresponds to ~ 3 pixels, which is the smallest spatial scale in our analysis. The largest considered scale, on the other hand, is limited by the size of the lensed images (gravitational arcs) in SDSS J0252+0039. Fig. 3.11 shows the resulting contour lines connecting matter-power-spectrum models with the same value of $\Delta_{\delta\kappa}^2$ inferred on a particular scale, overlaid on the original exclusion plot. Based on Fig. 3.11, we rule out matter-power-spectrum models with $\Delta_{\delta\kappa}^2$ larger than 1 on 0.5-kpc scale, larger than 0.1 on 1-kpc scale and larger than 0.01 on 3-kpc scale, at the 99 per cent confidence level.

These results can be additionally interpreted in terms of the standard deviation of the total convergence perturbation $\sigma_{\delta\kappa}(\lambda) \equiv \sqrt{\Delta_{\delta\kappa}^2(\lambda)}$ within the aperture diameter equal to the considered scale λ in an infinitely large sample of circular regions, randomly chosen in proximity of the Einstein radius. For the lensing-mass distribution of SDSS J0252+0039 we infer the following upper limits on this standard deviation (on the spatial scales between L^{-1} and pixel scale $^{-1}$, within the range of the power-law power-spectrum slope $3 \leq \beta \leq 8$): $\sigma_{\delta\kappa}(0.5 \text{ kpc}) < 1$ on 0.5-kpc scale, $\sigma_{\delta\kappa}(1 \text{ kpc}) < 0.3$ on 1-kpc scale and $\sigma_{\delta\kappa}(3 \text{ kpc}) < 0.1$ on 3-kpc scale. With the critical surface-mass density for SDSS J0252+0039 $\Sigma_{\text{cr}} \approx 4 \times 10^8 M_{\odot} \text{ kpc}^{-2}$, these constraints can be translated into upper limits on the integrated standard deviation $\sigma_{AM}(\lambda)$ in the aperture mass (within an aperture of diameter λ in the lens plane) in the inner region of the lens galaxy SDSS J0252+0039: $\sigma_{AM}(0.5 \text{ kpc}) < 0.8 \times 10^8 M_{\odot}$, $\sigma_{AM}(1 \text{ kpc}) < 1 \times 10^8 M_{\odot}$ for $\lambda = 1 \text{ kpc}$ and $\sigma_{AM}(3 \text{ kpc}) < 3 \times 10^8 M_{\odot}$ at the 99 per cent confidence level.

Finally, we provide a simple heuristic comparison with the Λ CDM predictions. For the sake of simplicity, we account solely for the contribution from CDM sub-haloes in the host halo of the lens galaxy and neglect both other fluctuations in the total (baryonic and dark) mass distribution of the lens galaxy and possible line-of-sight haloes. More specifically, we estimate their contribution to the dimensionless convergence power spectrum $\Delta_{\delta\kappa}^2(\lambda)$. Conservatively, we treat the sub-haloes as point masses and apply Poisson

Figure 3.11: Upper-limit constraints on the dimensionless-convergence power spectrum $\Delta_{\delta\kappa}^2$ and the dimensionless-differential-deflection power spectrum $\Delta_{\delta\alpha}^2$ in the lens galaxy SDSS J0252+0039, for three different sub-galactic scales. Each point of the exclusion plot corresponds to a particular sub-galactic matter-power-spectrum model $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$, as specified in Fig. 3.10. The *white contour lines* connect models with the same value of $\Delta_{\delta\kappa}^2$ (*left panel*) and $\Delta_{\delta\alpha}^2$ (*right panel*) on a particular scale. Overlaid in black is the 0.99-contour of the exclusion probability. Note that β represents the power-spectrum slope of the originally investigated potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$. The slopes of the associated power spectra of the convergence perturbations $\delta\kappa_{\text{GRF}}(\mathbf{x})$ and the deflection perturbations $\delta\alpha_{\text{GRF}}(\mathbf{x})$ decrease by 4 and 2, respectively (in terms of the absolute value).

statistics to analytically predict the number of such sub-haloes from the assumed CDM substructure-mass function of the form $dN/dM \propto M^{-1.9}$ (Springel et al., 2008) neglecting the suppression of the substructure population due to baryonic processes (Despali et al., 2018)). Furthermore, we assume the line-of-sight mass density near the Einstein radius of the lens halo to consist of dark matter with a projected substructure mass fraction of 0.005, for the sub-halo mass range between 4×10^6 and $4 \times 10^9 M_\odot$ (Vegetti & Koopmans, 2009b). Taking into consideration that the studied lens galaxy is well described by a Singular Isothermal Sphere model and, thus, its convergence $\kappa \sim 0.5$ in the proximity to the Einstein radius, we estimate the following upper limits on $\Delta_{\delta\kappa}^2(\lambda)$ due to CDM sub-haloes in the dark-matter halo of SDSS J0252+0039: $\Delta_{\text{CDM}}^2(0.5 \text{ kpc}) < 10^{-3}$ on 0.5-kpc scale, $\Delta_{\text{CDM}}^2(1 \text{ kpc}) < 4 \times 10^{-4}$ on 1-kpc scale and $\Delta_{\text{CDM}}^2(3 \text{ kpc}) < 10^{-4}$ on 3-kpc scale. While such a point-mass approach is solely an approximation, it provides a conservative upper-limit constraint on the contribution from CDM sub-haloes, any other density profile will only lower $\Delta^2(\lambda)$.

A comparison of this estimation with the observational constraints on $\Delta_{\delta\kappa}^2$ for a flat convergence-power spectrum (corresponding to the slope $\beta = 4$ of the potential-power spectrum depicted in Fig. 3.11) leads to the conclusion that the estimated effect of CDM sub-haloes lies significantly below the inferred observational upper-limits on all considered scales. Accounting for the more realistic NFW-density profile of CDM substructure (Navarro et al., 1996), instead of the assumed point-mass model, would lead to an even lower estimate for the convergence power spectrum caused by CDM substructure. This preliminary conclusion does not change even if we consider that in reality the total number of haloes perturbing the lensed images might be a few times higher than just the estimated number of sub-haloes in the lens galaxy, due to the possible presence of unbound haloes along the total line-of-sight (Despali et al., 2018). We attribute the above discrepancy mainly to the fact that, unlike our preliminary Λ CDM-based predictions, the observational upper-limit constraints inferred in this work refer to the total (dark and baryonic) mass distribution projected along the line-of-sight, including the complex baryonic and dark matter structure of the lens galaxy (e.g. Gilman et al., 2017b; Hsueh et al., 2017a) and the possible line-of-sight haloes. Future research will, therefore, adequately compare these observational results with the total-matter power spectra including the contributions from sub-haloes, line-of-sight haloes as well as

other fluctuations in the baryonic and dark-matter distribution of the lens galaxy, which can be obtained from hydrodynamical simulations.

3.9 Summary and conclusions

Alternative dark-matter models and galaxy-formation scenarios predict significantly different levels of mass structure on sub-galactic scales (e.g. Lovell et al., 2014). Observational constraints on the matter power spectrum on the critical 1-10 kpc-scales would make it possible to distinguish between these models. In this work, we have demonstrated a novel methodology allowing one to constrain the statistical nature of small-scale density fluctuations in the total projected mass distribution of massive elliptical lens galaxies (i.e. deviations from the smooth elliptical power-law density model with an external shear) based on the power spectrum of surface-brightness anomalies in highly magnified galaxy-scale Einstein rings and gravitational arcs. Our approach is based on the theoretical framework by Chatterjee & Koopmans (2018), who proposed to model such small-scale density fluctuations as Gaussian-Random-Field (GRF) potential perturbations, characterised by the power spectrum $P_{\delta\psi}$.

The pilot application of the presented methodology to the lens system SDSS J0252+0039 from the SLACS Survey leads to the following conclusions:

1. The enhanced intrinsic source-galaxy structure in the analysed U-band data requires a higher source-grid resolution and leads to more severe degeneracies between the source and lens models than it was the case for the I-band data previously modelled by Vegetti et al. (2014). Whereas this degeneracy is less problematic when trying to identify individual sub-haloes with masses above the detection limit, see Vegetti et al. (2014), alleviating it is crucial when performing a power-spectrum analysis. In this chapter, we have addressed this degeneracy by inferring solely the upper limits on the level of small-scale density fluctuations in the lens galaxy.
2. We rule out, at 99% C.L., potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ with variance $\sigma_{\delta\psi}^2$ exceeding $\sim 10^{-2.5}$ (on the spatial scales between L^{-1} and pixel scale $^{-1}$ and within the range of the power-law power-spectrum slope $3 \leq \beta \leq 8$).

3. We find that the fundamental quantity that determines the level of surface-brightness anomalies in the mock perturbed lensed images and, thus, the probability of the matter-power-spectrum model exclusion, is the total variance in the differential deflection angle $\sigma_{\delta\alpha}^2$ (on the spatial scales between L^{-1} and pixel scale $^{-1}$) resulting from the underlying potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$. Consequently, the exclusion probability is nearly insensitive to the slope β of the deflection-power spectrum $P_{\delta\alpha}(k)$, i.e. the distribution of $\sigma_{\delta\alpha}^2$ over the different length scales. Based on our analysis of SDSS J0252+0039, $\sigma_{\delta\alpha}^2 < 6 \times 10^{-3}$, independently of the slope β , within the range $3 \leq \beta \leq 8$. This insight is valuable for our future analysis of further galaxy-scale lens systems, which might be carried out by perturbing the deflection angle $\alpha(\mathbf{x})$ instead of perturbing the lensing potential $\psi(\mathbf{x})$. The threshold value itself, however, might vary for different lens systems and depend on the chosen field-of-view, the PSF and the signal-to-noise ratio of the analysed image.

4. In order to make our results less dependent on the chosen field-of-view, we infer the corresponding constraints on the dimensionless convergence-power spectrum $\Delta_{\delta\kappa}^2(\lambda)$ on three different sub-galactic scales and rule out matter-power-spectrum models with $\Delta_{\delta\kappa}^2(0.5 \text{ kpc}) > 1$ on 0.5-kpc scale, $\Delta_{\delta\kappa}^2(1 \text{ kpc}) > 0.1$ on 1-kpc scale and $\Delta_{\delta\kappa}^2(3 \text{ kpc}) > 0.01$ on 3-kpc scale (at the 99% C.L.).

5. Translating these constraints into the standard deviation $\sigma_{AM}(\lambda)$ of the aperture mass (integrated in a cylinder with the diameter λ in the lens plane, in the proximity to the Einstein radius of the lens galaxy) leads to the following limits: $\sigma_{AM}(0.5 \text{ kpc}) < 0.8 \times 10^8 M_\odot$, $\sigma_{AM}(1 \text{ kpc}) < 1 \times 10^8 M_\odot$ and $\sigma_{AM}(3 \text{ kpc}) < 3 \times 10^8 M_\odot$ (at the 99% C.L.).

In future work, we intend to investigate the modelling degeneracies in more detail, analyze a larger sample of lens systems, and infer more stringent constraints on the dark-matter and galaxy-formation models by comparing these results to hydrodynamical simulations.

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Appendix 3.A Noise power spectrum of the sky background

As discussed in Section 3.6.2, one of the crucial steps in our methodology is to account for the observational noise contributing to the measured residual surface-brightness fluctuations in the lensed images. Despite the fact that the pixels in the raw HST/WFC3/F390W-images can be considered to be independent, the observational noise in the drizzled images shows correlations caused by the drizzling procedure (see Casertano et al. (2000) for a more detailed explanation) as well as the charge-transfer inefficiency (CTI) in the HST/WFC3/UVIS-CCDs (see e.g. Baggett et al., 2015). The effect of this noise correlation is demonstrated in Fig. 3.12 showing a scale-dependent (one-dimensional azimuthally-averaged mean) power spectrum measured in an observed sample of 20 selected empty-sky regions located close to the studied lens system. For comparison, we additionally plot a flat power spectrum obtained for an uncorrelated Gaussian noise process with the same standard deviation. Both samples consist of images with the same size as the analysed image of the lens system (121 by 121 pixels).

In order to investigate and possibly reduce the noise correlation in our HST/WFC3/F390W images of SDSS J0252+0039, we test the effect of different settings for the drizzling parameters *final scale* and *final pixfrac* when running the ASTRODRIZZLE task from the DRIZZLEPAC package (Gonzaga et al., 2012), in comparison to performing the drizzling procedure using default parameter values, as chosen for our analysis in this paper. By default, HST/WFC3/F390W images are drizzled to the original output pixel scale of 0.0396 arcsec/pixel (*final scale* = 0.0396 arcsec) without

Figure 3.12: Noise correlation in the drizzled HST/WFC3/F390W-images: the mean azimuthally-averaged power spectrum measured in a sample of twenty selected blank-sky regions located in proximity to SDSS J0252+0039 vs. the flat power spectrum measured in a sample of twenty mock realisations of uncorrelated Gaussian noise with the same standard deviation as in the blank-sky cutouts.

Figure 3.13: The effect of the drizzling parameters *pixscale* (final scale of the output pixels) and *final pixfrac* (size of the *drops*) on the azimuthally-averaged power spectrum of surface-brightness fluctuations in a blank-sky field observed with HST/WFC3/F390W in proximity to SDSS J0252+0039.

shrinking the input pixels (referred to as *drops* in the drizzling procedure) before mapping them onto the final output grid (*final pixfrac* = 1). As can be seen in Fig. 3.13, shrinking the output pixel size (lowering the value of the *final scale*) and the size of the drops (lowering the value of *final pixfrac*) when combining the raw dithered images does not significantly affect the measured power spectrum of the chosen empty-sky cutout in the five lowest k -bins taken into account in our analysis. Taking into account that choosing a lower final output-pixel scale while maintaining the same field of view would substantially increase the number of pixels and, thus, the computational effort, we decide to proceed with the drizzling procedure in the default configuration.

In addition to exploring different settings in the drizzling procedure, we test the possibility of performing the analysis using the drizzled CTI-corrected science image (obtained by drizzling CTI-corrected calibrated `flc.fits` files) instead of the original drizzled HST science image (obtained by drizzling the calibrated `flt.fits` files). The CTI-corrected calibrated files for WFC3/UVIS data became available from the MAST archive only after the beginning of our analysis. In order to test whether the CTI-correction would have a substantial effect on our results, we compare the power spectrum of the surface-brightness fluctuations measured in an empty-sky cutout close to SDSS J0252+0039 before and after the CTI-correction. As demonstrated in Fig. 3.14, the CTI-correction does not affect the measured power spectrum on small scales, but it leads to a reduction in the noise correlation on the largest considered spatial scale - the power for $\lambda = 1.13$ arcsec is roughly 40 per cent lower for the CTI-corrected drizzled science image. To investigate the impact of this scale (i.e the smallest considered k) on our final exclusion probabilities, we carry out a test run of our methodology for which we exclude this scale from the analysis. This test proves that the exclusion probabilities inferred based on the reduced k -range do not differ significantly from the original ones. For this reason, we omit the CTI-correction and proceed using the original `flt.fits` files.

Appendix 3.B Dust analysis

Among all HST images available for SDSS J0252+0039 (U-, V-, I- and H-bands), the U-band photometry is most sensitive to the presence of dust. Thus, clumpy dust in the lens galaxy could potentially cause the small-scale variations in the surface brightness of the lensed images that, so far in this paper, have been interpreted as arising from density fluctuations in the lensing-mass distribution. In order to investigate this possible degeneracy, we compare co-aligned images of SDSS J0252+0039 in the U- and I-bands. For this comparison, the I-band image is drizzled to the reference frame and pixel size of the U-band image. The U-band image, on the other hand, is smoothed with a Gaussian profile to a larger blur (according to the Gaussian cascade smoothing) of the I-band image: the PSF of the F390W-filter is well described by a Gaussian profile with the FWHM 0.07 arcsec, whereas the PSF of the F814W filter has the FWHM 0.1 arcsec. Subsequently, both images are divided by the respective standard deviation of the photoelectron counts in the empty sky ($0.0012 \text{ e}^- \text{ sec}^{-1}$ for the smoothed U-band

Figure 3.14: The effect of the charge-transfer-efficiency (CTE) correction in the drizzling procedure on the measured power spectrum of surface-brightness fluctuations in HST/WFC3/F390W-images: the azimuthally-averaged power spectrum of a blank-sky cutout in proximity to SDSS J0252+0039 before vs. after CTE-correction (with the original pixel scale and the default *final pixfrac* = 1).

and $0.0045 \text{ e}^- \text{ sec}^{-1}$ for the I-band image drizzled to the pixel scale of the U-band) to obtain the signal-to-noise ratio for each pixel. Finally, to assess the effect of dust, we generate a ratio image between the I- and the U-band and conclude that the variations present across the lensed images can be attributed to differences in the corresponding Point Spread Functions of the compared filters. An additional visual inspection of the colour image in Fig. 3.1, based on observations in the UV (F390W), the visual (F814W) and the infrared bands (F160W), shows no indication for dust extinction either in the lens- or the source galaxy.

Chapter 4

Suppression of surface-brightness anomalies in the grid-based smooth lens modelling of SDSS J0946+1006

— Based on

**Suppression of surface-brightness anomalies in the
grid-based smooth lens modelling of SDSS J0946+1006**

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Abstract

The Λ CDM concordance cosmological model predicts galaxy-size haloes to be inhabited by an abundant population of dark-matter-dominated substructures. An observational proof of their existence would add another piece of evidence in favour of the Λ CDM model that seems to provide an excellent description of the large-scale matter distribution in the Universe, but at the same time is still challenged by tensions on sub-galactic scales. In this chapter, we infer observational constraints on the power spectrum of such small-scale mass structure in the massive elliptical lens galaxy SDSS J0946+1006 (the Double Einstein Ring). The constraints are derived from a statistical comparison of surface-brightness anomalies measured in the observed extended lensed images and in a set of simulated lensed images perturbed by Gaussian-Random-Field potential fluctuations. We improve on our earlier study of the lens system SDSS J0252+0039 by performing a grid-based smooth-lens modelling of the mock perturbed lensed images, in a way that is fully consistent with modelling of the real data. This allows us to account for all numerical effects inherent to the grid-based smooth lens modelling. Our feasibility study carried out for a single value ($\beta = 5$) of the power-law-power-spectrum slope characterising the potential perturbations leads to the conclusion that their variance $\sigma_{\delta\psi}^2$ in the lens galaxy SDSS J0946+1006 cannot exceed $10^{-3.0}$ ($p = 0.001$). Moreover, we demonstrate that the grid-based smooth lens modelling might significantly suppress the surface-brightness anomalies caused by small-scale mass structure in the lens galaxy, and incorrectly assign them to surface-brightness structure in the source galaxy. If not properly understood and accounted for, such signal suppression might result in severely biased constraints on the properties of the sub-galactic mass structure.

4.1 Introduction

The total (baryonic and dark) mass distribution inside massive elliptical galaxies is commonly assumed to be very smooth and well described by an isothermal gravitational potential (e.g. Gerhard et al., 2001; Treu et al., 2006; Churazov et al., 2010). However, according to our current understanding of galaxy formation and evolution in the framework of the concordance dark-energy-plus-cold-dark-matter (Λ CDM) cosmological model, galaxies form as a result of hierarchical mergers of low-mass structure into ever larger objects (e.g. White & Rees, 1978; White & Frenk, 1991; Kauffmann et al., 1993; Baugh et al., 1996). This hierarchical galaxy-formation process is supposed to leave its imprints on the mass distribution of the final merger products. Such imprints are indeed observed in the form of dwarf satellite galaxies orbiting the main galaxies, as well as other remnant structures, such as disks, shells, stellar streams or tidal debris. In addition to this observable baryonic structure, the Λ CDM model predicts galaxy-size haloes to be inhabited by a numerous population of dark-matter-dominated substructures (e.g. Zavala & Frenk, 2019). An observational proof of their existence would add another piece of evidence in favour of the Λ CDM model that seems to provide an excellent description of the large-scale matter distribution in the Universe (e.g. Vogelsberger et al., 2014; Schaye et al., 2015; Guo et al., 2016; Planck Collaboration et al., 2018), but at the same time is still challenged by tensions on sub-galactic scales (e.g. Bullock & Boylan-Kolchin, 2017).

Most of the developed techniques to search for the observational evidence of such extremely faint or even purely dark galactic substructures aim at detecting the resulting perturbations in the distribution of the chosen astrophysical observable (for example perturbations in the velocity field of stellar streams or the flux-ratio anomalies in the lensed images of background quasars) with respect to the predictions from a model assuming a smooth mass distribution (e.g. Koopmans, 2005; Vegetti & Koopmans, 2009a; Yoon et al., 2011; Birrer et al., 2017; Bonaca et al., 2019; Gilman et al., 2019). A particularly powerful approach in this regard makes use of the recent progress in the development of sophisticated methods to model galaxy-scale strong gravitational lens systems. The main idea of these lensing-based methods is to search for flux- or surface-brightness anomalies (with respect to the best-fitting smooth-lens model), possibly induced in the lensed images by the hypothetical substructures in the lens galaxy (e.g. Mao

& Schneider, 1998; Koopmans, 2005; Vegetti & Koopmans, 2009a; Vegetti et al., 2010a,d; Nierenberg et al., 2014). One such technique, *gravitational imaging* (Koopmans, 2005; Vegetti & Koopmans, 2009a), has lead to a detection of two dark substructures with the masses of $3.5 \times 10^9 M_\odot$ and $1.9 \times 10^8 M_\odot$ in massive elliptical lens galaxies at the redshifts $z = 0.2$ and $z = 0.881$, respectively, based on surface-brightness anomalies revealed in the extended lensed images of the background source galaxies (Vegetti et al., 2010c, 2012).

Building upon this gravitational-imaging technique, Bus (2012), Hezaveh et al. (2016), Diaz Rivero et al. (2018b) and Chatterjee & Koopmans (2018) have recently proposed a complementary approach aimed at studying the entire population of small-scale mass structure in lens galaxies in a statistical sense. In particular, within the theoretical framework introduced by Chatterjee & Koopmans (2018) and applied in this thesis, this small-scale mass structure is defined as deviations from a smooth power-law elliptical mass distribution model with an external shear and modelled in terms of Gaussian-Random-Field (GRF) potential perturbations superimposed on the best-fitting smoothly-varying lensing potential. The statistical properties of these potential perturbations are assumed to be well described by a power-law power spectrum $P_{\delta\psi}(\sigma_{\delta\psi}^2, \beta, k)$ with two free parameters – the total variance of the perturbations $\sigma_{\delta\psi}^2$ (within the considered field-of-view) and the power-law-power-spectrum slope β . Under this assumption, $\sigma_{\delta\psi}^2$ and β can possibly be constrained by comparing the ensemble properties of surface-brightness anomalies measured in observed lensed systems to a catalogue of mock lensed images perturbed by GRF potential fluctuations with known power spectra.

In Chapter 3, we applied this theoretical framework to high-resolution Hubble Space Telescope (WFC3/F390W) imaging of the galaxy-galaxy strong gravitational lens system SDSS J0252+0039 from the SLACS Survey (Bolton et al., 2006). This analysis allowed us to infer the first (conservative) upper-limit constraints on the power spectrum of small-scale structure in the total (dark and baryonic) mass distribution of a massive early-type (lens) galaxy. However, as was already discussed in Chapter 3, our methodology suffered from the degeneracy between the surface-brightness anomalies caused by mass structure in the lens galaxy and the intrinsic surface-brightness structure of the background source galaxy itself. This degeneracy turned out to be much more severe in our study than was the case in an earlier gravitational-imaging

analysis of 11 SLACS lens systems performed by Vegetti et al. (2014) in the I-band (HST/ACS/F814W), due to our choice of the ultra-violet observations (F390W). These enhanced the surface-brightness structure of the background galaxy, owing to the pronounced star-forming regions that have their maximal emissivity in the ultra-violet.

To alleviate this degeneracy and pave the way for a future unbiased *detection* of the hypothetical small-scale mass structure in massive elliptical lens galaxies, in this chapter we improve on our modelling techniques by thoroughly accounting for the bias inherent to our lens-modelling procedure. In particular, we use a new implementation of the Bayesian grid-based smooth-lens-modelling technique, developed by Vernardos et al. (in prep.), to model the mock perturbed lens systems in exactly the same manner as the real observations. This improvement allows us to narrow down the constraints on the matter power spectrum inferred in Chapter 3. Our analysis is based on deep HST/ACS/F814W-observations of the strong gravitational lens system SDSS J0946+1006, commonly known as the Double Einstein Ring (Gavazzi et al., 2008), with a substantially higher signal-to-noise ratio and less-structured lensed images than in the F390W-observations of SDSS J0252+0039 used in our pilot study (Chapter 3).

The present chapter is structured as follows. Section 4.2 provides a concise summary of our improved approach to constrain the power spectrum of small-scale mass structure in massive elliptical lens galaxies. In Section 4.3, we discuss the observations and data reduction, model the surface-brightness distribution of the lens galaxy, obtain the best-fitting smooth-lens model and perform the power-spectrum analysis of the residual surface-brightness fluctuations in the lensed images. Section 4.4 presents the generated catalogue of mock lensed images (mimicking the real observations of SDSS J0946+1006) perturbed with GRF potential fluctuations. In Section 4.5, we perform the full smooth lens modelling of the mock perturbed lens systems and infer constraints on the statistical properties of small-scale mass structure in the studied lens galaxy. In Section 4.6, we elaborate on the critical issue of signal suppression (also referred to as *bias*), inherent to the smooth lens modelling with a grid-based source reconstruction. Section 4.7 provides a final discussion of the modified methodology, the implications of the obtained results for our further research and, more generally, for the application of grid-based smooth-lens-modelling techniques in CDM-substructure studies.

For a consistent comparison of the lens models with earlier studies by Vegetti et al. (2014), throughout the chapter, we assume a concordance cosmology with $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.25$ and $\Omega_\Lambda = 0.75$. Given this cosmology, 1 arcsec corresponds to 3.46 kpc at the redshift of the lens galaxy $z_l = 0.222$ and 6.59 kpc at the redshift of the nearest source galaxy $z_s = 0.609$ in SDSS J0946+1006.

4.2 Analysis synopsis

The main goal of this chapter is to improve our methodology to search for gravitational signatures of small-scale mass structure in massive elliptical lens galaxies, introduced in our pilot study (Chapter 3). In short, following the original methodology, we first model the studied lens system assuming that the mass distribution in the lens galaxy is smooth and well described by the power-law elliptical mass-distribution model (PEMD, Barkana, 1998) in an external shear field. Unlike in Chapter 3, we use a new implementation of a Bayesian grid-based smooth-lens modelling code, developed by Vernardos et al. (in prep), see Section 4.3.3. The best-fitting smooth-lens model is subsequently subtracted from the real data to reveal residual surface-brightness fluctuations in the lensed images, possibly induced by the small-scale mass structure in the lens galaxy. These fluctuations are then quantified in terms of their power spectrum and compared to a set of simulated lensed images perturbed by GRF potential fluctuations with a priori known statistical properties. Contrary to our original approach, in this chapter we model the mock lensed images in *exactly* the same manner as the real observations (see Section 4.5.3), in order to account for the degeneracies, biases and other numerical issues encountered in our pilot study.

In the following sections, we discuss modifications in our methodology with respect to the original procedure introduced in Chapter 3. The reader is referred to Chapter 3 for a more detailed discussion of the analysis steps that were already included in our original methodology.

4.3 Observational measurement

We apply our modified methodology to deep high-signal-to-noise-ratio HST/ACS/F814W-observations (referred to as *I-band* in the remainder of

this chapter) of the Double Einstein Ring strong gravitational lens system (SDSS J0946+1006) from the SLACS sample (Bolton et al., 2006). This system has already been the subject of numerous detailed studies due to its double-source-plane nature (e.g. Gavazzi et al., 2008; Vegetti et al., 2010c). It is composed of a massive elliptical galaxy at the redshift $z_1 = 0.222$ acting as a strong gravitational lens on two almost co-aligned source galaxies located at the redshifts $z_{s1} = 0.609$ and $z_{s2} < 6.9$, respectively (Gavazzi et al., 2008), see also Collett & Smith (2020) for a recent discovery of the third doubly-imaged lensed (Ly- α) source at the redshift $z_{s3} = 5.975$.

However, in this work we do not take advantage of the additional constraints from the outer ring, corresponding to the second more distant source galaxy, which is faint and has no confirmed spectroscopic redshift, but focus solely on the analysis of the inner (partial) Einstein ring. We choose this particular lens system due to the high magnification of the lensed images and the availability of deep high signal-to-noise-ratio I-band observations with a less-structured source galaxy than in our pilot study performed in the U-band (HST/WFC3/F390W), see Chapter 3. This choice allows us to mitigate the degeneracy between the surface-brightness anomalies due to mass structure in the lens galaxy and the intrinsic surface-brightness structure of the background galaxy.

4.3.1 Data reduction

We apply the ASTRODRIZZLE task from the DRIZZLEPAC package (Gonzaga et al., 2012) in the default configuration (i.e. both the output-pixel size and the drop size are set to be equal to the original pixel size, i.e. 0.05 arcsec) to a set of four dithered charge-transfer-inefficiency-corrected flat-field calibrated F814W sub-exposures (i.e. flc.fits files) taken on November 03, 2006 (Program 10886, PI Bolton) and available for retrieval from the MAST archive¹. Fig. 4.1 presents the drizzled image with the total exposure time of 2096 s, clearly showing two concentric highly-magnified partial Einstein rings of radii 1.43 and 2.07 arcsec (Gavazzi et al., 2008), respectively, projected onto a massive elliptical lens galaxy. The very bright inner ring, which is the main target of our study, is an example of the *cusp configuration* – the longer gravitational arc consists of three merged lensed images observed at a larger projected distance from the lens centre than the fourth single counter-image.

¹<http://archive.stsci.edu/hst/search.php>

Figure 4.1: HST-imaging of the galaxy-galaxy strong gravitational lens system SDSS J0946+1006 taken with the Wide-Field Channel (WFC) of the Advanced Camera for Surveys (ACS) using the F814W-filter. The presented cutout corresponds to an area of 15.05 arcsec on a side in the original HST/ACS rotation. Both the inner and the (fainter) outer arc from a second source at a higher redshift (not used in our analysis) are clearly visible. The nearby (in projection) galaxies are modelled by an external shear.

4.3.2 Lens-galaxy subtraction

Before the lens modelling, the surface brightness of the inner partial Einstein ring needs to be corrected for the contribution from the foreground lens galaxy. With this aim, we model the global two-dimensional surface-brightness distribution of the lens galaxy using GALFIT (Peng et al., 2002), after masking the lensed features and other non-smooth components present in the considered field-of-view. The modeling is carried out based on a cutout with the size of 301 by 301 pixels (15.05 arcsec on a side), centered on the brightest pixel of the lens galaxy. The inferred best-fitting model consists of two Sérsic components combined with an exponential disk and a dust lane covering the very inner region (considerably smaller than the Einstein radius of the lens galaxy, thus not affecting the surface brightness of the lensed images). Fig. 4.2 presents the modelled cutout of the original image, the applied mask covering the features excluded from the fit, the best-fitting smooth model of the surface-brightness distribution in the lens galaxy, and the final residual image showing the lensed images corrected for the light contribution from the foreground lens galaxy.

4.3.3 Smooth lens modelling

As an initial step, we model the (lens-galaxy-subtracted) surface-brightness distribution of the inner partial Einstein ring in SDSS J0946+1006 under the assumption that the mass distribution in the lens galaxy is, to the first order, smooth and well described by the softened power law elliptical mass distribution model (SPEMD, Barkana, 1998) in an external shear field. To parameterize the mass distribution of the lens galaxy, we use the convergence κ , i.e. the mass density projected onto the lens plane (x, y) expressed in units of the critical mass density necessary for the occurrence of strong gravitational lensing, in the convention introduced by Vegetti & Koopmans (2009a):

$$\kappa(x, y) = \frac{b \left(2 - \frac{\gamma}{2}\right) q^{\gamma-3/2}}{2(x^2 q^2 + y^2)^{(\gamma-1)/2}}. \quad (4.1)$$

In the above equation, κ is a function of the lens strength b , the ratio of the minor to major axis q and the (three-dimensional) mass-density slope γ ($\gamma = 2$ in the isothermal case).

Figure 4.2: Best-fitting surface-brightness model of the lens galaxy SDSS J0946+1006 obtained using GALFIT. *Top row*: the HST/ACS/F814W-image of the lens system (*left panel*) and the applied mask covering features excluded from the fit (*right panel*). *Bottom row*: the best-fitting GALFIT model of the surface-brightness distribution in the lens galaxy (*left panel*) and the lens-galaxy-subtracted surface-brightness distribution of the lensed images (*right panel*).

Lens-modelling technique

The smooth lens modelling is performed by means of the adaptive grid-based Bayesian lens-modelling code by Vernardos et al. (in prep), which is based on the standard forward-reconstruction algorithm. As such, it fits the parameter values of the assumed parametric smoothly-varying mass-distribution model of the foreground lens galaxy (see equation (4.1)) and, simultaneously, reconstructs the unlensed surface-brightness distribution of the background source galaxy on a pixelized grid. The smooth-lens modelling procedure aims at minimizing the deviation of the measured flux values from those predicted by the model, which is quantified in terms of the Bayesian posterior probability. In addition, the implemented regularization of the source reconstruction penalises deviations from the parameterized covariance structure of the surface-brightness distribution in the source galaxy.

The non-linear search for the best-fitting smooth lens model is performed automatically via the Nested Sampling algorithm. However, the user is required to decide upon the type of the source-plane regularization (adaptive or non-adaptive, variance, gradient or curvature regularization) and the resolution of the background-source reconstruction on a pixelized grid (e.g. Suyu et al., 2006a; Vegetti et al., 2014). The source-grid resolution is controlled by a parameter, called n , that sets the linear scale (in pixels) of a square in the image plane out of which only the central pixel is cast back to the source plane. For example, if $n = 3$ only one pixel out of each contiguous 3×3 -pixel area is used to generate the source-reconstruction grid, while $n = 1$ corresponds to casting back every pixel. Another user-dependent choice is the size of the modelled image cutout and the shape of the mask including all the pixels that are taken into account during the modelling procedure. The impact of all these options on our final results is investigated and accounted for in our further analysis.

Compared to the smooth lens modelling technique by Vegetti & Koopmans (2009a), applied in the previous chapters, the code by Vernardos et al. (in prep) improves the lens modelling in two areas (but see also Ritondale et al. (2019) and Powell et al. (2020) for recently updated implementations of the original methodology by Vegetti & Koopmans (2009a), including the simultaneous modelling of the surface-brightness distribution in the lens galaxy and the lensed source as well as the modelling of gravitational lensing data in spectral and Fourier space,

within a fully parallelized Bayesian framework). First, the calculations are performed in a parallel way using multiple cores, which considerably increases the computational speed. Second, the regularization of the source reconstruction is performed in a more realistic and general way, using a more complex (parameterized) covariance-matrix structure that is optimized for in a Bayesian sense during the fitting procedure.

Best-fitting smooth-lens models and source reconstruction

In order to investigate the effect of the different settings, we perform the smooth lens modelling of SDSS J0946+1006 using two different masks and five different levels of the source-grid resolution, ranging from $n = 1$ to $n = 5$ (i.e. casting back every pixel or only one out of 25, respectively). In the entire analysis, we apply the curvature source-grid regularization as default. Our tests show that using more flexible covariance models does not alter the resulting smooth-lens models significantly.

Initially, we follow the original procedure from our pilot analysis, presented in Chapter 3, and use the smallest possible image cutout (81 pixels, or 4.05 arcsec, on a side, centred on the brightest pixel of the lens galaxy) together with a very tight mask covering solely the lensed images. However, as it was already the case in Chapter 3, the residual surface-brightness fluctuations in the lensed images turn out to be suppressed below the noise level when the smooth lens modelling is carried out with the highest source-grid resolution, corresponding to $n = 1$ and $n = 2$ (see also Section 4.3.4). In an attempt to alleviate this over-fitting problem, we apply a larger cutout (121 pixels, or 6.05 arcsec, on a side) and include additional regions close to the edge of the cutout (outside the second Einstein ring) in the fit. This modification allows us to increase the number of constraints and take into account additional information about the noise properties in the image.

Table 4.1 and Fig. 4.3 present the best-fitting smooth-lens models obtained based on this enlarged image cutout, for different levels of the source-grid resolution ($n = 1, 2, 3, 4, 5$) and with the regularization strength λ treated as a free parameter. As apparent from Table 4.1, the choice of the source-grid resolution does not alter the best-fitting parameter values significantly. Fig. 4.3 shows that the residual images obtained for $n = 2, 3, 4$ do not differ significantly either. However, in the reconstruction with $n = 1$,

the suppression of both the residual surface-brightness fluctuations and the observational noise can still clearly be seen.

Overall, we conclude that performing the smooth lens modelling using a larger image cutout together with a larger mask (and including additional regions outside the second Einstein ring) allows us to mitigate the signal suppression and the over-fitting of the observational noise down to $n = 2$ (but not for $n = 1$).

4.3.4 Power spectrum of the residual surface-brightness fluctuations

To quantify the residual surface-brightness fluctuations, remaining in the lensed images of SDSS J0946+1006 after the subtraction of the lens-galaxy light and the best-fitting smooth-lens model, we perform a power-spectrum analysis of the residual images (shown in Fig. 4.3). More specifically, following the original methodology introduced in Chapter 3, we set to zero the flux values of the pixels not included in the mask covering the modelled inner partial Einstein ring, and determine the one-dimensional azimuthally-averaged power spectrum of the whole Fourier-transformed residual image along a set of ten equidistant concentric annuli.

Fig. 4.4 illustrates the impact of the varying source-grid resolution ($n = 1, 2, 3, 4, 5$) on the power spectrum of the residual surface-brightness fluctuations (calculated within the original small cutout and mask) after subtracting the smooth-lens model obtained using the enlarged cutout and the corresponding mask. The over-fitting of the data in the case of the highest possible source-grid resolution ($n = 1$) can clearly be seen. Based on this comparison and taking into account the high computational costs of high-resolution reconstructions, we make the conservative choice of using the source-grid resolution corresponding to $n = 4$ in any further analysis performed in this chapter for modelling of both the data and the mock lens systems. Thus, the power spectrum of the residual surface-brightness fluctuations measured for $n = 4$ is the final statistics that will be used to compare observations to simulated lensed images.

For the sake of comparison, Fig. 4.5 depicts the power spectrum of the original HST-image, the lens galaxy-subtracted image before the smooth-lens modelling, the residual surface-brightness fluctuations after the smooth-lens modeling using alternatively the original narrow or the enlarged mask, as well as the power spectrum of the sky-background and

Table 4.1: Parameter values of the best-fitting smooth-lens models (corresponding to Fig. 4.3) for the lens galaxy SDSS J0946+1006 obtained in the I-band (F814W) for different levels of the source-grid resolution ($n = 2, 3, 4, 5$: casting back only one pixel out of each contiguous 2×2 -, 3×3 -, 4×4 - and 5×5 -pixel area, respectively). The mass distribution of the lens galaxy is modelled as SPEND with the following set of free parameters: the lens strength b , the position angle θ (with respect to the original telescope rotation), the axis ratio q , the (three-dimensional) mass-density slope γ , the coordinates of the centre of the mass distribution with respect to the central pixel of the image x_0 and y_0 , the external shear strength Γ , its position angle Γ_θ and the regularization strength λ . χ^2 quantifies the goodness of fit within the mask covering the lensed images. The reconstruction was carried out using the curvature source regularization and a fixed core radius $s = 0.001$. The typical statistical errors of the reconstructed parameter values are of the order 10^{-1} for the angles and 10^{-3} for the remaining parameters. The reconstruction with $n = 1$ (casting back each pixel from the lens plane to the source plane, not included in the table) leads to over-fitting.

n	b [arcsec]	θ [deg.]	q	γ	x_0 [arcsec]	y_0 [arcsec]	Γ	Γ_θ [deg.]	λ	χ^2
2	1.3614	18.9508	0.9213	2.1848	-0.0496	0.0068	0.0856	44.2683	0.0954	2076
3	1.3710	18.1241	0.9330	2.2500	-0.0429	0.0079	0.0848	45.6634	0.1004	2617
4	1.3668	15.9249	0.9224	2.2896	-0.0518	0.0099	0.0874	44.5500	0.0857	2868
5	1.3652	16.0453	0.9101	2.3202	-0.0473	0.0045	0.0893	43.6879	0.0759	3009

Figure 4.3: Best-fitting smooth-lens models of SDSS J0946+1006 in F814W (corresponding to Table 4.1), obtained with an increasing resolution of the source-grid reconstruction ($n = 4$ down to $n = 1$; *top to bottom*): the modelled image cutout showing the lens-galaxy-subtracted lensed images, overlaid with the applied mask (*left panel*), the best-fitting model of the lensed images (*middle panel*), and the residual (data-model) image revealing possible surface-brightness anomalies in the lensed images (*right panel*).

Figure 4.4: The effect of the source-grid resolution n on the power spectrum of the residual surface-brightness fluctuations in the lensed images of SDSS J0946+1006 (after the lens-galaxy subtraction and the smooth lens modeling), in comparison to the total noise power spectrum (including the sky background and the flux-dependent photon-shot noise). The presented power spectra correspond to the residual images in Fig. 4.3 (apart from the lowest resolution $n = 5$).

the estimated total noise power spectrum including the flux-dependent Poisson noise (see Appendix 4.A for the noise analysis). All the power spectra are calculated within the same original tight mask covering solely the analysed inner partial Einstein ring. As can be seen from Fig. 4.5, subtracting the light contribution from the foreground lens galaxy reduces the surface-brightness fluctuations down to the noise level on all spatial scales corresponding to $k > 8 \text{ arcsec}^{-1}$. For this reason, in what follows we consider only the first six k -bins ($k < 8 \text{ arcsec}^{-1}$), where the measured residual surface-brightness fluctuations are above the noise level. These correspond to spatial scales larger than 0.125 arcsec (or ~ 2.5 pixels).

4.4 Gaussian-Random-Field simulations of small-scale mass structure in the lens galaxy

In order to interpret the power spectrum of the residual surface-brightness fluctuations measured in the lensed images of SDSS J0946+1006, in this section we compare this measurement to simulated lens systems with known statistical properties of the small-scale mass structure in the lens galaxy. For this comparison, we generate a catalogue of mock lensed images, mimicking the real lens system, perturbed with a priori known Gaussian potential fluctuations.

4.4.1 GRF potential perturbations

We treat the small-scale sub-galactic mass structure as a statistical ensemble and model its statistical properties in terms of GRF potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$. The statistical behaviour of a GRF is fully described by its 2-point correlation function or, equivalently, its representation in the Fourier space – the power spectrum. The latter quantifies the variance of the GRF-fluctuations as a function of their spatial frequency k in the Fourier space. For our analysis, we choose the convention in which k is equal to the reciprocal wavelength λ of the corresponding frequency component:

$$k \equiv \lambda^{-1}, \quad (4.2)$$

and is measured in arcsec^{-1} .

Following Chatterjee & Koopmans (2018), we assume the power spectrum of the GRF potential perturbations $P_{\delta\psi}(k)$ in the lens galaxy

Figure 4.5: Power spectra of the original HST/ACS/F814W-image (*yellow line*), the lens-galaxy-subtracted image (*dark blue line*) and the residual surface-brightness fluctuations after the smooth lens modelling, performed using alternatively the original narrow (*red line*) or an enlarged mask including noisy pixels (*magenta line*), in comparison to the sky background (*black line*) and the total noise power spectrum (taking into account the flux-dependent Poisson noise, *green line*). All power spectra are consistently measured within the original narrow mask.

to be isotropic and obey a (piecewise) power law:

$$P_{\delta\psi}(k) = A \times k^{-\beta} \propto \sigma_{\delta\psi}^2 \times k^{-\beta}. \quad (4.3)$$

Under this assumption, a specific combination of the total variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β uniquely determines the statistical properties of the corresponding GRF potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$. In particular, the power-law power-spectrum slope β quantifies how the total variance $\sigma_{\delta\psi}^2$ is distributed over the different length scales and, thus, describes the spatial dependence of the investigated mass structure in the lens galaxy.

The integration of the power spectrum over all spectral components (i.e. k -modes) yields the total variance of the GRF-fluctuations in the considered field-of-view. Thus, for a given choice of $\sigma_{\delta\psi}^2$ and β , the amplitude A obeys the following normalization condition:

$$\int_{k_x} \int_{k_y} P_{\delta\psi}(A, \beta, \sqrt{k_x^2 + k_y^2}) dk_x dk_y = \sigma_{\delta\psi}^2. \quad (4.4)$$

More specifically, for the discrete pixel grid of the analysed image with a finite length $L = 4.05$ arcsec on a side, A can be computed as:

$$A(\sigma_{\delta\psi}^2, \beta, L) = \frac{L^2}{\sum_{k_x} \sum_{k_y} (\sqrt{k_x^2 + k_y^2})^{-\beta}} \times \sigma_{\delta\psi}^2. \quad (4.5)$$

Note that the integral in equation (4.4) is now replaced by a summation over discrete pixels in Fourier space with the linear size $dk_x = dk_y = L^{-1}$.

A particular realization of $\delta\psi_{\text{GRF}}(\mathbf{x})$ is related to the underlying realization of Gaussian convergence perturbations $\delta\kappa_{\text{GRF}}(\mathbf{x})$ via the Poisson equation:

$$\nabla^2 \delta\psi_{\text{GRF}}(\mathbf{x}) = 2\delta\kappa_{\text{GRF}}(\mathbf{x}). \quad (4.6)$$

The resulting deflection-angle perturbations $\delta\alpha_{\text{GRF}}(\mathbf{x})$ can be calculated as the gradient of the potential perturbations:

$$\delta\alpha_{\text{GRF}}(\mathbf{x}) = \nabla \delta\psi_{\text{GRF}}(\mathbf{x}). \quad (4.7)$$

Alternatively, the relations between the perturbations in the lensing potential, convergence and the deflection-angle field can be expressed in

Fourier space. The power spectrum of potential perturbations $P_{\delta\psi}(k)$ and the corresponding power spectrum of convergence perturbations $P_{\delta\kappa}(k)$ are linked via the following relation:

$$P_{\delta\kappa}(k) = 4\pi^4 k^4 P_{\delta\psi}(k). \quad (4.8)$$

The power spectrum $P_{\delta\alpha}(k)$ of the resulting deflection-angle perturbations is related to the $P_{\delta\psi}(k)$ as follows:

$$P_{\delta\alpha}(k) = 4\pi^2 k^2 P_{\delta\psi}(k). \quad (4.9)$$

4.4.2 Catalogue of simulated perturbed lensed images

To create a catalogue of simulated perturbed lensed images, for a comparison with the observations of SDSS J0946+1006, we generate realizations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ with different statistical properties (described by the power spectrum, as discussed in Section 4.4.1). Due to large computational costs, we consider only a set of discrete slope values $\beta = [3, 4, 5, 6, 7]$, and for each of those 25 different values of $\sigma_{\delta\psi}^2$, evenly spaced on the logarithmic scale. The exact range of $\sigma_{\delta\psi}^2$ is set individually for each β , such that it corresponds to a fixed set of values for the variance of the resulting deflection-angle perturbations $\sigma_{\delta\alpha}^2$. These are chosen to consist of 25 points evenly spaced in the logarithmic range $[10^{-5.7}, 10^{-0.7}]$. To increase the statistical robustness of our results, we generate 10 different realizations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ for all considered combinations of β and $\sigma_{\delta\psi}^2$.

We superpose each of these realizations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ on the best-fitting smooth lensing potential $\psi_0(\mathbf{x})$ (see Section 4.3.3) and use the resulting perturbed lensing potential $\psi_0 + \delta\psi_{\text{GRF}}(\mathbf{x})$ to repeat the lensing operation of the reconstructed background source (see Section 4.3.3). To account for the observational effects, the simulated lensed images are subsequently convolved with the PSF of the HST/ACS/F814W-optics and overlaid with a realistic noise realization. As an example, Fig. 4.6 depicts three different realizations of $\delta\psi_{\text{GRF}}(\mathbf{x})$, the corresponding convergence- and deflection-angle perturbations, $\delta\kappa_{\text{GRF}}(\mathbf{x})$ and $\delta\alpha_{\text{GRF}}(\mathbf{x})$, respectively, as well as the resulting mock perturbed lensed images. The most perturbed mock lens system (left) shows strongly disturbed lensed images, clearly distinguishable from both the real observations and the best-fitting smooth model. The second mock system (middle) is an example of moderately perturbed lensed images. Finally, the least perturbed mock lens system (right) can hardly be distinguished from the unperturbed case.

Figure 4.6: Mock lensed images mimicking SDSS J0946+1006, perturbed by Gaussian potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ with the power spectrum $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$. *Top row*: the superposed realizations of $\delta\psi_{\text{GRF}}(\mathbf{x})$ with different values of $\sigma_{\delta\psi}^2$ and a fixed value of $\beta = 5$. *Second row*: the corresponding convergence perturbations $\delta\kappa_{\text{GRF}}(\mathbf{x})$. *Third row*: the resulting perturbations in the deflection angle $\delta\alpha_{\text{GRF}}(\mathbf{x})$ (i.e. the magnitude of the deflection-angle vector). *Bottom row*: the resulting perturbed lensed images convolved with the PSF and overlaid with a realistic noise realization.

4.5 Constraints on the statistical properties of small-scale sub-galactic mass structure

In this section, we compare the catalogue of mock perturbed lensed images to the observational data. This comparison allows us to infer constraints on the statistical properties of small-scale mass structure in the lens galaxy SDSS J0946+1006. First, in Section 4.5.1, we follow the original methodology, introduced in Chapter 3, and compare our results to those obtained in the pilot study of SDSS J0252+0039 (see Chapter 3). Subsequently, we consider two alternative modifications to our methodology which improve the modelling procedure of the mock perturbed lensed images. These implement either the less computationally expensive *single-inversion* smooth lens modelling (see Section 4.5.2), suitable for an analysis of large samples, or the much more accurate but very time-consuming *full grid-based* smooth lens modelling, performed in a way that is fully consistent with the modelling of the real data (see Section 4.5.3).

4.5.1 Upper limits from a direct comparison to mock surface-brightness anomalies

In order to narrow down the upper-limit constraints inferred in our pilot study (Chapter 3), we now apply the same methodology to the current analysis. For each mock lens system from our catalogue, we compute the difference between the perturbed lensed images and the unperturbed smooth counterpart. The remaining residual surface-brightness fluctuations correspond to the surface-brightness anomalies induced in the lensed images by the superposed Gaussian potential perturbations. In an ideal case, these surface-brightness anomalies (overlaid with observational noise) could be fully recovered in a residual image if the perturbed mock lensed images were actually observed. This way, we mimic the smooth lens modelling of the perturbed mocks under the (not very realistic) assumption that the lens-modelling code is able to fully recover the true smooth lensing potential and the true surface-brightness distribution of the source galaxy. In this approximation, the statistical properties of the potential perturbations in the lens galaxy can be constrained by comparing the (one-dimensional azimuthally-averaged) power spectrum of the residual surface-brightness fluctuations measured in the HST-imaging of SDSS J0946+1006 with the residual power spectra calculated for the mocks (see Figs. 4.7 to 4.11).

As in Chapter 3, we quantify the deviation between the real measurement and the mock residual power spectra in terms of the exclusion probability, defined as the probability that the induced mock surface-brightness anomalies exceed the observed ones on *all* considered spatial scales (i.e. in all k -bins). Fig. 4.12 presents the exclusion probabilities inferred for different combinations of $\sigma_{\delta\psi}^2$ and β . As evident from Fig. 4.12, $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.5}$ is ruled out for all considered values of β . This corresponds to the upper bound of $\sigma_{\delta\psi} \sim 0.018$ (inside the field of view of the image cutout) on the standard deviation of the Gaussian potential perturbations in SDSS J0946+1006. For completeness, in Fig. 4.13 we show the corresponding constraints on the variance of the deflection-angle perturbations $\sigma_{\delta\alpha}^2$.

4.5.2 Upper limits from the source reconstruction with a fixed lensing potential

In this section, we improve on the original methodology by performing a simplified (single-inversion) grid-based smooth lens modelling for all combinations of $\sigma_{\delta\psi}^2$ and β included in our mock catalogue described in Section 4.4.2. In contrast to Section 4.5.1, we now reconstruct the unlensed image of the background source galaxy for each individual mock, but still do not optimize for the parameter values of the smooth lensing potential. Instead, we fix these parameter values to the solution obtained for the real data with the source-grid resolution $n = 4$ (see Table 4.1). To mimic the smooth lens modelling of real observational data, we simulate the mock perturbed lensed images with the highest resolution (corresponding to $n = 1$), but model them with $n = 4$ (as discussed in Section 4.3.4). Moreover, we set the strength of regularization to $\lambda = 0.06$, which was found to be optimal in the full smooth lens modelling of the unperturbed mock lensed images. Optimizing for the source, but not for the potential, allows us to account for the possible effect of signal absorption in the source structure, but is considerably less computationally expensive than the full smooth lens modelling (which includes the optimization of the smooth lensing potential).

For each mock, we subsequently subtract the respective smooth-lens model from the perturbed lensed images and compute the power spectrum of the residual surface-brightness fluctuations (including the observational noise). Figs. 4.14 to 4.18 present the resulting residual power spectra in comparison to the real measurement.

Figure 4.7: Power spectra of surface-brightness anomalies induced in the lensed images of SDSS J0946+1006 by GRF potential perturbations $\delta\psi(\mathbf{x})$ with $P_{\delta\psi}(k) \propto \sigma_{\delta\psi}^2 \times k^{-\beta}$ and $\beta = 3$, in comparison to HST-data. The anomalies are computed by subtracting the best-fitting smooth-lens model (inferred from the data) from the mock perturbed lensed images.

Figure 4.8: Idem as in Fig. 4.7 for the GRF power-spectrum slope $\beta = 4$.

Figure 4.9: Idem as in Fig. 4.7 for the GRF power-spectrum slope $\beta = 5$.

Figure 4.10: Idem as in Fig. 4.7 for the GRF power-spectrum slope $\beta = 6$.

Figure 4.11: Idem as in Fig. 4.7 for the GRF power-spectrum slope $\beta = 7$.

Figure 4.12: Upper-limit constraints on the variance of potential perturbations $\sigma_{\delta\psi}^2$ in the lens galaxy SDSS J0946+1006, for different values of the power-law power-spectrum slope β . The constraints are derived from a direct comparison between the power spectrum of the measured residual (noise-corrected) surface-brightness fluctuations and the power spectra of mock surface-brightness anomalies, induced in the lensed images by GRF potential perturbations with known $\sigma_{\delta\psi}^2$ and β (shown in Figs. 4.7 to 4.11).

Figure 4.13: Idem as in Fig. 4.12 for the corresponding constraints on the variance of the deflection-angle perturbations $\sigma_{\delta\alpha}^2$.

Based on this single-inversion smooth lens modelling of the perturbed mocks, we infer upper-limit constraints on the variance and the power-law power-spectrum slope of the hypothetical GRF potential perturbations in the lens galaxy SDSS J0946+1006. The exclusion probabilities are calculated in the same manner as in Chapter 3. Fig. 4.19 presents the inferred upper-limit constraints on the variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β of the hypothetical GRF potential perturbations in the studied lens galaxy SDSS J0946+1006. Fig. 4.20 shows the corresponding limits on the variance $\sigma_{\delta\alpha}^2$ of the deflection-angle perturbations.

As can be seen in Fig. 4.19, the analysis based on a single inversion of the perturbed mocks rules out $\sigma_{\delta\psi}^2 \gtrsim 10^{-2.5}$ for all considered values of β . The range of $\sigma_{\delta\psi}^2 \sim 10^{-3.5} - 10^{-2.5}$, ruled out based on our original methodology (Fig. 4.12), is not excluded when the mock perturbed lensed images are modelled. This discrepancy can be explained by a partial absorption of the induced mock surface-brightness anomalies in spurious source structure, which results in a suppression of the residual surface-brightness fluctuations. If such an absorption is observed in the modelling of the mocks, it is most probably present in the modelling of the real data as well. Thus, using the suppressed mock residual power spectra inferred here might actually be a more suitable way to compare the data and the mocks in a consistent way. This critical issue is discussed in more detail in Section 4.6.

4.5.3 Constraints from a simultaneous lens- and source reconstruction of the mock lensed images

Finally, in order to account for all the numerical effects and degeneracies present in the lens modelling of the real data, here we carry out the full smooth lens modelling for a sub-sample of the mock perturbed lens systems from our catalogue. The modelling is carried out using the code by Vernardos et al. (in prep., see Section 4.3.3), in exactly the same manner as for the HST-data (see Section 4.3.3).

We model each mock lens system individually allowing for the full optimization of all parameters characterising the smooth lensing potential and a grid-based reconstruction of the intrinsic surface-brightness distribution of the background source galaxy. The strength of the source-grid regularization is initially fitted as a free parameter. Due to high computational costs, we choose a single value for the power-law power-spectrum slope $\beta = 5$ and vary solely the total variance of the superposed

Figure 4.14: Catalogue of mock residual power spectra obtained from a single-inversion smooth lens modelling of the mock perturbed lensed images, in comparison to the real measurement based on the HST-data. The mock lensed images have been overlaid with a realistic noise realization and modelled with $n = 4$, while keeping the parameter values of the best-fitting smooth lensing potential fixed.

Figure 4.15: Idem as in Fig. 4.14 for the slope $\beta = 4$.

Figure 4.16: Idem as in Fig. 4.14 for the slope $\beta = 5$.

Figure 4.17: Idem as in Fig. 4.14 for the slope $\beta = 6$.

Figure 4.18: Idem as in Fig. 4.14 for the slope $\beta = 7$.

Figure 4.19: Exclusion probabilities for the variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β of the potential perturbations in the lens galaxy SDSS J0946+1006, inferred from a single inversion of the mock perturbed lensed images (i.e. reconstruction of the background source while keeping the parameter values of the best-fitting smooth lensing potential fixed).

Figure 4.20: Idem as in Fig. 4.19 for the variance of the deflection-angle perturbations $\sigma_{\delta\alpha}^2$.

potential perturbations $\sigma_{\delta\psi}^2$. Our test interval for the variance consists of 17 values evenly spaced in the logarithmic range $\sigma_{\delta\psi}^2 = [10^{-4.8}, 10^{-1.6}]$, roughly matching the range investigated in our pilot analysis discussed in Chapter 3. Moreover, we model only one GRF-realization for each considered variance level and perform the lens modelling with a relatively low source-grid resolution $n = 4$. To investigate the effect of reconstructing an intrinsically high-resolution (corresponding to $n = 1$) image of the lensed source galaxy using a lower source-grid resolution ($n = 4$), we additionally model an unperturbed mock.

Table 4.2 presents the parameter values of the best-fitting smooth lensing potential inferred for each of the mock lens systems. It is apparent that in some cases the reconstructed parameter values deviate considerably from the input values used to generate the mocks (i.e. the best-fitting values inferred for the real data). In Fig. 4.21, we present three examples of the best-fitting smooth-lens models obtained for the mocks perturbed with $\sigma_{\delta\psi}^2 = 10^{-2.0}, 10^{-3.2}$ and $10^{-4.4}$, respectively, in comparison to the best-fitting model for the real data (see Section 4.3.3). The inferred models for the remaining mock systems can be found in Figs. 4.32 to 4.36 in Appendix 4.B. A visual inspection of the residual images (Fig. 4.21 and Figs. 4.32 to 4.36) suggests that the residual surface-brightness fluctuations revealed in the HST-data most closely resemble the mildly perturbed mocks from our catalogue. At the same time, a distinct discrepancy between the data and both the least and the most perturbed mocks can be seen.

To quantify this discrepancy, we again compute the one-dimensional azimuthally-averaged power spectra $P_{\delta I}(k)$ of the mock residual surface-brightness fluctuations for all considered mock lens systems and compare them to the power spectrum actually measured in the observed lensed images, see Fig. 4.22. For completeness and the future comparison with theoretical predictions, we additionally express these results in terms of the dimensionless power spectrum:

$$\Delta_{\delta\psi}^2(k) \equiv 2\pi k^2 P_{\delta\psi}(k) \quad (4.10)$$

see Fig. 4.23. Furthermore, we test whether choosing a constant level of regularization, which has so far been treated as a free parameter and optimized for, affects the resulting residual power spectra. For this test, we set $\lambda = 0.06$, which was the optimal level found for the unperturbed mock. The comparison of these two options is presented in Fig. 4.24. As expected, the difference between the residual power spectra obtained with a

Table 4.2: Idem as in Table 4.1 for the *full grid-based smooth lens modelling* of mock lensed images perturbed by GRF potential fluctuations with the variance $\sigma_{\delta\psi}^2$ and the power-spectrum slope $\beta = 5$. The modelling was carried out using the source-grid resolution $n = 4$, a curvature source regularization and a fixed core radius $s = 0.001$. The typical statistical errors of the reconstructed parameter values are of the order 10^{-1} for the angles and 10^{-3} for the remaining parameters.

$\log_{10}\sigma_{\delta\psi}^2$	$b[arcsec]$	θ [deg.]	q	γ	$x_0[arcsec]$	$y_0[arcsec]$	Γ	Γ_θ [deg.]	λ	χ^2
-1.6	1.491	29.8	0.935	2.297	-0.035	-0.029	0.070	36.8	0.008	13635
-1.8	1.287	20.2	0.805	2.035	-0.092	-0.012	0.094	45.0	0.001	96
-2.0	1.316	29.1	0.993	2.259	-0.040	-0.010	0.071	30.0	0.000	1433
-2.2	1.310	13.2	0.891	2.002	-0.061	-0.080	0.094	40.5	0.003	447
-2.4	1.261	18.4	0.829	2.043	0.028	-0.019	0.094	39.4	0.014	219
-2.6	1.364	20.7	0.967	2.038	-0.074	-0.034	0.076	38.2	0.008	850
-2.8	1.324	24.4	0.839	2.050	0.025	0.055	0.090	47.3	0.008	733
-3.0	1.311	12.5	0.843	2.102	-0.028	-0.024	0.094	39.4	0.011	25
-3.2	1.313	14.9	0.871	2.003	-0.044	-0.015	0.094	35.1	0.010	892
-3.4	1.387	10.6	0.969	2.126	-0.022	-0.019	0.081	49.4	0.042	467
-3.6	1.349	12.2	0.897	2.171	-0.015	-0.002	0.087	43.4	0.082	791
-3.8	1.373	28.6	0.947	2.184	-0.003	-0.003	0.086	45.5	0.042	1375
-4.0	1.352	16.7	0.925	2.201	-0.002	-0.015	0.085	42.9	0.076	21
-4.2	1.347	24.9	0.897	2.164	-0.008	-0.007	0.090	46.0	0.073	526
-4.4	1.356	18.2	0.899	2.199	-0.012	-0.013	0.090	42.9	0.072	1155
-4.6	1.367	11.3	0.913	2.277	-0.014	-0.002	0.090	44.5	0.087	663
-4.8	1.365	14.2	0.925	2.215	-0.017	0.003	0.086	45.4	0.078	1650
unperturbed	1.359	20.8	0.935	2.018	-0.034	0.003	0.075	44.6	0.060	2009
input values	1.361	18.9	0.921	2.185	0.001	-0.007	0.086	44.3	0.002	—

Figure 4.21: Examples of best-fitting models inferred from the *full grid-based smooth lens modelling* of the mock perturbed lensed images (see Table 4.2), in comparison to the model obtained for the real data.

fixed versus free λ is negligible for the less perturbed mocks, since the values of the best-fitting λ are close to the fixed value. For the most perturbed mocks, however, a suitable smooth model is difficult to find. Thus, the lens-modelling code keeps lowering the regularization strength further and further, eventually reaching a non-regularized reconstruction. This leads to a stronger absorption of the residual surface-brightness fluctuations in the source structure and, thus, a suppression of the residual power spectra. We conclude that, in general, optimizing for the regularization strength allows us to find a better model and lower the residuals. However, for the most perturbed mocks the reconstruction might not properly converge if λ approaches zero. This is another possible source of bias in our analysis, which requires a more thorough consideration in future work.

As apparent from Fig. 4.22, the residual power spectra do not exactly correlate with the variance of the imposed GRF potential perturbations. This is due to numerical effects in the smooth-lens modelling and the random nature of the superposed GRF realizations. However, taking into account the sample variance allows us to draw the first conclusions. For a consistent comparison with the results obtained from both the original methodology (see Fig. 4.12 for $\beta = 5$) and the single-inversion modelling (see Fig. 4.19 for $\beta = 5$), we first use the computed mock power spectra to infer the upper-limit exclusion probabilities again, now based on the full modelling of the lens and the source. As can be seen in Fig. 4.25, the resulting exclusion probabilities do not allow us to exclude any variance values with a high statistical significance. Nevertheless, the exclusion plot still suggests a deviation between the data and the mocks for the variance level roughly above $\sigma_{\delta\psi}^2 \sim 10^{-3.5}$. This deviation becomes quite significant for $\sigma_{\delta\psi}^2 \gtrsim 10^{-2.5}$, which was already ruled out based on the single-inversion modelling. Following up the discussion on the signal suppression in the residual surface-brightness fluctuations encountered during the simplified single-inversion modelling of the mocks (Section 4.5.2), this effect turns out to be even stronger here, when the perturbed lensed images are fully modelled.

Assuming that the full modelling of the mock lensed images allows us to account for all the numerical effects present in the modelling of the real data in a fully consistent way, we now move away from our very conservative approach of calculating the exclusion probabilities. Instead, we use the χ^2 -statistic to quantify the goodness of the fit between the observed and the mock residual power spectra predicted for the specific values of $\sigma_{\delta\psi}^2$.

Figure 4.22: Power spectra of the residual surface-brightness fluctuations (including observational noise) revealed in simulated perturbed lensed images from our catalogue after subtracting the best-fitting smooth-lens model, in comparison to the real measurement from HST-observations of SDSS J0946+1006. The subtracted smooth-lens model is inferred for each mock individually from the *full grid-based lens modelling* of the perturbed lensed images, which is fully consistent with modelling of the real data.

Figure 4.23: The dimensionless power spectra $\Delta_{\delta\psi}^2(k)$ corresponding to Fig. 4.22.

Figure 4.24: The effect of modelling the mock perturbed lensed images with a free versus fixed regularization strength λ on the resulting residual power spectra. The strength of the regularization has been fixed to $\lambda = 0.06$ – the optimal level found for the best-fitting smooth-lens model of the unperturbed mock.

Figure 4.25: Exclusion probabilities for the variance of the potential perturbations $\sigma_{\delta\psi}^2$ in the lens galaxy SDSS J0946+1006, inferred from the *full smooth-lens modelling* of mock lensed images perturbed by Gaussian potential perturbations with the power-law power-spectrum slope $\beta = 5$.

and $\beta = 5$. Fig. 4.26 presents the obtained χ^2 -values. The statistical significance of our findings is indicated by the p -value, which corresponds to the probability of observing a particular (extreme) value of the χ^2 -statistic even if the null hypothesis is true (i.e. the observed and the mock perturbed lensed images are statistically indistinguishable). Based on this test, we conclude that for the variance values $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.0}$ there is a statistically significant (p -value of $p = 0.001$) difference between the observed and the predicted power spectra of the residual surface-brightness fluctuations in the lensed images. Thus, even based on a single GRF-realization for each considered variance level, we have sufficient evidence to reject the variance values $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.0}$ with a very high level of statistical significance.

4.6 Suppression of surface-brightness anomalies in the grid-based smooth lens modelling

As was discussed already in Chapter 3, grid-based smooth lens modelling seems to absorb the surface-brightness anomalies and partially even the observational noise in a spurious source structure. This effect has been found in the analysis of both SDSS J0252+0039 and SDSS J0946+1006 (see Chapter 3 and Section 4.3.4, respectively). In particular, the grid-based source modelling with the highest possible resolution (i.e. $n = 1$) has been found to result in an absorption of all residual surface-brightness fluctuations (including the observational noise) in a spurious background-source structure. The smooth lens modelling of mock perturbed lensed systems with an a priori known level of surface-brightness anomalies, performed in Section 4.5, provides us with the opportunity to quantify this effect for the first time.

Figs. 4.27 and 4.28 show the power spectra of the induced mock surface-brightness anomalies computed before and after accounting for the observational effects (i.e. PSF and observational noise), for a set of 17 mocks perturbed by GRF potential fluctuations with different levels of the total variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope $\beta = 5$. As can be seen from these plots, the convolution with the PSF enhances the signal for large spatial scales (i.e. low k -values), while pushing it down to the noise level on the small spatial scales (i.e. high k -values). The effect is particularly strong for the least perturbed mocks.

Figure 4.26: Constraints on the variance of the hypothetical Gaussian potential perturbations $\sigma_{\delta\psi}^2$ in the lens galaxy SDSS J0946+1006, inferred from the full grid-based smooth lens modelling (i.e. simultaneous lens- and source reconstruction) of mock perturbed lensed images. The discrepancy between the observed and the mock power spectra of the residual surface-brightness fluctuations in the lensed images is expressed in terms of the χ^2 -statistic. The statistical significance of the χ^2 -goodness-of-fit test is indicated by the p -value. Based on this plot, we rule out potential perturbations with $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.0}$ in SDSS J0946+1006 ($p = 0.001$).

Figs. 4.29 and 4.30 depict the power spectra of the residual surface-brightness fluctuations (anomalies and noise) recovered from the same set of 17 mock perturbed lensed images when performing a single-inversion or the full smooth-lens modelling, respectively. A step-wise suppression of the induced surface-brightness anomalies can clearly be seen for all investigated mocks. The single-inversion modelling suppresses the true mock anomalies by ~ 1 order of magnitude. The suppression in the full smooth-lens modelling is even more striking: up to ~ 2 orders of magnitude for the most perturbed mock lens systems. The effect is particularly prominent for the lowest k -values (i.e. largest spatial scales) and becomes less severe with increasing k -values (i.e. smaller spatial scales).

4.7 Summary and conclusions

Our original methodology, introduced in Chapter 3, turned out to suffer from a degeneracy between the surface-brightness anomalies caused by the small-scale mass structure in the lens galaxy and the intrinsic surface-brightness fluctuations in the background source galaxy. Due to numerical effects, such signal suppression is particularly severe when the smooth-lens modelling is performed in a narrow field-of-view and using the highest possible source-grid resolution ($n = 1$ or even $n = 2$). Choosing a lower source-grid resolution (e.g. $n = 4$ used in this work) alleviates this signal suppression, however at the expense of a decreased accuracy of the source reconstruction, which potentially leads to a biased lens model with enhanced residuals. Finding the optimal source-grid resolution for the analysis of surface-brightness anomalies is therefore crucial but very challenging. Quantifying and alleviating this issue has been the key question addressed in the present chapter.

With this in mind, we have introduced and tested a more accurate version of our pilot methodology allowing us to take this effect into account. The major improvement has been the modelling of the perturbed mock lensed images in exactly the same manner as it is carried out for the real data. The modification is introduced gradually in two steps – the less computationally expensive single-inversion modelling (suitable for studies of large statistically significant samples of mock perturbed lens systems) and the much more accurate but very time-consuming full grid-based smooth-lens modelling. The smooth-lens modelling performed for a set of 17 mock lensed systems with a priori known surface-brightness anomalies has given

Figure 4.27: Power spectra of mock surface-brightness anomalies calculated *before* accounting for the observational effects (i.e. a convolution with the PSF and adding a realistic noise realization) for a set of 17 mocks perturbed by GRF potential fluctuations with different levels of the total variance $\sigma_{\delta\psi}^2$ and a fixed power-law power-spectrum slope $\beta = 5$. The mock power spectra are computed based on a single GRF-realization of potential perturbations for each $\sigma_{\delta\psi}^2$ -value. The error bars quantify the estimated noise and the sample variance.

Figure 4.28: Idem as in Fig. 4.27, *after* the convolution with the PSF and adding a realistic noise realization.

Figure 4.29: Idem as in Fig. 4.28, after the grid-based smooth lens modelling of the mock lensed images with the parameter values of the best-fitting lensing potential kept fixed (i.e. single-inversion reconstruction of the source galaxy).

Figure 4.30: Idem as in Fig. 4.29, for the *full* grid-based smooth lens modelling of the mock lensed images (i.e. the optimization of all parameter values of the lensing potential and the reconstruction of the background source on a grid, performed simultaneously).

us the unique opportunity to quantify the effect of the signal suppression in the grid-based smooth-lens modelling for the first time.

Based on this study, we draw two main conclusions.

- First, from the performed feasibility study including the full smooth-lens modelling of the mock perturbed lensed images, we infer statistically significant constraints on the variance of the hypothetical Gaussian potential perturbations in the studied lens galaxy SDSS J0946+1006. This analysis proves that GRF potential perturbations with the variance $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.0}$ are highly improbable ($p = 0.001$). However, so far, the investigation is based on one single value $\beta = 5$ for the power-spectrum slope of the potential perturbations and a single GRF-realization for a range of different variance values. Future extended analysis encompassing larger sets of GRF-realizations might push these limits even further down. Moreover, in order to study the impact of the power-spectrum slope on the inferred constraints, it is necessary to include different β -values in our analysis.
- Second, our analysis clearly demonstrates a striking suppression of the induced mock surface-brightness anomalies in the course of the full grid-based smooth-lens modelling. This means that the measurements obtained for real data might be severely suppressed as well and the studied lens system could be considerably more perturbed than what we reconstruct. From this, we conclude that signal suppression in the grid-based smooth lens modelling is a real and important issue that requires more attention from the lensing community. If not properly understood and accounted for, it might lead to severely biased results with respect to both the mass distribution in the lens galaxies and the reconstruction of the lensed high-redshift sources. In particular, inferring meaningful constraints on the sub-galactic mass structure from observations of galaxy-galaxy strong gravitational lens systems requires a full grid-based smooth-lens modelling of the mock perturbed lensed images in a way that is consistent with the modelling of the real data.

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Appendix 4.A Noise power spectrum

In order to account for random surface-brightness fluctuations present in the observed image due to the presence of observational noise, we first quantify the properties of the pure sky background in HST/ACS/F814W-observations in terms of the power spectrum. For this, we create a sample of 20 blank-sky regions located in proximity to SDSS J0946+1006 with the same cutout size as the analyzed image of the lens (81 by 81 pixels), calculate the one-dimensional azimuthally-averaged power spectrum of each region and, finally, average it over the entire sample. As in Chapter 2, we perform the same procedure for both the original and the CTE-corrected drizzled images to investigate the effect of the CTE correction on our results. Fig. 4.31 shows the average power spectra obtained in these two samples. For comparison, we additionally plot the average power spectrum for a sample of twenty mock realizations of uncorrelated Gaussian noise with the same standard deviation as in the blank-sky cutouts (white noise with $\text{RMS}=0.008$). Based on the non-flatness of the power spectrum measured in both sets of blank-sky images as compared to the flat power spectrum of the white noise, we conclude that the CTE-correction does not significantly alleviate the effect of correlations between adjacent pixels introduced by the drizzling procedure (Fruchter & Hook, 2002). Similar correlations were also found in the HST/WFC3/F390W-data of the SLACS sub-sample analysed in Chapters 2 and 3.

In addition to the revealed correlation pattern in the drizzled sky background, the noise power spectrum needs to include the specific spatially-varying flux-dependent photon-shot noise present in the analyzed image of SDSS J0946+1006. Following the methodology presented in

Figure 4.31: Power spectrum of the sky background in HST/ACS/F814W-observations of the Double Ring lens system and the effect of the charge-transfer-efficiency (CTE) correction: the mean azimuthally-averaged power spectrum measured in a sample of twenty drizzled blank-sky regions located in proximity to the Double Ring lens system before (*green line*) vs. after CTE-correction (*red line*). The measured power spectra are additionally compared to the flat power spectrum of an uncorrelated Gaussian noise with the same standard deviation as in the blank-sky cutouts (i.e. RMS=0.008, *black line*). We conclude that the CTE-correction has a very limited effect on the noise power spectrum and does not alleviate the effect of correlations between adjacent pixels introduced by the drizzling procedure.

Chapter 3, we estimate the total noise power spectrum based on modified blank-sky cutouts with the flux values adjusted according to the noise-sigma map of the analysed science image, see Fig 4.5. Before calculating the power spectrum, we overlay them with the same mask as the one used in the analysis of the residual surface-brightness fluctuations and set to zero all pixels outside the mask. This estimate can be directly subtracted from the measured power spectrum of the total residual surface-brightness fluctuations to reveal possible surface-brightness anomalies due to mass structure in the lens galaxy.

Appendix 4.B Smooth-lens models of the perturbed mock lens systems

Figure 4.32: Best-fitting smooth-lens model for SDSS J0946+1006 and the models inferred for comparable mock lens systems perturbed by Gaussian potential fluctuations with variance $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope $\beta = 5$.

Figure 4.33: Idem as in Fig. 4.32 for decreasing $\sigma_{\delta\psi}^2$.

Figure 4.34: Idem as in Fig. 4.33 for decreasing $\sigma_{\delta\psi}^2$.

Figure 4.35: Idem as in Fig. 4.34 for decreasing $\sigma_{\delta\psi}^2$.

Figure 4.36: Idem as in Fig. 4.35 for decreasing $\sigma_{\delta\psi}^2$.

Chapter 5

Conclusions and future prospects

The main goal of this thesis has been to develop a methodology allowing us to infer observational constraints on the statistical properties of hypothetical small-scale mass structure in massive elliptical lens galaxies from the power spectrum of surface-brightness anomalies measured in highly magnified galaxy-scale Einstein rings and gravitational arcs. The major problem in developing such a methodology turned out to be the degeneracy between the true surface-brightness anomalies caused by the small-scale mass structure in the lens galaxy and the intrinsic surface-brightness fluctuations of the background-source galaxy. The most important results and conclusions are summarised below.

5.1 Summary of the main results

In **Chapter 2**, we have presented a novel observational methodology making it possible to reliably measure the power spectrum of surface-brightness anomalies in deep HST imaging, possibly imprinted on the extended lensed images (i.e. Einstein rings or gravitational arcs) by small-scale mass structure in the lens galaxies. By applying this methodology to a SLACS sub-sample observed with HST/WFC3/F390W in the ultra-violet, we have performed the first power-spectrum analysis of surface-brightness anomalies in extended lensed images of observed galaxy-galaxy strong gravitational lens systems. In order to illustrate the feasibility of this statistical approach, we have shown that the developed methodology is able to recover a-priori known statistical properties of mock sub-galactic mass structure (modelled in terms of Gaussian potential perturbations) from perturbed lensed images of the SLACS lens system SDSS J0252+0039.

However, the introduced approach faces numerous modelling challenges and is sensitive to several model degeneracies. One of the main challenges has been to distinguish between the real anomalies induced by mass structure and the spurious surface-brightness fluctuations present in the observational data due to observational noise. This issue requires a particularly careful consideration of noise correlations caused by the combined effect of the drizzling procedure and the charge transfer inefficiency of the HST/WFC3/UVIS-CCDs. Therefore, an essential part of the presented methodology deals with the question of how to quantify the noise properties of our observational data and perform the noise correction of the measured residual surface-brightness fluctuations. To this end, we have developed a methodology to estimate the noise-power spectrum in the analysed

science image, based on blank-sky fields from the full field-of-view HST-observations which are modified to account for the Poisson noise of the specific observation.

Another critical issue in our methodology has been an over-fitting problem (i.e. modelling the inherent noise present in the data) in the lens-modelling procedure. Modelling the lensed images with the highest possible source-grid resolution ($n = 1$) results in the power spectra of the residual surface-brightness fluctuations lying at or below the estimated noise power-spectrum level, despite having the largest value of the Bayesian evidence. This means that the hypothetical low-mass structure in the lens galaxy, and even a substantial fraction of the background noise, have been interpreted as spurious structure in the surface-brightness distribution of the background-source galaxy. A practical solution to the over-fitting problem was to lower the resolution of the source reconstruction, while keeping fixed the best-fitting parameter values obtained in the highest-resolution model. Later, in Chapter 4, we have found that this problem can be partly alleviated by the choice of a larger mask covering not only the lensed images but also additional pixels located close to the edge of the analysed image. These pixels carry additional information about the noise level in the observed image and provide more constraints for the Bayesian lens-modelling procedure, which improves the quality of the resulting smooth-lens models.

In **Chapter 3**, we have extended the methodology such that the revealed residual surface-brightness fluctuations can be modelled and related to the underlying low-mass structure in the lens galaxy, treated in a statistical sense. With this goal in mind, we performed a systematic study of mock surface-brightness anomalies caused by small-scale mass structure with varying statistical properties, which can be compared to the real measurement. The small-scale structure in the lens galaxy is modelled by a Gaussian-Random-Field (GRF) potential perturbations superposed on the best-fitting smoothly-varying lensing potential. Following the formalism introduced by Chatterjee & Koopmans (2018), we assume these potential perturbations (on the considered 1-10 kpc scales) to be to first order characterized by a power-law power spectrum $P_{\delta\psi}(\sigma_{\delta\psi}^2, \beta, k)$ with two free parameters: the integrated variance of the GRF-fluctuations $\sigma_{\delta\psi}^2$ and the power-law power-spectrum slope β . The study is performed on a grid in the parameter space $\sigma_{\delta\psi}^2 - \beta$. For each point of this grid, referred to as *matter-power spectrum model*, we generate realizations of the corresponding mock

Gaussian potential fluctuations, superpose them on the best-fitting lensing potential, compute the resulting perturbed lensed images and, finally, the induced mock surface-brightness anomalies. The power spectra of such mock surface-brightness anomalies are directly comparable to the power spectrum of the measured surface-brightness fluctuations.

The pilot application of the complete methodology to the lens system SDSS J0252+0039 from the SLACS Survey has allowed us to infer the first observational constraints on the statistical properties of low-mass structure on the sub-galactic scales. Based on this pilot study, we have ruled out potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ with variance $\sigma_{\delta\psi}^2$ exceeding $\sim 10^{-2.5}$ in the analysed field-of-view ≈ 20 kpc on a side (with the exclusion probability at least 99 per cent for a wide range of power-spectrum slopes).

Moreover, we have found that the fundamental quantity that determines the level of the surface-brightness anomalies in the lensed images and, thus, the exclusion probability of a specific matter-power-spectrum model, is the variance of the perturbations in the deflection angle $\sigma_{\delta\alpha}^2$ (in the analysed field-of-view) resulting from the associated potential perturbations $\delta\psi_{\text{GRF}}(\mathbf{x})$ in the lens galaxy. A valuable insight for future analysis is that the exclusion probability is almost insensitive to the slope of the power spectrum $P_{\delta\alpha}(k)$, i.e. the distribution of the total variance in the deflection angle over the different length scales. Based on our analysis of the lens system SDSS J0252+0039, a matter power-spectrum model is excluded (with 99 per cent probability) if the variance of the corresponding deflection-angle perturbations $\sigma_{\delta\alpha}^2 > 6 \times 10^{-3}$ (in the studied field-of-view), independently of the slope β . However, this threshold value might vary for different lens systems.

In order to make the results less dependent on the chosen field-of-view, we have derived the corresponding constraints on the dimensionless convergence-power spectrum $\Delta_{\delta\kappa}^2(\lambda)$ on three different sub-galactic scales and rule out matter power-spectrum models with $\Delta_{\delta\kappa}^2(0.5 \text{ kpc}) > 1$ on 0.5-kpc scale, $\Delta_{\delta\kappa}^2(1 \text{ kpc}) > 0.1$ on 1-kpc scale and $\Delta_{\delta\kappa}^2(3 \text{ kpc}) > 0.01$ on 3-kpc scale (at the 99 per cent confidence level). These constraints on the dimensionless convergence-power spectrum $\Delta_{\delta\kappa}^2(\lambda)$ can be expressed with respect to the standard deviation of the average convergence $\sigma_{\delta\kappa}(\lambda) \equiv \sqrt{\Delta_{\delta\kappa}^2(\lambda)}$ in an infinitely large sample of randomly chosen circular regions (in proximity to the Einstein radius of the studied lens system) with an aperture diameter equal to the considered scale λ . Based on our analysis, we have inferred the following upper-limit constraints on this standard

deviation: $\sigma_{\delta\kappa}(0.5 \text{ kpc}) < 1$ on 0.5-kpc scale, $\sigma_{\delta\kappa}(1 \text{ kpc}) < 0.3$ on 1-kpc scale and $\sigma_{\delta\kappa}(3 \text{ kpc}) < 0.1$ on 3-kpc scale (at the 99 per cent confidence level).

Alternatively, these constraints can be translated into upper limits on the standard deviation $\sigma_{AM}(\lambda)$ in the aperture mass (total integrated mass within a cylinder with the diameter λ projected onto the lens plane) in the proximity to the Einstein radius of the lens galaxy SDSS J0252+0039: $\sigma_{AM}(0.5 \text{ kpc}) < 0.8 \times 10^8 M_\odot$ inside the circular aperture of diameter $\lambda = 0.5 \text{ kpc}$, $\sigma_{AM}(1 \text{ kpc}) < 1 \times 10^8 M_\odot$ for $\lambda = 1 \text{ kpc}$ and $\sigma_{AM}(3 \text{ kpc}) < 3 \times 10^8 M_\odot$ for $\lambda = 3 \text{ kpc}$ (at the 99 per cent confidence level).

The problem in applying the proposed methodology to the U-band imaging of SDSS J0252+0039 turned out to be the encountered degeneracy between the real surface-brightness anomalies due to mass-density fluctuations in the lens galaxy and the complex intrinsic structure of the star-forming background-source galaxy itself. Grid-based source modelling can model away some of the surface-brightness anomalies caused by mass structure in the lens galaxy and partially even the observational noise. Due to numerical effects, such signal suppression is particularly severe when the modelling is performed in a narrow field-of-view with the highest possible source-grid resolution. For this reason, the constraints on the level of sub-galactic mass structure in the lens galaxy SDSS J0252+0039, inferred in this chapter, can only be expressed in terms of an upper limit.

Chapter 4 has been fully devoted to alleviating the aforementioned degeneracy between the real surface-brightness anomalies due to mass structure in the lens galaxy and the complex intrinsic surface-brightness distribution of the background-source galaxy itself. With this goal in mind, we have introduced and tested a more accurate version of the methodology. The major modification consists of performing the full smooth lens modelling of perturbed mock lensed images in exactly the same manner as it is carried out for the real data. The modification is introduced gradually in two steps – the less computationally expensive single-inversion modelling (i.e. reconstructing the pixelized surface-brightness distribution of the source while keeping fixed the parameter values of the lensing potential), suitable for studies of large samples of mock perturbed lens systems, and the much more accurate but time-consuming full grid-based smooth-lens modelling (i.e. both lens and source are modelled). This approach allows us to compare the observed and the mock perturbed lens systems in a fully consistent way, however, it is significantly more time-consuming

and introduces another layer of complexity and modelling choices to be made. To test the feasibility of this more accurate methodology, we have applied it to deep high-signal-to-noise HST/ACS/F814W observations of the strong gravitational lens system SDSS J0946+1006, commonly known as the Double Einstein Ring.

Based on the performed feasibility study including the full smooth-lens modelling of the mock perturbed lensed images, we have inferred statistically significant constraints on the variance of the hypothetical Gaussian potential perturbations in the studied lens galaxy SDSS J0946+1006. This analysis demonstrates that GRF potential perturbations with the variance $\sigma_{\delta\psi}^2 \gtrsim 10^{-3.0}$ are highly improbable ($p = 0.001$). However, so far, the investigation is based on one single value $\beta = 5$ for the power-spectrum slope of the GRF potential perturbations and a single GRF-realization for a range of different variance values. Future extended analysis encompassing larger sets of GRF-realizations might push these limits even further down. Moreover, in order to study the impact of the power-spectrum slope on the inferred constraints, it is necessary to include different values for β in the analysis.

Finally, the modelling performed for a set of 17 mock lensed systems perturbed with a priori known surface-brightness anomalies has provided us with the unique opportunity to quantify the effect of signal suppression in the grid-based smooth-lens modelling for the first time. The results clearly show a striking suppression of the true surface-brightness anomalies (up to ~ 2 orders of magnitude for the most perturbed mock lens systems) in the course of the full grid-based smooth-lens modelling. The effect has been particularly prominent for the lowest k -values (largest scales). This implies that the measurement obtained for the real data could in some cases be suppressed as well. Thus, in reality, the analysed lens system (and other galaxy-galaxy strong gravitational lens systems), could be considerably more perturbed than what follows from the residual surface-brightness fluctuations recovered in the grid-based smooth lens modelling.

Based on this finding, we conclude that the signal suppression in the grid-based smooth-lens modelling is a real and important issue that requires more attention from the lensing community. If not properly understood and accounted for, it might lead to severely biased results with respect to both the mass distribution in the lens galaxies and the reconstruction of the lensed high-redshift sources. Inferring meaningful constraints on the sub-galactic mass structure from surface-brightness anomalies in the

lensed images requires a full grid-based smooth-lens modelling of the mock perturbed lensed images in a way that is fully consistent with the modelling of the real data, and no shortcuts can be made.

5.2 Future prospects

Comparison with hydrodynamical simulations of the cosmological structure formation

The observational constraints on the power spectrum of small-scale subgalactic mass structure, inferred in Chapters 3 and 4, can be directly compared with predictions from hydrodynamical simulations of the cosmological structure-formation process for both different galaxy-formation scenarios within the Λ CDM model and the various alternative dark-matter models. To achieve this, we intend to study mass-power spectra in a sample of massive elliptical galaxies from state-of-the-art hydrodynamical simulations, comparable to the SLACS lens galaxies, such as e.g. those created in the framework of the Simulating EAGLE Lenses programme (SEAGLE, Mukherjee et al., 2018), based on the EAGLE hydrodynamical simulations (Schaye et al., 2015). This comparison will potentially allow us to distinguish between the various cosmological models and, finally, constrain the nature of dark matter.

Large samples of galaxy-scale strong gravitational lenses

While the observational constraints inferred in this thesis are currently based on the analysis of two single lens systems, they can potentially be narrowed down by deriving analogous results for a larger sample of galaxy-galaxy strong gravitational lens systems. Recent astronomical surveys, such as the Kilo-Degree Survey (KiDS) or the Dark Energy Survey (DES), and the upcoming facilities, such as the James Webb Space Telescope (JWST), the Euclid Mission, the Nancy Grace Roman Space Telescope (WFIRST) or the Vera C. Rubin Observatory Legacy Survey of Space and Time (LSST), are expected to increase the number of known galaxy-galaxy strong gravitational lens systems by at least two orders of magnitude (Treu, 2010; Collett, 2015; Drlica-Wagner et al., 2019). This development will significantly change the entire field of substructure lensing and provide a new opportunity to tackle the problem discussed in this thesis. Two possible

ways of taking advantage of these emerging large samples of galaxy-scale strong gravitational lenses are the development of dedicated lens-modelling pipelines and the application of machine-learning techniques to search for imprints of sub-galactic mass structure in a large number of gravitational lens systems in a fully automatic manner.

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Samenvatting

Het donkere materie probleem

Bijna honderd jaar geleden werden astronomen geconfronteerd met een onverwacht raadsel. De metingen van de snelheden waarmee de sterren in ons eigen sterrenstelsel - de Melkweg - rond het centrum draaien, suggereerden dat deze draaiing veel sneller was dan wat verwacht werd van berekeningen op basis van de standaard Newtoniaanse zwaartekrachttheorie. Later zijn in veel meer sterrenstelsels en melkwegclusters herhaaldelijk zulke onverwacht hoge omloopsnelheden gemeten. De meest prominente en meest bekende van deze afwijkende metingen zijn de *vlakke draaikrommen* die in spiraalvormige sterrenstelsels zijn waargenomen: in plaats van een sterke afname als functie van de afstand tot het centrum, zoals voorspeld wordt door de Newtoniaanse zwaartekrachttheorie en waargenomen wordt voor de planeten in ons eigen Zonnestelsel, lijken de werkelijke baansnelheden van sterren en gas in spiraalvormige sterrenstelsels constant te blijven over het gehele waargenomen bereik van de afstanden tot het centrum van het sterrenstelsel.

Na lang te hebben nagedacht over deze verwarrende metingen kwamen astronomen tot de conclusie dat de totale massa van sterrenstelsels en melkwegclusters aanzienlijk groter moet zijn dan wat kan worden geschat op basis van de waarnemingen van hun lichtgevende componenten, zoals sterren, gas en stof. Uiteindelijk leidde deze conclusie tot de *donkere materie hypothese*, die stelt dat de ontbrekende massa wordt geleverd door de aanwezigheid van een extra niet-lichtgevende component – de *donkere materie*.

In de afgelopen eeuw heeft de astronomische gemeenschap een overtuigende hoeveelheid bewijs verzameld die de donkere materie hypothese ondersteunt en het bestaan van donkere materie is tegenwoordig een algemeen geaccepteerde aanname. Volgens ons huidige begrip maakt donkere materie ongeveer 84 procent deel uit van alle bestaande materie. Als dominante materiecomponent wordt aangenomen dat het de drijvende kracht is achter de vorming van alle structuren in het heelal, inclusief sterrenstelsels, sterren, planeten, en dus ook al het leven op Aarde. De zichtbare sterrenstelsels en melkwegclusters worden verondersteld omgeven te zijn door beduidend grotere klompen donkere materie – de *donkere materie halo's* – waarvan de zwaartekracht de samenstellende sterren en gas bij elkaar houdt.

Echter, ondanks het sterke empirische bewijs en de algemene acceptatie door de wetenschappelijke gemeenschap, is de aard van donkere materie nog steeds onbekend en moeten de bestanddelen ervan nog worden ontdekt. In de afgelopen veertig jaar zijn er talloze kandidaten voor donkere materie in overweging genomen. In wezen kunnen al deze kandidaten worden onderverdeeld in twee verschillende categorieën. De eerste categorie omvat de *MACHO's* (*Massive Compact Halo Objects* / *Massieve Compacte Halo Objecten*) – een verscheidenheid van massieve niet-lichtgevende astronomische lichamen, zoals zwarte gaten, zwakke sterren of stellaire overblijfselen. Hoewel de meeste hiervan zijn uitgesloten als de belangrijkste bestanddelen van donkere materie, blijven de hypothetische oeroude zwarte gaten (die in het vroege universum worden gevormd door de directe ineenstorting van de zwaartekracht in gebieden met te hoge dichtheden) nog steeds een levensvatbare kandidaat voor donkere materie. De tweede, en meest waarschijnlijke, categorie van kandidaten omvat de *donkere materie deeltjes* – nog onbekende elementaire deeltjes die niet wederzijds beïnvloed lijken te worden door de atomen die de bekende gewone materie vormen en dus extreem moeilijk te detecteren zijn.

Tal van experimenten zijn toegewijd aan de zoektocht, maar tot nu toe zijn er geen donkere materiedeeltjes op overtuigende wijze gedetecteerd. Daarom blijft het onthullen van de aard van donkere materie een van de belangrijkste nog onopgeloste problemen in de kosmologie en de fundamentele fysica. Het onderzoek in dit proefschrift is bedoeld om bij te dragen aan de zeer uiteenlopende inspanningen van de wetenschappelijke gemeenschap om dit *donkere materie probleem* op te lossen.

Substructuren in sterrenstelsels als alternatief venster tot donkere materie

Naast de inspanningen om het hypothetische donkere materiedeeltje daadwerkelijk te detecteren en de eigenschappen ervan te karakteriseren, hebben kosmologen een meer algemene fenomenologische benadering bedacht om donkere materie te onderzoeken, met als doel de aard ervan en het algemene gedrag op basis van astronomische waarnemingen te beperken. Dit is vergelijkbaar met de studie van de macroscopische eigenschappen van een gas, zoals de temperatuur of de druk, zonder iets te weten over de microscopische eigenschappen van de samenstellende gasdeeltjes. In dit kader kunnen de kandidaten voor de donkere materie in drie bredere categorieën worden onderverdeeld, de *hete*, *koude* en *warmer* donkere materie, die afhankelijk zijn van de gemiddelde snelheid van hun willekeurige beweging in de vroege tijden net na de oerknal. Deze snelheid speelt een cruciale rol omdat het de gemiddelde afstand bepaalt die de donkere materiedeeltjes in het vroege heelal afleggen voordat ze uiteindelijk samenklonteren en kosmologische structuren vormen. Deze afstand komt dus overeen met de kleinste ruimtelijke schalen waarop zich kosmologische structuren zouden kunnen vormen. Alle bestaande structuren die kleiner zijn dan deze afstand werden afgevlakt, terwijl grotere schommelingen intact bleven.

Hoewel alle bovengenoemde donkere materie-modellen zeer vergelijkbare eigenschappen van de kosmologische structuren op ruimtelijke schalen groter dan individuele sterrenstelsels voorspellen, blijkt dat hun voorspellingen significant verschillen met betrekking tot de hoeveelheid substructuren binnen de melkwegstelsels. Zoals geïllustreerd in Fig. 5.1, die gesimuleerde melkwegstelsels laat zien, voorspelt het huidige favoriete koude donkere materie-model donkere materie om talrijke substructuren binnen melkwegstelsels te vormen. De alternatieve modellen, zoals warme of hete donkere materie, voorspellen daarentegen een gebrek aan dergelijke sub-galactische donkere materie structuren. Daarom is het zeer waarschijnlijk dat de werkelijk waargenomen klonterigheid van echte sterrenstelsels belangrijke aanwijzingen bevat over de aard en eigenschappen van donkere materie.

Figure 5.1: Verdeling van donkere materie binnen een gesimuleerd sterrenstelsel dat koude donkere materie bevat (*linkerpaneel*), versus warme donkere materie (*rechterpaneel*) als scenario. Met dank aan: Lovell et al. (2014).

Het observeren van substructuren in verre sterrenstelsels door lenzen

Het opsporen van substructuren in verre sterrenstelsels, ver van onze eigen Melkweg, is een veeleisende onderneming. Volgens ons huidige begrip zijn dergelijke substructuren waarschijnlijk voornamelijk van donkere materie gemaakt, of kunnen ze zelfs helemaal uit donkere materie bestaan (d.w.z. volledig verstoken van sterren) en dus intrinsiek onzichtbaar zijn. Zelfs wanneer deze substructuren licht geven, zijn ze te zwak om met de momenteel beschikbare telescopen direct te worden waargenomen. Daarom is het zoeken naar tekenen van de zwaartekracht-interactie van substructuren, met de bekende gewone materie of met licht, de enige manier om ze te bestuderen. In de afgelopen jaren is het fenomeen van lichtafbuiging, veroorzaakt door de zwaartekracht van zogenaamde *sterke zwaartekrachtlenzen*, naar voren gekomen als het krachtigste middel om substructuren in verre sterrenstelsels in kaart te brengen en zo hun klonterigheid te bepalen.

Het fenomeen van *sterke lenswerking tussen sterrenstelsels* treedt op wanneer, vanuit het perspectief van een waarnemer op de Aarde, twee (of meer) sterrenstelsels langs dezelfde gezichtslijn liggen. Als

gevolg van deze toevallige uitlijning worden lichtstralen die van het achtergrond-sterrenstelsel richting de waarnemer reizen afgebogen door het tussenliggende voorgrond-sterrenstelsel, in overeenstemming met de Algemene Relativiteitstheorie. Zoals geïllustreerd in het linkerpaneel van Fig. 5.2 leidt deze lichtafbuiging door zwaartekracht tot het ontstaan van meerdere langwerpige *vervormde afbeeldingen* van hetzelfde achtergrond-sterrenstelsel. Dit kan zijn in de vorm van *zwaartekrachtsbogen* of zelfs, in het geval van een perfecte uitlijning en een bijna sferische vorm van de betrokken sterrenstelsels, tot een volledige *Einsteinring*.

Alle substructuren die aanwezig zijn in het voorgrond-sterrenstelsel worden verondersteld een klein verstrend effect te hebben op de totale lichtafbuiging die door de zwaartekracht van dit sterrenstelsel wordt veroorzaakt. Dit leidt tot een afwijking tussen het waargenomen uiterlijk van de vervormde afbeeldingen en de voorspellingen van een rekenmodel dat uitgaat van een sterrenstelsel zonder substructuren. De techniek achter het afbeelden van zwaartekracht, die de basis vormt van dit proefschrift, maakt het mogelijk om deze afwijkingen aan het licht te brengen en ze te herleiden tot de onderliggende substructuren in het lens-sterrenstelsel. Het rechterpaneel van Fig. 5.2 laat een voorbeeld zien van een zwaartekrachtsboog die buigt en splitst in de buurt van een relatief zware lichtgevende substructuur in het lens-sterrenstelsel. Dankzij de geavanceerde modellerings-software die de afgelopen jaren voor zwaartekrachtlenzen is ontwikkeld kunnen dergelijke verstoringen in de vervormde afbeeldingen worden gedetecteerd en gemodelleerd, zelfs als ze worden veroorzaakt door substructuren die aanzienlijk minder zwaar zijn en volledig verstoken van sterren.

Dit proefschrift

Hoewel de voorspellingen van de fenomenologische modellen voor donkere materie praktisch niet te onderscheiden zijn wat betreft het verwachte aantal individuele zware sub-galactische structuren (zoals de zware lichtgevende substructuur in het rechter paneel van Fig. 5.2), is er een cruciaal verschil in zowel de hoeveelheid als populatiekenmerken van substructuren met een lagere massa. Het verschijnsel van sterke lenswerking tussen sterrenstelsels biedt een unieke kans om te zoeken naar dergelijke lichte substructuren binnen sterrenstelsels die zich gedragen als gravitatielenzen voor verder weg gelegen sterrenstelsels langs dezelfde gezichtslijn. In dit

Figure 5.2: Een visualisatie van de sterke lenswerking tussen sterrenstelsels, met in het *linkerpaneel* een bijna volledige Einsteinring, en in het *rechterpaneel* een zwaartekrachtsboog die ernstig wordt verstoord door een massieve lichtgevende substructuur in het voorgrond-sterrenstelsel. Dankzij de geavanceerde modellerings-software die de afgelopen jaren voor zwaartekrachtlenzen is ontwikkeld kunnen dergelijke verstoringen in de vervormde afbeeldingen worden gedetecteerd en gemodelleerd, zelfs als ze worden veroorzaakt door substructuren die aanzienlijk minder zwaar zijn en volledig verstoken van sterren. Met dank aan: ALMA (ESO/NRAO/NAOJ), L. Calçada (ESO), Y. Hezaveh et al., Vegetti et al. (2010b).

proefschrift presenteren we een nieuwe statistische benadering om het gezamenlijke verstorende effect van deze lichte sub-galactische structuren op de lensbeelden van de achtergrond-sterrenstelsels, zoals Einsteinringen of zwaartekrachtbogen, te onderzoeken. Onze aanpak is gebaseerd op waarnemingen van sterke lenswerking tussen sterrenstelsels door de Hubble Space Telescope (HST), de daaropvolgende computermodelering van het waargenomen gravitatielenseffect en, ten slotte, de Fourieranalyse van de anomalieën die in de vervormde lensbeelden naar voren komen. Het proefschrift is als volgt opgebouwd.

In **hoofdstuk 2** presenteren we onze observatiestrategie en gaan we dieper in op alle stappen die nodig zijn om de anomalieën in de lensbeelden veroorzaakt door substructuren in het lenssterrenstelsel op een betrouwbare manier te extraheren. Bovendien onderzoeken we andere effecten die deze anomalieën kunnen nabootsen, en de modelleringsuitdagingen die inherent zijn aan onze aanpak. Tot slot testen we de haalbaarheid van onze methodologie door deze toe te passen op een steekproef uit de tot nu toe grootste catalogus van sterrenstelsels met een sterke zwaartekracht: het Sloan Lens ACS Survey.

In **hoofdstuk 3** bespreken we hoe de statistische eigenschappen van de blootgelegde anomalieën in de lensbeelden ons kunnen helpen om inzicht te krijgen in de populatiekenmerken van de onderliggende lichte substructuren in de lenssterrenstelsels. Met dit doel voor ogen, voeren we een systematische studie uit van gesimuleerde mock-anomalieën die door lichte substructuren met variërende populatiekenmerken op de lensbeelden zouden worden ingeprent. Een vergelijking van deze simulaties met onze HST-waarnemingen leidt tot de allereerste statistische karakterisering van lichte substructuren binnen een echt, ver weg gelegen sterrenstelsel.

In **hoofdstuk 4**, verbeteren we onze oorspronkelijke methodologie door alle modelleringsuitdagingen en biases die we in de vorige twee hoofdstukken tegenkwamen te mitigeren. Om dit te bereiken, simuleren we een set van realistischer mock-lenssystemen die lichte substructuren met verschillende populatiekenmerken bevatten, en modelleren ze op precies dezelfde manier als de echte gegevens. Deze analyse stelt ons in staat om onze studie verder te verfijnen en een nauwkeuriger karakterisering te verkrijgen van lichte substructuren in het gekozen lenssterrenstelsel.

In toekomstig onderzoek zal deze nieuwe benadering worden toegepast op een grotere steekproef van lenssterrenstelsels en zullen de waargenomen populatiekenmerken van lichte substructuren in echte sterrenstelsels worden

vergeleken met de voorspellingen van de fenomenologische modellen voor donkere materie, zoals de hete, warme en koude donkere materie. Deze vergelijking zou ons eindelijk in staat kunnen stellen om de ware aard van donkere materie te onthullen.

Summary for non-experts

The dark-matter problem

Nearly one hundred years ago, astronomers found themselves faced with an unexpected puzzle. The measurements of the velocities at which the stars in our own galaxy - the Milky Way - rotate around its centre suggested this rotation to be much faster than what was expected from calculations based on the standard Newtonian theory of gravity. Later on, such unexpectedly high orbital velocities have been repeatedly measured in many more galaxies, and galaxy clusters. The most prominent and widely known among these anomalous measurements are the *flat rotation curves* observed in spiral galaxies: instead of exhibiting a steep decline as a function of the distance from the centre, as predicted from the Newtonian theory of gravity and observed for the planets in our own Solar System, the actual orbital velocities of stars and gas in spiral galaxies seem to remain constant over the entire observed range of distances from the galactic centre.

After puzzling over these confusing measurements for a long time, astronomers came to the conclusion that the total mass of galaxies and galaxy clusters must be considerably larger than what can be estimated from the observations of their luminous constituents, such as stars, gas and dust. Finally, this conclusion led to the *dark-matter hypothesis*, postulating that the missing mass is provided by the presence of a supplemental non-luminous matter component – the *dark matter*.

Over the last century, the astronomical community has accumulated a compelling body of evidence supporting the dark-matter hypothesis and the existence of dark matter is nowadays a widely accepted assumption. Ac-

cording to our current understanding, dark matter makes up approximately 84 per cent of all the existing matter. As a dominant matter component, it is thought to constitute the driving force behind the formation of all structures in the Universe, including galaxies, stars, planets, and, consequently, all the life on Earth. The visible galaxies and galaxy clusters are believed to be embedded in significantly larger clumps of dark matter – the *dark-matter haloes* – whose gravitational pull holds the constituent stars and gas together.

However, despite the strong empirical evidence and the wide acceptance by the scientific community, the nature of dark matter is still unknown and its constituents are yet to be discovered. Over the last forty years, numerous dark-matter candidates have been considered. Essentially, all these candidates can be divided into two distinct categories. The first category includes the *massive compact halo objects (MACHOs)* – a variety of massive non-luminous astronomical bodies, such as black holes, faint stars or stellar remnants. While most of these have been ruled out as the main constituents of dark matter, the hypothetical primordial black holes (formed in the early Universe through the direct gravitational collapse of highly overdense regions) still remain a viable dark-matter candidate. The second, and most probable, category of candidates encompasses the *particle dark matter* – as-yet unknown elementary particles that do not seem to interact with the atoms making up the familiar ordinary matter and are, thus, extremely difficult to detect.

Numerous experiments have been dedicated to the search, however, no dark-matter particles have been convincingly detected so far. Therefore, unveiling the nature of dark matter remains one of the major still unsolved problems in cosmology and the fundamental physics. The research carried out in this thesis is intended to contribute to the very diverse efforts of the scientific community aiming at solving this *dark-matter problem*.

Substructures in galaxies as an alternative window on dark matter

In parallel with the efforts to actually detect the hypothetical dark-matter particle and characterise its properties, cosmologists have come up with a more general phenomenological approach to investigate dark matter, aiming at constraining its nature and the overall behaviour based on astronomical observations. This is similar to the study of the macroscopic properties

of a gas, such as its temperature or pressure, without knowing anything about the microscopic properties of the constituent gas particles. In this framework, the dark-matter-particle candidates can be divided into three broader categories – the *hot*, *cold* and *warm* dark matter – depending on the average velocity of their random motion in early times just after the Big Bang. This velocity plays a crucial role, since it determines the average distance covered by the dark-matter particles in the early Universe before they eventually clumped together and formed cosmological structures. Consequently, this distance corresponds to the smallest spatial scales on which cosmological structures could form. All existing structures smaller than this distance were smoothed out, whereas larger fluctuations remained intact.

While all the mentioned dark-matter models predict very similar properties of the cosmological structures on spatial scales larger than individual galaxies, it turns out that their predictions vary significantly with respect to the abundance of substructures within galaxies. As is illustrated in Fig. 5.3, showing simulated galaxies, the currently favoured cold-dark-matter model predicts dark matter to form numerous substructures within galaxies. To the contrary, the alternative models, such as hot or warm dark matter, predict a dearth of such sub-galactic dark-matter structures. Therefore, the actual clumpiness of real galaxies is very likely to hold important clues about the nature and properties of dark matter.

Observing substructures in distant galaxies through a lens

Detecting substructures in distant galaxies, far away from our own Milky Way, is a demanding undertaking. According to our current understanding, such substructures are likely predominantly made of dark matter or might even be purely dark (i.e. completely devoid of stars) and, thus, intrinsically invisible. Even if luminous, they would be too faint to be observed directly with the currently available telescopes. For this reason, the only way to study them is to search for signatures of their gravitational interaction with the familiar ordinary matter or light. Over the recent years, the phenomenon of the gravitational light deflection, called *strong gravitational lensing*, has emerged as the most powerful tool to map out substructures in distant galaxies and, thus, probe their clumpiness.

In particular, the phenomenon of the *galaxy-galaxy strong gravitational lensing* occurs when, from the perspective of an observer on the Earth,

Figure 5.3: Distribution of dark matter within a simulated galaxy in the cold-dark-matter (*left panel*) vs. warm-dark-matter (*right panel*) scenario. Credits: Lovell et al. (2014).

two (or more) galaxies happen to lie along the same line of sight. As a consequence of this serendipitous alignment, the light rays propagating from the background *source galaxy* towards the observer are deflected by the intervening foreground *lens galaxy*, in accordance with the theory of General Relativity. As is illustrated in the left panel of Fig. 5.4, this gravitational light deflection leads to the emergence of multiple elongated *lensed images* of the same source galaxy, either in the form of *gravitational arcs* or, in the case of a perfect alignment and a nearly spherical shape of the involved galaxies, even a complete *Einstein ring*.

Any substructures present in the foreground lens galaxy are supposed to have a tiny perturbative effect on the overall gravitational deflection of light caused by this galaxy. This leads to a deviation between the observed appearance of the lensed images and the predictions from a computational model assuming no substructures. The *gravitational-imaging technique*, which builds the basis for this thesis, allows one to uncover these anomalies and trace them back to the underlying substructures in the lens galaxy. The right panel of Fig. 5.4 shows an example of a gravitational arc that splits and bends around a relatively massive luminous substructure in the lens galaxy. Thanks to sophisticated lens modelling codes developed in the recent years, such effects in the lensed images can be detected and

Figure 5.4: Visualisation of the phenomenon of galaxy-galaxy strong gravitational lensing with a nearly complete Einstein ring (*left panel*) and an example of a gravitational arc severely perturbed by a massive luminous substructure in the foreground lens galaxy (*right panel*). Thanks to sophisticated lens modelling codes developed in the recent years, such anomalies in the lensed images can be detected and modelled even if caused by substructures that are significantly less massive and completely devoid of stars. Credits: ALMA (ESO/NRAO/NAOJ), L. Calçada (ESO), Y. Hezaveh et al., Vegetti et al. (2010b).

modelled even if caused by substructures that are significantly less massive and completely devoid of stars.

This thesis

Whereas the predictions from the phenomenological dark-matter models are practically indistinguishable with respect to the expected number of individual massive sub-galactic structures (such as the massive luminous substructure shown in the right panel of Fig. 5.4), the critical difference lies in the abundance and the population characteristics of substructures with lower masses. The phenomenon of the galaxy-galaxy strong gravitational lensing provides a unique opportunity to search for such low-mass substructures inside galaxies that happen to act as gravitational lenses on more distant galaxies, serendipitously located along the same line of sight. In this thesis, we present a novel statistical approach to investigate the collective perturbative effect of these low-mass sub-galactic structures on the lensed images of the background galaxies, such as Einstein rings or gravitational arcs. Our approach is based on Hubble Space Telescope (HST) observations of galaxy-galaxy strong gravitational lens systems, the

subsequent computational modelling of the observed gravitational-lensing effect and, finally, the Fourier analysis of the anomalies revealed in the lensed images. The thesis is structured as follows.

In **Chapter 2**, we present our observational strategy and elaborate on all the steps that are necessary to reliably extract the anomalies in the lensed images caused by substructures in the lens galaxy. Moreover, we investigate other effects that can mimic these anomalies, and the modelling challenges inherent in our approach. Finally, we test the feasibility of our methodology by applying it to a sample from the so far largest catalogue of galaxy-galaxy strong gravitational lens systems – the Sloan Lens ACS Survey.

In **Chapter 3**, we discuss how the statistical properties of the revealed anomalies in the lensed images can help us gain insight into the population characteristics of the underlying low-mass substructures in the lens galaxies. With this goal in mind, we perform a systematic study of simulated mock anomalies that would be imprinted on the lensed images by low-mass substructures with varying population characteristics. A comparison of these simulations to our HST observations leads to the first ever statistical characterisation of low-mass substructures within a real distant galaxy.

In **Chapter 4**, we improve on our original methodology by mitigating all the modelling challenges and biases that we encountered in the previous two chapters. To achieve this, we simulate a set of more realistic mock lens systems containing low-mass substructures with varying population characteristics and model them in exactly the same manner as the real data. This analysis allows us to further refine our study and obtain a more accurate characterisation of low-mass substructures in the chosen lens galaxy.

Future work will apply this novel approach to a larger sample of lens galaxies and compare the observed population characteristics of low-mass substructures in real galaxies to the predictions from the phenomenological dark-matter models, such as the hot, warm and cold dark matter. This comparison might finally allow us to reveal the true nature of dark matter.

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"It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of light, it was the season of darkness, it was the spring of hope, it was the winter of despair."

Charles Dickens, *A Tale of Two Cities*

Very quickly after starting the PhD, I realised that its true value lies in the journey itself, the personal growth, all the lessons to be learned from the experiences and obstacles along the way and, most of all, the opportunity to walk the path accompanied by exceptional people. I would never have imagined that the journey will finally take me to a place so distant from Groningen, and this makes me feel even more grateful to all of you who contributed to this thesis in one way or another!

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