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Bergshoeff, E.; Sezgin, E.; Townsend, P.K.

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SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

E. BERGSHOEFF¹, E. SEZGIN

International Centre for Theoretical Physics, I-34100 Trieste, Italy

and

P.K. TOWNSEND

DAMPT, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

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We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of n -dimensional extended objects to d -dimensional supergravity.

1. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu-Schwarz-Ramond formulation of the spinning string theory.

More progress towards the construction of a membrane theory was made by Sugamoto [4] in 1983. More recently, Hughes, Li and Polchinski [5] have constructed a Green-Schwarz-type action for a three-extended object propagating in flat six-dimensional spacetime. The consistency of the action requires the

existence of a closed superspace five-form, in analogy with the Henneaux and Mezincescu [6] construction for the Green-Schwarz superstring action [7], where a closed superspace three-form is required. The novel feature of the theory of Hughes et al. is that the parameter of the Siegel-type transformation [8] is a scalar rather than a vector on the world volume.

The generalization of the Hughes et al. model to n -extended objects propagating in flat d -dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$ -form given by

$$H = E^\beta E^\alpha E^{a_n} \dots E^{a_1} (\gamma_{a_1 \dots a_n})_{\alpha\beta}, \quad (1)$$

where (E^α, E^a) are the basis one-forms in superspace. This form is closed provided that the following Γ -matrix identity holds:

$$(\gamma^a)_{(\alpha\beta} (\gamma^{aa_1 \dots a_{n-1}})_{\gamma\delta)} = 0. \quad (2)$$

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an n -extended object in d -dimensional curved superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$ -form in curved superspace. Thus we

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expect that the n -extended objects under consideration can consistently propagate only in $d \leq 11$ supergravities whose superspace formulation involves a closed $(n+2)$ -form. We further expect that such forms exist in supergravity theories in which a closed bosonic $(n+2)$ -form occurs. As far as we know, the following possibilities exist (we include the Yang-Mills couplings whenever possible):

The dual formulation of $d=10, N=1$ supergravity involves a closed seven-form. Its dimensional reduction on a $(10-d)$ -dimensional torus leads to real closed $(d-3)$ -forms in d -dimensional supergravities. (These are $N=1$ supergravities in $d=8, 9, 10$; $N=2$ in $d=7$ and $N=2$ or 4 in $d=6$) [9]. Apart from these, there is: (i) *A real closed four-form in $d=11, N=1$ supergravity*, (ii) *a real closed three-form in non-chiral $d=10, N=2$ supergravity*, (iii) *a complex closed three-form in chiral $d=10, N=2$ supergravity*.

Excluding Yang-Mills coupling, as is well known, closed super three-forms exist in $d=3, 4, 6$, and 10 .

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11 . Since the superspace formulation of $d=7, N=2$ supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of $d=11$ supergravity to a supermembrane. In particular the Kalb-Ramond-like third rank antisymmetric tensor field of $d=11$ supergravity couples to the supermembrane via a Wess-Zumino term.

In the following we focus our attention on the description of the supermembrane action in $d=11$. The extension to the case of n -extended objects is given in the appendix.

2. We propose the following action for a supermembrane coupled to $d=11$ supergravity:

$$S = \int d^2 \xi \left(\frac{1}{2} \sqrt{-g} g^{ij} E_i^a E_j^b \eta_{ab} + \epsilon^{ijk} E_i^A E_j^B E_k^C B_{CBA} - \frac{1}{2} \sqrt{-g} \right). \quad (3)$$

Here $i=0, 1, 2$ labels the coordinates $\xi^i = (\tau, \sigma, \rho)$ of the world volume with metric g_{ij} and signature $(-, +, +)$. The super three-form B is needed for the superspace description of $d=11$ supergravity [10]. For the Levi-Civita symbol ϵ^{ijk} we use the same con-

ventions as in ref. [11]. In (3) we have used the notation

$$E_i^A = (\partial_i Z^M) E_M^A, \quad (4)$$

where $Z^M(\xi)$ are the superspace coordinates, and $E_M^A(Z)$ is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric g_{ij} gives the embedding equation

$$g_{ij} = E_i^a E_j^b \eta_{ab} \equiv T_{ij}. \quad (5)$$

We require that the action S is invariant under a fermionic gauge transformation of the form [5]

$$\delta E^a = 0, \quad \delta E^\alpha = (1 + \Gamma)^\alpha_\beta \kappa^\beta,$$

$$\delta g_{ij} = 2[X_{ij} - g_{ij} X^k{}_k / (n-1)] \quad (n=2 \text{ for membrane}), \quad (6)$$

where the transformation parameter $\kappa^\alpha(\xi)$ is a 32 component Majorana spinor, and a world-volume scalar, and

$$\delta E^A = \delta Z^M E_M^A, \quad (7)$$

$$\Gamma^\alpha_\beta = (1/6\sqrt{-g}) \epsilon^{ijk} E_i^a E_j^b E_k^c (\gamma_{abc})^\alpha_\beta. \quad (8)$$

Here $\gamma^a (a=0, 1, \dots, 10)$ are the Dirac matrices in eleven dimensions. X_{ij} is a function of E_i^A which will be determined by the invariance of the action. The choice of δg_{ij} is due to the fact that, given a variation of the action of the form $\delta S = T_{ij} X^{ij}$, and writing this variation as

$$T_{ij} X^{ij} = g_{ij} X^{ij} + (T_{ij} - g_{ij}) X^{ij}, \quad (9)$$

the second term on the right-hand side cancels $(\delta S / \delta g_{ij}) \delta g_{ij}$. Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form $T_{ij} X^{ij}$, we can use eq. (5), provided that we add X^{ij} to δg_{ij} as in (6).

The matrix Γ^α_β occurring in (8) satisfies the property

$$\Gamma^\alpha_\beta \Gamma^\beta_\delta = (\Gamma^i{}_i \Gamma^j{}_j \Gamma^k{}_k) \delta^\alpha_\delta \equiv \Gamma^2 \delta^\alpha_\delta. \quad (10)$$

The normalization in (8) is chosen such that upon the use of the equation $T_{ij} = g_{ij}$, the matrix Γ^α_β satisfies the relation $\Gamma^\alpha_\beta \Gamma^\beta_\gamma = \delta^\alpha_\gamma$.

Now using (6) the variation of the action (3) is (we consider a closed supermembrane and therefore discard the surface terms)

$$\begin{aligned} \delta S = & \int d^2 \xi \left[\sqrt{-g} g^{ij} (-\delta E^\beta E_i{}^\gamma T^\alpha{}_{\gamma\beta}) E_{ja} \right. \\ & + \sqrt{-g} g^{ij} (-\delta E^\beta E_i{}^c T^a{}_{c\beta}) E_{ja} \\ & + \epsilon^{ijk} E_i{}^A E_j{}^B E_k{}^C \delta E^\alpha H_{\alpha CBA} \\ & \left. - \frac{1}{2} \sqrt{-g} \delta g^{ij} (T^{ij} - \frac{1}{2} g^{ij} T - \frac{1}{2} g^{ij}) \right]. \end{aligned} \quad (11)$$

The torsion two-form T^A and the four-form field strength H are defined by (our superspace conventions are those of Howe [12])

$$\begin{aligned} T^A &= dE^A + E^B \Omega_B{}^A = \frac{1}{2} E^B E^C T_{CB}{}^A, \\ H &= dB = \frac{1}{24} E^D E^C E^B E^A H_{ABCD}. \end{aligned} \quad (12)$$

We now organize the terms in (11) according to the number of one-forms E^α they contain. Those with three E^α and two E^α come only from the Wess-Zumino term. They must vanish separately, and this requires the constraints

$$H_{\alpha\beta\gamma\delta} = H_{\alpha\beta\gamma d} = 0. \quad (13)$$

The cancellation of the terms linear in E^α lead to the constraints

$$T^a{}_{\alpha\beta} = (\gamma^a)_{\alpha\beta}, \quad (14)$$

$$H_{\alpha\beta ab} = -\frac{1}{6} (\gamma_{ab})_{\alpha\beta}, \quad (15)$$

while the cancellation of the terms not containing E^α require the constraint

$$\eta_{c(a} T^c{}_{b)\alpha} = \eta_{ab} \Lambda_\alpha, \quad (16)$$

$$H_{\alpha abc} = -\frac{1}{2} A_\beta (\gamma_{abc})^\beta{}_\alpha. \quad (17)$$

Here Λ_α is an arbitrary spinor superfield which is vanishing in $d=11$ [10].

It is important to realize that in obtaining (14)-(17) we have used the identity

$$\delta E^\alpha = \Gamma^\alpha{}_\beta \delta E^\beta + (1 - \Gamma^2) \kappa^\alpha. \quad (18)$$

Using this identity in the variation of the kinetic term, the terms arising from $\Gamma^\alpha{}_\beta$ in (18) can be shown to cancel similar terms coming from the variation of the Wess-Zumino term, modulo terms which cancel by

an appropriate variation of g_{ij} . [Using the argument below (8) once.] In the remaining terms coming from $(1 - \Gamma^2)$, we use the argument given below (8) repeatedly to compute further contributions to δg_{ij} . Thus we find the result

$$\begin{aligned} X_{ij} = & -\frac{1}{4} \epsilon_i{}^{kl} E_k{}^a E_l{}^b (\gamma_{ab})_{\alpha\beta} \delta E^\beta E_j{}^\alpha \\ & + \frac{1}{2} \kappa^\beta E_n{}^\alpha (\gamma^d)_{\alpha\beta} E^{nd} g_{i|j} (T^k{}_k T^l{}_{|l} + \delta^k{}_k T^l{}_{|l}) \\ & + i \leftrightarrow j. \end{aligned} \quad (19)$$

In summary, the action (3) is invariant under (6) provided that (13)-(17) hold, and X^{ij} is given by (19). In addition, the following Bianchi identities must hold:

$$DT^A = -E^B \wedge R_B{}^A, \quad DH = 0. \quad (20)$$

The generalization of the results of this section to the general case of n -extended objects in d -dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of $d=11$ supergravity given by Cremmer and Ferrara [10] and Brink and Howe [10] do provide a solution to (13)-(17) and the Bianchi identities (20), with $\Lambda_\alpha = 0$.

In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a *closed* supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few.

Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane [13], it is encouraging that, here, we have a spacetime supersymmetric membrane action.

Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, and at the same time from three-dimensional world volume to a two-dimensional

world sheet. It would be interesting to see what kind of $d=10$ string theories could possibly emerge in an infinitely thin membrane limit.

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Appendix. In this appendix we construct the action for an n -extended object propagating in d -dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

$$S = \int d^2\xi \left[\frac{1}{2} \sqrt{-g} g^{ij} E_i^a E_j^b \eta_{ab} + \epsilon^{i_1 \dots i_n} E_{i_1}^{A_1} \dots E_{i_{n+1}}^{A_{n+1}} B_{A_{n+1} \dots A_1} - \frac{1}{2} (n-1) \sqrt{-g} \right]. \quad (A1)$$

The transformation rules are those in (6), where the matrix Γ^α_β is now given by

$$\Gamma^\alpha_\beta = [\eta / (n+1)! \sqrt{-g}] \times \epsilon^{i_1 \dots i_{n+1}} E_{i_1}^{a_1} \dots E_{i_{n+1}}^{a_{n+1}} (\gamma_{a_1 \dots a_{n+1}})^\alpha_\beta, \quad (A2)$$

where η is given by

$$\eta = (-1)^{(n+1)(n-2)/4}. \quad (A3)$$

Invariance of the action (A1) is ensured by imposing the following set of constraints:

$$T^a_{\alpha\beta} = (\gamma^a)_{\alpha\beta}, \quad \eta_{c(a} T^c_{b)\alpha} = \eta_{ab} A_\alpha, \quad (A4)$$

$$H_{\alpha a_{n+1} \dots a_1} = (\eta / n!) A_\beta (\gamma_{a_1 \dots a_{n+1}})^\beta_\alpha,$$

$$H_{\alpha\beta a_n \dots a_1} = [\eta (-1)^n / (n+1)!] (\gamma_{a_1 \dots a_n})_{\alpha\beta}, \quad (A5)$$

$$H_{\alpha\beta\gamma A_1 \dots A_{n-1}} = 0, \quad (A5\text{cont'd})$$

and by taking X_{ij} occurring in (6) to be

$$X_{ij} = (-\eta / 2n!) \times \epsilon_i^{k_1 \dots k_n} E_{k_1}^{a_1} \dots E_{k_n}^{a_n} (\gamma_{a_1 \dots a_n})_{\alpha\beta} \delta E^\beta E_j^\alpha + \frac{1}{2} \kappa^\beta [E_n^\alpha (\gamma^a)_{\alpha\beta} E^{na} + (n+1) A_\beta] \times g_{ij} (T^{k_1}_{k_1} \dots T^{k_n}_{k_n}) + \delta^{k_1}_{k_1} T^{k_2}_{k_2} \dots T^{k_n}_{k_n} + \delta^{k_1}_{k_1} \dots \delta^{k_{n-1}}_{k_{n-1}} T^{k_n}_{k_n} - \frac{1}{2} \delta E^\beta A_\beta g_{ij} + (i \leftrightarrow j). \quad (A6)$$

References

- [1] P.A.M. Dirac, Proc. R. Soc. A 268 (1962) 57.
- [2] P.A. Collins and R.W. Tucker, Nucl. Phys. B 112 (1976) 150.
- [3] P.S. Howe and R.W. Tucker, J. Phys. A 10 (1977) L155.
- [4] A. Sugamoto, Nucl. Phys. B 215 (1983) 381.
- [5] J. Hughes, J. Liu and J. Polchinski, Supermembranes, University of Texas preprint (1986).
- [6] M. Henneaux and L. Mezincescu, Phys. Lett. B 152 (1985) 340.
- [7] M.B. Green and J.H. Schwarz, Nucl. Phys. B 243 (1984) 285.
- [8] W. Siegel, Phys. Lett. B 128 (1983) 397.
- [9] See e.g. A. Salam and E. Sezgin, eds., Supergravity in diverse dimensions, Vol. 1 (World Scientific, Singapore, 1987).
- [10] E. Cremmer and S. Ferrara, Phys. Lett. B 91 (1980) 61; L. Brink and P. Howe, Phys. Lett. B 91 (1980) 384.
- [11] E. Bergshoeff, E. Sezgin and P.K. Townsend, Phys. Lett. B 169 (1986) 191.
- [12] P.S. Howe, Nucl. Phys. B 199 (1982) 309.
- [13] K. Kikkawa and M. Yamasaki, Can the membrane be a unification model?, Osaka University preprint (August 1986).