12 Summary

One of the projects undertaken in the 1970s by IOWO (the Institute for the Development of Mathematics Education) was an attempt at developing a new arithmetic/mathematics curriculum for the primary school (K - 6), called Wiskobas (Mathematics in Primary Education). For grade 1 its prototype included an initiating course of addition and subtraction where the package "Buses" played a central part.

Realistic and mechanistic views on learning and teaching arithmetic

During the developmental activities at the Dr.W.Dreesschool (the Wiskobas design school in Arnhem) certain views emerged on how children are learning and how instruction can take advantage on it. These so-called "realistic" views differed globally on three points from the ideas prevailing in traditional mechanistic instruction. The three points, defined by a reinterpretation of what is learning, were related to the use of contexts and of arithmetical languages, and to modes of training.

Contexts

It is a principle of realistic instruction in general, and in particular with regard to arithmetic, to have children "realising" their own ideas. Following their own ideas, experiences, and imaginations children create visual or mental situations matching the arithmetical tasks they are given.

In the setting of 'Buses' this means plays, scenes, and stories about people who are boarding and getting off buses, in which the learner performs one time as an actor or story-teller and another time as a spectator or listener - a variety of ways of learning.

Mechanistic instruction of arithmetic makes use of drawings and objects, as are Cuisenaire rods, in order to imitate the arithmetical operations. One agrees on certain actions - so-called arithmetical actions - with the rods, whereas no attention is paid to the learner's factual arithmetical knowledge. The protracted process of training to master the arithmetic operations goes on along a road of stepwise increased difficulties, where mistakes are anxiously tried to be circumvented. This graduality is one more characteristic that distinguishes mechanistic from realistic instruction, where learners "realise" their own ideas in clashing with those of others or with unexpected mathematical facts, which is organised to happen in dramatised confrontations and conflicts.
Arithmetical languages
The second point on which realistic and mechanistic instruction differ from each other is languages. In our approach the main part is played by the bus-arrow language. An important feature of this language is decorating arrows by little wheels and stop posts, which are to refer to the bus context. This feature invites the children to apply decorating and decorated arrows in other contexts, and as a consequence, to generalize the meaning of the arrows. Even the equality sign can serve as an ornament, which allows switching to and fro between tasks in the bus-arrow and in the traditional arithmetic language (that of the equality sign).

Mechanistic instruction has numbers and rods precisely tuned to each other in order to accentuate their accordance. There are even plus- and minus-signs placed on cards between the rods. In order to be generalised, the only figurative context (rods) present has entirely to be abandoned.

Modes of training
By long series of mechanical exercises with a great diversity of problems one tries to strengthen the bonds of similarity between numbers and rods, in order afterwards to teach arithmetic rules, and to get knowledge and abilities memorised and automatised. The aim of training is bare arithmetic, while in realistic instruction its scope is much broader, as the learner is expected to experience the various modes of training as meaningful as such. They include among others the learners' own productions in behalf of their peers, applications of arithmetic in contexts of an earlier date, and arithmetical reasoning in arrow system puzzles.

Four research projects
In order to check whether the course developed for adding and subtracting justified one's expectations, the package "Buses" has in the course of time been subjected to four research projects.

- The Cito-Iowo-project was a formalistic research based on what external analysts had come to believe to represent the objectives of "Buses". It didn't work out as well as expected: the report didn't include any details on how the children arrived at their solutions, target objectives were interpreted as minimum objectives, and the previous instruction was not taken into account.

- In the next project "Evaluation of instruction at the Dreesschool" the teachers of the design school got the floor. It became clear that the package "Buses" had been implemented in a variety of ways.
New supplements were thought up, certain subjects were rejected, others were emphasised or modified. Teachers could not agree on the kind of instruction that is required to decently implement the arithmetical concepts.

- In the project 'Hypothesis-forming research' it was the turn of the children to display their ideas on buses. Here the method of mutual observation was initiated and applied: The researcher gave the subject all details of the report he was writing down during the observation; so while being dealt with as a research subject one discovered that the text as it was written concerned oneself, and thus started meddling into the research itself, as it were as a co-researcher. It appeared that young children don't think the way adults do about buses and in other contexts. Even on counting and numbers they nourished quite different ideas.

- The fourth project, which is the mainstay of the present thesis, is a 'Comparative research' between realistic arithmetic instruction as provided at the Dreesschool, and instruction at another school (the Nieuwlandschool at Dieren) with a traditional mechanistic curriculum. The project was meant to project the two schools upon each other: on which points did they distinguish themselves mutually, and was the one better than the other?

Regularly during one year test talks of ten minutes were carried on with each pupil, while using the method of mutual observation. The detailed day by day registration of the instruction as actually administered at both schools, including data on the time, spent on instruction and preparation, and numbers of arithmetical tasks set, made it possible to explain the subjects' performances by means of the instruction they had received.

The package 'Buses' which was used only at the Dreesschool was the proper target of the research. Three criterium measures were applied to test all subjects: the general arithmetical abilities, indirect sums, and the so-called own productions.

Conclusions and recommendations on behalf of arithmetic instruction

The detailed picture of the factual instruction and the learning performances at both schools made it possible to indicate a number of preconditions for higher quality instruction, which in turn regarded the used contexts, arithmetical languages, and modes of training.
**Contexts**

Plays and stories around buses and other people-and-animals-related contexts resulted in a convincingly faster process of learning the operations of adding and subtracting than the traditional mechanistic approach by means of illustrating objects.

**Arithmetical languages**

Wherever context tasks are set, the bus-arrow language can create awareness with regard to such situations as cannot possibly be described in the equality sign language. The latter applies only in a small minority of contexts. The multi-interpretability of the bare arrow, however, if concretised by decorations, favoured both generalisation and applicability. Decoration by means of the equality sign was an intermediary between appealing contexts and the equality language.

**Modes of training**

Applying arithmetic language in appealing contexts is something that has to be learned. It does not develop automatically as apparently presupposed in mechanistic instruction. Moreover the number of bare arithmetic sums dealt with in the last three months of mechanistic instruction can drastically be reduced in favour of, for instance, applications. A meaningful frame for children's own production was essential as a condition for correct practice. In the role play of authoring textbooks, children made less mistakes than in the more formal tasks.

In general we may conclude that the subject area oriented innovation as designed by realistic arithmetic instruction has rightly and successfully appealed to young children's playful faculties.