University of Groningen

Modeling and analysis of a hysteretic deformable mirror with electrically coupled actuators

Schmerbauch, Anja E. M.; Vakis, Antonis I.; Huisman, Robert; Jayawardhana, Bayu

Published in:
2020 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)

DOI:
[10.1109/AIM43001.2020.9158999](https://doi.org/10.1109/AIM43001.2020.9158999)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2020

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Schmerbauch, A. E. M., Vakis, A. I., Huisman, R., & Jayawardhana, B. (2020). Modeling and analysis of a hysteretic deformable mirror with electrically coupled actuators. In *2020 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)* (pp. 1929-1934). [9158999] (IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM; Vol. 2020-July). IEEE.
<https://doi.org/10.1109/AIM43001.2020.9158999>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Modeling and analysis of a hysteretic deformable mirror with electrically coupled actuators

A.E.M. Schmerbauch^{1,*}, A.I. Vakis², R. Huisman³ and B. Jayawardhana¹

Abstract—In this paper, we present our novel concept of deformable mirror which enables a high density 2D actuation array through multiplexing. We investigate the resulting electrical coupling effect in the neighboring actuators through finite element analysis. These insights are used to consider the behavior within the least-square fitting performed to achieve wavefront error corrections.

I. INTRODUCTION

Deformable mirrors (DMs) are instruments used for the correction of light wavefront errors in telescope images and are applied in adaptive optical systems. In the search for planets outside our solar system (so called exoplanets), a critical technology development requirement is to advance DM technology. Astronomers have been able to find a few thousand exoplanets in orbit of distant stars, typically using indirect methods, where variations in the light coming from the star reveal the presence of an accompanying planet. To be able to characterize the exoplanet atmosphere, it is necessary to directly observe the light coming from the planet. In this case, the extreme contrast between the brightness of the star and the planet requires unprecedented wavefront control. Future large space telescopes, dedicated to direct imaging of exoplanets like LUVOIR [1], will rely on DMs to achieve clean and stable wavefronts over long periods of time, while reducing the negative effect of star light speckles originating from small surface shape errors of the optics. By increasing the number of actuators used to deform the mirror surface, the higher the quality of the obtained images. Current developments can reach 100 to 6000 actuators but rarely higher [2]. Nevertheless, scaling up the number of actuators will lead to design challenges because one of the major limitations for employing DMs on a space mission is the associated cable harness, electronics and related reliability issues. If every actuator has to be driven continuously to hold a particular position, an assigned channel with high voltage amplifier and converter is required which leads to a massive hardware of electronical components. To reduce the number of wires needed to connect and actuate, a new concept of DM called Hysteretic Deformable Mirror (HDM) is presented

in [3] which relies on multilayered piezoelectric actuators with high hysteresis that are electronically addressed by time-division multiplexing. A schematic representation of the HDM can be found in Figure 1. The actuators are realized

Fig. 1. A schematic representation of the hysteretic deformable mirror which shows the concept in 3D (left) and exploded view (right). The integral components are the mirror surface, an isolation layer, parallel electrodes, piezoelectric layers and perpendicular electrodes.

with a purposely designed piezoelectric material to exhibit asymmetric butterfly loops with remnant deformation, and intersecting electrodes to address the particular areas. Figure 2 shows an example of such a butterfly loop collected from previous material tests.

Fig. 2. Asymmetric butterfly hysteresis loop with remnant deformation. The axial displacement was measured while a certain voltage was applied.

¹A.E.M. Schmerbauch and B. Jayawardhana are with the Engineering and Technology Institute Groningen - Discrete Technology & Production Automation, Faculty of Science and Engineering, University of Groningen, 9747AG Groningen, The Netherlands

²A.I. Vakis is with the Engineering and Technology Institute Groningen - Computational Mechanical and Materials Engineering, Faculty of Science and Engineering, University of Groningen, 9747AG Groningen, The Netherlands

³R. Huisman is with the Netherlands Institute for Space Research, Groningen, The Netherlands

*Corresponding author: a.e.m.schmerbauch@rug.nl

for creating the desired wavefront correction. The modeling and control for the associated butterfly hysteresis behavior is presented in [4] and [5]. In the following, we will present finite element method (FEM) simulations that show the electrical coupling between the actuators, and propose one approach how to consider this behavior in the control system and in the surface fitting process to achieve a wavefront error correction.

This paper is organized as follows. In Section II we explain the electrical coupling between certain actuators while actuating the mirror, and validate the behavior with FEM simulations. We introduce a plate model in Section III for modeling the reflective mirror surface by consideration of the particular interconnection layout of the electrodes. In Section IV we use this plate model to finally fit the mirror to Zernike polynomials, which are commonly used to represent wavefronts for aberration correction in optical systems, and consider the electrical coupling behavior between the actuators. All theoretic aspects are elucidated by means of a 3×3 actuator array. Conclusions are given in Section V.

II. ELECTRICAL COUPLING BETWEEN ACTUATORS

Fig. 3. Hysteric deformable mirror for a 3×3 actuator array visualized as electrical circuit.

A. The origin of the electrical coupling

The HDM benefits from time-division multiplexing and subsequently, provides a very simple electrode layout. The top and bottom electrodes lie perpendicular to each other to form intersecting areas of the electrodes presenting an actuator. The mirror addressing is bundled by sharing the same electrodes for actuators along a line. The voltage is transmitted over a shared top electrode while the corresponding bottom electrode for the desired actuator is grounded. Surrounding electrodes are floating, meaning free of potential and able to let a potential appear on a single electrode when all other potentials on the other electrodes are held constant. Considering this configuration, the neighboring actuators which share the same electrode with the actuated one will be affected due to the spillover of the electrical charges and also experience mechanical deformation that diminishes with increasing distance due to interaction effects. We refer to this effect as the electrical coupling of the actuators. To exemplify the electrical coupling, we can consider a simple 3×3 array

of our HDM and its associated electrical circuit diagram which is shown in Figure 3. Each actuator is represented as a parallel-plate capacitor having a capacitance C . When one actuator is addressed by application of an electrical potential V , the quantity of charge density on the plates Q is given by

$$Q = CV. \quad (1)$$

Due to constant thickness and size ratios, we assume that every actuator possesses the same C and without loss of generality, let us consider the case when $C = 1$. Following from this, Equation (1) simplifies to $Q = V$. Due to the fact that certain actuators share either top or bottom electrodes, it is implied that neighboring electrodes along a line share the introduced charges on the plates. An equilibrium of charges will build up and at this equilibrium, the plates of neighboring actuators become charged with the charges $-Q/2$ and $+Q/2$; this results in an additional deformation on the intersecting areas. Electrical coupling can be further illustrated by considering the 3×3 actuator array. Addressing the central actuator affects the direct neighbors (midline actuators) as well as the remaining ones (diagonal actuators). Nevertheless, with diagonal actuators the transferred charges are much smaller due to the high number of intersections between electrodes, and consequently the electrical coupling here is negligible.

B. Simulation results

FEM simulations have been performed to simulate the electric field in the piezoelectric array and the actuator response. When an electric potential is placed on one of the top electrodes, while one of the bottom electrodes is connected to the ground, there is a strong electric field at the interconnection (Figure 4), which will yield a piezoelectric response and deformation of the central actuator (Figure 5 and Figure 6).

The FEM simulation and the model are set up with the dimensions, material characteristics, and boundary conditions summarized in Table I. Analysis of the results shows that the electrical coupling is present. The midline electrodes which surround the central actuator reveal a smaller deformation as well. Diagonal actuators are minimally affected and exhibit a very low deformation.

The behavior of electrically coupled actuators has an impact on accurate shape control and needs to be considered in the fitting process. Although it can be expected that due to the limited field strength observed at the neighboring actuators, together with the strongly nonlinear nature of the remnant deformation, unacceptable high shape changes at other actuators are avoided, but nevertheless we resolve this and continue with modeling the facesheet and describe how to consider the electrical coupling in the determination of required pressures to fit the mirror surface to Zernike polynomials.

III. PLATE MODEL WITH INTERCONNECTION LAYOUT

An influence function defines the deformed mirror shape resulting by the actuation of each actuator. Besides their

Fig. 4. FEM simulation showing the distribution of the electric field in the xy-midplane of a 3×3 actuator array. An electrical potential of 100 V was applied to the mid-top electrode while the mid-bottom electrode was grounded.

Fig. 5. FEM simulation showing the deformation of the central actuator after actuation and the electrical coupling to the neighboring actuators in an isometric view. The maximum displacement of the center amounts to 31 nm. This deformation was achieved by applying an electrical potential of 100 V to the mid-top electrode while the mid-bottom electrode was grounded.

Fig. 6. FEM simulation showing the deformation of the central actuator and its two neighboring actuators in a cross-section view of the xz-plane. The maximum displacement of the center amounts to 31 nm. This deformation was achieved by applying an electrical potential of 100 V to the mid-top electrode while the mid-bottom electrode was grounded.

TABLE I
PARAMETERS AND BOUNDARY CONDITIONS FOR THE FEM
SIMULATION.

Electrodes	
Width	1 mm
Spacing (center to center)	2 mm
Piezoelectric layer	
Material	PZT-5H
Thickness	0.5 mm
Diameter	10 mm
Boundary conditions	
Electrodes	Thin layer boundary condition
Center top electrode	Grounded
Center bottom electrode	Electrical potential of 100 V
Remaining electrodes	Floating potential
Mesh	
Domain elements	100248
Boundary elements	32836
Edge elements	3456
Number of DOFs solved for	1247337

characterization by means of radially symmetric Gauss functions and splines [6], [7], influence functions can be modeled by application of Kirchhoff or van Kármán theory [8]–[10] for plate deformations smaller than the plate thickness. Furthermore, models based on the Kirchhoff plate use assumptions how actuator forces act on the mirror facesheet, which either presuppose the exerted force as point load or approximated electrode areas with constantly distributed loads, and how facesheets are attached, which is introduced by boundary conditions presenting circularly clamped DMs [11], [12] or facesheets with a free outer edge [13].

In order to increase the modeling accuracy for the HDM and give an exact definition of the electrode shape that exerts a pressure on the mirror surface, we briefly explain our model for calculating the influence functions whose complete description is out of the scope of this work and refer the interested reader to [14].

Given the concept of the HDM, the electrodes have an interconnection layout creating square areas lying under a thin circular facesheet. The actuators are separated by a specified distance. To model this, we consider the Poisson equation and incorporate the square shape of the electrodes during integration. Therefore, each actuator is separated into several areas which can be described by a coordinate transformation using the Cartesian coordinates as well as the radial and angular limits. It is assumed that the thickness of electrodes can be neglected and the actuators modeled as springs in parallel to a force source which creates pressure on the facesheet.

A. Determination of influence matrix

We obtain a mechanical model to describe the relation between the facesheet deflection z and the pressures q applied by the actuators. We consider the Poisson equation

[15]

$$\nabla^2 z = -\frac{q}{T} \quad (2)$$

for small surface displacements for a membrane under tension T . Incorporating the HDM's particular arrangement of the electrodes into the solution to Poisson's equation in polar coordinates (r, ϕ) leads to

$$z(r, \phi, \bar{r}, \bar{\phi}) = D \iint_A \mathcal{F}(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) q(\bar{r}, \bar{\phi}) d\bar{\phi} d\bar{r} \quad (3)$$

with

$$A = \{(\bar{r}, \bar{\phi}) | \phi_1(\bar{r}) \leq \bar{\phi} \leq \phi_2(\bar{r}), 0 \leq \bar{r} \leq 1\} \quad (4)$$

where

$$\mathcal{F}(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) = \begin{cases} f_1(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) & \text{if } 0 < \bar{r} < r \\ f_2(r, \phi, \bar{r}, \bar{\phi}(\bar{r})) & \text{if } r < \bar{r} < 1 \end{cases} \quad (5)$$

and $z(r, \phi)$ is the out-of-plane displacement of the thin facesheet, $(\bar{r}, \bar{\phi})$ are the integration variables, $q(\bar{r}, \bar{\phi})$ are the distributed forces over the particular electrode area, and constant $D = a^2/T$ contains the relation between the facesheet radius a and the surface tension. Edge deflection and slopes are both equal to zero.

The solutions of Equation (3) can be written in matrix form [12] and the surface deflection on a specific point on the facesheet calculated according to

$$z(r, \phi) = \sum_{j=1}^{N_e} \mathcal{M}_{(r, \phi)j} q_{(r, \phi)j} \quad (6)$$

where $\mathcal{M}$ represents the coefficients derived from the solutions of the Poisson equation, $q_{(r, \phi)j}$ are piecewise constant pressures exerted on the respective j -th electrode, and N_e is the total number of electrodes.

B. Actuator model

The stiffness of the facesheet is responsible that actuators become coupled. Generally, a mechanical coupling between neighboring actuators, i.e. inter-actuator coupling, can improve the surface accuracy and smooth the deformation on the facesheet [16]. We consider the inter-actuator coupling by modeling each actuator simplified as a spring in parallel with a force source that interact with the facesheet over a specified area (Figure 7).

Fig. 7. Simplified actuator model, modeled by a stiffness k in parallel to a force source over an area Φ .

It is assumed that all the actuators are identical. The pressure term $q(r, \phi)$ can be split so that it captures both

components in terms of stiffness and force source over that area. Consequently, the relation from Equation (6) may be described by

$$z(r, \phi) = \sum_{j=1}^{N_e} \mathcal{M}_{(r, \phi)j} \left(\Phi_{Pj}(V) - k_j \tilde{z}_j \right) \quad (7)$$

with

$$\Phi_{Pj}(V) = Y_j \Phi_{Tj}(V) \quad (8)$$

and

$$\tilde{z}_j := \frac{\sum_{i \in E_j} z_i / n_e}{A_e} \quad (9)$$

where $\Phi_{Pj}(V)$ denotes the Preisach operator capturing the highly nonlinear hysteresis of the actuators in regard to the total deformation in relation of the initial thickness dimension, Y_j the diagonal matrix containing the Young's modulus, $\Phi_{Tj}(V)$ the longitudinal elongations of the actuators, k_j the diagonal stiffness matrix containing the actuators' stiffness, and $\tilde{z}_j$ the mean surface deflection above the respective electrode with area A_e calculated by means of n_e surface displacement points z_i on a specific position within the electrode area. A model for describing the electric-field dependence on the strain in piezoelectric materials purposely designed to exhibit loops with remnant deformation was presented in [4] based on the use of the Preisach operator. The complete definition of this operator can be found in [5].

IV. RESULTS

A. Least-square fitting with consideration of electrical coupling

To represent the optical imperfections which can occur in adaptive optical systems, we use a common representation of wavefront aberrations which is a series of Zernike polynomials. Zernike polynomials are defined on a unit circle as functions of azimuthal frequency m and radial degrees n where $m \leq n$ using polar coordinates (r, θ) . The set of polynomials [17] can be given by

$$\begin{aligned} Z_n^m(r, \theta) &= R_n^m(r) \cos(m\theta) & \text{for } m \geq 0 \\ Z_n^{-m}(r, \theta) &= R_n^m(r) \sin(m\theta) & \text{for } m < 0 \end{aligned} \quad (10)$$

where

$$R_n^m(r) = \sum_{S=0}^{(n-m)/2} \frac{(-1)^S (n-S)! r^{n-2S}}{S! [(n+m)/2 - S]! [(n-m)/2 - S]!} \quad (11)$$

Each displacement of a respective point on the facesheet which is defined by (r, ϕ) is fit to the corresponding point on the Zernike polynomial so that we can calculate the required pressure terms which can generate the desired shape. An over-determined set of equations is solved in the least-square sense resulting in

$$\Phi = (\mathcal{M}^T \mathcal{M})^{-1} \mathcal{M}(z_d + \mathcal{M} k \tilde{z}_d). \quad (12)$$

Fig. 8. Mirror surface of a 3×3 actuator array from a top-view perspective fitted to the Zernike polynomial Z_1^1 with an amplitude of $1 \mu\text{m}$.

Fig. 9. Mirror surface of a 3×3 actuator array from a top-view perspective fitted to the Zernike polynomial Z_2^0 with an amplitude of $1 \mu\text{m}$.

which aims at minimizing the root-mean-square deviation (RMSD) between the two quantities.

The obtained results are generalized measures to the desired pressures for each actuator of the HDM. Taking into account the existing electrical coupling, a separation between direct and indirect introduced pressures has to be performed. The direct pressure is exerted by the addressed actuator, while the indirect pressure is caused by the neighboring actuators which are electrically coupled. Therefore, following the proposed array of actuators, the proportion of direct and indirect pressures can be broken down as

$$\Phi_{P_j}(V) = \underbrace{\Phi_{P_j}^*(V)}_{\text{direct pressure}} + \underbrace{\sum_{j=1}^{N_e} \lambda_n \Phi_{P_n}^*(V)}_{\text{indirect pressure}} \quad (13)$$

where $\Phi_{P_j}^*(V)$ denotes the actual pressure exerted by the addressed actuator and the summation of $\Phi_{P_n}^*(V)$ describing the influence of the electrically coupled neighbors with coupling factor λ_n . This coupling factor refers to a constant of a particular actuator in connection with the addressed actuator. We assume that the actuators are addressed one by one due to the concept of remnant deformation which is

proposed by the HDM. To determine final values, we can set up a linear system of equations and, by solving it, determine the proportions for direct and indirect pressure. The system of equations in matrix form can be given by

$$\underbrace{\begin{bmatrix} \Phi_{P_1} \\ \Phi_{P_2} \\ \vdots \\ \Phi_{P_{N_e}} \end{bmatrix}}_{:=\Phi} = \underbrace{\begin{bmatrix} \lambda_{P_1} & \lambda_{12} & \dots & \lambda_{1N_e} \\ \lambda_{21} & \lambda_{P_2} & \dots & \vdots \\ \vdots & \dots & \lambda_{P_{j-1}} & \lambda_{N_e-1N_e} \\ \lambda_{N_e1} & \lambda_{N_e2} & \dots & \lambda_{P_j} \end{bmatrix}}_{:=\Lambda} \underbrace{\begin{bmatrix} \Phi_{P_1}^* \\ \Phi_{P_2}^* \\ \vdots \\ \Phi_{P_{N_e}}^* \end{bmatrix}}_{:=\Phi^*} \quad (14)$$

where Φ is a vector containing the constant terms from the least-square fitting, Λ a $N_e \times N_e$ matrix including the coupling factors (λ_{P_j} is 1 if the actuator is actuated directly) and Φ^* a vector with the unknown pressures. If the matrix Λ is square and has full rank, then the system has a unique solution given by

$$\Phi = \Lambda^{-1} \Phi^* \quad (15)$$

where Λ^{-1} is the inverse of Λ .

As an example according to the 3×3 actuator array, with numbering beginning at the center and continuing with the right upper corner clockwise from 1 to 9, we achieve the following matrix

$$\begin{bmatrix} \Phi_{P_1} \\ \Phi_{P_2} \\ \Phi_{P_3} \\ \Phi_{P_4} \\ \Phi_{P_5} \\ \Phi_{P_6} \\ \Phi_{P_7} \\ \Phi_{P_8} \\ \Phi_{P_9} \end{bmatrix} = \begin{bmatrix} \lambda_{P_1} & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n & 0 & \hat{\lambda}_n \\ 0 & \lambda_{P_2} & \hat{\lambda}_n & \hat{\lambda}_n & 0 & 0 & 0 & \hat{\lambda}_n & \hat{\lambda}_n \\ \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{P_3} & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & 0 & 0 \\ 0 & \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{P_4} & \hat{\lambda}_n & \hat{\lambda}_n & 0 & 0 & 0 \\ \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & \lambda_{P_5} & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n \\ 0 & 0 & 0 & \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{P_6} & \hat{\lambda}_n & \hat{\lambda}_n & 0 \\ \hat{\lambda}_n & 0 & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & \lambda_{P_7} & \hat{\lambda}_n & 0 \\ 0 & \hat{\lambda}_n & 0 & 0 & 0 & \hat{\lambda}_n & \hat{\lambda}_n & \lambda_{P_8} & \hat{\lambda}_n \\ \hat{\lambda}_n & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & 0 & 0 & \hat{\lambda}_n & \lambda_{P_9} \end{bmatrix} \begin{bmatrix} \Phi_{P_1}^* \\ \Phi_{P_2}^* \\ \Phi_{P_3}^* \\ \Phi_{P_4}^* \\ \Phi_{P_5}^* \\ \Phi_{P_6}^* \\ \Phi_{P_7}^* \\ \Phi_{P_8}^* \\ \Phi_{P_9}^* \end{bmatrix} \quad (16)$$

The coupling behavior is known from the presented FEM simulations in Section II-B and the coupling factors $\hat{\lambda}_n$ can be extracted as estimation for solving this system. The resulting pressures denoted with Φ^* are needed to control the mirror because they assume the initial state for the input. Together with the electrical coupling behavior the actuators will create the Zernike polynomial. As an example, Figure 8 shows the fitted mirror surface to the Zernike polynomial Z_1^1 with a RMSD of 7.17%. The calculation of coefficients and the simulations were performed using MATLAB R2019a. The optically active region of the mirror was restricted to avoid edge effects due to the clamped facesheet. The stiffness of the actuators were estimated based on general available information of piezoelectric actuators. Low order Zernike polynomials can still be fitted with a low density array but the higher the degree of the polynomial the higher the resulting RMSDs. This can be demonstrated by Figure 9 which shows the fitted mirror surface to the Zernike polynomial Z_2^0 with a

RMSD of 28.63%. By means of high density arrays a higher fitting accuracy and reduction of the RMSD can be reached.

V. CONCLUSION

We discussed the electrical coupling between the actuators in DMs which underly the concept of the hysteretic deformable mirror and have validated this behavior with FEM simulations. An approach was presented to model the mirror facesheet including the particular interconnection layout of the electrodes. The electrical coupling was analysed and integrated based on the known information. With this, it was possible to simulate the fitting of the mirror surface to Zernike polynomials. It serves as the basis method to calculate the initial pressure of each actuator for controlling and creating desired wavefront error corrections with this mirror.

ACKNOWLEDGMENT

The authors would like to thank M.A. Vasquez-Beltran, Dr. M. Acuautila and Prof. B. Noheda of the University of Groningen, and C. de Jonge, Dr. S.N.R. Kazmi, M. Eggens and H. Smit of the Netherlands Institute for Space Research for their valuable input on the Hysteretic Deformable Mirror project.

REFERENCES

- [1] LUVOIR Team, "The LUVOIR Mission Concept Study Final Report." arXiv preprint, arXiv:1912.06219, 2019.
- [2] P.-Y. Madec, "Overview of deformable mirror technologies for adaptive optics and astronomy," *Adaptive Optics Systems III*, vol. 8447, edited by Brent L. Ellerbroek, Enrico Marchetti and Jean-Pierre Vran, International Society for Optics and Photonics, SPIE, pp. 22-39, 2012.
- [3] R. Huisman et al., "Deformable mirror concept utilizing piezoelectric hysteresis for stable shape configurations," in preparation, 2020.
- [4] B. Jayawardhana, M.A. Vasquez-Beltran, W.J. van de Beek, C. de Jonge, M. Acuautila, S. Damerio, R. Peletier, B. Noheda and R. Huisman, "Modeling and Analysis of Butterfly Loops via Preisach Operators and its Application in a Piezoelectric Material," 2018 IEEE Conference on Decision and Control, pp. 6894-6899, 2018.
- [5] M.A. Vasquez-Beltran, B. Jayawardhana and R. Peletier, "Asymptotic Stability Analysis of Lur'e Systems With Butterfly Hysteresis Nonlinearities," in *IEEE Control Systems Letters*, vol. 4, no. 2, pp. 349-354, April 2020.
- [6] R.K. Tyson, "Adaptive Optics Engineering Handbook," CRC Press, p. 352, 1999.
- [7] R.K. Tyson and B.W. Frazier, "Field Guide to Adaptive Optics," SPIE Press, p. 82, 2004.
- [8] R.P. Grosso and M. Yellin, "The membrane mirror as an adaptive optical element," *J. Opt. Soc. Am.* 67, pp. 399-406, 1977.
- [9] L. Arnold, "Optimized axial support topologies for thin telescope mirrors," *Optical Engineering*, pp. 567-574, 1995.
- [10] S.K. Ravensbergen, R.F.M.M. Hamelinck, P.C.J.N. Rosielle and M. Steinbuch, "Deformable mirrors: Design fundamentals for force actuation of continuous facesheets," *SPIE Optics & Photonics; Advanced Wavefront Control: Methods, Devices, and Applications VII*, vol. 7466, p. 74660G, 2009.
- [11] K. Bush, D. German, B. Klemme, A. Marrs and M. Schoen, "Electrostatic membrane deformable mirror wavefront control systems: design and analysis," *SPIE Optics & Photonics; Advanced Wavefront Control: Methods, Devices, and Applications II*, vol. 5553, pp. 28-38, 2004.
- [12] E. Scott Claflin and Noah Bareket, "Configuring an electrostatic membrane mirror by least-squares fitting with analytically derived influence functions," *J. Opt. Soc. Am. A* 3, pp. 1833-1839, 1986.
- [13] A. Menikoff, "Actuator influence functions of active mirrors," *Appl. Opt.* 30, pp. 833-838, 1991.
- [14] A.E.M. Schmerbauch, M.A. Vasquez-Beltran, A.I. Vakis, R. Huisman and B. Jayawardhana, "Influence functions for a novel hysteretic deformable mirror with a high density 2D array of actuators," arXiv preprint, arXiv:2005.07418, 2020.
- [15] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics, Part II," McGraw-Hill, New York, p. 1191, 1953.
- [16] John W. Hardy, "Adaptive Optics for Astronomical Telescopes," *Oxford Series in Optical and Imaging Sciences*, Oxford University Press, p. 448, 1998.
- [17] V. Lakshminarayanan and A. Fleck, "Zernike polynomials: A guide," *Journal of Modern Optics - J MOD OPTIC*, vol 58, pp. 1678-1678, 2011.