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Bergshoeff, E.; Sezgin, E.; Townsend, P.K.

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SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

E. BERGSHoeff, E. SEZGIN
International Centre for Theoretical Physics, I-34100 Trieste, Italy

and

P.K. TOWNSEND
DAMPT, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

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We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of $n$-dimensional extended objects to $d$-dimensional supergravity.

I. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu–Schwarz–Ramond formulation of the spinning string theory.


The generalization of the Hughes et al. model to $n$-extended objects propagating in flat $d$-dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$-form given by

$$H = E^a E^a' E^a'' \ldots E^a_{(n+1)},$$

where $(E^a, E^a')$ are the basis one-forms in superspace. This form is closed provided that the following $\Gamma$-matrix identity holds:

$$(\gamma^a)_{a\beta} (\gamma^{a_2 \ldots a_{n+1}})_{\gamma^a} = 0.$$  

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an $n$-extended object in $d$-dimensional curved superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$-form in curved superspace. Thus we
expect that the \( n \)-extended objects under consideration can consistently propagate only in \( d \leq 11 \) supergravities whose superspace formulation involves a closed \((n+2)\)-form. We further expect that such forms exist in supergravity theories in which a closed bosonic \((n+2)\)-form occurs. As far as we know, the following possibilities exist (we include the Yang-Mills couplings whenever possible):

The dual formulation of \( d=10, N=1 \) supergravity involves a closed seven-form. Its dimensional reduction on a \((10-d)\)-dimensional torus leads to real closed \((d-3)\)-forms in \( d \)-dimensional supergravities. (These are \( N=1 \) supergravities in \( d=8, 9, 10; N=2 \) in \( d=7 \) and \( N=2 \) or \( 4 \) in \( d=6 \)) [9]. Apart from these, there is: (i) A real closed four-form in \( d=11, N=1 \) supergravity, (ii) a real closed three-form in non-chiral \( d=10, N=2 \) supergravity, (iii) a complex closed three-form in chiral \( d=10, N=2 \) supergravity.

Excluding Yang-Mills coupling, as is well known, closed super three-forms exist in \( d=3, 4, 6, \) and \( 10 \).

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11. Since the superspace formulation of \( d=7, N=2 \) supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of \( d=11 \) supergravity to a supermembrane. In particular the Kalb-Ramond-like third rank antisymmetric tensor field of \( d=11 \) supergravity [10].

In the following we focus our attention on the description of the supermembrane action in \( d=11 \). The extension to the case of \( n \)-extended objects is given in the appendix.

2. We propose the following action for a supermembrane coupled to \( d=11 \) supergravity:

\[
S = \int d^2 \xi \left( \frac{1}{2} \sqrt{-g} g^\alpha a E_i^a E_j b \eta_{ab} + \epsilon^{ijk} E_i^a E_j b E_k c B_{BCA} - \frac{1}{2} \sqrt{-g} \right). \tag{3}
\]

Here \( i=0, 1, 2 \) labels the coordinates \( \xi^i = (\tau, \sigma, \rho) \) of the world volume with metric \( g_{ij} \) and signature \((-,, +, +)\). The super three-form \( B \) is needed for the superspace description of \( d=11 \) supergravity [10]. For the Levi-Civita symbol \( \epsilon^{ijk} \) we use the same conventions as in ref. [11]. In (3) we have used the notation

\[
E_i^a = (\partial_i Z^M) E_M^a, \tag{4}
\]

where \( Z^M(\xi) \) are the superspace coordinates, and \( E_M^a(Z) \) is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric \( g_{ij} \) gives the embedding equation

\[
g_{ij} = E_i^a E_j b \eta_{ab} \equiv T_{ij}. \tag{5}
\]

We require that the action \( S \) is invariant under a fermionic gauge transformation of the form [5]

\[
\delta E^a = 0, \quad \delta E^a = (1 + \Gamma)^{\alpha}_\beta \kappa^\alpha, \tag{6}
\]

\[
\delta g_{ij} = 2 [X_{ij} - g_{ij} X^k \eta_{k} / (n-1)]
\]

\[ (n=2 \text{ for membrane}) \tag{6},
\]

where the transformation parameter \( \kappa^\alpha(\xi) \) is a 32 component Majorana spinor, and a world-volume scalar, and

\[
\delta E^a = \delta Z^M E_M^a, \tag{7}
\]

\[
\Gamma^{\alpha}_\beta = (1/6 \sqrt{-g}) \epsilon^{ijk} E_i^a E_j b E_k c (\gamma_{abc})^{\alpha}_\beta. \tag{8}
\]

Here \( \gamma^a (a=0, 1, ..., 10) \) are the Dirac matrices in eleven dimensions. \( X_{ij} \) is a function of \( E_i^a \) which will be determined by the invariance of the action. The choice of \( \delta g_{ij} \) is due to the fact that, given a variation of the action of the form \( \delta S = T_{ij} X^j \), and writing this variation as

\[
T_{ij} X^i = g_{ij} X^i + (T_{ij} - g_{ij}) X^i, \tag{9}
\]

the second term on the right-hand side cancels \( \delta S / \delta g_{ij} \). Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form \( T_{ij} X^j \), we can use eq. (5), provided that we add \( X^j \) to \( \delta g_{ij} \) as in (6).

The matrix \( \Gamma^{\alpha}_\beta \) occurring in (8) satisfies the property

\[
\Gamma^{\alpha}_\beta \Gamma^\beta_\delta = (T_i^j T_i^j T_k^l) \delta^{\alpha}_\delta \equiv \Gamma^2 \delta^{\alpha}_\delta. \tag{10}
\]

The normalization in (8) is chosen such that upon the use of the equation \( T_{ij} = g_{ij} \), the matrix \( \Gamma^{\alpha}_\beta \) satisfies the relation \( \Gamma^{\alpha}_\beta \Gamma^\beta_\gamma = \delta^{\alpha}_\gamma \).
Now using (6) the variation of the action (3) is 
(we consider a closed supermembrane and therefore 
discard the surface terms)
\[ \delta S = \int d^2 \xi \left\{ \sqrt{\alpha} g^{ij}(-\delta E^\beta E_i \gamma T^a g_{ij})E_{ja} \right\} \]
\[ + \sqrt{\alpha} g^{ij}(-\delta E^\beta E_i \gamma T^c g_{ij})E_{ja} \]
\[ + \epsilon^{ijk} E_i^A E_j^B E_k^C \delta E^\alpha H_{alpha} \]
\[ - \frac{1}{2} \sqrt{\alpha} \delta g^{ij}(T^{ij} - \frac{1}{2} g^{ij} T - \frac{1}{2} g^{ij}) \] .
\[ (11) \]
The torsion two-form \( T^A \) and the four-form field strength \( H \) are defined by (our superspace conventions are those of Howe \[12\])
\[ T^A = dE^a + \epsilon^{a} E^a g^{2B} A = \frac{1}{2} E^a E^c T_{cB}^A , \]
\[ H = dB = E^a E^d E^e A^f = \frac{1}{2} E^a E^c T_{cB}^A . \]
\[ (12) \]
We now organize the terms in (11) according to the number of one-forms \( E^a \) they contain. Those with three \( E^a \) and two \( E^a \) come only from the Wess-Zumino term. They must vanish seperately, and this requires the constraints
\[ H_{a\beta\gamma} = H_{a\beta\gamma} = 0 . \]
\[ (13) \]
The cancellation of the terms linear in \( E^a \) lead to the constraints
\[ T^a_{\alpha\beta} = (\gamma^a)_{\alpha\beta} , \]
\[ H_{a\beta\gamma\delta} = -\frac{1}{2}(\gamma_{ab})_{\alpha\beta} , \]
\[ (14) \]
while the cancellation of the terms not containing \( E^a \) require the constraint
\[ \eta_{(a} T^{b)\alpha} = \eta_{ab} A^a , \]
\[ (15) \]
\[ \eta_{(a} T^{b)\alpha} = \eta_{ab} A^a . \]
\[ (16) \]
Here \( A^a \) is an arbitrary spinor superfield which is vanishing in \( d=11 \) \[10\].
It is important to realize that in obtaining (14)–(17) we have used the identity
\[ \delta E^a = \Gamma^a_{\beta} \delta E^\beta + (1 - \Gamma^2) \kappa^a . \]
\[ (18) \]
Using this identity in the variation of the kinetic term, the terms arising from \( \Gamma^a_{\beta} \) in (18) can be shown to cancel similar terms coming from the variation of the Wess-Zumino term, modulo terms which cancel by an appropriate variation of \( g_{ij} \). [Using the argument below (8) once.] In the remaining terms coming from \((1 - \Gamma^2) \), we use the argument given below (8) repeatedly to compute further contributions to \( \delta g_{ij} \). Thus we find the result
\[ X_{ij} = -\frac{1}{4} \epsilon^{ijkl} E_k^a E_l^b (\gamma^{ab})_{\alpha\beta} \delta E^\beta E_j^{\alpha} \]
\[ + \frac{1}{4} \kappa^b E_n^{\alpha}(\gamma^{d})_{\alpha\beta} E^{md} g_{ij}(T^{k}_{k} T^{l}_{l} + \delta^{k}_{k} T^{l}_{l}) \]
\[ + i \leftrightarrow j . \]
\[ (19) \]
In summary, the action (3) is invariant under (6) provided that (13)–(17) hold, and \( X_{ij} \) is given by (19). In addition, the following Bianchi identities must hold:
\[ DT^A = -E^a \wedge R^A_{a} , \quad DH = 0 . \]
\[ (20) \]
The generalization of the results of this section to the general case of \( n \)-extended objects in \( d \)-dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of \( d=11 \) supergravity given by Cremmer and Ferrara \[10\] and Brink and Howe \[10\] do provide a solution to (13)–(17) \( and \) the Bianchi identities (20), with \( A^a = 0 \).
In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a \textit{closed} supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few.
Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane \[13\], it is encouraging that, here, we have a spacetime supersymmetric membrane action.
Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, \textit{and} at the same time from three-dimensional world volume to a two-dimensional
world sheet. It would be interesting to see what kind of \(d=10\) string theories could possibly emerge in an infinitely thin membrane limit.

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Appendix. In this appendix we construct the action for an \(n\)-extended object propagating in \(d\)-dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

\[
S = \int d^2 \xi \left[ \frac{1}{2} \sqrt{-g} \left( g^{ij} E_i^a E_j^b \eta_{ab} + \epsilon^{i_1 \ldots i_{n+1}} E_{i_1}^{a_1} \ldots E_{i_{n+1}}^{a_{n+1}} B_{A_0 \ldots A_1} ight) ight]
\]

\[+ \frac{1}{2} (n-1) \sqrt{-g} \right] .
\]

The transformation rules are those in (6), where the matrix \(I^\alpha_\beta\) is now given by

\[
I^\alpha_\beta = \left[ \eta/(n+1) \right] \sqrt{-g} \times \epsilon^{i_1 \ldots i_{n+1}} E_{i_1}^{a_1} \ldots E_{i_{n+1}}^{a_{n+1}} (\gamma_{a_1 \ldots a_{n+1}})^\alpha_\beta ,
\]

where \(\eta\) is given by

\[
\eta = (-1)^{(n+1)(n-2)/4} .
\]

Invariance of the action (A1) is ensured by imposing the following set of constraints:

\[
T^a_{\alpha \beta} = (\gamma^a)^{\alpha \beta} , \quad \eta_{(a} T^c_{b)\gamma} = \eta_{\gamma a} A_{\alpha} ,
\]

\[
H_{a \gamma a_1 \ldots a_{n-1}} = (\eta/(n+1)!) (\gamma_{a_1 \ldots a_{n-1}})^a_{\alpha} ,
\]

\[
H_{a \beta a_1 \ldots a_{n-1}} = [\eta (-1)^n/(n-1)!] (\gamma_{a_1 \ldots a_{n-1}})^a_{\gamma} ,
\]

and by taking \(X_{ij}\) occurring in (6) to be

\[
X_{ij} = (-\eta/2n!)
\]

\[
\times \epsilon_{i_1 \ldots i_{k_i}} E_{i_1}^{a_1} \ldots E_{i_{k_i}}^{a_{k_i}} (\gamma_{a_1 \ldots a_{k_i}}) \delta E^\alpha E_j^\beta
\]

\[
+ \frac{1}{2} \kappa^\beta \left[ E_n^{a_\gamma} (\gamma^a)^{\alpha \beta} E^{\alpha a} + (n+1) A_{\beta} \right]
\]

\[
\times g_{ij} \left( T^{k_1} \ldots T^{k_{k_i}} \right) + \delta^{k_1 \gamma k_2} \ldots T^{k_{k_i} k_i}
\]

\[- \frac{1}{2} \delta E^\beta A_{\gamma} g_{ij} + (i\leftrightarrow j) .
\]

References


