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The stability of exchange networks

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ABSTRACT

Economic and sociological exchange theories predict divisions of exchange benefits given an assumed fixed network of exchange relations. Since network structure has been found to have a large impact on actors' payoffs, actors have strong incentives for network change. We answer the question what happens to both the network structure and actor payoffs when myopic actors change their links in order to maximize their payoffs. We investigate the networks that are stable, the networks that are efficient or egalitarian with varying tie costs, and the occurrence of social dilemmas. Only few networks are stable over a wide range of tie costs, and all of them can be divided into two types: efficient networks consisting of only dyads and at most one isolate, and Pareto efficient and egalitarian cycles with an odd number of actors. Social dilemmas are observed in even-sized networks at low tie costs.

1. Introduction

An exchange situation can broadly be defined as a situation involving actors who have the opportunity to collaborate for the benefits of all actors involved. While exchange has been intensively studied in economics for more than a century (e.g., Coddington, 1968; Edgeworth, 1881; Young, 1975), exchange entered the fields of social psychology and sociology only in the second half of the twentieth century. Homans (1958, p. 606) introduced the idea that 'social behavior is an exchange of goods, material goods but also non-material ones, such as the symbols of approval and prestige'. The conception of social behavior as exchange was also used by other prominent social scientists in the same time period, such as Thibaut and Kelley (1959) and Blau (1964). After these important works research on exchange as a model of social behavior gained a prominent position in social psychology and sociology.

Since Stolte and Emerson (1977) and Cook and Emerson's (1978) seminal studies, sociologists have focused on the effect of social structures on exchange outcomes. The basic idea of this research is that social behavior is shaped by the social relations in which it occurs, which are in return conditioned by the structures within which they are embedded (Willer, 1999, p. xiii). Where social behavior is conceived of as exchange, the social relation is dubbed an 'exchange relation' and the structure is denoted an 'exchange network'. If two persons have an exchange relation, this means that both persons have the opportunity to exchange, but they need not do so. If they do not have an exchange relation, they have no opportunity to exchange. These opportunities and restrictions to exchange arise naturally in many real-life situations. Three of the most common causes for the absence of an exchange relation between two persons are natural barriers, non-matching preferences, and the decreasing marginal utility of relations. Examples of barriers are not knowing each other, or not being able to contact or meet each other. Also, two people might not have an exchange relation because one of them has nothing to offer that is valuable enough to the other. Finally, if maintaining each tie is costly and the marginal benefit of a relation decreases in the number of relations one already has, it could be optimal to forego some relations.

In almost all sociological studies on exchange, both theoretical and empirical, the social structure is the independent variable, i.e., what was studied was the effect of the network structure on outcomes of persons in different network positions. These studies show that network structure has a large impact on what actors earn in their exchange relations (e.g., Willer, 1999; special issue Social Networks, June 1992; special issue Rationality and Society, January 1997). The well-documented tendency for exchange experiments with small sums of money to yield rather egalitarian outcomes largely independent of experimental conditions (e.g., Roth, 1995) makes this result all the more pervasive. Since in different networks actors obtain different exchange benefits, there is an incentive for actors to change the network structure. Important questions are, therefore, how exchange networks evolve in the first place, and which networks are stable or resistant to change. Note that, although we attempt to answer these questions in the context of exchange networks, these questions are relevant to any network in which benefit differences between actors are substantial and

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depend on the network structure. Examples are communication, knowledge, and friendship networks.

In our study the structure of the exchange network is the dependent variable. We ask what structure can be expected to emerge if actors have the opportunity to choose with whom to maintain exchange relations, as a function of tie costs. We assume that the tie costs are constant and both actors involved in a tie pay the tie costs. Actors add and keep ties only if they are marginally beneficial. We investigate the networks that are stable, and the networks that are efficient or egalitarian with varying tie costs. We have two main results. First, sparse networks consisting of only dyads and odd-sized cycles are both stable and egalitarian over a wide range of tie costs. Second, we find that at low tie costs no even-sized network exists that is both stable and efficient; we call this situation a 'social dilemma', i.e., actors end up in networks where actors are unwilling to delete a tie to reach a network in which all actors are better off.

We proceed as follows. In Section 2 we review the sociological exchange literature and explain why we use Friedkin's Expected Value Theory for determining exchange benefits in exchange networks. We also outline the rules pertaining how actors engage in exchange. In Section 3, we define network stability, efficiency, and equality. In Section 4, we analyze the efficiency and equality of stable networks as a function of tie costs. We present simulation results for networks up to size 8 and prove three general theorems. These general theorems hold for networks of any size and hold for all published theories of network exchange. We conclude with a discussion in Section 5.

2. Theoretical background

An exchange network is a set of actors and their exchange relations within this set. Fig. 1 depicts some examples of exchange networks. In these networks, connected actors can exchange with each other. In sociological research on exchange networks, two assumptions have been commonly made that we also make in the present study (e.g., Willer, 1999; special issue Social Networks, June 1992; special issue Rationality and Society, January 1997). First, an exchange relation is represented as an opportunity to divide an exchange benefit of 24. Exchange occurs if two connected actors can agree on a division. If they do not agree they obtain nothing in that relation. Second, actors can only engage in one exchange, the so-called one-exchange rule. Applied to the Line 3 (see Fig. 1), the one-exchange rule implies that the central actor can exchange with one of the two peripheral actors, but not with both.

Theories of network exchange predict how actors divide exchange benefits given the network structure. They specify the expected exchange benefits for each actor in the network. Many theories of network exchange have been developed and tested in the last three decades; power-dependence theory (e.g., Cook and Emerson, 1978; Cook and Yamagishi, 1992), exchange-resistance theory (e.g., Skvoretz and Willer, 1993), a graph analytic theory using the graph-theoretic power index (GPI) (e.g., Markovsky et al., 1988), core theory (e.g., Bienenstock and Bonachich, 1992), optimal seek theory (Willer and Simpson, 1999), identity theory (Burke, 1997), Yamaguchi's (1996, 2000) rational choice model, expected value theory (e.g., Friedkin, 1992), non-cooperative bargaining models (Berg and Panther, 1998; Braun and Gautschi, 2006), and a recent model that takes sequentiality of exchange into account (Buskens and Van de Rijt, 2008a). Four theories have received more attention than the other theories (Willer, 1999; special issue Social Networks, June 1992; special issue Rationality and Society, January 1997); core theory, power-dependence theory, expected value theory, and NET, which is a combination of exchange-resistance, GPI, and optimal seek theories.

We use one of these exchange theories to determine what exchange benefits actors obtain in a given network. Once exchange benefits are determined we can compare the differences in benefits of actors across different networks. We can then compute the exact effect of adding and deleting ties on actors' exchange benefits. Hence, an exchange theory is required that generates a unique point prediction for the exchange benefits of each actor in each exchange network. Since core theory and power-dependence theory do not provide unique point predictions they are not suitable

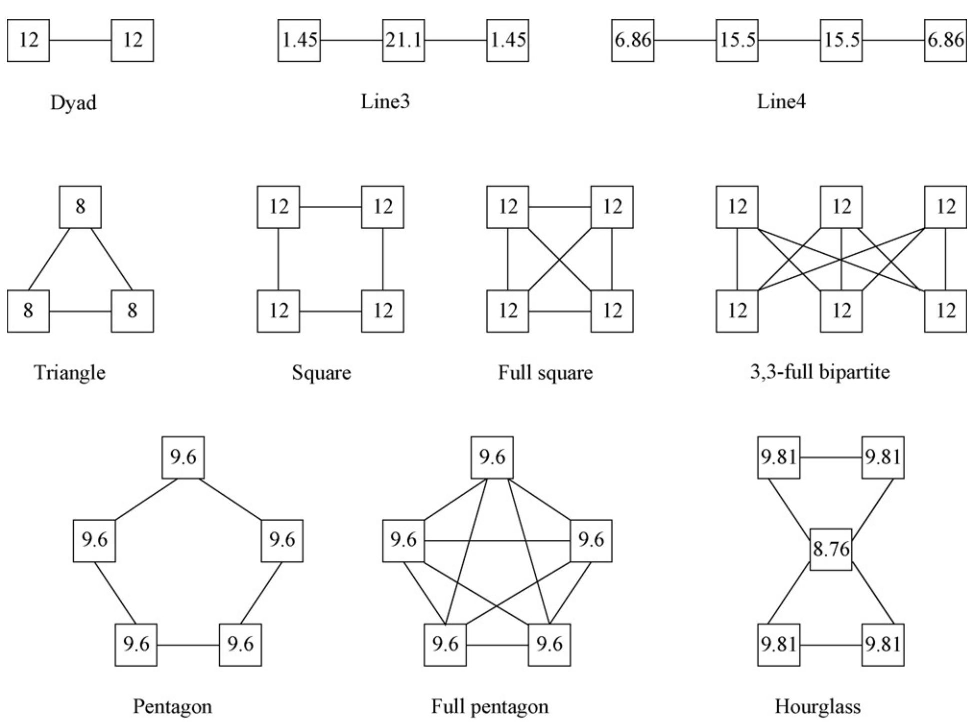

Fig. 1. Networks with expected exchange benefits based on EVT.

for our investigation. We did not select NET because there are several different versions of the theory, and the most recent version of NET advocated by its developers has not been computerized yet (Emanuelson, 2005; Willer and Emanuelson, 2008). Consequently, we select the remaining theory: Friedkin's (1992, 1993, 1995) expected value theory (EVT). Although uniqueness and existence for all networks is not proven, for every network up to 8 actors, the algorithm of EVT generates a point prediction. Previous research suggests that EVT, like other theories of network exchange, predict the outcomes of the exchange networks that are realized in the lab with reasonable accuracy (e.g., Friedkin, 1995; Van de Rijt and Van Assen, 2008).

After establishing the exchange benefits associated with each position in each network by using EVT, differences in benefits of actors across networks can be assessed. A number of studies have examined the effects of adding and deleting ties to one actor on his expected benefits, his neighbors' expected benefits, and the overall power differences or variance in benefits in the network (Leik, 1991, 1992; Willer and Willer, 2000; Van Assen and Van de Rijt, 2007). These studies examine the effect of exogenous network changes, whereas we focus on endogenously stable networks. That is, the actors in their studies are passive with respect to changing ties, while the actors in our study actively consider deleting and adding ties to maximize their expected benefits.

In our model, while the exact strategy space of the actors depends on the stability concept employed, the unilateral deletion and bilateral addition of a tie is shared by all stability concepts considered in this paper. An actor deletes a tie if the deletion results in a network in which he obtains a larger payoff. A pair of actors adds a tie between them if a network results in which at least one of them obtains a better payoff and none of them fares worse. Actors thus only care about immediate improvements in expected payoffs that are a direct consequence of their tie change. They do not take potential future rewards into account that could result from subsequent tie changes by other actors. They are *myopic* maximizers. Since both actors in a tie pay the tie costs, establishing a link between two actors requires mutual consent, whereas deletion of a tie is unilateral.

In this study it is assumed that *equal* tie costs are incurred for *both* actors involved in a tie. Tie costs apply for each tie that an actor has. We vary the tie cost from 0 to 12 to analyze the impact of tie costs on the stable, efficient, and egalitarian networks. Since the amount to be divided in an exchange is 24, there always exists an actor who wants to delete a tie for tie costs higher than 12. Such costs need not be considered. In this paper, we use the term expected benefits to refer to the expected benefits from exchange minus the tie costs incurred.

2.1. Expected value theory

Building upon the theory of social power proposed by French (1956), Friedkin (1986) first suggested the idea of using expected values to predict the outcomes in a power structure. Friedkin (1992, 1993) extended the idea of expected values to analyze outcomes in an exchange network. Friedkin's model predicts the probability with which each maximal exchange pattern occurs, and the distribution of outcomes in each one of these patterns. A maximal exchange pattern is maximal in the sense that no further feasible transaction exists between the actors that have not exchanged yet. For example, the Line 4 (see Fig. 1) has two maximal exchange patterns: each peripheral actor exchanges with one of the central actors, or the two central actors exchange with each other. Using an iterative algorithm, each actor's expected exchange benefits are calculated as the expected value of his exchange benefits over all possible maximal exchange patterns.

Despite what the name suggests, EVT is not a theory based on actors rationally maximizing their benefits. The algorithm generating the predictions assumes that both actors' claim of their share in their relation increases non-linearly in the probability that each of them is excluded in any exchange. Three rules determine the final allocation in the relation. Which rule is applied depends on the sum of both actors' claims and their claims relative to half of the exchange benefits to be divided. See Friedkin (1995) for details of the EVT model. An inconvenience of the EVT model is the analytical intractability of the algorithm because of the non-linear function and the three rules embedded in it. Another characteristic of EVT is that its predictions satisfy symmetry (i.e., automorphically equivalent positions get the same payoff), however, invariance under scalar multiplication is violated (relative share is affected if all resource pools are multiplied with the same constant). EVT implies that the relative share of an actor decreases if the resource pool is increased. See Van de Rijt and Van Assen (2008) for an overview of properties of EVT and other theories of network exchange.

Fig. 1 shows expected exchange benefits predicted by EVT for several networks. For example, in the Line 3, it is expected that the peripheral actor who exchanges with the central actor receives 2.9 out of 24, while the central actor obtains 21.1. Because peripheral actors are only expected to exchange half of the time, their expected benefit is 1.45.

3. Definitions of stability, equality, and efficiency

We borrow our definitions of what constitutes a stable network from a rapidly growing literature on network formation in economics (e.g., Dutta and Jackson, 2003; Demange and Wooders, 2005; Goyal, 2007). Some networks considered in this literature are similar to exchange networks. By doing so, we bring together research on network exchange in sociology and research on network formation in economics.

3.1. Stability

Jackson and Wolinsky (1996) introduced the *pairwise stability* concept. In his survey on network formation Jackson (2003) argues that pairwise stability might be considered a necessary condition for network stability. It is the weakest notion of stability that allows for tie formation while providing narrow predictions about the set of stable networks. An exchange network is *pairwise stable* if (i) adding a currently *absent* tie is costly to at least one of the two actors involved or leaves both actors equally well off, (ii) removing a *present* tie does not benefit either of the two actors it currently connects.

As an example of how to use the notion of pairwise stability, consider an 8-actor network where one actor is connected to three other actors, but not to the remaining four actors. This can be denoted by an 'adjacency row' 1110000, where the 1's indicate the three existing relations and the 0's the four absent relations. Then, with regard to the focal actor, pairwise stability holds if no single change from 1 to 0 (the deletion of one tie) increases this actor's expected benefits, and if no single change from 0 to 1 (the addition of one tie) increases the expected benefits of the focal actor while not decreasing the expected benefits of the other actor in this relation. A network is pairwise stable if the conditions above hold for each actor in the network.

In our analysis we also use two stronger stability concepts that are refinements of pairwise stability: pairwise Nash and unilateral stability. Pairwise Nash is a refinement of pairwise stability, and unilateral stability is a refinement of pairwise Nash. We use these two additional concepts for two reasons. First, we want to examine whether our results are robust against alternative stability concepts. Second, we are able to prove two general theorems using pairwise Nash, which we cannot by using pairwise stability.

A network is *strongly pairwise stable* (Gilles and Sarangi, 2004) or *pairwise Nash* (Calvó-Armengol and İlkılıç, 2005) if (i) adding a presently *absent* tie is costly to at least one of the two actors or leaves both actors equally well off; and (ii) removing a subset of an actor's *present* ties does not benefit this actor. Note that condition (i) of pairwise Nash is identical to condition (i) of pairwise stability. The difference between pairwise stability and pairwise Nash is that pairwise Nash allows for simultaneous deletions of ties. Continuing our previous example, consider again the 8-actor network with adjacency row equal to 1110000 for the focal actor. Pairwise Nash holds for this actor if each change of *one* 0 to 1 does not increase his expected benefits and the other actor is not worse off, and each change of a subset of 1s to 0s does not increase the focal actor's expected benefits. A network is pairwise Nash if this condition holds for each actor in the network.

A network is *unilaterally stable* (Van de Rijt and Buskens, 2007; Buskens and Van de Rijt, 2008b) if no actor can profitably reconfigure his ties without objection by his *new* contacts. Different from pairwise stability, unilateral stability allows for simultaneous addition and deletion of ties such that actors can replace one tie with another as long as the addition of each new tie does not make the actor in the new tie worse off.[1] In our example, unilateral stability holds for the focal actor if no adjacency row other than 1110000 simultaneously (i) increases this actor's expected benefits, *and* (ii) makes no actor who is connected to the focal actor in the new network but disconnected in the old network worse off. Only the actors newly connected have to agree, because only for creating new ties mutual consent is required. A network is unilaterally stable if this condition holds for each actor in the network.

3.2. Egalitarian networks

Networks are defined as egalitarian if all actors in the network obtain the same expected benefits in which the subtraction of the tie costs is included. Some examples of egalitarian networks are complete networks, cycles, and even-sized networks consisting of only dyads. We will categorize stable networks that are egalitarian.

3.3. Efficiency

With the introduction of tie costs inefficient pairwise stable networks might arise. We distinguish two forms of efficiency. First, a network is *socially efficient* if, given tie costs, there is no other network in which the sum of the expected benefits is larger than in this network. Second, a network is *Pareto efficient* if there exists no other network in which no actor earns less and at least one actor earns more than in the given network. We will investigate if there are tie cost levels for which no pairwise stable network of a given size is Pareto efficient. If such tie cost levels exist, we say that there is a tension between efficiency and stability and say that a *social dilemma* exists. It is easy to see that social efficiency implies Pareto efficiency.

4. Results

Friedkin's EVT is a system of many assumptions and equations that makes a formal analysis very difficult. However, we prove some general results on stability that concern networks of any size and do not depend on the use of EVT as a prediction method of exchange benefits. In addition, to get more insight into the properties of stable networks, we analyze a subset of exchange networks; all 13,597 non-isomorphic exchange networks of size 2 through 8. We first present simulation results for these small networks obtained with EVT, and then prove our general results.

4.1. Simulation results

For all exchange networks of size 2 through 8, we computed at what tie cost levels each exchange network is pairwise stable, pairwise Nash, or unilaterally stable. In order to find these cost levels, we calculated for each network, the highest cost level h for which some pair of actors still wants to add a tie. In addition, we calculated the lowest cost level l for which some actor still wants to remove one or more ties (depending on the stability concept). Whenever the resulting interval $[h, l]$ is not empty, the network is stable for the cost levels in this interval. The reason is that for these cost levels, tie costs are too high to add ties and too low to remove ties. Because we also need to check for simultaneous removal and addition of ties to establish unilateral stability, some more conditions need to be checked for unilateral stability. The details of this procedure can be obtained from the authors.[2]

A first general observation is that there are rather few networks (at least for not too small network size) that are stable within some tie cost range, and that the proportion of stable networks decreases quickly with network size. If network size is 6, 17 out of 156 networks are pairwise stable of which 16 are pairwise Nash and 13 are unilaterally stable. This is about 10% of the total number of networks of size 6. When network size is 7, 40 out of 1044 networks are pairwise stable, of which 34 are pairwise Nash and 30 unilaterally stable. Here the proportion of stable networks is around three percent. Finally, for network size 8, 105 out of 12,346 networks (less than 1%) are pairwise stable of which 93 are pairwise Nash and 72 are unilaterally stable.

Table 1 provides detailed information for a subset of stable networks, namely those that are pairwise stable over a tie cost range spanning at least 1 exchange point (out of 24). We use the names triangle, square, pentagon, hexagon, etc., for cycles of 3, 4, 5, 6, etc., actors. The word "full" is added if we refer to the complete network with the same number of actors (see also Fig. 1). Table 1 reports lower and upper bounds on the tie cost intervals for which the networks are stable. The lower bound for pairwise Nash is not reported because it coincides with the lower bound for pairwise stability. The reason is that they both refer to the highest cost level for which some pair of actors still wants to add a tie. The table also reports the density, tie costs for which the network is egalitarian, and tie costs for which the network is both pairwise stable and Pareto efficient. Stable networks in Table 1 consist of either one component or of multiple disconnected components in which everybody has an automorphically equivalent position and therefore expects the same benefits. Between components there can be differences in expected benefits. Note that if all components are identical, stable networks are egalitarian irrespective of the tie costs.

Social efficiency is not included in the table because for positive tie costs the only socially efficient networks that exist are the dyads, possibly in combination with one isolate if the number of actors is odd. We call the unconnected dyads and at most one isolate M(inimal) networks. Given nonzero tie costs M networks are the unique socially efficient networks. In M networks the maximum number of exchanges is realized with a minimal number of ties.

[1] If a network is unilaterally stable, then it is also pairwise stable, but the opposite is not true. Hence it is possible that a network is pairwise stable at some cost level but not unilaterally stable; even if it is not profitable to add a tie, or delete ties, an actor might profitably reconfigure his network by adding more than one tie, or replacing some of his ties.

[2] All calculations were programmed in Borland Delphi. The source code is available from the authors upon request.

Table 1
Pairwise stable networks that are stable in a tie cost range larger than one.

Network	Size	Lower bound for pairwise stability	Upper bound for pairwise stability	Upper bound for pairwise Nash	Lower bound for unilateral stability	Upper bound for unilateral stability	Density	Tie costs for which egalitarian	Tie costs for which pairwise stable and Pareto efficient
Dyad	2	−∞	12	12	−∞	12	1	Always	≤12
Dyad, isolate	3	1.449	12	12	1.449	12	0.333	12	1.449–12
Triangle		−∞	6.551	4	−∞	4	1	Always	<4
Two dyads	4	3.482	12	12	3.482	12	0.333	Always	3.482–12
Triangle, isolate		4.114	6.551	Not pairwise Nash	Not unilaterally stable		0.5	4	Never
Square		2.811	5.138	5.138	2.811	5.138	0.667	Always	Never
Full square		−∞	4.026	3.943	−∞	3.943	1	Always	≤0
Two dyads, isolate	5	3.482	12	12	3.482	12	0.2	12	3.482–12
Square, isolate		2.811	5.138	5.138	Not unilaterally stable		0.4	6	Never
Triangle, dyad		−0.536	6.551	4	0.488	4	0.4	Never	−0.536 to 4
Pentagon		0.124	5.95	4.8	0.366	4.8	0.5	Always	0.124–4.8
Full pentagon		−∞	0.839	0.839	−∞	0.839	1	Always	≤0
Three dyads	6	3.482	12	12	3.482	12	0.2	Always	3.482–12
Triangle, dyad, isolate		4.114	6.551	Not pairwise Nash	Not unilaterally stable		0.267	Never	Never
Pentagon, isolate		3.326	5.95	4.8	Not unilaterally stable		0.333	4.8	Never
Square, dyad		2.811	5.138	5.138	2.811	5.138	0.333	0	Never
Hexagon		2.723	5.15	5.15	2.723	5.15	0.4	Always	Never
3,3-Full bipartite		1.941	3.007	3.007	1.941	3.007	0.6	Always	Never
Two triangles		1.553	6.551	4	Not unilaterally stable		0.4	Always	Never
Full square, dyad		1.178	4.026	3.943	1.178	3.943	0.467	0	Never
Full hexagon		−∞	2.104	2.033	−∞	2.033	1	Always	≤0
Three dyads, isolate	7	3.482	12	12	3.482	12	0.143	12	3.482–12
Square, dyad, isolate		2.811	5.138	5.138	Not unilaterally stable		0.238	Never	Never
Hexagon, isolate		2.78	5.15	5.15	Not unilaterally stable		0.286	6	Never
Triangle, two dyads		3.482	6.551	4	3.482	4	0.238	Never	3.482–4
Pentagon, dyad		0.124	5.95	4.8	0.622	4.8	0.286	Never	0.124–4.8
Two triangles, isolate		4.114	6.551	Not pairwise Nash	Not unilaterally stable		0.286	4	Never
Heptagon		0.796	5.769	5.143	0.796	5.143	0.333	Always	0.796–5.143
Square, triangle		2.811	5.138	4	2.811	4	0.333	Never	Never
Full square, triangle		−0.042	4.026	3.943	2.977	3.752	0.429	4	−0.042 to 0
Full pentagon, dyad		−0.662	0.839	0.839	0.432	0.839	0.524	Never	−0.662 to 0
Full heptagon		−∞	0.395	0.395	−∞	0.395	1	Always	≤0
Four dyads	8	3.482	12	12	3.482	12	0.143	Always	3.482–12
Triangle, two dyads, isolate		4.114	6.551	Not pairwise Nash	Not unilaterally stable		0.179	Never	Never
Pentagon, dyad, isolate		3.326	5.95	4.8	Not unilaterally stable		0.214	Never	Never
Square, two dyads		3.482	5.138	5.138	3.482	5.138	0.214	0	Never
Heptagon, isolate		3.059	5.769	5.143	Not unilaterally stable		0.25	5.143	Never
Hexagon, dyad		2.723	5.15	5.15	2.723	5.15	0.25	0	Never
Square, triangle, isolate		4.114	5.138	Not pairwise Nash	Not unilaterally stable		0.25	Never	Never
3,3-Full bipartite, dyad		1.941	3.007	3.007	1.941	3.007	0.357	0	Never
Two squares		2.811	5.138	5.138	2.811	5.138	0.286	Always	Never
Octagon		1.635	5.282	5.2	1.944	5.2	0.286	Always	Never
Two triangles, dyad		1.553	6.551	4	Not unilaterally stable		0.25	Never	Never
Pentagon, triangle		0.774	5.95	4	2.47	4	0.286	Never	Never
Square, full square		2.811	4.026	3.943	2.811	3.943	0.357	0	Never
Two full squares		1.456	4.026	3.943	1.456	3.876	0.429	Always	Never
Full hexagon, dyad		0.594	2.104	2.033	0.594	2.033	0.571	0	Never
Full octagon		−∞	1.326	1.254	−∞	1.254	1	Always	≤0

The number of networks that are pairwise stable in some cost range smaller than 1 and hence not included in the table for size 2 to 8 are respectively, 0, 0, 0, 2, 7, 28, and 88. There is one other network for size 6 that is egalitarian and stable in a small range and one that is egalitarian but not stable. There is also one other egalitarian network with 7 actors that is stable in a small range. There are 7 other networks of size 8 that are egalitarian from which 6 fulfill all stability criterions for a small range. The remaining network is unstable.
There are more other networks that are pairwise stable and Pareto efficient especially for sizes equal to 5 and 7 for small cost ranges. Examples are the hourglass and the hourglass + dyad, which are both pairwise stable and Pareto efficient for cost = 1. All these other networks are non-egalitarian and have a subset of actors who are relatively well off.
A complete overview of the ranges for which networks of sizes 2–8 are stable is available from the authors.

Total exchange benefits can never be larger and costs are minimized in *M* networks. In Table 1 we see that all socially efficient *M* networks are stable if tie costs are large enough, larger than 3.48 in the case of EVT. For even-sized networks, these networks are also egalitarian, and Pareto efficient. For odd-sized networks, there is some inequality in these networks because one actor obtains nothing.

The density of stable networks in general decreases with tie costs, although not monotonically. As an example, consider the pairwise stable networks of size 4. All pairwise stable networks of size 4 are listed in Table 1. For tie costs in the interval [0, 4.026], the complete network with density 1 is pairwise stable. The square (4-cycle) with density 2/3 is pairwise stable for tie costs in [2.811, 5.138]. The *M* network (2 dyads) with density 1/3 is pairwise stable when tie costs are in [3.482, 12]. A triangle and an isolate, with density 1/2, is pairwise stable for tie costs in [4.114, 6.551]. Interestingly, the often investigated Line 4 network is not stable at any tie cost; for tie costs larger than 3.482 the actors with two ties prefer to delete their connection in order to form a dyad, and for tie costs smaller than 5.138 the two peripherals prefer to be connected to each other. Hence, our analysis suggests that in real-life exchange settings that resemble the point of departure of our analysis, the Line 4 network will not occur often.

Because stable networks at low tie costs have a higher density than the socially efficient *M* networks, a social dilemma situation might exist. Indeed, if tie costs are smaller than 3.48 the even-sized *M* network is not stable (see also Theorem 2 below) but Pareto dominates the stable networks for such tie costs. Hence EVT predicts social dilemma situations for tie costs smaller than 3.48 in even-sized exchange networks of size 4, 6, 8; the networks will evolve into stable networks that are 'overconnected' and Pareto dominated by the *M* networks. In odd-sized networks, there exist other Pareto efficient stable networks in addition to the *M* network because all actors gain more than the isolate in the *M* network.

Although most networks listed in Table 1 as well as the other stable networks are not egalitarian, differences in exchange benefits across actors are mostly not large. First, no pairwise stable network is a so-called 'strong power' network (Willer, 1999, pp. 109–111), i.e., there is no network up to size 8 with a minority of actors earning almost all points in their exchange relations. The largest benefit differences arise in odd-sized *M* networks. Within components some actors might earn a bit more than others, the hourglass network with five actors (see Fig. 1) is one example, but if power difference becomes too large within components, there are always actors in these components who do seek other exchange relations, rendering the network unstable.

4.2. Analytic results

The analysis of networks up to size 8 demonstrates that *M* networks are important: they are efficient, stable for a large tie cost interval, and egalitarian if the network size is even. Here we show some general results on the efficiency (Theorem 1) and stability (Theorems 2 and 3) of *M* networks of any size. Throughout the remainder of the section, the only assumptions made about the distribution of exchange benefits with no side payments are[3] (i) if two actors can exchange with one another and do not exchange with third parties then they will exchange with one another, and (ii) if they both have only one exchange relation they divide exchange benefits in that relation equally. With the exception of core theory – which leaves exchange divisions in isolated dyads unspecified – all exchange theories that have been proposed in the sociological literature satisfy these requirements.

As we have already argued in the previous subsection, *M* networks are socially efficient networks and, therefore, they are also Pareto efficient as is summarized in the first theorem.

Theorem 1. *An M network is socially efficient (and therefore also Pareto efficient) for any tie cost c in the closed interval [0, 12].*

Proof of Theorem 1. Total benefits are maximal because the maximum number of exchanges, $n/2$ (rounded down if n is odd), is always carried out. Any additional tie reduces the total profit by $2c$. Also, the marginal benefit of each tie in the *M* network is 24, which is the highest possible marginal benefit a tie can have. For c in [0, 12], the increase in net total benefit from each of these ties is nonnegative. □

In particular, *M* networks are the *only* socially efficient networks for tie costs in the interval (0, 12). There are other socially efficient networks if tie costs are equal to 0, because in that case any network that guarantees the maximal number of exchanges is socially efficient. There are also other Pareto efficient networks. For even-sized networks, these Pareto efficient networks are non-egalitarian. By definition an *M* network has the minimum number of ties with the maximum number of exchanges. Hence, any Pareto efficient network other than the *M* network has more ties than the *M* network. If another Pareto efficient network is also egalitarian, then the *M* network Pareto dominates it for any tie cost larger than zero, which is a contradiction. None of the other Pareto efficient even-sized stable networks are included in Table 1, since these networks are stable only for narrow tie cost ranges. For odd-sized networks, many Pareto-efficient stable networks exist. One of these networks is the cycle with all actors involved, which is egalitarian and stable over a wide range of tie costs.

M networks are unilaterally stable (and hence pairwise Nash, and pairwise stable) for high tie costs. For pairwise Nash and pairwise stability, this result can be generalized to networks of any size and any exchange theory, as Theorem 2 shows. Consider a value $d = \max(e - 12, \min(f - 12, g))$, where e is the exchange benefit of a central actor in the Line 4, f the exchange benefit of the central actor in the Line 3, and g the exchange benefit of the peripheral actor in the Line 3. For example, $d = 3.48$ for EVT, since $e = 15.48$, $f = 21.1$, and $g = 1.45$.

Theorem 2. *An M network of any size is pairwise Nash (and therefore also pairwise stable) for tie costs c in (d, 12)*

Proof of Theorem 2. Because everyone with a tie obtains benefits from the exchange equal to 12 in an *M* network, no one wants to remove a tie as long as the tie costs are below 12. Since actors can only add one tie, there are only two possible additions. First, if two dyads connect with each other the connecting actors see their exchange benefits raise to e. Thus, to make this change unprofitable, tie costs should exceed the marginal benefits of these actors, which equal $e - 12$. In odd-sized networks the isolate can try to connect to a dyad, raising his partner's expected benefits to f, obtaining g himself. For this tie addition to be unprofitable, tie costs should exceed the lowest marginal benefit across the two actors, which is $\min(f - 12, g)$. Therefore, pairwise Nash stability is established for the interval $(d, 12)$. □

Table 1 suggests that given EVT benefits, the *M* network is the only pairwise stable network at tie costs larger than 6.55. Networks other than *M* networks that are pairwise stable at, e.g., $c = 6.5$, consist of combinations of triangles, dyads, and at most one isolate. We were unable to prove that *M* networks are the only pairwise stable networks for any tie costs c in (6.55, 12) using EVT. Nonetheless, we can prove that with tie costs c in (6, 12), the only pairwise Nash

[3] The assumptions are related only to the distribution of exchange benefits. Hence the results apply to any exchange distribution mechanism given that (i) the networks are undirected under the 1-exchange rule, (ii) there is mutual consent for tie addition, (iii) tie deletion is unilateral, and (iv) equal tie costs are incurred for both parties in the tie.

and unilaterally stable networks are *M* networks regardless of the theory used. Note that triangles are not pairwise Nash for $c > 4$ since one actor can improve his payoffs with $2c - 8$ by deleting two links at the same time.

Theorem 3. *The only pairwise Nash (and unilaterally stable) networks with c in (6, 12) are M networks.*

Proof of Theorem 3. The networks with fewer ties than *M* networks are not stable since tie addition is profitable for two isolates. Hence, it suffices to show that in any network with more ties than an *M* network with *c* in (6, 12) at least one actor wants to delete at least one of his ties. Such a network has at least one actor with two ties. Let us first assume everyone has more than one tie. Then everyone pays more than 12 for the ties. However, not everybody can earn more than 12. Thus, someone has negative net earnings and is better off deleting all ties. Hence, there should be at least one actor with one tie who has a neighbor with at least two ties (otherwise we have the *M* network). Let us label the actor with one tie as A, A's neighbor with at least two ties as B, and one of B's neighbors other than A as C. Actor A should have at least an expected benefit *c* from exchange, otherwise he would remove the tie. Actor B should have at least $12 + c$ as exchange benefit from his two ties, otherwise he would remove all his ties except with A, and get an exchange benefit of 12. But actor B can earn at most $24 - c$ from his exchange with A which is less than $12 + c$ for *c* in (6, 12). So actor B should earn more than $24 - c$ in at least one of his other exchanges, and without loss of generality this neighbor is C. However, now actor C earns less than *c* from his exchange with B, and hence actor C prefers to delete the tie. □

5. Discussion

Research on network exchange in sociology has focused on the effect of the social structure on outcomes of exchange. In almost all of this research, the exchange network has been exogenous and the independent variable. The main result of this research is that network structure has a large impact on what actors earn in their exchange relations. Because positions in different networks obtain different benefits, there exist incentives for actors to change the network. Subsequently, the questions of how these networks evolve, and which networks are stable arise. In the current article we study what the network structure looks like if actors have the opportunity to choose with whom they have an exchange relation, that is, the network structure is our dependent variable.

We investigate the networks that are stable, the networks that are efficient or egalitarian with varying tie costs, and the occurrence of social dilemmas. We employed the assumptions mostly used in the sociological literature on exchange networks, including the one-exchange rule and that the value of each exchange relation is the same. Additionally, we assume that actors delete and add ties in order to maximize their expected benefits; for adding ties mutual consent is needed, whereas tie deletion is unilateral. Ties are costly and the costs are the same for both actors in the tie. To assess the stability of exchange networks we employ three stability concepts from the economic literature on networks: pairwise stability, pairwise Nash, and unilateral stability. In addition, social efficiency and Pareto efficiency are used as efficiency measures, and the networks are considered egalitarian if all actors in the network expect the same benefits. To calculate the actors' expected benefits we use Friedkin's Expected Value Theory. First, we investigate all networks up to size 8. Then we prove three general results.

Few networks are stable over a wide range of tie costs, and all of them can be divided into two types: *M* networks consisting of only dyads and at most one isolate, and cycles with an odd number of actors. Even-sized *M* networks and odd cycles are egalitarian. As we have shown, *M* networks of any size are the only *socially* efficient networks for any tie costs but they are only stable starting from intermediate cost levels. Hence, any stable network is socially *in*efficient at low tie costs. Finally, we observe social dilemmas at low tie costs in even-sized networks; that is, none of the (*socially* inefficient) stable networks are *Pareto* efficient. Using pairwise Nash or unilateral stability does not change any of these results.

A remarkable result is that almost all egalitarian networks up to size 8 are stable at some tie costs. Although actors in our model maximize their own expected benefits that do not include equality preferences, they can be "satisfied" in egalitarian networks. Nonetheless, many non-egalitarian networks are stable as well. Thus, it is an interesting empirical question whether exchange networks evolve into non-egalitarian networks in real world settings. To what extent actors' preferences for equality [as in Fehr and Schmidt, 1999] *do* affect the stability of networks, and whether these preferences are observed in the evolution of (exchange) networks remains to be examined.

Although there exist stable networks that are also efficient at some tie costs, most stable networks are not efficient. Hence, tensions between stability and efficiency arise over a wide range of tie costs. This tension is strongest in even-sized networks at low tie costs where social dilemmas occur, because at such tie costs no stable network is Pareto efficient. It is an interesting empirical question whether actors resist the immediate temptation and end up in the both socially and Pareto efficient but not stable *M* network. Preferences for efficiency can be incorporated in the actors' utility function as in Charness and Rabin (2002), and the effect of efficiency and equality preferences on the evolution and stability of networks can thus be examined.

The results of our study can be compared to those of Bonacich (2001), and Van de Rijt and Buskens (forthcoming) who studied the dynamics of sociological exchange networks. Bonacich (2001) simulated exchange network evolution and found that, in equilibrium, differences in benefits were small. One important difference between our analysis and Bonacich's analysis is that in his simulation the actors are assumed to be myopic *satisficers* such that they change the network if their earnings drop below a certain level. In our study however, the actors are assumed to be myopic *maximizers* such that they change the network as long as marginal benefits outweigh marginal tie costs. Another difference is that instead of adding or deleting ties, Bonacich' actors move to another cell on a checkerboard, where they can exchange with actors in adjacent squares. Interestingly, despite these different assumptions, he also arrives at the conclusion that in equilibrium networks are egalitarian. Bonacich (2004) provides some intuitions for how this approach can be extended to more general structures. Van de Rijt and Buskens (forthcoming) analyzed the stability of exchange networks employing the same stability concepts, but using another exchange theory, and without focusing on efficiency and equality. Our stability results are very similar to the ones reported in their paper, and confirm the robustness of the dynamics results across exchange theories.

The focus of the present article is on the possible path ends of the evolution of exchange networks, i.e., the stable networks. Consequently, a natural extension is investigating how to get to these possible endpoints, i.e., the evolution of the network. The probability of reaching each stable network could also be investigated. Factors that could determine these probabilities are the initial network configuration, tie costs, and preferences for equality and efficiency. It might also be that stable networks differ in their robustness to the errors of actors. In some stable networks one error might be enough to move to another stable network, whereas other stable networks might be stable after one or more accidental errors. Such issues remain to be investigated both in the laboratory, and by theory or simulation.

In our analysis we assume that actors are myopic; they only take their immediate benefits into account but not the consequences of their behavior on future benefits. The justification of this assumption is that not only it makes the analysis more tractable, but also that actor behavior in the laboratory seems to be rather myopic as well. Most actors are found to think two, or at most three steps ahead in many different experimental games (Camerer, 2003, Chapter 5). Nevertheless, in their book on network formation Dutta and Jackson (2003, p. 13) argue that allowing for farsighted actors in models of network evolution is "perhaps the most important (and possibly the hardest) issue regarding modeling the formation of networks". Actors taking the future into account are likely to affect the evolution and stability of exchange networks. To mention two examples, taking the future into account gives opportunities to solve social dilemmas and the problems of inefficiency of stable networks as described above. An even-sized M network that is efficient but not stable can be sustained as equilibrium if the actors are rational and the 'network evolution game' is indefinitely repeated. Similarly, an odd-sized M network that is efficient but neither stable at low tie costs nor egalitarian can be sustained as equilibrium if the shadow of the future is sufficiently long, and if the actors coordinate on alternating the excluded actor. Alternating the excluded actor ensures that both efficiency and equality are satisfied.

The implications of the current study extend to other social networks among which are communication, knowledge, and friendship networks. In these networks similar questions of stability are of interest such as whether only few networks are stable, and whether stable networks tend to be egalitarian. As Jackson and Wolinsky (1996) showed with a theorem in their seminal paper, there exist a tension between stability and efficiency in different economic and social network contexts. Thus, analyzing whether actors form socially inefficient but stable networks that correspond to social dilemma situations is particularly appealing. For example, Buskens and Van de Rijt (2008b) show that efficiency, stability, and equal divisions of outcomes can also go hand in hand even in competitive network formation settings in which everybody strives for a central network position. Considering the importance of network studies in aforementioned fields, more of such analyses are to be expected.

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