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Bergshoeff, E.; Berman, D.; Schaar, J.P. van der; Sundell, P.

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The M5-brane and non-commutative loop space

E Bergshoeff, D Berman, J P van der Schaar and P Sundell

Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands

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Abstract

We investigate, in a certain decoupling limit, the effect of having a constant C -field on the M-theory 5-brane using an open-membrane probe. We define an open-membrane metric for the 5-brane that remains non-degenerate in the limit. The canonical quantization of the open-membrane boundary leads to a non-commutative loop space which is a functional analogue of the non-commutative geometry that occurs for D -branes.

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1. Introduction

Before considering the M2/M5 system it is instructive to first briefly review the relation between D -branes and non-commutative geometry [1]. Consider a fundamental string F1 ending on a Dp -brane via a point or 0-brane. The effective tensions τ of the string and the Dp -brane behave like $\tau_{F1} \sim 1$, $\tau_{Dp} \sim 1/g_s$. Therefore, for small g_s , the string is much lighter than the Dp -brane and can be treated as a test string probing the Dp -brane. Furthermore, the effective gravitational couplings $G_N \tau$ (Newton's constant times tension) behave like $G_N \tau_{F1} \sim g_s^2$, $G_N \tau_{Dp} \sim g_s$ and therefore we can assume that the spacetime background is approximately flat. The open-string action reads

$$S = \frac{1}{2\alpha'} \int_{M^2} d^2\sigma \bar{\partial} X^M \partial X^N \eta_{MN} + \frac{1}{2} \int_{\partial M^2} d\tau \mathcal{F}_{\mu\nu} X^\mu \dot{X}^\nu, \quad (1)$$

where $\mathcal{F}_{\mu\nu}$ is the constant background field strength on the Dp -brane. We assume that the only non-vanishing components of $\mathcal{F}$ are $\mathcal{F}_{r's'}$, where we have decomposed the worldvolume index μ as $\mu = (r, r')$ with $r = 0, 1, \dots, p - \text{rank } \mathcal{F}$ and $r' = p + 1 - \text{rank } \mathcal{F}, \dots, p$:

$$\mathcal{F} = \begin{pmatrix} \mathcal{F}_{rs} = 0 & 0 \\ 0 & \mathcal{F}_{r's'} \end{pmatrix}. \quad (2)$$

We consider now the following decoupling limit (see, e.g., [2]). We take $\epsilon \rightarrow 0$ such that¹

$$\eta_{r's'} \sim \epsilon \eta_{r's'}, \quad \alpha' \sim \epsilon^{1/2} \alpha', \quad (3)$$

¹ In (3) it is understood that the $\eta_{r's'}$ and α' occurring on the right-hand side are ϵ -independent.

while keeping all other quantities fixed. The open string action scales as follows:

$$S \sim \frac{1}{2\epsilon^{1/2}\alpha'} \int_{M^2} d^2\sigma \bar{\partial} X^m \partial X^n \eta_{mn} + \frac{1}{2\epsilon^{1/2}\alpha'} \int_{M^2} d^2\sigma \bar{\partial} X^r \partial X^s \eta_{rs} \\ + \frac{\epsilon^{1/2}}{2\alpha'} \int_{\Sigma} d^2\sigma \bar{\partial} X^{r'} \partial X^{s'} \eta_{r's'} + \frac{1}{2} \int_{\partial M^2} d\tau \mathcal{F}_{r's'} X^{r'} \dot{X}^{s'}. \quad (4)$$

One may now argue (see, e.g., [2, 3]) that the dynamics of the $F1/Dp$ system is dominated by the last so-called Wess–Zumino term, i.e.

$$S \sim \frac{1}{2} \int_{\partial M^2} d\tau \mathcal{F}_{r's'} X^{r'} \dot{X}^{s'}. \quad (5)$$

Moreover, the open string metric is finite in this limit and is given by the maximal rank matrix

$$G_{\mu\nu} = \begin{cases} \eta_{\mu\nu} & \text{for } \mu, \nu = r, s \\ -\alpha'^2 \mathcal{F}_{\mu\rho} \eta^{\rho\sigma} \mathcal{F}_{\sigma\nu} & \text{for } \mu, \nu = r', s'. \end{cases} \quad (6)$$

The equations of motion corresponding to the Wess–Zumino term read

$$\dot{X}^{r'} = 0, \quad (7)$$

i.e. there are additional Dirichlet conditions: the endpoint of the string is not allowed to move in the r' directions. The non-commutative nature of the D -brane arises from quantizing the Wess–Zumino term (5). Applying the standard canonical quantization procedure leads to the following non-zero Dirac brackets:

$$\{X^{r'}(\sigma), X^{s'}(\sigma')\} = (\mathcal{F}^{-1})^{r's'} \delta(\sigma - \sigma'). \quad (8)$$

We thus conclude that the string probing the Dp -brane sees a non-commutative geometry in the r' directions of the Dp -brane worldvolume.

2. The M2/M5 system

The M-theory origin of the $F1/Dp$ system is a M2/M5 system, i.e. an open membrane ending on a 5-brane in an 11-dimensional supergravity background. The membrane boundary is a string that is constrained to lie within the 5-brane. The action for the open bosonic membrane is as follows:

$$S = S_k + \int_{M^3} f_2^* C + \int_{\partial M^3} f_1^* b, \quad (9)$$

where the kinetic term can be written in Polyakov form as

$$S_k = \frac{1}{2(\ell_p)^2} \int_{M^3} d^3\sigma \sqrt{-\det \gamma} (-\gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N \hat{g}_{MN} + \ell_p^2). \quad (10)$$

Here ℓ_p is the $D = 11$ Planck constant, $\hat{g}_{MN}$ is the $D = 11$ spacetime metric and $\gamma_{\alpha\beta}$ is the auxiliary worldvolume metric. The maps f_2 and f_1 denote the embedding of the membrane and its boundary into the spacetime and the 5-brane, respectively. The worldvolume 3-form $f_2^* C$ is the pull-back of the $D = 11$ 3-form potential C to the membrane worldvolume and, similarly, $f_1^* b$ is the pull-back of the 5-brane 2-form potential b to the boundary of the membrane. In terms of components, we write

$$(f_2^* C)_{\alpha\beta\gamma} = \partial_\alpha X^M \partial_\beta X^N \partial_\gamma X^P C_{MNP}, \quad (f_1^* b)_{ij} = \partial_i X^\mu \partial_j X^\nu b_{\mu\nu}, \quad (11)$$

where $M = 0, 1, \dots, 9, 11$ are spacetime indices, $\mu = 0, 1, \dots, 5$ are 5-brane worldvolume indices, $\alpha = 0, 1, 2$ are membrane worldvolume indices and $i = 0, 1$ are indices on the boundary of the membrane.

The coupling of b to the boundary of the membrane ensures that the open-membrane action is invariant under the spacetime gauge transformations $\delta C = d\Lambda$ provided that $\delta b = -f_5^* \Lambda$, where f_5 denotes the embedding of the 5-brane into spacetime. The 2-form b satisfies the 5-brane field equations. These are equivalent to a nonlinear self-duality condition on the following gauge invariant 3-form field strength of b :

$$\mathcal{H} = db + f_5^* C. \quad (12)$$

Here the last term is the pull-back of the spacetime 3-form potential to the 5-brane:

$$(f_5^* C)_{\mu\nu\rho} = \partial_\mu x^M \partial_\nu x^N \partial_\rho x^P C_{MNP}, \quad (13)$$

where $x^M(X^\mu)$ are local embedding functions satisfying the 5-brane equations of motion.

We shall consider backgrounds where $\mathcal{H}_{\mu\nu\rho}$ is constant. This is only consistent with (12) provided we require that the pull-back of the spacetime 4-form field strength $F = dC$ to the 5-brane vanishes, i.e. $f_5^* F = 0$. It is convenient to write $C = \tilde{C} + dC_2$ with $f_5^* \tilde{C} = 0$ and $f_5^* C_2 = c$. This enables us to rewrite the following bulk term as a boundary term:

$$\int_{M^3} f_2^* C = \int_{\partial M^3} f_2^* C_2 = \int_{\partial M^3} f_1^* f_5^* C_2 = \int_{\partial M^3} f_1^* c, \quad (14)$$

where we have applied Stoke's theorem. Finally, since $f_5^* C = dc$ we have that $\mathcal{H} = d(b + c)$ or

$$(b + c)_{\mu\nu} = \mathcal{H}_{\mu\nu\rho} X^\rho. \quad (15)$$

This enables us to rewrite the Wess–Zumino term as

$$S_{WZ} = \frac{1}{3} \int_{\partial M^3} d^2\sigma \mathcal{H}_{\mu\nu\rho} X^\mu \dot{X}^\nu X'^\rho. \quad (16)$$

A complicating feature of the M5-brane is that the 3-form curvature $\mathcal{H}$ satisfies a nonlinear self-duality condition [4]:

$$\frac{\sqrt{-\det g}}{6} \epsilon_{\mu\nu\rho\sigma\lambda\tau} \mathcal{H}^{\sigma\lambda\tau} = \frac{1+K}{2} (G^{-1})_\mu{}^\lambda \mathcal{H}_{\nu\rho\lambda}, \quad (17)$$

where $\epsilon^{012345} = 1$ and the scalar K and the tensor $G_{\mu\nu}$ are given by

$$K = \sqrt{1 + \frac{\ell_p^6}{24} \mathcal{H}^2}, \quad (18)$$

$$G_{\mu\nu} = \frac{1+K}{2K} \left(g_{\mu\nu} + \frac{\ell_p^6}{4} \mathcal{H}_{\mu\nu}^2 \right). \quad (19)$$

In [3] it was argued that the tensor $G_{\mu\nu}$ is the metric on the 5-brane seen by an open membrane in the presence of a background 3-form field strength $\mathcal{H}$. It is understood that in the above three equations the indices are contracted with the induced 5-brane metric:

$$g_{\mu\nu} = \partial_\mu x^M \partial_\nu x^N \hat{g}_{MN}. \quad (20)$$

It will be useful to introduce a specific parametrization of the solutions of the self-duality condition (17) as follows²:

$$\mathcal{H}_{\mu\nu\rho} = \frac{h}{\sqrt{1 + \ell_p^6 h^2}} \epsilon_{\alpha\beta\gamma} v_\mu^\alpha v_\nu^\beta v_\rho^\gamma + h \epsilon_{abc} u_\mu^a u_\nu^b u_\rho^c, \quad (21)$$

$$G_{\mu\nu} = \frac{\left(1 + \sqrt{1 + h^2 \ell_p^6}\right)^2}{4} \left(\frac{1}{1 + h^2 \ell_p^6} \eta_{\alpha\beta} v_\mu^\alpha v_\nu^\beta + \delta_{ab} u_\mu^a u_\nu^b \right). \quad (22)$$

Here h is a real field of dimension (mass)³ and (v_μ^α, u_μ^a) , $\alpha = 0, 1, 2$, $a = 3, 4, 5$, are sechsbein fields in the nine-dimensional coset $SO(5, 1)/SO(2, 1) \times SO(3)$ satisfying

$$g^{\mu\nu} v_\mu^\alpha v_\nu^\beta = \eta^{\alpha\beta}, \quad g^{\mu\nu} u_\mu^a v_\nu^\beta = 0, \quad g^{\mu\nu} u_\mu^a u_\nu^b = \delta^{ab}, \quad (23)$$

$$g_{\mu\nu} = \eta_{\alpha\beta} v_\mu^\alpha v_\nu^\beta + \delta_{ab} u_\mu^a u_\nu^b. \quad (24)$$

3. Limits on M5

We will now consider a limit of the open-membrane/5-brane system with the main property that the boundary string that lives in the 5-brane is governed solely by the Wess–Zumino term (16). Compared with the case of a string ending on a Dp -brane we are faced with two problems.

- (1) The decoupling limit must be consistent with the nonlinear self-duality condition (17).
- (2) Since both $\tau_{M2} \sim 1$ and $\tau_{M5} \sim 1$ we cannot use the membrane as a probe to study the worldvolume geometry of the M5-brane.

In this paper we will discuss a particular limit that avoids these two problems. Other limits were discussed at this conference by Per Sundell. Problem (1) is circumvented by using the explicit solution for $\mathcal{H}$ given by (21) and (24). To take care of problem (2) we consider, instead of a flat background, a $D = 11$ background consisting of a stack of N parallel 5-branes, given by the solution

$$ds^2(\hat{g}) = H^{-1/3} (dx^\mu)^2 + H^{2/3} (dy^m)^2, \quad H = 1 + \frac{N_5 \ell_p^3}{r^3}, \quad F = N_5 \epsilon_4, \quad (25)$$

where $\mu = 0, 1, \dots, 5$; $m = 6, 7, 8, 9, 11$, N_5 is the number of stacked 5-branes and ϵ_4 is the volume form on the transverse S^4 . We let the open membrane end on one of these 5-branes removed from the stack and placed at radius r_0 . If $N_5 \gg 1$ and r_0 is small, then the interactions between the stack and the separated 5-brane effectively stiffens the latter so that the membrane can probe it without deforming it. Under these conditions the induced metric on the 5-brane (20) is given by

$$g_{\mu\nu} = H^{-1/3}(r_0) \eta_{\mu\nu}. \quad (26)$$

Moreover, from (25) it follows that the $D = 11$ background 4-form field strength satisfies $f_5^* F = 0$. From the discussion in section 2 it follows that we may consider an open-membrane action given by

$$S = \frac{1}{2(\ell_p)^2} \int_{M^3} d^3\sigma \sqrt{-\det \gamma} \left(-H^{-1/3} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} - H^{2/3} \gamma^{\alpha\beta} \partial_\alpha Y^m \partial_\beta Y^n \delta_{mn} + \ell_p^2 \right) \\ + N_5 \int_{M^3} f_2^* \tilde{C} + \frac{1}{3} \int_{\partial M^3} d^2\sigma \mathcal{H}_{\mu\nu\rho} X^\mu \dot{X}^\nu X'^\rho, \quad (27)$$

² This parametrization has been derived independently by [5].

where the $D = 11$ background 3-form potential $\tilde{C}$ obeys $d\tilde{C} = \epsilon_4$ and $f_5^* \tilde{C} = 0$ and the background 3-form field strength $\mathcal{H}_{\mu\nu\rho}$ on the 5-brane is constant.

We now propose the following decoupling limit obtained by taking $\epsilon \rightarrow 0$ such that

$$\ell_p \sim \epsilon \ell_p, \quad (28)$$

$$\frac{N_5}{r_0^3} \sim \epsilon^{-3\delta} \frac{N_5}{r_0^3}, \quad (29)$$

$$h \sim \epsilon^{-\lambda} h. \quad (30)$$

For simplicity we shall assume that $\delta > 1$ such that we may drop the 1 from the harmonic function in the metric (25). It then follows from (26) that the induced 5-brane metric and the sechsbein fields in (24) scale as

$$g_{\mu\nu} \sim \epsilon^{\delta-1} g_{\mu\nu}, \quad u_\mu^a \sim \epsilon^{\frac{1}{2}(\delta-1)} u_\mu^a, \quad v_\mu^\alpha \sim \epsilon^{\frac{1}{2}(\delta-1)} v_\mu^\alpha. \quad (31)$$

Furthermore, we assume that $\lambda \leq 3$. This implies that $h\ell_p^3$ remains finite which enables us to keep the 3-form field strength and the open-membrane metric (22) non-degenerate in the limit. Thus we find that the open-membrane action (27) scales as

$$S \sim \epsilon^{-\Delta} \left[\frac{1}{2\ell_p^2} \int_{M^3} d^3\sigma \sqrt{-\gamma} (-\epsilon^{\Delta+\delta-3} H^{-1/3} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} - \epsilon^{\Delta-2\delta} H^{2/3} \gamma^{\alpha\beta} \partial_\alpha Y^m \partial_\beta Y^n \delta_{mn} + \epsilon^\Delta \ell_p^2) + \frac{1}{3} \int_{\partial M^3} d^2\sigma \mathcal{H}_{\mu\nu\rho} X^\mu \dot{X}^\nu X'^\rho \right], \quad (32)$$

where we have defined

$$\Delta = \lambda - \frac{3}{2}(\delta - 1). \quad (33)$$

We now impose the following requirements for our decoupling limit (for a more detailed discussion, see [3]).

- (a) All interactions on the M5-brane worldvolume must vanish.
- (b) The bulk modes must decouple.
- (c) The open-membrane metric must remain non-degenerate after taking the decoupling limit.

Given these assumptions we find the following restrictions on our parameters [3]:

$$\Delta + \frac{3}{2}(\delta - 1) \leq 3 < \Delta + \delta, \quad \Delta < 2\delta, \quad 1 < \delta < 3. \quad (34)$$

These conditions are solved by (Δ, δ) in a finite size region. For instance, $\delta = \frac{5}{3}$, $\lambda = 3$ and $\Delta = 2$ leads to a decoupled 5-brane theory in a background with a nonlinearly self-dual field strength, while $\delta = \frac{4}{3}$, $\lambda = 2$ and $\Delta = 2$ yields a linearly self-dual field strength.

A noteworthy feature is that (34) implies $\Delta > 0$, such that there is necessarily an overall scaling of the action in (32). Such a scaling was not required in the string case. A crucial difference between the string and the membrane is that only the string action (1) has a microscopic interpretation. On the other hand, the membrane action (10) should be seen as an effective action. One interpretation of the scaling (32) with $\Delta > 0$ is that actually we are taking a semiclassical limit.

Summarizing, in order to understand the geometry of the 5-brane worldvolume we are led to study the quantization of the Wess–Zumino action (16) with $\mathcal{H}$ constant.

4. Canonical analysis

We now will canonically quantize the action (16) with constant field strength $\mathcal{H}_{\mu\nu\rho}$. For convenience, we assume that the field strength can be diagonalized as follows [2]:

$$\mathcal{H}_{012} = -\frac{h}{\sqrt{1 + \ell_p^6 h^2}}, \quad \mathcal{H}_{345} = h, \quad (35)$$

where the dimensionless combination $h\ell_p^3$ is non-vanishing provided the decoupling limit (30) has been taken with $\lambda = 3$.

In the parametrization (35) the action (16) splits into two independent Lagrangians for the two sets of coordinates $X^{0,1,2}$ and $X^{3,4,5}$:

$$S = \frac{h}{3\sqrt{1 + \ell_p^6 h^2}} \int_{\partial M^3} d^2\sigma \epsilon_{\alpha\beta\gamma} X^\alpha \dot{X}^\beta X'^\gamma + \frac{h}{3} \int_{\partial M^3} d^2\sigma \epsilon_{abc} X^a \dot{X}^b X'^c, \quad (36)$$

where $\alpha = 0, 1, 2$ and $a = 3, 4, 5$. The action is invariant under worldsheet reparametrizations:

$$\delta_\xi X^\alpha = \xi^i \partial_i X^\alpha, \quad \delta_\eta X^a = \eta^i \partial_i X^a, \quad i = 0, 1. \quad (37)$$

Note that, due to the absence of a worldsheet metric, there is no need to identify the vector fields ξ and η .

The equations of motion are

$$\epsilon_{\alpha\beta\gamma} \dot{X}^\beta X'^\gamma = 0, \quad \epsilon_{abc} \dot{X}^b X'^c = 0. \quad (38)$$

Assume now that the string boundary inside the M5-brane has a non-compact extension in the time direction. In that case we can impose the gauge choice $X^0 = \tau$. Substituting this into the equations of motion we obtain

$$X'^\alpha = 0, \quad (39)$$

which means that the spatial extension of the string must be in the a direction. Assuming that $|\vec{X}'| \neq 0$ we obtain

$$\dot{X}^a = 0, \quad (40)$$

which implies additional Dirichlet conditions in the a directions.

Let us continue by analysing the phase space dynamics of the three coordinates $\vec{X} = (X^0, X^1, X^2)$. The canonical momenta are given by

$$\Pi_\alpha(\sigma) := \frac{\delta S}{\delta \dot{X}^\alpha(\sigma)} = -\frac{1}{3} h \epsilon_{\alpha\beta\gamma} X^\beta X'^\gamma, \quad (41)$$

indicating that there are three primary constraints $\phi_\alpha(\sigma)$:

$$\phi_a := \Pi_a + \frac{1}{3} h \epsilon_{abc} X^b X'^c \approx 0. \quad (42)$$

The non-trivial canonical Poisson brackets are

$$\{X^a(\sigma), \Pi_b(\sigma')\} = \delta_b^a \delta(\sigma - \sigma') \quad (43)$$

and the non-zero Hamiltonian is given by

$$H = \int d\sigma \lambda^a(\sigma) \phi_a(\sigma), \quad (44)$$

where $\lambda^a(\sigma)$ are three Lagrange multipliers. To proceed with the canonical analysis we study the consistency conditions

$$\dot{\phi}_a(\sigma) = \lambda^b(\sigma) M_{ba}(\sigma) \approx 0, \quad (45)$$

where

$$\{\phi_a(\sigma), \phi_b(\sigma')\} = M_{ab}(\sigma) \delta(\sigma - \sigma'), \quad M_{ab} = h \epsilon_{abc} X'^c. \quad (46)$$

Note that in the α space we can impose $X^0 = \tau$ and, via the equations of motion, $X'^\alpha = 0$. This implies that $M_{\alpha\beta} = 0$. In other words, the three primary constraints $\phi_\alpha(\sigma)$ are all first class.

In contrast, let us now consider the canonical analysis of the three Euclidean coordinates $\vec{X} = (X^3, X^4, X^5)$. A similar analysis as above leads to the same result except that in this case we have assumed that $|\vec{X}'| \neq 0$ and therefore

$$M_{ab} X'^b = 0. \quad (47)$$

The matrix M_{ab} is thus non-degenerate in the two-dimensional subspace orthogonal to $\vec{X}'$. It is convenient to introduce a projection onto this subspace as follows ($I = 1, 2$):

$$P_I^a(\sigma) P_J^b(\sigma) \delta_{ab} = \delta_{IJ}, \quad (48)$$

$$\delta^{IJ} P_I^a(\sigma) P_J^b(\sigma) = \delta^{ab} - \frac{X'^a X'^b}{|\vec{X}'|^2}, \quad (49)$$

$$\epsilon^{IJ} P_I^a(\sigma) P_J^b(\sigma) = \frac{\epsilon^{abc} X'^c}{|\vec{X}'|}. \quad (50)$$

The three constraints ϕ_a now split into the two second-class constraints

$$\chi_I := P_I^a \phi_a, \quad (51)$$

with the now non-degenerate matrix

$$\{\chi_I(\sigma), \chi_J(\sigma')\} := M_{IJ}(\sigma) \delta(\sigma - \sigma'), \quad M_{IJ} = P_I^a P_J^b M_{ab}, \quad (52)$$

and one first-class constraint

$$\phi := X'^a \phi_a \equiv X'^a \Pi_a, \quad (53)$$

which acts as the generator of σ reparametrizations.

The presence of the two second-class constraints leads to a non-trivial Dirac bracket between the X^a coordinates given by

$$[X^a(\sigma), X^b(\sigma')]^D = -\frac{1}{h} \frac{\epsilon^{abc} X'^c(\sigma)}{|\vec{X}'(\sigma)|^2} \delta(\sigma - \sigma'). \quad (54)$$

The conclusion is that the membrane probe sees a so-called non-commutative loop space geometry in the a directions of the M5-brane worldvolume.

5. Non-commutative loop space

The main conclusion of this paper is that, whereas D -branes lead to a non-commutative geometry of points, the M5-brane seems to lead to a non-commutative geometry of loops. To the best of our knowledge, such a non-commutative loop space geometry has not been considered before in the literature.

As a historical note, it is perhaps of interest to note that, whereas the idea of lightlike integrability applied to a superspace geometry naturally leads to the superspace constraints of Yang–Mills [6], the same idea when applied to a loop superspace geometry leads to the constraints of supergravity coupled to Yang–Mills [7]. In the latter work the definition of a loop space covariant derivative plays a central role. The gauge field part of this covariant derivative is given by the pull-back of the self-dual antisymmetric tensor, i.e.

$$\mathcal{D}_\mu(\sigma) = \frac{\delta}{\delta X^\mu(\sigma)} + b_{\nu\mu} X'^\nu. \quad (55)$$

Through this paper we are naturally led to consider a non-commutative version of loop superspace. One of the open questions is how to exactly construct a covariant derivative corresponding to such a non-commutative loop space. It suggests that this problem is related to the problem of how to construct a field theory for a set of $D = 6$ (2, 0) non-Abelian tensor multiplets. The analogy is as follows. On the one hand, in the non-commutative case, one must replace the term $b_{\nu\mu} X'^\nu$, present in the covariant derivative, by some non-commutative generalization with

$$\{X^\mu, X^\nu\} \neq 0. \quad (56)$$

On the other hand, in the non-Abelian case, one must replace this term by some non-Abelian generalization, i.e. $b_{\nu\mu}^I X'^\nu T^I$ with

$$[T^I, T^J] \neq 0. \quad (57)$$

More generally, the suggestion is that, in order to describe a set of $D = 6$ (2, 0) tensor multiplets, it will not suffice to work with a local field theory but instead, one should work with a non-local loop space where in the covariant derivative one makes the replacement:

$$b_{\nu\mu}(X) X'^\nu \implies A_\mu(X(\sigma)). \quad (58)$$

The antisymmetric tensor is just one of the many components of the gauge field $A_\mu(X(\sigma))$. In this way one would also circumvent the no-go theorem of [8]. It would be of interest to investigate these issues in more detail.

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