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Published in:
Astronomy and astrophysics

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2008

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

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A clumpy-cloud PDR model of the global far-infrared line emission of the Milky Way

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Received July 12, 2008; accepted

ABSTRACT

Context. The fractal structure of the interstellar medium suggests that the interaction of UV radiation with the ISM as described in the context of photon-dominated regions (PDR) dominates most of the physical and chemical conditions, and hence the far-infrared and submm emission from the ISM in the Milky Way.

Aims. We investigate to what extent the Galactic FIR line emission of the important species CO, C, C^+, and O, as observed by the Cosmic Background Explorer (COBE) satellite can be modeled in the framework of a clumpy, UV-penetrated cloud scenario.

Methods. The far-infrared line emission of the Milky Way is modeled as the emission from an ensemble of clumps with a power law clump mass spectrum and mass-size relation with power-law indices consistent with the observed ISM structure. The individual clump line intensities are calculated using the KOSMA-γ PDR-model for spherical clumps. The model parameters for the cylindrically symmetric Galactic distribution of the mass density and volume filling factor are determined by the observed radial distributions. A constant FUV intensity, in which the clumps are embedded, is assumed.

Results. We show that this scenario can explain, without any further assumptions and within a factor of about 2, the absolute FIR-line intensities and their distribution with Galactic longitude as observed by COBE.


1. Introduction

The importance of the interaction between the interstellar UV-radiation and dense clouds in the ISM, determining the physical and chemical conditions in the molecular clouds, which are modelled as so-called photon-dominated regions (PDRs), has been recognized since the first observations of the dominant cooling lines in the FIR, [C ii] 158 µm (Russell et al. 1980, 1981; Stutzki et al. 1988; Mizutani et al. 1994), [O i] 63 µm (Melnick et al. 1979; Stacey et al. 1983), and the mid- and high-J CO lines (Storey et al. 1981; Watson et al. 1983; Jaffe et al. 1987; Boreiko & Betz 1991). Starting from the first PDR-models, tailored to explain the FIR line emission from massive star forming regions such as Orion (Stanimirović & Dalgarno 1989), the inclusion of more details of the heating and cooling mechanisms as well as of the chemical network, nowadays allows the modeling over a wide range of physical parameters and has resulted in successful modeling of many detailed aspects of observed photon-dominated regions (review by Hollenbach & Tielens 1999). PDRs thus have proven to be a very useful concept in understanding the mutual interaction between star formation and the structure of the ISM, both in individual regions in the Milky Way but also in star forming regions in external galaxies (Kramer et al. 2005; Contursi et al. 2002; Madden et al. 2001; Unger et al. 2000; Nikola et al. 1998; Schilke et al. 1993). An overview of the different models is given by the recent comparison study of PDR codes (Röllig et al. 2007).

The importance of the PDR scenario is related to the fact that the ISM is fractal and thus most of the material is close to surfaces and hence affected by UV radiation, i.e. is located in PDRs. This was realized early on by comparing the spatial distribution of observed PDR tracers (Stutzki et al. 1988; Howe et al. 1991) with simple models of homogeneous or clumpy cloud structures. The observed relatively uniform line ratios of low-J 13CO and 12CO in molecular clouds, inconsistent with simple, uniform cloud models, are shown to be naturally explained if the emission is assumed to originate in many, relatively small clumps (Störzer et al. 1996). The necessity of high densities in order to explain the observed large 13CO brightnesses in the mid-J lines in the submm independently indicate a clumpy structure (Graf et al. 1990; Wolfire et al. 1989).

The fractal structure of the ISM is well represented by an ensemble of clumps with a power law clump mass distribution, and a power law clump mass-size relation (Stutzki et al. 1998). Power law mass spectra have been derived by decomposing the observed 3D-datacubes
of emission of CO isotopologues into clumps by various methods (Stutzki & Güsten 1990, Kramer et al. 1998, Williams & Blitz 1993, Williams et al. 1994). The identified clumps typically show also a power law mass-size relation, although obtaining significant coverage over more than 1.5 to 2 orders of magnitude in length scale is difficult, combining low and high angular resolution observations of the Polaris Flare, Habing units (Heithausen et al. 1998). The paper is organized as follows: In Section 2 we shortly summarize the concepts to quantify the clumpy, fractal cloud structure and introduce the clumpy cloud PDR model. Section 3 discusses the line emissivity of the clump ensemble, in particular the dependence on varying mass limits of the clump distribution. In Section 4 the model results are compared with the observational data from COBE. A short conclusion with outlook is given in Section 5.

1 Habing unit corresponds to the integrated flux in the wavelength range from 91.2 to 111.0 nm of $1.6 \times 10^{-3}$ erg s$^{-1}$ cm$^{-2}$ (Habing 1968).

Fig. 1. Brightness temperature vs. impact parameter $p$ of the CO 7–6 rotational transition for selected clumps with mass and density values following the clump-mass spectrum and mass-size relation. $n_s$ is the gas density at the surface of the clumps in $\text{cm}^{-3}$ (see Eq. 1), $M_\odot$ the clump mass in $\text{M}_\odot$, and the incident FUV-flux is $\chi = 10^2 \chi_\odot$.

2. The galaxy model: an UV-illuminated clump ensemble

Modeling the large-scale Galactic FIR line emission as observed by COBE is done in the following steps:

- generate a large parameter grid for the emission of individual spherical clumps with the KOSMA-τ PDR-model, where the individual clumps are characterized by their mass, density (resp. size), and the UV-intensity that they are embedded in;
- generate a clump mass and size distribution at each Galactocentric radius from the observed mass density and volume filling factor distribution in the Milky Way, assuming an universal power law index for the clump mass and size spectra and applying adequate high and low mass cutoffs;
- estimate the Galactic radial distribution of the effective mean UV-field in which the dense ISM clumps are embedded and the average clump density in the ensemble respectively the volume filling factor of the clump ensemble;
- by combining the above, calculate the volume emissivity of the ISM originating from the UV illuminated clump ensemble as a function of Galactocentric radius in the Milky Way;
- calculate the line emission versus Galactic longitude by line-of-sight integration over this emissivity in the direction towards the Earth.

We describe each of these steps in detail in the following:

2.1. The KOSMA-τ PDR model: line intensities of an individual clump

The KOSMA-τ PDR-model describes spherical clumps characterized by their mass, density (or size, respectively), and the incident UV field.

We treat the mass of the clumps $M_\odot$ as the total hydrogen gas mass. To obtain the total gas mass of a clump the...
Table 1. Properties of the KOSMA-τ PDR-model clumps with given masses and densities at an incident FUV-flux of $10^5 \chi_D$. Orders of magnitude are given in parentheses.

<table>
<thead>
<tr>
<th>Density</th>
<th>Mass</th>
<th>Column densities</th>
<th>Temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface</td>
<td>Center</td>
<td>$M_{\odot}$</td>
<td>$N_{H_2}$</td>
</tr>
<tr>
<td>$n_s$ (cm$^{-3}$)</td>
<td>$n_p$ (cm$^{-3}$)</td>
<td>$M_{cl}$ (M$_{\odot}$)</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>4.55</td>
<td>3.0</td>
<td>1.41(22)</td>
</tr>
<tr>
<td>4.0</td>
<td>5.05</td>
<td>1.5</td>
<td>9.74(21)</td>
</tr>
<tr>
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<td>6.69(21)</td>
</tr>
<tr>
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<td>6.05</td>
<td>-1.5</td>
<td>4.57(21)</td>
</tr>
<tr>
<td>5.5</td>
<td>6.55</td>
<td>-3.0</td>
<td>3.12(21)</td>
</tr>
</tbody>
</table>

Table 2. The parameter values of the precomputed KOSMA-τ PDR-model clumps are equidistant on logarithmic scale in steps of half an order of magnitude.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Parameter range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $n_s$ (cm$^{-3}$)</td>
<td>$10^2$, $10^6$</td>
</tr>
<tr>
<td>Mass $M_{cl}$ (M$_{\odot}$)</td>
<td>$10^{-3}$, $10^3$</td>
</tr>
<tr>
<td>FUV flux $\chi/\chi_D$</td>
<td>$10^0$, $10^6$</td>
</tr>
</tbody>
</table>

mass of helium and heavier elements has to be added. This is a factor of about 1.4 higher (Anders & Grevesse 1989). Due to the fact, that the massive clumps dominate the ensemble mass (see Section 2.2) and contain only a few percent of their gas mass in the atomic phase, we can neglect the contribution of the atomic gas so that the ensemble mass is basically given by the mass of molecular hydrogen.

The radial density structure $n(r)$ of the model clumps with radius $R_{cl}$ is given by an inner core region with constant density for $r < 0.2 R_{cl}$ and a power law with index $-1.5$ for $0.2 R_{cl} < r < R_{cl}$:

$$n(r) = n_s \cdot \begin{cases} \left( \frac{r}{R_{cl}} \right)^{-1.5}, & \text{for } 0.2 R_{cl} < r < R_{cl} \\ \frac{5}{1.5}, & \text{for } r \leq 0.2 R_{cl} \end{cases} \quad (1)$$

Note that $n_s = n_{H,s} + 2 n_{H_2,s}$ is the total hydrogen nucleus number density at the clump surface. By neglecting the presence of helium and metals in the number density, we underestimate the number of collision partners by a factor of 1.2. The effect on the collision rates is, however, smaller due to the larger mass and thus lower velocity of helium. This does not change the results significantly. With the clump mass $M_{cl}$, the clump volume $V_{cl} = \frac{4}{3} \pi R_{cl}^3$, and the mass of the hydrogen atom $m_{H}$, the average clump density $n_{cl} = M_{cl}/(m_{H} V_{cl})$, used in the following to characterize each clump, is given by $n_{cl} = (2 - 5^{-3/2}) n_s = 1.91 n_s$.

The FUV flux illuminating each clump is assumed to be isotropic. It is given in units of the Draine field (Draine 1978): $\chi_D = 2.7 \times 10^{-3}$ erg s$^{-1}$ cm$^{-2}$. This is the flux of the local interstellar radiation field (ISRF) integrated over the wavelength interval from 91.2 to 111.0 nm (Draine 1978).

In the KOSMA-τ PDR-model, a constant flux of cosmic rays and the incident FUV radiation are the only heating sources of the clumps. The temperature and chemical structure of the clumps are derived from a radiative transfer computation including line-shielding for the incident FUV radiation and an escape probability approximation for the emitted FIR line radiation. The efficiency of the line cooling depends on the average line width $\delta v$ which is assumed to be 1.2 km s$^{-1}$ (see Table 3). More details are described by Gierens et al. (1992), Köster et al. (1994), Störzer et al. (1996), and Röllig et al. (2000).

In order to calculate the average line intensity $I_{cl}$ of each clump, first a line-of-sight integration of the source function and the attenuation along parallel paths through the clump is performed to calculate the resulting intensities $I(p)$ for different impact parameters $p$. As an example the line integrated CO 7–6 intensities are shown as a function of impact parameter in Fig. 1. Note the strong emission of the small, low mass, high density clumps. The physical reason for this is the enhanced population of the $J = 7$ rotational excited state (i.e., a higher excitation temperature) of the CO molecule in the smaller and denser clumps, as only for these the density in the clump center falls near or above the critical density of the CO 7–6 rotational transition of about $10^6$ cm$^{-3}$ (Kaufman et al. 1999, Kramer et al. 2004). The less dense but massive clumps have much larger column densities, partially compensating for the lower excitation temperature and increasing the optical depth. This leads to the apparently constant central brightness temperatures of the bigger clumps in Fig. 1. Detailed information about these clumps is compiled in Table 4.

Second, $I_{cl}$ is obtained by averaging over the projected clump surface via

$$I_{cl} = \frac{2}{R_{cl}} \int_0^{R_{cl}} I(p) p \, dp. \quad (2)$$

The results of these computations are used for the clump ensemble intensities.

We generate a large grid of intensities for the three independent clump parameters mass, density and FUV intensity, with grid points equidistant on a logarithmic scale in steps of half an order of magnitude. The range covered by the parameter cube of the KOSMA-τ PDR-model is shown in Table 2 for clumps with intermediate parameter values, the KOSMA-τ PDR-model intensities are interpolated between the grid points using logarithmic differences. Further parameters for the PDR model, held fixed for the present work, are listed in Table 3.

2.2. Clump mass distribution and geometry

Observations of the ISM show structure at all scales down to the resolution limit, as studied by e.g. Stutzki et al. (1988), Stutzki & Güsten (1990), Heithausen et al. (1998), Kramer et al. (1998), Simon et al. (2001). Quantitative analysis of the observed intensity distribution shows that it can be described as a self-similar, i.e. fractal structure. Alternatively, the observed 3d spectral line data cubes can be decomposed into clump ensembles. The “clump-mass spectrum”, i.e. the number of clumps d$N_{cl}$ per mass bin...
The “clump mass density spectrum”, i.e. the mass per volume element, per mass interval of the clump ensemble is correspondingly given by
\[
\frac{dN_{cl}(r)}{dM_{cl}} M_{cl} = \tilde{A}(r) M_{cl}^{1-\alpha},
\] (8)
and the mass differential clump volume per volume element, i.e. the “clump filling factor spectrum”, is
\[
\frac{d\tilde{V}_{cl}(r)}{dM_{cl}} V_{cl} = \tilde{A}(r) C^{-3/\gamma}(r) M_{cl}^{3/\gamma-\alpha}.
\] (9)

Integration over the clump mass leads to the “volume mass density” \(\rho(r)\) and the “volume filling factor” \(f_{V}(r)\), respectively, given by
\[
\rho(r) = \frac{\tilde{A}(r)}{2-\alpha} \left( \frac{m_{a}^{2-\alpha} - m_{i}^{2-\alpha}}{m_{a}^{1+3/\gamma-\alpha} - m_{i}^{1+3/\gamma-\alpha}} \right), \quad \text{and}
\] (10)
\[
f_{V}(r) = \frac{4\pi}{3} \frac{\tilde{A}(r)}{C^{3/\gamma}(r)} \left( \frac{m_{a}^{1+3/\gamma-\alpha} - m_{i}^{1+3/\gamma-\alpha}}{1 + 3/\gamma - \alpha} \right)
\] (11)

The “ensemble averaged clump mass density” \(\rho_{ens}(r)\) is given by
\[
\rho_{ens}(r) = \frac{3}{4\pi} C^{3/\gamma}(r) \left( \frac{m_{a}^{1+3/\gamma-\alpha} - m_{i}^{1+3/\gamma-\alpha}}{1 + 3/\gamma - \alpha} \right)
\] (12)

As discussed in detail in subsection 2.4 below, the distribution of the dense ISM in the Milky Way in our model is specified by the volume mass density \(\rho(r)\) and the (higher) ensemble averaged clump mass density \(\rho_{ens}(r)\) (respectively the volume filling factor of the molecular material). For given values of the low and high mass cutoffs of the clump distribution, equations (10) and (12) then determine the pre-factor of the clump number density spectrum and its spatial variation,
\[
\tilde{A}(r) = \frac{2-\alpha}{m_{a}^{2-\alpha} - m_{i}^{2-\alpha}} \rho(r),
\] (13)
through the volume mass density profile of the Milky Way, and the pre-factor of the mass-size relation and its spatial variation,
\[
C(r) = \left( \frac{4\pi}{3} \frac{2-\alpha}{1 + 3/\gamma - \alpha} \right) \left( \frac{m_{a}^{1+3/\gamma-\alpha} - m_{i}^{1+3/\gamma-\alpha}}{m_{a}^{2-\alpha} - m_{i}^{2-\alpha}} \right) \rho_{ens}(r)^{\gamma/3}
\] (14)
through the ensemble averaged clump mass density.

A further quantity of interest is the beam filling factor \(f_{B}\), as the ratio of the solid angle filled by the clumps of the ensemble \(\Omega_{ens}\) and the solid angle of the beam \(\Omega_{B}\). The solid angle of a specific clump with mass \(M_{cl}\) and Radius \(R_{cl}\) at a distance \(s = |s|\) (considering \(s\) as given in Galactic coordinates centered at the sun) is given by
\[
\Omega_{cl}(s) = 2\pi(1 - \sqrt{1 - (R_{cl}/s)^2}) \approx \pi \frac{R_{cl}^2}{s^2}.
\] (15)
for $R_\odot \ll s$. Then, the “solid angle density spectrum”, i.e. the solid angle per volume and clump mass element of the ensemble, is

$$\omega_{\text{cl}}(s) = \frac{d\tilde{N}_{\text{cl}}}{dM_{\text{cl}}} \Omega_{\text{cl}} \approx \frac{\pi}{C^2/\gamma(s)s^2} M_{\text{cl}}^{2/\gamma - \alpha}. \quad (16)$$

For the adopted values of $\alpha$ and $\gamma$ the exponent $2/\gamma - \alpha$ has a value of $-0.93$, implying that on a logarithmic mass scale, $\omega_{\text{cl}}(s)dM_{\text{cl}}/d(\ln M_{\text{cl}}) = M_{\text{cl}}\omega_{\text{cl}}(s) \propto M_{\text{cl}}^{-0.97}$, each logarithmic mass interval contributes about equal to 2.3. Volume emissivity and intensity of the clumpy medium

For the values of $\alpha$ and $\gamma$ adopted above, the numerator shows again a very weak mass dependence of $\propto m_{\text{cl}}^{-0.97}$. In the denominator in both factors the upper- and lower mass cut-off, $m_{\text{cl}}^{-1/\gamma} \approx m_{\text{cl}}^{-0.4}$, i.e. weakly decreasing with increasing upper mass cut-off and basically independent on the lower mass cut-off. Integration over the beam volume $V_B$ gives the total solid angle filled by the clump ensemble

$$\Omega_{\text{ens}} = \int_{V_B} \omega_{\text{ens}}(s) dV$$

$$= \frac{\pi}{4} \int_{V_B} \frac{\rho(s)}{\rho_{\text{ens}}(s)^{2/3}s^2} dV$$

$$= \frac{\pi}{4} \frac{(2 - \alpha)^{1/3}(1 + 3/\gamma - \alpha)^{2/3}}{2/\gamma - \alpha + 1}$$

$$\frac{m_{\text{cl}}^{2/\gamma - \alpha + 1} - m_{\text{cl}}^{2/\gamma - \alpha + 1}}{(m_{\text{cl}}^{2 - \alpha} - m_{\text{cl}}^{2 - \alpha})^{1/3}(m_{\text{cl}}^{1 + 3/\gamma - \alpha} - m_{\text{cl}}^{1 + 3/\gamma - \alpha})^{2/3}} \quad (18)$$

with the same dependence on upper and lower mass cut-off as for the volume specific solid angle.

### 2.4. Galactic parameter distributions

The Galactic parameter distributions of mass, density, and UV-intensity are assumed to vary only in Galactocentric radial direction within a cylindrically symmetric Galactic disk of constant half-height $h$. The values of the different parameters (at the solar circle at $R = R_\odot = 8.5$ kpc) are compiled in Table 3. We adopt a half-height of the molecular disk of 59 pc and for the atomic gas layer of 115 pc (Wolfire et al. [2003]).

In this paper we focus on the simplified structure of the Galactic parameter distributions in our model disk. The Galactic mass distribution of the neutral ISM is a key parameter of the model, because the modeled line intensities are basically proportional to the mass of the molecular gas assumed to be completely represented by PDRs (see Section 2.2). The Galactic mass distribution has been discussed in the literature e.g. by Clemens [1985], Rohlfs & Kreitschmann [1987], Williams & McKee [1997], Bronfman et al. [2001]. We use an approximation of the radial Galactic H$_2$ mass distribution presented by Wolfire et al. [2003] for $3 < R_\odot$/kpc $< 18$. According to Williams & McKee [1997], Fig. 3 the H$_2$ mass surface density decreases inwards from 3 to 1.7 kpc Galactocentric radius exponentially by a factor of about 3/8. For $0.6 < R_\odot$/kpc $< 1.7$ the molecular hydrogen gas mass distribution is set constant. In the Galactic center region ($R_G < 0.6$
kpc), the molecular hydrogen gas mass surface density is assumed to have a constant value of 88.4 M⊙ pc⁻² corresponding to an H₂ mass of 10⁶ M⊙ within the central 600 pc of the Galaxy (Dahmen et al. 1998; Gusten & Philipp 2004). This translates into a mean H₂ gas mass density of 0.75 M⊙ pc⁻³, corresponding to an H₂ number density of about 15 cm⁻³. The assumed mass distributions are visualized in Fig. 2 a). The neutral atomic hydrogen mass distribution is used to estimate the contribution of diffuse atomic phases to the [CII] 158µm fine structure line emission (as shown in Section 4.2.1). The contribution of the diffuse WIM is estimated by Reynolds (1990) to 37% of the total H₁ mass, which is slightly less than the WNM mass.

We adopt a lower clump mass limit m₁ of 10⁻³ M⊙ corresponding to the observational lower clump mass limit from Heithausen et al. (1998), as well as an upper clump mass limit m_u of 10² M⊙. Gorti & Hollenbach (2002) show an decreasing lifetime of clumps with decreasing mass and increasing density and argue against the existence of stable, small, and dense PDR clumps. This indicates a transient nature of these clumps.

The observed mean gas density of molecular clouds shows variation with Galactocentric radius (e.g. Brand & Wouterloot 1997). We adopt the Galactic radial distribution of the CNM density provided by Wolfire et al. (2003) for the ensemble averaged clump density n_ens = ρ_ens/n_H₁, scaled to an absolute value of 10³.8 cm⁻³ at the solar circle (Fig. 2 b)), obtained by a χ² fit to the intensity distributions as observed by COBE (see section 4.1). As shown in Section 2.2 the volume filling factor is determined by the underlying mass and density distributions. The result is shown in Fig. 2 c). The computed beam filling factors from the solar point of view along the Galactic plane are shown in Fig. 2 d). The density and temperature distributions with Galactocentric radius of the CNM and WNM, used in the following to estimate the [CII] emission from these phases, are adopted from Wolfire et al. (2003) (extrapolated inwards of 3 kpc Galactocentric radius) and the WIM distributions are assumed to equal the WNM distributions (McKee & Ostriker 1977; Reynolds 1991; Cox 2003; Hill et al. 2007).

The FUV flux in the Galaxy is mainly produced by OB stars. Observations show, that OB associations are associated with giant molecular clouds (GMCs) (e.g. Stark & Blitz 1978; McKee & Williams 1997) and Williams & McKee (1997) show, that the distributions of Galactic OB associations and GMCs can be described by truncated power laws with the same slope, i.e. show a strong correlation. Misiriotis et al. (2006) derive a nearly constant dust temperature from the COBE observations. Assuming a proportionality between dust continuum emission tracing the molecular material and the incident FUV flux from the stellar population this leads to a rather constant average FUV-flux impinging on the molecular gas within the Galactic disk. According to McKee & Williams (1997, and references therein) there are 6-7 OB associations with more than 30 massive stars within 1 kpc distance from the sun, dominating the local interstellar FUV radiation field. Blaauw (1955) states a mean scale height of about 50 pc of the local OB associations. Therefore we assume one layer of surrounding OB associations with 3 next neighbors. This leads to an average radius of major influence for each OB association of R_{OB} ≈ 7⁻¹/² to 6⁻¹/² kpc or 378 to 408 pc. The actual distance measurement to the Orion Nebula Cluster resulting in 414 ± 8 pc (Menten et al. 2007) corresponds well to this range. Together with the local interstellar FUV-flux of 1 χ_D (Draine 1978) and average distances d_{MC-OB} of about 20 to 50 pc of the GMCs to their associated OB associations (estimated from the sizes and shapes given in Stark & Blitz 1978, Fig. 1 and Table 1) we obtain FUV-fluxes impinging the molecular clouds of χ_MC ≈ (R_{OB}/d_{MC-OB})² χ_D/3 with values of 19 to 139 χ_D or log(χ_MC/χ_D) ≈ 1.3 to 2.1. In Section 4 we find a best fitting value of log(χ/χ_D) ≈ 1.8 for our model, falling well within this range.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial extension of Galactic disk</td>
<td>R_G</td>
<td>kpc</td>
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<td>[1]</td>
</tr>
<tr>
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<tr>
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<td>FUV-flux **</td>
<td>χ</td>
<td>χ_D</td>
<td>10¹.8</td>
<td></td>
</tr>
<tr>
<td>Metallicity **</td>
<td>Z</td>
<td>Z⊙</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Clump mass spectral index **</td>
<td>α</td>
<td></td>
<td>1.8</td>
<td>[3]</td>
</tr>
<tr>
<td>Mass-size relation index **</td>
<td>γ</td>
<td></td>
<td>2.3</td>
<td>[3]</td>
</tr>
<tr>
<td>Upper clump mass limit **</td>
<td>m_u</td>
<td>M⊙</td>
<td>10²</td>
<td>[3]</td>
</tr>
<tr>
<td>Lower clump mass limit **</td>
<td>m₁</td>
<td>M⊙</td>
<td>10⁻³</td>
<td>[3]</td>
</tr>
</tbody>
</table>
3. The clump-ensemble emissivity

To discuss and understand the behavior of the clump ensemble emissivity it is useful to first look at the features of individual clumps and the mass-differential emissivity of the ensemble. As explained in Section 2.3 the contribution to the volume emissivity spectrum of the ensemble $d\eta/dM_{cl}$ in the various line transitions is dominated by the clump intensity $I_{cl}$, as the filling factors do not significantly depend on the clump mass. The logarithmic volume emissivity spectrum $d\eta/d\log(M_{cl}) = M_{cl} d\eta/dM_{cl}$ for an ensemble averaged clump density of $n_{ens} = 10^{3.8} \text{ cm}^{-3}$ and an FUV-flux of $\chi = 10^{1.8} \chi_D$ is shown in Fig. 3. The symbols show the sampling points on the model grid. The upper plot shows the behavior for the different CO transitions. Note the trend towards larger contributions of the lower clump masses for higher transition numbers. The ensemble emission of the higher CO transitions ($J > 5$) almost exclusively stem from the low mass clumps. The bottom panel shows $M_{cl} d\eta/dM_{cl}$ for the fine structure cooling lines. The [CII] 158 $\mu$m and [CII] 609 $\mu$m emissions show an increased contribution of high mass clumps.
The CO emission of a single clump of one solar mass and a density of $n_{cl} = 10^{3.8}$ cm$^{-3}$ compared to the emission of an ensemble of clumps with the same total mass and ensemble averaged clump density, embedded in an isotropic FUV-flux of $10^{1.8}$ $\chi_D$.

In Fig. 4 a direct comparison of the emission of an individual clump with the emission of a clump ensemble with $1$ $M_\odot$ is shown for the different CO transitions. For $n_{ens} = 10^{3.8}$ cm$^{-3}$ and $\chi = 10^{1.8}$ $\chi_D$ we find a significantly higher intensity in the mid-$J$ CO transitions for the clump ensemble, which amounts to 2.5 orders of magnitude for the CO 8–7 line. This is due to the fact, that the mid-$J$ CO emission shows a steep decrease with clump mass (see Fig. 3).

Fig. 5 illustrates the dependence of the normalized emissivity of the lower and upper clump mass limits of the ensemble for different tracers. Again, the mid-$J$ CO emission shows a strong dependence on the lower clump mass limit, corresponding to the strong intensity enhancement with decreasing clump mass in these tracers. The CO 8–7 emissivity is nearly proportional to the lower clump mass limit. The [C ii] and [O i] emissivities show variations of one order of magnitude over an upper clump mass limit range of three orders of magnitude. This is due to the increasing surface to volume ratio of the clump ensemble with decreasing upper clump mass limit (cf. Section 2.2) and the fact that the [C ii] and [O i] emissions arise mainly in shells in the outer parts of the clumps.

4. Model results and comparison with observations

The resulting emissivity distribution inherits the intensity dependence of the clumps on the Galactic mass and density distributions as shown in Fig. 6.

The beam averaged intensity is determined according to the derivation in Section 2.2. For the comparison of our model result with the COBE FIRAS observations we calculate the intensities in a beam with an extent of $5^{\circ}$ in Galactic longitude and $1^{\circ}$ in Galactic latitude (Fixsen et al. 1999). All main PDR cooling lines in the COBE FIRAS spectral range including [C ii] 158 $\mu$m, [C i] 609 and 370 $\mu$m, CO $J = 1–0$ to $J = 8–7$, and [O i] 146 $\mu$m are used for the comparison.

To overview the emission features along the Galactic plane we labeled the main features visible in the [C ii] intensity cut in Fig. 7. One can easily identify the Galactic large scale structure, dominated by the spiral arms and the Galactic molecular ring.

4.1. Quantitative comparison

To quantify the quality of the model result we derive the reduced $\chi^2$ of the intensity distributions. The $\Delta I_i$ are the uncertainties of the observed intensities due to the calibration of the COBE FIRAS instrument and the data reduction. The number of degrees of freedom is given by $N_{dof} = N_l N_{spec} - 2$, with $N_l$ the number of bins in Galactic longitude and $N_{spec}$ the number of species used for the fit. The subtraction of 2 is due to the 2 fitted parameters $n_{ens}$ and $\chi$. Hence, the reduced $\chi^2$ is given by

$$\chi^2 = \frac{1}{N_{dof}} \sum_{i=1}^{N_l N_{spec}} \left( \frac{I_{obs,i} - I_{mod,i}}{\Delta I_i} \right)^2.$$ (24)

The resulting $\chi^2$-distribution of the intensity distributions is plotted as contours onto the density-FUV-plane in Fig. 8. The minimum $\chi^2$ has a value of 16.5 at an ensemble averaged clump density at the solar circle of $n_{ens} = 10^{3.8}$ cm$^{-3}$.
Fig. 6. Calculated CO emissivity distribution vs. Galactocentric radius for $n_{\text{ens}} = 10^{3.8} \text{ cm}^{-3}$ and $\chi = 10^{1.8} \chi_D$ at the solar circle.

Fig. 7. The [C\text{\tiny II}] 158 $\mu$m intensity distribution with Galactic longitude as published by Fixsen et al. (1999). The annotated source names are identified using the CO 1–0 map published by Dame et al. (2001).

and an FUV-flux of $\chi = 10^{1.8} \chi_D$. The $\chi^2$ distribution obviously confines the density to FUV-flux ratio $n_{\text{ens}}/\chi$ very well, as this ratio determines the molecular gas fraction within a PDR (Hollenbach & Tielens 1999). In contrast, the product $n_{\text{ens}}\chi$ is hardly constrained.

4.2. Discussion

The results of the modeled line intensities are overlaid onto the observed intensities in Figs. 9 and 10. We note that the relatively high value of the minimum $\chi^2$ is due to the pronounced Galactic large scale structure in the observed [C\text{\tiny II}] emission (which is not contained in the smooth model distribution) and the high signal to noise ratio of these data. In fact, it is surprising that one can find average values for these parameters, which allow us to reproduce the bulk of the FIR line emission of the Milky Way. The partly significant local deviations of the model results from the observations are due to the strongly simplifying assumptions. Except CO $J = 1–0, 6–5,$ and $8–7$, which have large ob-

Fig. 8. The reduced $\chi^2$ plotted as contours in the ensemble averaged clump density-FUV-flux-plane. The contours are given as factors of the minimum $\chi^2$ value. The solid line as well as the color code represents the $\chi^2$ distribution including all species, while the dotted lines are the $\chi^2$ contours neglecting the [C\text{\tiny II}] intensity. The $\chi_{\text{min}}$ minima are given by the diamond and triangle symbols, respectively.

4.2.1. Contributions to the Galactic [C\text{\tiny II}] 158 $\mu$m emission

We now turn to the comparison of the COBE observed [C\text{\tiny II}] intensities with the model results (see Fig. 9). Additional flux is measured mainly from the Galactic ring, the Cyg X region and other star-forming regions in the spiral arms. This is in contrast to the picture of [C\text{\tiny II}]-emission stemming to a large part from the CNM (Hollenbach & Tielens 1999). Based on direct UV absorption measurements of the excited state of C$^+$, Pottasch et al. (1979) and Gry et al. (1992) derive an average emissivity of the CNM, in particular diffuse H\text{\tiny I}-clouds, of $5 \times 10^{-26}$ erg s$^{-1}$ H-atom$^{-1}$. Using this number the contribution from PDRs to the Galactic [C\text{\tiny II}]-emission should be very small. This has been questioned by Petuchowski & Bennett (1993) who accounted for about half of the [C\text{\tiny II}]-emission from PDRs, the other half from the WIM and only a very small fraction from the CNM. In contrast, Heiles (1994) found that the CNM ranks second after the WIM while PDR contributions should be small. Carral et al. (1994) state that up to 30% of the interstellar [C\text{\tiny II}] emission may stem from H\text{\tiny II}-regions. Kramer et al. (2005) used ISO/LWS observations of the [N\text{\tiny II}] 122 $\mu$m line in the spiral arms of M83 and M51 to estimate that between 15% and 30% of the observed [C\text{\tiny II}] emission originates from H\text{\tiny II}-regions. Computations by Abel et al. (2005) indicate that the main [C\text{\tiny II}]-emission arises from PDRs and a fraction of 10% up to 60 % from H\text{\tiny II}-regions. Calculations by Kaufman et al. (2006) find similar [C\text{\tiny II}]-emission fractions, deviating by less than a factor of 2.
Fig. 10. The modeled intensities of the rotational transitions $J = 1-0$ (upper left) to $J = 8-7$ (lower right) of CO overlaid the observational distributions along the Galactic plane. Note that the combined CO $J = 7-6$ and [C$\text{i}$] 370 $\mu$m line intensities are shown in the lower left as they are not spectrally resolved by the COBE FIRAS instrument.

We can test this result from the viewpoint of the excitation of C$^+$ which is a simple problem because the fine structure line arises basically from a two-level system. Using the excitation rates for collisions with H, H$_2$, He, and electrons from Flower & Launay (1977), Launay & Roueff (1977), and Wilson & Bell (2002) we can compute an upper limit to the emissivity of C$^+$ as a function of density under the temperature and ionization conditions of the different phases of the ISM via

$$\eta_{\text{esc}} = \frac{1}{4\pi} \frac{n_{\text{C}^+} A_{\text{ul}} E_{\text{ul}} \beta}{1 + \frac{2n_c}{3n_e} \exp \left( \frac{E_{\text{ul}}}{k_B T} \right) \left( 1 + \frac{n_{\text{C}^+} \beta}{n_e} \right)}$$  \hspace{1cm} (25)
with \( n \) the total number density of the collision partners (mainly hydrogen atoms or electrons), \( A_{ul} \) the Einstein coefficient for spontaneous emission, \( g_u, g_l \) the statistical weights for the lower and upper state, \( n_{C^+} \) the number density of \( C^+ \), \( E_{ul} \) the energy spacing between the upper and lower state, \( n_{cr} \) the critical density of the \([C\text{\textsc{ii}}] \, ^2P_{3/2}-^2P_{1/2} \) transition with respect to neutral particles or \( n_{cr,e} \) for electrons, \( k_B \) the Boltzmann constant, and \( T \) the temperature. The escape probability \( \beta \) describes the probability of a photon of the considered transition to escape the cloud from its position within, assuming constant excitation conditions allover the cloud. To obtain an upper limit of the emission of the atomic gas components, we assume optically thin emission, i.e. \( \beta = 1 \). The values for the different parameters are summarized in Table 5.

### Table 5. The parameter values used for the escape probability calculation of the \([C\text{\textsc{ii}}] \, ^2P_{3/2}-^2P_{1/2} \) emissivity from the CNM and WNM. Powers of ten are given in parentheses.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>1.0 ± 0.2</td>
</tr>
<tr>
<td>( g_u )</td>
<td>4/3</td>
</tr>
<tr>
<td>( g_l )</td>
<td>2/3</td>
</tr>
<tr>
<td>( A_{ul} , [s^{-1}] )</td>
<td>2.4 ± 0.6</td>
</tr>
<tr>
<td>( n_{cr} , [\text{cm}^{-3}] )</td>
<td>4.3</td>
</tr>
<tr>
<td>( n_{cr,e} , [\text{cm}^{-3}] )</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Fig. 11 shows the \([C\text{\textsc{ii}}] \) emission from the different phases. All carbon is considered to be singly ionized, using an elemental abundance of \( 1.4 \times 10^{-4} \) (see Table 4). Upper limits are also guaranteed by assuming an ionization degree, i.e. an electron density relative to the total density of protons, of \( 10^{-3} \) in the CNM, \( 10^{-2} \) in the WNM, and 1 in the WIM (see Table 4 for the local Galactic parameter values of the different phases). We find that the “standard value” of \( 5 \times 10^{-26} \text{erg s}^{-1} \text{H-atom}^{-1} \) from Hollenbach & Tielens (1999) is in agreement with the density range of the CNM given in Wolfire et al. (2003). At low densities an efficient excitation of \( C^+ \) is only possible if a large fraction of the collision partners is charged, i.e. in the ionized medium.

The results of Petuchowski & Bennett (1993) and Abel (2006), who found, that the \([C\text{\textsc{ii}}] \) and \([N\text{\textsc{ii}}] \) emission are strongly correlated, indicate that main contributions (60% maximum) are expected from low density \( \text{H}_2 \)-regions. Using this result, we rerun the \( \chi^2 \) fit of the lines but increase the computed \([C\text{\textsc{ii}}] \)-intensity by a global factor 2.5 before fitting the model results to the observations. This fit provides a lower limit of the FUV-flux of \( \chi = 10^{1.2} \chi_D \), which is slightly below the estimated range from Section 2.4 with a lower limit of \( \chi = 10^{1.3} \chi_D \). The best fitting ensemble averaged clump density is derived to \( n_{\text{ens}} = 10^{3.9} \text{cm}^{-3} \) in this case. Additionally we check the fit results completely ignoring the \([C\text{\textsc{ii}}] \) intensity distribution. In this case we obtain parameter values of \( n_{\text{ens}} = 10^{3.8} \text{cm}^{-3} \) and \( \chi = 10^{1.0} \chi_D \).
The $\chi^2$-distribution with a minimum of 5.40 for this case is also shown in Fig. 8. The resulting low FUV-flux is due to the low signal to noise ratio in the [O\textsc{i}] and upper CO transitions, so that this result is biased by the low-J CO and [C\textsc{ii}] emission and lies below our estimate for the average FUV-flux seen by the Galactic molecular gas. The confinement of the $\chi^2$ distribution ignoring the [C\textsc{ii}] intensity distribution is weak in either direction. In particular at $n_{\text{ms}} = 10^{3.8}$ cm$^{-3}$ and $\chi = 10^{1.8} \chi_D$ we find a $\chi^2$ value of 5.75 – only about 7% more than the minimum value, indicating that the [C\textsc{ii}] emission fixes the resulting FUV-flux value of the model. The computed PDR [C\textsc{ii}] intensity distributions for these cases are shown in Fig. 12.

There are smaller, comparable contributions from the diffuse WIM, which has an average density well below 1 cm$^{-3}$ (McKee & Ostriker 1977; Reynolds 1991; Cox 2005; Hill et al. 2007), and from the CNM, and a vanishing contribution of the WNM. However, the much larger scale height of the diffuse WIM leads to a reduced contribution in comparison to the CNM, in agreement with our result for the intensity distribution along the Galactic plane shown in Fig. 8.

We conclude that our model can reproduce the Galactic [C\textsc{ii}] emission with the main contribution from clumpy PDRs, but we cannot exclude a major contribution from low-density H\textsc{ii} regions.

4.2.2. The FIR continuum emission

Only 51% of the Galactic FIR emission in the COBE FIRAS spectral range is reproduced by a separate dust radiative transfer calculation for the KOSMA-τ PDR-model clumps (R. Szczepańska, priv. comm.) as shown in Fig. 13. This is expected, as the dust is also heated by less energetic radiation than the FUV from the older stellar population. Mochizuki & Nakagawa (2000) argue that about half of the total stellar flux lies in the FUV spectral range in agreement with our result.

5. Summary and Conclusions

The bulk of the FIR and submillimeter line emission in [C\textsc{ii}] 158 \mu m, [C\textsc{iii}] 609 \mu m and 370\mu m, [O\textsc{i}] 146 \mu m, and CO 1–0 to 8–7 of the Milky Way as observed by COBE FIRAS can be reproduced by a clumpy PDR-model, which takes the fractal structure of the dense ISM as a clump ensemble with a power law clump mass and size distribution into account. This is a remarkable result, as the model contains essentially no free parameters; all parameters being constrained by independent knowledge about the structure, kinematics of, and star formation rate in the disk of the Milky Way.

The result suggests that the bulk of the Galactic FIR line emission stems from fractally structured PDRs, illuminated by the FUV radiation field emitted by the Galactic population of massive stars in adjacent star forming regions. The UV-penetrated, clump-cloud PDR scenario applied here to the Milky Way line emission as observed by COBE is a versatile approach, which can be applied in the future to model the emission from individual Galactic star forming regions as well as the line maps of spatially resolved nearby galaxies.

Future studies and observations should address the validity of particular assumptions, like the limits of the clump mass distribution. Observations with high spatial and/or spectral resolution, that will be possible with upcoming missions as SOFIA, Herschel, and ALMA, will give further insights in the nature of the ISM.

Acknowledgements. We are grateful to D. J. Fixsen for providing us the results of Fixsen et al. (1999) in electronic form. We thank the anonymous referee for her/his helpful comments. This work is supported by the Deutsche Forschungsgemeinschaft (DFG) via Grant SFB 494.

References
