Chapter 4

Possible mechanisms generating the characteristic toroidal flow profile of pellets in a high-shear mixer

Abstract

In this chapter the torus profile of the high-shear pelletisation process is discussed. Possible mechanisms that cause the torus profile are given. From these mechanisms, the propulsive mechanism (combination of the centrifugal movement, accumulation of the pellets at the wall, and gravity) was considered to be the most probably mechanism for the generation of the torus. Also the internal collisions between the pellets are very important, this was demonstrated in experiments in a coffee grinder. Dry pellets did not form a torus, and were thrown out of the bowl if the lid was removed. The flow characteristics of the dry pellets changed drastically as soon as a small amount of wet pellets of different properties was added. A small percentage of wet pellets caused almost all pellets to stay inside the mixer even without the lid. The effect of the pellet-wall collision was less pronounced than the visco-elastic properties of the pellets involved.
4.1. Introduction

The coffee grinder can be used for downscaling purposes. Previous studies showed that the coffee grinder is a suitable and realistic small-size model of larger high-shear mixer granulators like the Gral 10 (chapter 3). There are many reasons why this particular coffee grinder was used:

1. This coffee grinder also has been used by a number of industrial laboratories in the Netherlands as a first check on the agglomeration behaviour of a powder in production-scale apparatus. Obviously there is a relation between the agglomeration behaviour of these two kinds (scales) of apparatus.

2. A number of phenomena have been observed in a coffee grinder as well as in a full-scale high-shear mixer. Similarities can be found in the existence of a torus, the influence of visco-elastic properties of the pellets on the process, and the narrow particle size distributions obtained after pelletisation.

3. The amount of material used in a coffee grinder is very small (about 25 grams). Because of this, a large number of experiments can be performed, in which the whole contents can be taken as one representative sample.

4. The coffee grinder can easily be operated and cleaned.

5. The processing time in a coffee grinder is short, 2 minutes.

6. It is very easy to change the internals of the coffee grinder, like lining of the wall.

7. The coffee grinder is very cheap.

8. We are only interested in the elucidation of mechanisms. In that case knowing the order of magnitude of the operational variables at the scale involved is usually sufficient.

There are some disadvantages of the coffee grinder: The geometry of the coffee grinder differs from the Gral; the velocity of the impeller in the coffee grinder is very high, resulting in a higher tip-speed. Also, there is no chopper in the coffee grinder. This has to be kept in mind when comparing results of the coffee grinder and the Gral.

The interest in the coffee grinder increased further when we observed the formation of:

- a characteristic stable flow pattern (torus) during pelletisation;
- almost spherical pellets;
- almost mono-sized pellets i.e. with a very narrow pellet size distribution\(^1\);
- a reproducible relationship between the impeller rotational speed and the pellet size distribution (chapter 3).

Further, the strong influence of the material properties and the moisture content arose our curiosity. Why can only certain materials like microcrystalline cellulose be used in the pelletisation process, and many others not. And why is the moisture content such a critical factor on the pellet growth behaviour and the final pellet size.
The aim of this study is to give a semi quantitative and mechanistic model for the flow profile of pellets in a high-shear mixer.

4.2. Literature

During pelletisation a characteristic flow profile of pellets can be observed. All pellets move in such a way that a torus is formed. This torus (figure 4.1) can be described as a doughnut-shaped ring, in which the bottom of the bowl is always visible in the centre of the torus.

Figure 4.1. Schematically drawing of the flow profile of the pellets in the torus.

The influence of the design of the impeller and the chopper (angle of inclination of the impeller as well as the size of the chopper) on granulation of dicalcium phosphate in high-shear mixers has been described by Holm. The effect of the impeller design could be explained in terms of the relative swept volume \((RSV)\). A high \(RSV\) resulted in more densification, causing narrow granule size distributions. No effect of the chopper design and its rotational speed on granule size distributions was observed. It was suggested that the primary function of the chopper was to disturb the characteristic flow-pattern of the total granule mass. Disturbances in the granule flow pattern in the surroundings of the chopper were found. Because of these disturbances (even if the chopper was set off, disturbances were observed), the effect of the chopper on granule growth was suggested to be mainly due to its presence. But what about the high impact forces of the chopper on the rotating granules? Because granules are broken by the impact of the impeller, it is reasonable to expect that the effect of the chopper is twofold:

1. breakage of the large nuclei and large lumps, due to its high impact, and
2. disturbances in granule flow pattern due to its presence. Thus inducing granule-granule collisions, which also may affect granule growth and breakage (chapter 6).

The influence of the flow profile of pellets on the sphericity of pellets, made by melt pelletisation in a high-shear mixer, was mentioned before by Schæfer et al. Improvement of the flow profile of pellets by changing the impeller design from plane blades towards curved blades, caused more rounded pellets, a more reproducible process, and a lower porosity of the pellets.
Measurements of the velocity of granules in a high-shear mixer with the PEPT-technique were performed by Wellm. Although some improvements on this method have to be made (only velocities smaller than 2 m/s could be measured, while the tip velocity of the impeller arm was 14 m/s), the results give some information about the flow profile of pellets in a high-shear mixer. The pellets move in a mixer with a spiralling motion. This motion is outwards in the lower regions of the mixer and inwards in the upper regions of the mixer. The mean velocities of the granules, in the lower regions of the torus, were about 100 times lower than the tip velocity of the mixing arms.

The observed torus in the high-shear mixer shows similarities with the flow profile observed during spheronisation in a marumerizer. In the early seventies, Reynolds described the spheronisation process and mentioned the existence of the rope-like motion, which - as believed by many people - is a must for good spheronisation practices.

The toroidal flow pattern has also been mentioned for the rotary-processor by several authors. The upward flow at the wall was mentioned to be a result of the airflow at the bottom of the rotary-processor. Because there is no airflow in the high-shear mixer, except for the turbulence as a result of the intense mixing, another factor but the airflow must cause the upward flow of pellets at the wall-site.

### 4.3. Forces and mechanisms

The flow-characteristics can be observed (figure 4.1):

- At the bottom, a forward flow can be observed, i.e. an angular motion of the pellets in the direction of the running impeller blades.
- At the wall, an upward flow is observed.
- From the top-view, pellets move from the upper side of the torus at the wall to the inner side of the torus near the impeller, along a path that is curved like the strands in a rope. From the top-view, the pellets are also moving in the direction of the rotating impeller with a velocity of about 1 m/s (chapter 3).

Pellets in a torus continuously collide. Different kinds of collisions are possible during the process:

1. pellet-wall collisions;
2. pellet-impeller collisions;
3. pellet-pellet collisions.

All these collisions determine the final flow-profile of the pellets. The influence of each of these collisions was investigated separately.

Consider a pellet moving in a coffee grinder with a rotating impeller. The characteristics of the system are: pellet diameter $d_p = 1$ mm; pellet density $\rho_p = 1500$
kg/m³; pellet velocity \( v_p = 1 \) m/s; pellet angular velocity \( \omega = 1000 \) s\(^{-1}\); diameter bowl \( D = 8 \) cm; impeller rotational speed \( N = 80 \) s\(^{-1}\); tip velocity impeller \( v_{tip} = 20 \) m/s; density of the air \( \rho_i = 1.2 \) kg/m³; viscosity of the air \( \eta = 1.8 \cdot 10^{-5} \) Pa·s; velocity of the air \( v_i = 0 \) m/s. The velocity of the pellet relative to the air (\( v_r \)) is given by: \( v_r = v_p - v_i \).

In order to clarify the mechanisms responsible for the trajectory of the pellet, some order of estimate calculations have been made.

### 4.3.1. Forces acting on a pellet

There are a number of forces acting on a pellet. These forces are:

a. lifting force, the so-called Magnus force due to spin;

b. drag force due to friction with air;

c. gravity force due to gravity;

d. centrifugal force due to circulation;

e. deformation due to collisions with the impeller and between pellets (which will be discussed in a greater depth in chapter 5).

Spin is the rotation of a particle around its own centre, circulation is the rotation of a pellet in an orbit around the impeller shaft.

#### 4.3.1.1. Lifting force

The lifting force, also called the Magnus force, is given by:

\[
F_{lift} = C_L \frac{1}{2} \rho_i v_r^2 A_{\perp}
\]

with
\[
C_L = 0.4 \Gamma \quad \text{and} \quad \Gamma = \frac{d_p \omega}{2v_i}.
\]

Here, \( C_L \) is the lift coefficient, \( \rho_i \) is the density of the air, \( v_r \) is the velocity of the pellet relative to the air, \( \omega \) is the angular velocity of the pellet, and \( \Gamma \) is the non-dimensional angular velocity, i.e. the ratio of the surface velocity of the pellet and the relative velocity of the pellet (\( v_r \)). The direction of the Magnus forces is perpendicular to the relative velocity of the pellet.

#### 4.3.1.2. Drag force

The drag force is a friction force of the pellet with the air. The drag force is given by:

\[
F_{drag} = C_D \frac{1}{2} \rho_i v_r^2 A_{\perp}
\]

Where \( A_{\perp} \) is the cross sectional area of the pellet, and \( C_D \) is the coefficient of friction for a pellet. At high Reynolds the drag coefficient is 0.43 for pellets; at lower Reynolds numbers, as in our experiments, the drag coefficient is a function of the Reynolds number:

\[
C_D = \frac{24}{Re} + 0.4 Re^{-0.5} + 0.124 Re^{0.5}
\]

\( Re \) is the Reynolds number, defined as:

\[
Re = \frac{v_i D}{\nu}
\]

where \( \nu \) is the kinematic viscosity of the air.

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where \( \nu \) is the kinematic viscosity of the air.
\[ C_D = \frac{24}{Re} + \frac{0.4}{\sqrt{Re}} + 0.43 \] and
\[ Re = \frac{\rho_f v_f d_p}{\eta_f} \] (4.4)

4.3.1.3. Gravity force

The force due to gravity is given by:
\[ F_g = \frac{\pi}{6} d_p^3 (\rho_p - \rho_f)g \] (4.6)
where \( \rho_p \) is the density of the pellet, and \( g \) the acceleration due to the force of gravity.

4.3.1.4. Centrifugal force

In the high-shear mixer, the centrifugal force pushes the pellets towards the wall.
The centrifugal force is given by:
\[ F_c = \frac{m_p v_p^2}{R_{loop}} \] (4.7)
where \( m_p \) is the mass of the pellet, and \( R_{loop} \) is the radius of the loop as given in figure 4.2b.

4.3.2. Possible mechanisms

Several possible mechanisms might explain the toroidal flow profile. The possible mechanisms for the generation of the torus are spin, drag, and propulsive forces (figure 4.2). These mechanisms exist of one or more forces mentioned in the previous section. With schematically pictures and order of magnitude calculations, these mechanisms are illustrated and compared.

Figure 4.2. Possible mechanisms generating the torus profile: a. spin mechanism; b. drag mechanism; c. propulsive mechanism.

4.3.2.1. Spin mechanism

A possible explanation of the flow profile of pellets can be the spin mechanism. Due to the internal rotation of pellets, the pellets are lifted upwards after impact with the impeller (figure 4.2a).
The influence of the spin of a pellet on its trajectory can be calculated using the lifting force (eq. 4.1), drag force (eq. 4.3), and gravity (eq. 4.6). The total force is given by:

\[ F_{\text{tot}} = F_{\text{lift}} - F_{\text{drag}} - F_g \]  

(4.8)

The derivatives of distance and velocity are given by:

\[ \frac{dx}{dt} = v \]  

(4.9)

\[ \frac{dv}{dt} = \frac{F_{\text{tot}}}{m_p} \]  

(4.10)

The angular velocity changes according to the equation:

\[ \frac{d\omega}{dt} = \frac{T}{I} = \frac{T}{\frac{1}{3}m_p d_p^2} \]  

(4.11)

where \( I \) is the moment of inertia of the sphere and \( T \) the torque acting on the pellet. In this work, the result by Dennis et al.\(^\text{10}\) was applied to \( T \):

\[ T = C_T \frac{1}{2} \rho_f \left(0.5d_p\right)^2 \omega^2 \]  

(4.12)

with \( C_T \) is

\[ C_T = \frac{6.45}{\sqrt{\text{Re}_\omega}} + 32.1 \]  

(4.13)

and \( \text{Re}_\omega \) is

\[ \text{Re}_\omega = \frac{\rho_f \left(0.5d_p\right)^2 \omega}{\nu_f} \]  

(4.14)

The results of some order of estimate calculations for a normal situation (\( v_r = 1 \text{ m/s, and } \omega = 1000 \text{ s}^{-1} \)) and for an extreme situation (\( v_r = 10 \text{ m/s, and } \omega = 20000 \text{ s}^{-1} \)) are given in table 4.1. For the normal situation, the force due to gravity is two orders of magnitude larger than the lift force. Even for the extreme situation, the trajectory calculations did not show a circular movement. Moreover, when the pellet hits the wall of the bowl, the accumulated upward movement is about 0.06 mm. Therefore, the movement of the particles to the top of the torus can thus not be explained by the spin mechanism.

<table>
<thead>
<tr>
<th>( F_g )</th>
<th>normal, ( v_r = 1 \text{ m/s, } \omega = 1000 \text{ s}^{-1} )</th>
<th>extreme, ( v_r = 10 \text{ m/s, } \omega = 20000 \text{ s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{\text{lift}} )</td>
<td>( 9 \times 10^{-8} \text{ N} )</td>
<td>( 2 \times 10^{-5} \text{ N} )</td>
</tr>
<tr>
<td>( F_{\text{drag}} )</td>
<td>( 4 \times 10^{-7} \text{ N} )</td>
<td>( 2 \times 10^{-5} \text{ N} )</td>
</tr>
</tbody>
</table>

### 4.3.2.2. Drag mechanism

Another possible mechanism to explain the flow profile of pellets is the drag mechanism. The turbulent airflow inside the apparatus causes specific air streams during pelletisation (figure 4.2b). How large is the radius of the loop of the trajectory of the pellet, caused by the
centrifugal force and the drag force?

There are two important forces acting on a pellet that can cause the drag of this pellet. These forces are the drag force (eq. 4.3) and the centrifugal force (eq. 4.7).

For $F_{\text{drag}} = F_c$, the radius of the loop of the trajectory of the pellets ($R_{\text{loop}}$, figure 4.2b) can be calculated by:

$$R_{\text{loop}} = \frac{m_pv_p^2}{C_D \frac{1}{2} \rho_f v_r^2 A_\perp} = \frac{4d_p \rho_p v_p^2}{3C_D \rho_f v_r^2}$$

(4.15)

For $C_D = 0.43$, $v_p = 1 \text{ m/s}$, $v_l = 10 \text{ m/s}$, and $v_r = 9 \text{ m/s}$, the radius of the loop is 4.8 cm. With a radius $R_{\text{loop}}$ of 4.8 cm, the diameter of two loops will be at least 19.2 cm, which is too large compared to the dimensions of the coffee grinder ($D_{\text{cg}} = 8 \text{ cm}$). Therefore, the torus profile can not be explained by the drag mechanism.

### 4.3.2.3. Propulsive mechanism

The propulsive mechanism exists of the summation of three parts. The centrifugal movement is induced by the impact of the impeller and causes the pellets to move in the direction of the wall. This is followed by an accumulation of the pellets near the wall, resulting in an upward transport. Gravity then causes the pellets to fall downwards, back to the impeller (figure 4.2c). The kinetic energy generated by the impeller arm can partly be transferred into potential energy (height) of the pellets in the torus. The following equations allow us to calculate if the kinetic energy is large enough to lift the pellets to the top of the torus.

$$E_{\text{kin}} e^2 = E_{\text{pot}}$$

$$\frac{1}{2}mv^2 e^2 = mgh$$

$$h = \frac{\frac{1}{2}v^2 e^2}{g}$$

(4.16)

(4.17)

(4.18)

where $e$ is the coefficient of restitution based on linear velocity differences, which is about 0.3 as measured with the horizontal-impact method (chapter 5). The coefficient of restitution is sometimes based on kinetic energy. We relate the coefficient of restitution on the linear velocity, which is explained in more detail in the appendix to chapter 4.

The maximum height the pellets can reach by one impact of the impeller ($v_{\text{tip}} = 20 \text{ m/s}$, $e = 0.3$) is 1.8 m. A velocity of 3 m/s is needed to lift the pellets to the top of the torus in the coffee grinder (4 cm). Compared to the tip velocity of the impeller arm at 80 rps, this is only possible if a considerable part of the kinetic energy is dissipated in some way or another, like frictional loss. Otherwise the pellets will always be thrown-out of the bowl if no lid is present. A possible explanation is internal cohesion.

Internal cohesion can be described as impulse transfer between the pellets causing the pellets to behave as a coherent substance with a quasi viscosity (figure 4.3).
Momentum transfer between the colliding pellets can be described as:

\[ m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \]  

(4.19)

where the apostrophe indicates the situation after collision. In case of equal masses \( m_1 = m_2 \) and \( v_2 = 0 \) equation 4.19 can be simplified to:

\[ v_2' = v_1 - v_1' \]  

(4.20)

The coefficient of restitution based on linear velocity differences (see also appendix) is given by:

\[ e = \frac{v_1' - v_2'}{v_1 - v_2} \]  

(4.21)

In case of \( v_2 = 0 \) and substitution eq. 4.20 in eq. 4.21, the following equation is found for \( v_1' \) :

\[ v_1' = \frac{1}{2} v_1 (1 + e) \]  

(4.22)

Take for example one pellet that hits the impeller and reaches a velocity of 20 m/s (equal to the tip velocity of the impeller arm of the coffee grinder). The velocity of the pellet becomes 13 m/s after the first pellet-pellet collision \( (v_2 = 0, \text{ and thus } e = 0.3) \). The velocity becomes 8.5 m/s after a second collision, 5.5 m/s after a third collision, etc. After seven collisions a velocity of 1 m/s is reached.

So, seven collisions of the initial pellet with non-moving pellets are needed to reach a velocity of 1 m/s, which is comparable to the mean velocity in the torus. The internal pellet-pellet collisions thus have a pronounced effect on the pellet velocity inside the torus.

4.3.3. Tentative conclusions

Some first conclusions based on the preliminary calculations can be made. The movement of pellets in the torus can not be explained by the drag mechanism or the spin mechanism. The propulsive mechanism (combining the centrifugal forces, accumulation and gravity) seems to explain a part of the flow profile. Furthermore, the interactions between pellets (e.g. internal cohesion) seem to be a very important factor explaining the velocities in the torus.

4.4. Small-scale experiments

To illustrate the importance of the propulsive mechanism and the interactions between the pellets, several small-scale experiments were performed in a Moulinex coffee grinder. Pellets were made of an equal mass mixture of lactose and MCC as described in chapter 2. The
experiments were performed with dry pellets and with (freshly prepared) wet pellets. Sometimes a mixture of dry and wet pellets was used. In one experiment, some adaptations were made regarding the kind of material on the wall of the bowl.

4.4.1. Effects of gravity

Gravity causes all pellets to fall downwards. Although there seems to be some logic behind this, it also could be possible for the pellets to be dragged downwards as a consequence of a circular movement of the air. To be sure that only gravity causes the pellets to fall, the coffee grinder was tilted. This resulted in the disappearance of the torus. Therefore it can be concluded that gravity is an important factor for the presence of the torus.

4.4.2. Pellet-lid interactions

In order to investigate the influence of the lid, freshly prepared pellets were mixed in the coffee grinder without the presence of the lid. All pellets stayed in the coffee grinder and moved in the torus. Therefore, the pellet-lid collisions obviously do not contribute to the formation of the torus.

Dry pellets, on the contrary, do not flow in a torus. When the lid of the bowl is removed during pelletisation, all pellets will be thrown out of the bowl almost immediately. This indicates immediately the importance of the material properties of the pellets for the presence of the torus.

4.4.3. Pellet-wall interactions

The effect of the pellet-wall collisions can be investigated by changing the roughness of the wall. The wall was covered with different materials like felt, paper, and sandpaper. After adding 24 grams of dry pellets (elastic, having a high coefficient of restitution) the amount of material left in the bowl was measured as a function of the mixing time (at 82 rps). The results are given in figure 4.4. As reference-value the amount of wet pellets in the bowl without any cover on the wall is also given. From these pellets, 98% stayed in the bowl after 30 seconds of mixing.

When mixing dry pellets without any cover on the wall, all pellets were removed from the bowl within 15 seconds. The collisions with the wall could be dimmed (become less elastic) by covering the bowl with felt or paper. But now, after 25 seconds of mixing, the whole bowl was empty. Covering the wall with sandpaper, 75% of the dry pellets stayed in the bowl after 30 seconds of mixing. There obviously is a pronounced pellet-wall interaction. This interaction depends on the wall properties.
Figure 4.4. Influence of the pellet-wall interactions on the amount of dry pellets staying in the torus. The wall was covered with: (▲) nothing, (■) felt, (♦) paper, (☆) fine sandpaper, and (--) coarse sandpaper. The results of wet pellets and no cover on the wall (●) are given as a reference.

4.4.4. Pellet-pellet interactions

The influence of pellet-pellet interactions was investigated by changing the mass fractions of dry pellets (elastic, with a high coefficient of restitution) and wet pellets (plastic (inelastic) with a low coefficient of restitution). In this way the elastic behaviour of the material was controlled. As the content of the bowl exist of dry pellets only, all pellets were thrown out of the bowl within 15 seconds of mixing (figure 4.5). By adding 20 percent of wet pellets and mixing for 30 seconds, more than 60 percent of the pellets remained in the bowl. When the whole content of the bowl existed of wet pellets only, all pellets stayed inside the bowl during 30 seconds of mixing.

Wet pellets can deform plastically and show a low coefficient of restitution ($e$ is 0.3, chapter 5). Dry pellets can not deform plastically and have a higher coefficient of restitution than the wet pellets.

Because of the high coefficient of restitution, the velocity of dry pellets after impact with the impeller is much higher compared to the wet pellets, resulting in more pellets being thrown out of the bowl.

Figure 4.5 clearly illustrates the fact that plastic (or inelastic) collisions are required to keep the pellets in the bowl. Figure 4.5 also illustrates that, as soon as the amount of pellets in the bowl reaches a critical (minimal) value of about 50%, all pellets are immediately thrown out of the bowl. This indicates that, besides the inelastic pellet-wall collisions, also the pellet-pellet collisions are important to generate the stable torus profile.
Figure 4.5. Influence of the pellet-pellet interactions on the amount of pellets left in the bowl after 30 seconds of mixing. For two situations, the bowl was empty even within the 30 seconds of mixing, and the time until an empty bowl given in the figure.

4.5. Conclusions

With some calculations and some small-scale experiments in a coffee grinder, the flow pattern of the pellets inside the high-shear mixer was examined. The observed flow-pattern is similar to the flow pattern as observed in the large-scale high-shear mixer (Gral 10), the spheroniser and the centrifugal equipment, like the roto-granulator. As soon as the pellets are formed from the loose agglomerates, the torus profile is observed. The torus is characterised by the regular and stable movement of the pellets. From top-view, the pellets move in a rope-like motion, where the centre of the bowl remains visible during mixing.

Different mechanisms were considered causing the generation of the torus. With some calculations, the spin-mechanism, and the drag-mechanism could be excluded. The propulsive mechanism in combination with the pellet-pellet interactions explains the generation and presence of the torus. These findings were illustrated with small-scale experiments.

The effect of the wall of the mixer was investigated by covering the wall with different materials. Increasing the roughness of the wall by covering it with (sand)paper increased the stability of the torus.

More effect on the stability of the torus was found by changing the elastic properties of the pellets. With highly elastic pellets (using only dry pellets), no stable flow profile was observed, and all pellets were thrown out of the bowl within a few seconds after removal of the lid. Addition of wet (and thus plastic or inelastic) pellets increased the stability of the torus. More insight in the generation of the torus can be gained by modelling the flow profile as a whole, and using the relevant material properties and the forces involved. This still remains to be investigated.
4.6. Nomenclature

\( A \)    pellet surface area (m\(^2\))
\( C_{f0} \) friction coefficient
\( C_L \)   lift coefficient
\( d \)    diameter (m)
\( D \)    diameter (m)
\( e \)    coefficient of restitution based on linear velocity differences
\( E \)    energy (J)
\( F \)    force (N)
\( g \)    acceleration (m/s\(^2\))
\( h \)    height (m)
\( I \)    moment of inertia (kg·m\(^2\))
\( m \)    mass (kg)
\( N \)    number of rotations per second (1/s)
\( R \)    radius (m)
\( Re \)   Reynolds number
\( t \)    time (s)
\( T \)    torque (N·m)
\( v \)    velocity (m/s)
\( v' \)   velocity after collision (m/s)
\( x \)    distance (m)

Greek symbols
\( \Gamma \) non-dimensional angular velocity
\( \eta \)   viscosity (Pas)
\( \rho \)   density (kg/m\(^3\))
\( \omega \) angular velocity (1/s)

Subscripts
\( cg \) coffee grinder
\( f \)    fluid, air
\( p \)    pellet
\( r \)    relative
4.7. References


Appendix to chapter 4

Formal definition of the coefficient of restitution

Foerster et al.\(^1\) presented a collision model based with a coefficient of restitution based on the linear velocity. This model considers two particles with different masses \(m\), each with a linear velocity \(v\) and a rotational velocity \(\omega\). The diameter of the particles is given with \(d\).

At collision, the particles have the following velocity difference \(g\):

\[
g = (v_1 - v_2) - \left(\frac{d_1}{2} \omega_1 + \frac{d_2}{2} \omega_2\right) \times n
\]

where \((n)\) is the normal vector.

The coefficient of restitution based on the differences in linear velocity \(e\) is defined as:

\[
e = -\frac{n \cdot g'}{n \cdot g}
\]

The apostrophe is used to indicate the velocity difference after collision.

The coefficient of restitution can be used to calculate the velocities after collision by using the momentum balance. The momentum balance without rotation is given by:

\[
m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'
\]

The dimensionless momentum balance is given by:

\[
1 + M_2 V_2 = V_1' + M_2 V_2'
\]

using \(M_2 = \frac{m_2}{m_1}, V_2 = \frac{v_2}{v_1}, V_2' = \frac{v_2'}{v_1},\) and \(V_1' = \frac{v_1'}{v_1}\).

The expression of \(V_2'\) is given by:

\[
V_2' = \frac{1}{M_2} \left(1 - V_1' + M_2 V_2\right)
\]

For simple collisions without any rotation in a two dimensional situation \((n'/n=1)\), the coefficient of restitution based on linear velocity differences can be described as:

\[
e = -\frac{n' \cdot g'}{n \cdot g} = \frac{V_2' - V_1'}{1 - V_2}
\]

Substitution of equation A4 in equation A5 gives the general expression for the coefficient of restitution:

\[
e = \frac{1}{M_2} \left(1 - V_1' + M_2 V_2\right) - V_1'
\]

\[
e = \frac{1}{1 - V_2}\left(1 - V_1' + M_2 V_2\right) - V_1'
\]
Coefficient of restitution for equal masses

Considering a particle colliding with another particle of the same mass \( M_2 = 1 \), the general equation for the coefficient of restitution (eq. A6) can be simplified to:

\[
e = \frac{1 - 2V_1' + V_2}{1 - V_2}
\]  

(A7)

More simplification in case of \( V_2 = 0 \) results into:

\[
e = 1 - 2V_1'
\]  

(A8)

Coefficient of restitution for a particle – wall collision

Considering a particle colliding with a wall \( (v_2 = v_2' = 0, m_2 = \infty) \), the general equation for the coefficient of restitution (eq. A6) can be simplified to:

\[
e = -V_1' = \frac{v_1}{v_1}
\]  

(A9)

This is also the coefficient of restitution as used by Ennis et al.\(^2\), Tsjui et al.\(^3\), and Tashiro et al.\(^4\).

Coefficient of restitution based on kinetic energy

Iveson et al.\(^5\) and Beekman et al.\(^6\) used the coefficient of restitution based on the fraction of kinetic energy which is maintained by one particle after collision \( (e_{\text{kin}}) \), thus

\[
e_{\text{kin}} = \left(\frac{v_1}{v_1}\right)^2
\]  

(A10)

It is obvious that this coefficient of restitution is the square of the coefficient of restitution based on linear velocity differences (equation A9).
Nomenclature to appendix

\(d\)  diameter (m)  
\(e\)  coefficient of restitution based on linear velocity differences  
\(e_{\text{kin}}\)  coefficient of restitution based on kinetic energy  
\(g\)  velocity difference between two particles (m/s)  
\(m\)  mass (kg)  
\(M\)  dimensionless mass (-)  
\(n\)  normal vector  
\(v\)  linear velocity before collision (m/s)  
\(v'\)  linear velocity after collision (m/s)  
\(V\)  dimensionless velocity (-)  

Greek symbols  
\(\omega\)  angular velocity (1/s)

References to appendix
