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Bergshoeff, E.; Sezgin, E.; Townsend, P.K.

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SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

E. BERGSHOEFF[1], E. SEZGIN
International Centre for Theoretical Physics, I-34100 Trieste, Italy

and

P.K. TOWNSEND
DAMPT, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

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We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of n-dimensional extended objects to d-dimensional supergravity.

1. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu–Schwarz–Ramond formulation of the spinning string theory.

More progress towards the construction of a membrane theory was made by Sugamoto [4] in 1983. More recently, Hughes, Li and Polchinski [5] have constructed a Green–Schwarz-type action for a three-extended object propagating in flat six-dimensional spacetime. The consistency of the action requires the existence of a closed superspace five-form, in analogy with the Henneaux and Mezincescu [6] construction for the Green–Schwarz superstring action [7], where a closed superspace three-form is required. The novel feature of the theory of Hughes et al. is that the parameter of the Siegel-type transformation [8] is a scalar rather than a vector on the world volume.

The generalization of the Hughes et al. model to n-extended objects propagating in flat d-dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$-form given by

$$H=E^{\beta}E^{\alpha}E^{a_n}\dots E^{a_1}(\gamma_{a_1\dots a_n})_{\alpha\beta}\,, \tag{1}$$

where (E^{α}, E^{a}) are the basis one-forms in superspace. This form is closed provided that the following Γ-matrix identity holds:

$$(\gamma^{a})_{(\alpha\beta}(\gamma^{aa_1\dots a_{n-1}})_{\gamma\delta)}=0\,. \tag{2}$$

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an n-extended object in d-dimensional *curved* superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$-form in curved superspace. Thus we

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expect that the n-extended objects under consideration can consistently propagate only in $d \leqslant 11$ supergravities whose superspace formulation involves a closed $(n+2)$-form. We further expect that such forms exist in supergravity theories in which a closed *bosonic* $(n+2)$-form occurs. As far as we know, the following possibilites exist (we include the Yang–Mills couplings whenever possible):

The dual formulation of $d=10$, $N=1$ supergravity involves a closed seven-form. Its dimensional reduction on a $(10-d)$-dimensional torus leads to real closed $(d-3)$-forms in d-dimensional supergravities. (These are $N=1$ supergravities in $d=8$, 9, 10; $N=2$ in $d=7$ and $N=2$ or 4 in $d=6$) [9]. Apart from these, there is: (i) *A real closed four-form in $d=11$, $N=1$* supergravity, (ii) a real closed three-form in non-chiral $d=10$, $N=2$ supergravity, (iii) a complex closed three-form in chiral $d=10$, $N=2$ supergravity.

Excluding Yang–Mills coupling, as is well known, closed super three-forms exist in $d=3$, 4, 6, and 10.

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11. Since the superspace formulation of $d=7$, $N=2$ supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of $d=11$ supergravity to a supermembrane. In particular the Kalb–Ramond-like third rank antisymmetic tensor field of $d=11$ supergravity couples to the supermembane via a Wess–Zumino term.

In the following we focus our attention on the description of the supermembane action in $d=11$. The extension to the case of n-extended objects is given in the appendix.

2. We propose the following action for a supermembrane coupled to $d=11$ supergravity:

$$S=\int d^2\xi\,\left(\tfrac{1}{2}\sqrt{-g}\,g^{ij}E_i{}^a E_j{}^b \eta_{ab} + \epsilon^{ijk}E_i{}^A E_j{}^B E_k{}^C B_{CBA} - \tfrac{1}{2}\sqrt{-g}\right) . \qquad (3)$$

Here $i=0, 1, 2$ labels the coordinates $\xi^i=(\tau, \sigma, \rho)$ of the world volume with metric g_{ij} and signature $(-, +, +)$. The super three-form B is needed for the superspace description of $d=11$ supergravity [10]. For the Levi-Civita symbol ϵ^{ijk} we use the same conventions as in ref. [11]. In (3) we have used the notation

$$E_i{}^A=(\partial_i Z^M)E_M{}^A , \qquad (4)$$

where $Z^M(\xi)$ are the superspace coordinates, and $E_M{}^A(Z)$ is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric g_{ij} gives the embedding equation

$$g_{ij}=E_i{}^a E_j{}^b \eta_{ab} \equiv T_{ij} . \qquad (5)$$

We require that the action S is invariant under a fermionic gauge transformation of the form [5]

$$\delta E^a=0, \quad \delta E^\alpha=(1+\Gamma)^\alpha{}_\beta \kappa^\beta ,$$

$$\delta g_{ij}=2[X_{ij}-g_{ij}X^k{}_k/(n-1)] \quad (n=2 \text{ for membrane}) , \qquad (6)$$

where the transformation parameter $\kappa^\alpha(\xi)$ is a 32 component Majorana spinor, and a world-volume scalar, and

$$\delta E^A=\delta Z^M E_M{}^A, \qquad (7)$$

$$\Gamma^\alpha{}_\beta=(1/6\sqrt{-g})\epsilon^{ijk}E_i{}^a E_j{}^b E_k{}^c(\gamma_{abc})^\alpha{}_\beta . \qquad (8)$$

Here $\gamma^a (a=0, 1, ..., 10)$ are the Dirac matrices in eleven dimensions. X_{ij} is a function of $E_i{}^A$ which will be determined by the invariance of the action. The choice of δg_{ij} is due to the fact that, given a variation of the action of the form $\delta S=T_{ij}X^{ij}$, and writing this variation as

$$T_{ij}X^{ij}=g_{ij}X^{ij}+(T_{ij}-g_{ij})X^{ij} , \qquad (9)$$

the second term on the right-hand side cancels $(\delta S/\delta g_{ij})\delta g_{ij}$. Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form $T_{ij}X^{ij}$, we can use eq. (5), provided that we add X^{ij} to δg_{ij} as in (6).

The matrix $\Gamma^\alpha{}_\beta$ occurring in (8) satisfies the property

$$\Gamma^\alpha{}_\beta \Gamma^\beta{}_\delta=(T^i{}_{[i}T^j{}_j T^k{}_{k]})\delta^\alpha{}_\delta \equiv \Gamma^2 \delta^\alpha{}_\delta . \qquad (10)$$

The normalization in (8) is chosen such that upon the use of the equation $T_{ij}=g_{ij}$, the matrix $\Gamma^\alpha{}_\beta$ satisfies the relation $\Gamma^\alpha{}_\beta \Gamma^\beta{}_\gamma=\delta^\alpha{}_\gamma$.

Now using (6) the variation of the action (3) is (we consider a closed supermembrane and therefore discard the surface terms)

$$\delta S=\int d^2\xi\,[\sqrt{-g}\,g^{ij}(-\delta E^\beta E_i{}^\gamma T^a{}_{\gamma\beta})E_{ja}$$
$$+\sqrt{-g}\,g^{ij}(-\delta E^\beta E_i{}^c T^a{}_{c\beta})E_{ja}$$
$$+\epsilon^{ijk}E_i{}^A E_j{}^B E_k{}^C\delta E^\alpha H_{\alpha CBA}$$
$$-\tfrac{1}{2}\sqrt{-g}\,\delta g^{ij}(T^{ij}-\tfrac{1}{2}g^{ij}T-\tfrac{1}{2}g^{ij})]\,. \qquad (11)$$

The torsion two-form T^A and the four-form field strength H are defined by (our superspace conventions are those of Howe [12])

$$T^A=dE^A+E^B\Omega_B{}^A=\tfrac{1}{2}E^BE^CT_{CB}{}^A\,,$$
$$H=dB=\tfrac{1}{24}E^DE^CE^BE^AH_{ABCD}\,. \qquad (12)$$

We now organize the terms in (11) according to the number of one-forms E^α they contain. Those with three E^α and two E^α come only from the Wess–Zumino term. They must vanish seperately, and this requires the constaints

$$H_{\alpha\beta\gamma\delta}=H_{\alpha\beta\gamma d}=0\,. \qquad (13)$$

The cancellation of the terms linear in E^α lead to the constraints

$$T^a{}_{\alpha\beta}=(\gamma^a)_{\alpha\beta}\,, \qquad (14)$$
$$H_{\alpha\beta ab}=-\tfrac{1}{6}(\gamma_{ab})_{\alpha\beta}\,, \qquad (15)$$

while the cancellation of the terms not containing E^α require the constraint

$$\eta_{c(a}T^c{}_{b)\alpha}=\eta_{ab}\Lambda_\alpha\,, \qquad (16)$$
$$H_{\alpha abc}=-\tfrac{1}{2}\Lambda_\beta(\gamma_{abc})^\beta{}_\alpha\,. \qquad (17)$$

Here Λ_α is an arbitrary spinor superfield which is vanishing in $d=11$ [10].

It is important to realize that in obtaining (14)–(17) we have used the identity

$$\delta E^\alpha=\Gamma^\alpha{}_\beta\delta E^\beta+(1-\Gamma^2)\kappa^\alpha\,. \qquad (18)$$

Using this identity in the variation of the kinetic term, the terms arising from $\Gamma^\alpha{}_\beta$ in (18) can be shown to cancel similar terms coming from the variation of the Wess–Zumino term, modulo terms which cancel by an appropriate variation of g_{ij}. [Using the argument below (8) once.] In the remaining terms coming from $(1-\Gamma^2)$, we use the argument given below (8) repeatedly to compute further contributions to δg_{ij}. Thus we find the result

$$X_{ij}=-\tfrac{1}{4}\epsilon_i{}^{kl}E_k{}^aE_l{}^b(\gamma_{ab})_{\alpha\beta}\delta E^\beta E_j{}^\alpha$$
$$+\tfrac{1}{2}\kappa^\beta E_n{}^\alpha(\gamma^d)_{\alpha\beta}E^{nd}g_{i[j}(T^k{}_kT^l{}_{l]}+\delta^k{}_kT^l{}_{l]})$$
$$+i\leftrightarrow j\,. \qquad (19)$$

In summary, the action (3) is invariant under (6) provided that (13)–(17) hold, and X^{ij} is given by (19). In addition, the following Bianchi identities must hold:

$$DT^A=-E^B\wedge R_B{}^A\,,\quad DH=0\,. \qquad (20)$$

The generalization of the results of this section to the general case of n-extended objects in d-dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of $d=11$ supergravity given by Cremmer and Ferrara [10] and Brink and Howe [10] do provide a solution to (13)–(17) *and* the Bianchi identities (20), with $\Lambda_\alpha=0$.

In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a *closed* supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few.

Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane [13], it is encouraging that, here, we have a spacetime supersymmetric membrane action.

Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, *and* at the same time from three-dimensional world volume to a two-dimensional world sheet. It would be interesting to see what kind of $d=10$ string theories could possibly emerge in an infinitely thin membrane limit.

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Appendix. In this appendix we construct the action for an n-extended object propagating in d-dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

$$S=\int d^2\xi\,[\tfrac{1}{2}\sqrt{-g}\,g^{ij}E_i{}^aE_j{}^b\eta_{ab}$$
$$+\epsilon^{i_1\dots i_n}E_{i_1}{}^{A_1}\dots E_{i_{n+1}}{}^{A_{n+1}}B_{A_{n+1}\dots A_1}$$
$$-\tfrac{1}{2}(n-1)\sqrt{-g}]\,. \tag{A1}$$

The transformation rules are those in (6), where the matrix $\Gamma^\alpha{}_\beta$ is now given by

$$\Gamma^\alpha{}_\beta=[\eta/(n+1)!\sqrt{-g}]$$
$$\times\epsilon^{i_1\dots i_{n+1}}E_{i_1}{}^{a_1}\dots E_{i_{n+1}}{}^{a_{n+1}}(\gamma_{a_1\dots a_{n+1}})^\alpha{}_\beta\,, \tag{A2}$$

where η is given by

$$\eta=(-1)^{(n+1)(n-2)/4}\,. \tag{A3}$$

Invariance of the action (A1) is ensured by imposing the following set of constraints:

$$T^a{}_{\alpha\beta}=(\gamma^a)_{\alpha\beta}\,,\quad \eta_{c(a}T^c{}_{b)\alpha}=\eta_{ab}\Lambda_\alpha\,, \tag{A4}$$

$$H_{\alpha a_{n+1}\dots a_1}=(\eta/n!)\Lambda_\beta(\gamma_{a_1\dots a_{n+1}})^\beta{}_\alpha\,,$$

$$H_{\alpha\beta a_n\dots a_1}=[\eta(-1)^n/(n+1)!](\gamma_{a_1\dots a_n})_{\alpha\beta}\,, \tag{A5}$$

$$H_{\alpha\beta\gamma A_1\dots A_{n-1}}=0\,, \tag{A5cont'd}$$

and by taking X_{ij} occurring in (6) to be

$$X_{ij}=(-\eta/2n!)$$
$$\times\epsilon_i{}^{k_1\dots k_n}E_{k_1}{}^{a_1}\dots E_{k_n}{}^{a_n}(\gamma_{a_1\dots a_n})_{\alpha\beta}\delta E^\beta E_j{}^\alpha$$
$$+\tfrac{1}{2}\kappa^\beta[E_n{}^\alpha(\gamma^a)_{\alpha\beta}E^{na}+(n+1)\Lambda_\beta]$$
$$\times g_{i[j}(T^{k_1}{}_{k_1}\dots T^{k_n}{}_{k_n]}+\delta^{k_1}{}_{k_1}T^{k_2}{}_{k_2}\dots T^{k_n}{}_{k_n]}$$
$$+\delta^{k_1}{}_{k_1}\dots\delta^{k_{n-1}}{}_{k_{n-1}}T^{k_n}{}_{k_n]})$$
$$-\tfrac{1}{2}\delta E^\beta\Lambda_\beta g_{ij}+(i\leftrightarrow j)\,. \tag{A6}$$

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