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GAUGED $N = 4$ MATTER COUPLINGS

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The $N = 4$ Yang–Mills multiplet is coupled to $N = 4$ conformal supergravity. The action has a local $U(4) \times G$ symmetry, where G is the Yang–Mills gauge group. The action and supersymmetry transformation rules are presented in the Poincaré gauge, and properties of the scalar potential are discussed.

In a recent paper [1], the complete lagrangian for an arbitrary number of $N = 4$ abelian vector multiplets coupled to $N = 4$ conformal and Poincaré supergravity was established. If the number of vector multiplets is $n+6$, then six of these multiplets are used in the transition from $N = 4$ conformal to Poincaré supergravity, and the remaining n correspond to physical vector multiplets. The scalar fields in such theories parametrize the manifold $M = [SO(n, 6)/SO(n) \times SO(6)] \times [SU(1, 1)/U(1)]$. In the present work, I extend these results to the non-abelian $N = 4$ matter multiplet.

The $N = 4$ Yang–Mills multiplet [2] has the following transformation rules under global supersymmetry

$$\begin{aligned}\delta A_\mu &= \bar{\epsilon}^i \gamma_\mu \psi_i + \bar{\epsilon}_i \gamma_\mu \psi^i, \\ \delta \psi_i &= -\sigma_{\mu\nu} \epsilon_i F_{\mu\nu}^+ - 2 \not{\partial} \phi_{ij} \epsilon^j - 2 \epsilon_j [\phi_{ik}, \phi^{kj}], \\ \delta \phi_{ij} &= \bar{\epsilon}_{[i} \psi_{j]} - \epsilon_{ij} \bar{\epsilon}^k \psi^l.\end{aligned}\tag{1}$$

The fields take values in the Lie-algebra of a local symmetry group G , with generators T_I ($I = 1, \dots, P$)

$$[T_I, T_J] = f_{IJ}^K T_K, \quad \text{Tr } T_I T_J = \eta_{IJ},\tag{2}$$

with real structure constants f . The metric η can be taken diagonal. The fields ϕ_{ij} satisfy

$$\phi_{ij} \equiv \phi_{ij}^I T_I = -\phi_{ji}, \quad \phi^{ij} \equiv (\phi_{ij}^I)^* = -\frac{1}{2} \epsilon^{ijk} \phi_{kl}^I.\tag{3}$$

I follow the conventions of ref. [1].

In this paper I couple the multiplet (1) to $N = 4$ conformal supergravity [3]. The superconformal multiplet has an $SU(1, 1) \times U(1)$ symmetry, which in ref. [1] played a crucial role in the construction of the transformation rules and action. The scalar fields of the Weyl multiplet, ϕ_α ($\alpha = 1, 2$), which satisfy

$$\phi^\alpha \phi_\alpha \equiv \phi_1 \phi_1^* - \phi_2 \phi_2^* = 1,\tag{4}$$

transform as a doublet under $SU(1, 1)$. For abelian vector multiplets in a superconformal background the $SU(1, 1)$ symmetry also acts as duality transformations on the field strengths. In the non-abelian case this symmetry will be broken due to the appearance of the vector potential A_μ in covariantizations. Nevertheless, the result of ref. [1] is a useful starting point for the present work.

The Q - and S -supersymmetry transformations (with parameters ϵ^i and η^i) of the $N = 4$ Yang–Mills multiplet in a superconformal background turn out to be

$$\begin{aligned}
\delta A_\mu &= \Phi(\bar{\epsilon}^i \gamma_\mu \psi_i - 2\bar{\epsilon}^i \psi_\mu^j \phi_{ij} + \bar{\epsilon}_i \gamma_\mu \Lambda_j \phi^{ij}) + \Phi^*(\bar{\epsilon}_i \gamma_\mu \psi^i - 2\bar{\epsilon}_i \psi_{\mu j} \phi^{ij} + \bar{\epsilon}^i \gamma_\mu \Lambda^j \phi_{ij}), \\
\delta \psi_i &= -\sigma_{ab} \epsilon_i \hat{F}_{ab}^+ - 2\gamma^j \phi_{ij} e^j + E_{ij} \phi^{jk} \epsilon_k + \frac{1}{2} \epsilon_i \bar{\Lambda}_j \psi^j - \epsilon_j \bar{\Lambda}_i \psi^j + \frac{1}{2} \gamma^a \epsilon^k \bar{\Lambda}^l \gamma_a \Lambda_l \phi_{kl} - 2\epsilon_j [\phi_{ik}, \phi^{kj}] \Phi^* - 2\phi_{ij} \gamma^j, \\
\delta \phi_{ij} &= \bar{\epsilon} [i \psi_j] - \epsilon_{ijk} \bar{\epsilon}^k \psi^l.
\end{aligned} \tag{5}$$

All derivatives are covariant with respect to Yang–Mills and all superconformal symmetries, and $\hat{F}_{\mu\nu}$ is the completely covariant Yang–Mills field strength. In (5) $\hat{F}$ appears in the form

$$\hat{F}_{ab}^+ = (1/\Phi) \hat{F}_{ab}^+ + (\Phi^*/\Phi) K_{ab}^+, \quad K_{ab}^+ = -\frac{1}{2} \bar{\Lambda}^i \sigma_{ab} \psi_i + T_{abij} \phi^{ij}. \tag{6}$$

The fields Λ_i, E_{kl} and T_{abij} are matter fields of the $N=4$ Weyl multiplet [3]. The scalars ϕ_α of the Weyl multiplet appear in the combination

$$\Phi = \phi^1 + \phi^2, \quad \Phi^* = \phi_1 - \phi_2. \tag{7}$$

The commutator of two transformations (5) closes on the superconformal algebra [3], a Yang–Mills gauge transformation and the fermion field equation.

As in ref. [1], the field equations determine a superconformally invariant action, which now contains a number of new terms due to the non-abelian structure. I will present the result only in the Poincaré gauge, in which the superconformal D and S symmetries are broken by the conditions

$$\text{Tr } \phi_{ij} \phi^{ij} = -6, \quad \text{Tr } \phi_{ij} \psi^j = 0. \tag{8}$$

The action can be simplified considerably by using some information from the equations of motion. It contains terms which are linear in the fields D_{kl}^{ij} and χ_{kl}^i of the Weyl multiplet, and the equations of motion of these fields imply a more stringent condition on ϕ_{ij} and ψ^j [1]:

$$\text{Tr } \phi_{ij} \phi^{kl} = -\frac{1}{2} \delta_{[i}^k \delta_{j]}^l, \quad \text{Tr } \phi_{ij} \psi^k = 0. \tag{9}$$

On using the conditions (9) the Poincaré action takes on the following form:

$$\begin{aligned}
e^{-1} \mathcal{L} &= \text{Tr} \left[-\frac{1}{4} F_{\mu\nu}^+ F^{\mu\nu+} (1/\Phi) (\phi^1 - \phi^2) - \frac{1}{2} \bar{\psi}^i \gamma^\mu \mathcal{D}_\mu \psi_i - \frac{1}{4} \mathcal{D}_\mu \phi_{ij} \mathcal{D}^\mu \phi^{ij} - (1/\Phi) \hat{F}_{\mu\nu}^+ K^{\mu\nu} - \frac{1}{4} E^{kl} \bar{\psi}_k \psi_l \right. \\
&\quad + \frac{1}{4} \epsilon^{ijkl} \bar{\psi}_i \sigma \cdot T_{jk} \psi_l - \frac{1}{2} (\Phi^*/\Phi) K_{\mu\nu}^+ K^{\mu\nu+} \left. \right] - \frac{1}{4} R(\omega) - \frac{1}{2} e^{-1} \epsilon^{\mu\nu\lambda\rho} \bar{\psi}_\mu^i \gamma_\nu \mathcal{D}_\lambda \psi_{\rho i} \\
&\quad + \frac{1}{8} (E_{kl} E^{kl} + 4D_a \phi^\alpha D_a \phi_\alpha - 2\bar{\Lambda}^k \gamma^\mu \mathcal{D}_\mu \Lambda_k) + \text{Tr} \left[-\Phi \phi^{ij} [\bar{\psi}_i, \psi_j] + \Phi \bar{\Lambda}_k \psi^i W_i^k + \frac{1}{4} |\Phi|^2 W_i^j W_j^i \right] + \frac{1}{3} \Phi^* E^{kl} X_{kl} \\
&\quad + \text{Tr} \left[-\bar{\psi}^i \gamma^\mu \gamma^\nu \psi_\mu^j \mathcal{D}_\nu \phi_{ij} - \Phi \bar{\psi}_\mu^k \gamma^\mu \psi_j W_k^j - (1/2\Phi) F_{ab}^+ (\bar{\psi}_{i\mu} \sigma_{ab} \gamma^\mu \psi^i - \bar{\psi}_\mu^i \gamma^\mu \sigma_{ab} \Lambda^j \phi_{ij} + 2\bar{\psi}_{ai} \psi_{bj} \phi^{ij}) \right] \\
&\quad + \frac{1}{3} \Phi \bar{\psi}_{\mu k} \gamma^\mu \Lambda_l X^{kl} - \frac{2}{3} \Phi^* \bar{\psi}_{\mu k} \sigma^{\mu\nu} \psi_{\nu l} X^{kl} + \frac{1}{4} e^{-1} \epsilon^{\mu\nu\lambda\rho} \bar{\psi}_{\mu k} \gamma_\nu \psi_\lambda^j \phi^{ik} \hat{\mathcal{D}}_\rho \phi_{ij} + \frac{1}{4} \epsilon_{\alpha\beta} \bar{\psi}_\mu^k \gamma^\mu \phi^\alpha \not{D} \phi^\beta \Lambda_k \\
&\quad + \text{h.c.} + \text{four-fermion terms}.
\end{aligned} \tag{10}$$

I have introduced the following tensors which depend on the scalar fields ϕ_{ij} :

$$W_i^j = [\phi_{ik}, \phi^{kj}], \quad X_{ij} = \text{Tr } \phi^{kl} [\phi_{ik}, \phi_{lj}]. \tag{11}$$

They transform under $SU(4)$ as the 15- and 10-dimensional representations, respectively. In (9) the derivatives $\mathcal{D}_\mu$ contains local Lorentz, $SU(4)$, $U(1)$ and G covariantizations. The four-fermion terms are the same as in the abelian case [1].

In the Poincaré gauge the supersymmetry transformations are a combination of (5), and a compensating transformation required to maintain the conditions (8). In the present form the invariance of (10) under supersymmetry requires the use of the equations of motion of E_{ij}, T_{abij} and the $SU(4)$ gauge field $V_\mu^i{}_j$. This is due to the fact that I have used (9), which are equations of motion and not invariant under supersymmetry, to simplify the action. Only after elimination of E, T and V is the action fully invariant. These fields are then replaced by

$$E_{ij} = \text{Tr } \bar{\psi}_i \psi_j - \frac{4}{3} \Phi^* X_{ij}, \quad T_{abij} = (1/\Phi^*) \text{Tr } \phi_{ij} \hat{F}_{ab}^+ + \frac{1}{2} (\Phi/\Phi^*) \text{Tr } \bar{\psi}_i \sigma_{ab} \psi_j, \quad (12, 13)$$

$$V_{\mu j}^i = \text{Tr } [A_{\mu} W_j^i + \frac{1}{2} \phi^{ik} \hat{\partial}_{\mu}^{\leftrightarrow} \phi_{jk} - \frac{1}{2} \bar{\psi}^i \gamma^{\mu} \psi_j] - \frac{1}{4} \bar{\Lambda}^i \gamma^{\mu} \Lambda_j - \text{traces}. \quad (14)$$

Compared to ref. [1], the final form of the action now contains a scalar potential $\mathcal{L}_V$ and a collection of mass-like terms $\mathcal{L}_M$. These are easily derived from (10) and (12)–(14):

$$e^{-1} \mathcal{L}_V = |\Phi|^2 (-\frac{4}{9} X^{kl} X_{kl} + \frac{1}{2} \text{Tr } W_l^j W_j^i), \quad (15)$$

$$e^{-1} \mathcal{L}_M = \text{Tr} [\frac{1}{3} \Phi X^{kl} \bar{\psi}_k \psi_l - \Phi \phi^{ij} [\bar{\psi}_i, \psi_j] + \Phi \bar{\Lambda}_k \psi^l W_l^k - \Phi \bar{\psi}_{\mu}^l \gamma^{\mu} \psi_k W_l^k] \\ + \frac{1}{3} \Phi \bar{\psi}_{\mu k} \gamma^{\mu} \Lambda_l X^{kl} - \frac{2}{3} \Phi^* \bar{\psi}_{\mu k} \sigma^{\mu\nu} \psi_{\nu l} X^{kl} + \text{h.c.} \quad (16)$$

The final form of the supersymmetry transformations is

$$\delta \phi_{\alpha} = -\bar{\epsilon}^i \Lambda_i \epsilon_{\alpha\beta} \phi^{\beta}, \quad \delta \phi_{ij} = \bar{\epsilon}_i \psi_j - \epsilon_{ijkl} \bar{\epsilon}^k \psi^l, \\ \delta \Lambda_i = 2\epsilon^{\alpha\beta} \phi_{\alpha} \not{D}_{\beta} \epsilon_i - \frac{4}{3} \Phi^* X_{ij} \epsilon^j + \frac{1}{2} \epsilon_{ijkl} \sigma_{ab} \epsilon^j (1/\Phi) \text{Tr } \phi^{kl} \hat{F}_{ab}^- + \psi^2\text{-terms}, \\ \delta \psi_i = -2 \not{D}_{\mu} \phi_{ij} \epsilon^j - 2\Phi^* \epsilon_j [W_i^j + \frac{2}{3} (\phi_{ik} X^{kj} + X_{ik} \phi^{kj})] - \sigma_{ab} \epsilon_i (1/\Phi) (\hat{F}_{ab}^+ + \phi^{kl} \text{Tr } \phi_{kl} \hat{F}_{ab}^+) \\ - \gamma^{\mu} \epsilon^j \text{Tr } (\phi^{kl} \hat{D}_{\mu}^{\leftrightarrow} \phi_{[il]} \phi_{kj}) + \Lambda \psi, \psi^2\text{-terms}, \\ \delta A_{\mu} = \Phi (\bar{\epsilon}^i \gamma_{\mu} \psi_i - 2\bar{\epsilon}^i \psi_{\mu}^j \phi_{ij} + \bar{\epsilon}_i \gamma_{\mu} \Lambda_j \phi^{ij}) + \Phi^* (\bar{\epsilon}_i \gamma_{\mu} \psi^i - 2\bar{\epsilon}_i \psi_{\mu j} \phi^{ij} + \bar{\epsilon}^i \gamma_{\mu} \Lambda^j \phi_{ij}), \\ \delta \psi_{\mu}^i = 2 \not{D}_{\mu} \epsilon^i - \frac{2}{3} \gamma_{\mu} \Phi^* X^{ij} \epsilon_j - \sigma_{ab} \gamma_{\mu} \epsilon_j (1/\Phi) \text{Tr } \phi^{ij} \hat{F}_{ab}^- \\ - \text{Tr } (\phi^{ik} \hat{D}_{\mu}^{\leftrightarrow} \phi_{jk}) \epsilon^j - \frac{1}{2} \phi^{\alpha} \hat{\partial}_{\mu}^{\leftrightarrow} \phi_{\alpha} \epsilon^i - \epsilon^{ijkl} \bar{\epsilon}_j \psi_{\mu k} \Lambda_l + \psi^2, \Lambda^2\text{-terms}, \\ \delta e_{\mu}^a = \bar{\epsilon}^i \gamma_a \psi_{\mu i} + \text{h.c.} \quad (17)$$

In this form the theory has local G, SU(4) and U(1) symmetry. However, these will in general not be preserved by solutions of the constraints (4) and (9).

Let us therefore consider the restrictions on G, which follow from the constraint (9). The condition on the scalar field can only be solved if the metric $\eta_{IJ}(2)$ has at least six negative eigenvalues. If furthermore one requires that the scalar kinetic terms all have the canonical sign, then there can be no more than six negative eigenvalues. In particular, for $P=6$ all eigenvalues of η_{IJ} must be negative, and if one normalizes to $\eta_{II} = -1$ the constraint (9) is solved by (T_I are represented by 4×4 matrices, $a, b = 1, \dots, 4$)

$$(\phi_{ij})_{ab} = \frac{1}{4} (1+i) \delta_a [i \delta_j]_b - \frac{1}{4} (1-i) \epsilon_{ijab}, \quad (\psi_i)_{ab} = 0. \quad (18)$$

This solution preserves a local SO(4) group, and leads to the gauged $N=4$ supergravity theory of Freedman and Schwarz [4]. The tensors W and X are now given by

$$(W_i^j)_{ab} = \frac{1}{2} \delta_a [i \delta_j]_b, \quad X_{ij} = -\frac{3}{4} (1+i) \delta_{ij}, \quad (19)$$

and the scalar potential term (15) is

$$e^{-1} \mathcal{L}_V = +|\Phi|^2. \quad (20)$$

Eq. (20) takes on a more familiar form if the constraint on ϕ_{α} , eq. (4), is solved by setting

$$\phi_1 = (1 - |Z|^2)^{-1/2}, \quad \phi_2 = Z(1 - |Z|^2)^{-1/2}, \quad (21)$$

thus breaking the local U(1) symmetry. In this parametrization (20) reads

$$e^{-1} \mathcal{L}_V = |1 - Z|^2 / (1 - |Z|^2). \quad (22)$$

The corresponding potential is unbounded below, and has no critical points. Nevertheless, the equations of motion have a stable electrovac solution [5].

In the general case one is interested in critical points of the potential (15) in the ϕ_{ij} directions, in view of the possibility of supersymmetry breaking. This aspect is presently being investigated. Variation of (15) yields

$$e^{-1} \delta \mathcal{L}_V = |\Phi|^2 \text{Tr} \delta \phi_{ij} Z^{ij}, \quad (23)$$

with

$$Z^{ij} = \frac{2}{3} X^{[ik} W_k^{j]} - \frac{2}{3} \epsilon^{ijk} X_{lm} W_k^m + [W_k^{[i}, \phi^{j]k}], \quad (24)$$

Since $\delta \phi_{ij}$ can be restricted to those variations which preserve the constraint (9), a sufficient condition for a critical point is that Z^{ij} is proportional to ϕ^{ij} . One easily verifies that Z^{ij} vanishes for the solution (18). By contracting with ϕ_{ij} one finds that $Z^{ij} = 0$ implies

$$\frac{4}{3} X^{ij} X_{ij} = \text{Tr} W_j^i W_i^j, \quad (25)$$

so that in this class of critical points the value of the potential $V = -\mathcal{L}_V$ is negative. This is a necessary condition for unbroken supersymmetry [6].

After completion of this work I learned that similar results have been obtained by Bergshoeff, Koh and Sezgin [7], using a different method.

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