SUPERMEMBRANES AND ELEVEN-DIMENSIONAL SUPERGRAVITY

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We construct an action for a supermembrane propagating in $d=11$ supergravity background. Using the constraints of $d=11$ curved superspace, we show that the action is invariant under Siegel-type transformations recently generalized by Hughes, Li and Polchinski. The transformation parameter is a world-volume scalar and $d=11$ spacetime spinor. We also discuss the general problem of the coupling of $n$-dimensional extended objects to $d$-dimensional supergravity.

1. Now that we have become accustomed to the notion that strings should replace particles, it is natural to investigate the properties of higher-dimensional extended objects, in particular of membranes since they are the simplest extended objects, and they might describe strings in an appropriate limit.

In 1962 Dirac [1] put forward a theory of an extended electron based on the idea of a relativistic membrane. In 1976, Collins and Tucker [2] studied the classical and quantum mechanics of free relativistic membranes. A year later a locally supersymmetric and reparametrization-invariant action for a spinning membrane was constructed by Howe and Tucker [3]. The action describes anti de Sitter supergravity coupled to a number of scalar multiplets in three dimensions. It is the membrane analog of the Neveu–Schwarz–Ramond formulation of the spinning string theory.


The generalization of the Hughes et al. model to $n$-extended objects propagating in flat $d$-dimensional superspace is evident. All that is required is the existence of a closer super $(n+2)$-form given by

$$H = E^a E^a E^a \ldots E^a (\gamma_{a_{1} \ldots a_{n}})_{\alpha \beta},$$

where $(E^a, E^\alpha)$ are the basis one-forms in superspace. This form is closed provided that the following $\Gamma$-matrix identity holds:

$$\gamma^{a_{1} \ldots a_{n+1}} (\gamma_{a_{2} \ldots a_{n+1}})_{\alpha \beta} = 0.$$

The purpose of this note is to construct Hughes et al. -type actions describing the propagation of an $n$-extended object in $d$-dimensional curved superspace. We give a general formula for the action and the transformation rules, whose consistency requires, among other things (see below), the existence of a closed $(n+2)$-form in curved superspace. Thus we
expect that the \( n \)-extended objects under consideration can consistently propagate only in \( d \leqslant 11 \) supergravities whose superspace formulation involves a closed \((n+2)\)-form. We further expect that such forms exist in supergravity theories in which a closed \textit{bosonic} \((n+2)\)-form occurs. As far as we know, the following possibilities exist (we include the Yang-Mills couplings whenever possible):

The dual formulation of \( d = 10, N = 1 \) supergravity involves a closed seven-form. Its dimensional reduction on a \((10-d)\)-dimensional torus leads to real closed \((d-3)\)-forms in \( d \)-dimensional supergravities. (These are \( N = 1 \) supergravities in \( d = 8, 9, 10 \); \( N = 2 \) in \( d = 7 \) and \( N = 2 \) or \( 4 \) in \( d = 6 \)) [9]. Apart from these, there is: (i) A real closed four-form in \( d = 11, N = 1 \) supergravity, (ii) a real closed three-form in non-chiral \( d = 10, N = 2 \) supergravity, (iii) a complex closed three-form in chiral \( d = 10, N = 2 \) or \( 4 \) supergravity.

Excluding Yang-Mills coupling, as is well known, closed super three-forms exist in \( d = 3, 4, 6, \) and \( 10 \).

Considering the case of the membranes, from the above list it follows that the candidate dimensions are 7 and 11. Since the superspace formulation of \( d = 7, N = 2 \) supergravity is not known at present, we are led to consider the supermembrane propagating in eleven-dimensional spacetime.

Our main result is the construction of an action which describes a consistent coupling of \( d = 11 \) supergravity to a supermembrane. In particular the Kalb-Ramond-like third rank antisymmetric tensor field of \( d = 11 \) supergravity [10] is needed for the superspace description of \( d = 11 \) supergravity [10]. For the Levi-Civita symbol \( \varepsilon^{\alpha\beta} \) we use the same conventions as in ref. [11]. In (3) we have used the notation

\[ E_i^A = (\partial_i Z^M) E_M^A, \]

where \( Z^M(\xi) \) are the superspace coordinates, and \( E_M^A(Z) \) is the supervielbein.

Note that the action has a cosmological constant with a fixed magnitude. This is so that the field equation of the metric \( g_{ij} \) gives the embedding equation

\[ g_{ij} = E_i^a E_j^b \eta_{ab} \equiv T_{ij}. \]

We require that the action \( S \) is invariant under a fermionic gauge transformation of the form [5]

\[ \delta E^a = 0, \quad \delta E^a = (1 + \Gamma)^{\alpha\beta} \kappa^{\delta}, \]

\[ \delta g_{ij} = 2[X_{ij} - g_{ij} \kappa^k \gamma^{kl}] \]

\(n=2\) for membrane, (6)

where the transformation parameter \( \kappa^{\alpha}(\xi) \) is a 32 component Majorana spinor, and a world-volume scalar, and

\[ \delta E^A = \delta Z^M E_M^A, \]

\[ \Gamma^{\alpha\beta} = (1/6 \sqrt{g}) \varepsilon^{\alpha\beta\gamma} E_i^a E_j^b E_k^c \gamma^{abc}. \]

Here \( \gamma^{ij} = 0, 1, \ldots, 10 \) are the Dirac matrices in eleven dimensions. \( X_{ij} \) is a function of \( E_i^a \) which will be determined by the invariance of the action. The choice of \( \delta g_{ij} \) is due to the fact that, given a variation of the action of the form \( \delta S = \delta g_{ij} + \delta g_{ij} \), and writing this variation as

\[ T_{ij} X^{ij} = g_{ij} X_{ij} + (T_{ij} - g_{ij}) X_{ij}, \]

the second term on the right-hand side cancels \( \delta S / \delta g_{ij} \delta g_{ij} \). Thus we are left with the first term on the right-hand side, which equals the left-hand side upon the use of (5). Effectively, this means that whenever we encounter a variation of the form \( T_{ij} X^{ij} \), we can use eq. (5), provided that we add \( X_{ij} \) to \( \delta g_{ij} \) as in (6).

The matrix \( \Gamma^{\alpha\beta} \) occurring in (8) satisfies the property

\[ \Gamma^{\alpha\beta} \Gamma^{\beta\delta} = (T^{i j}_{k l} T^{j i}_{k l} T^{k i}_{j l}) \delta^{\alpha\delta} \equiv \Gamma^{2} \delta^{\alpha\delta}. \]

The normalization in (8) is chosen such that upon the use of the equation \( T_{ij} = g_{ij} \), the matrix \( \Gamma^{\alpha\beta} \) satisfies the relation \( \Gamma^{\alpha\beta} \Gamma^{\beta\gamma} = \delta^{\alpha\gamma}. \)
Now using (6) the variation of the action (3) is (we consider a closed supermembrane and therefore discard the surface terms)

$$\delta S = \int d^2 \xi \left[ \sqrt{-g} g^{ij} \left( \delta E^\beta E_i^a T^a_{\alpha \beta} \right) E_{ja} + \sqrt{-g} g^{ij} \left( \delta E^\beta E_i^a T^a_{\alpha \beta} \right) E_{ja} + \epsilon^{ijk} E_i^a E_j^b E_k^c \delta E^\alpha A_{ABC} + \sqrt{-g} \delta g^{ij} \left( T^{ij} - \frac{1}{3} g^{ij} T - \frac{1}{3} g^{ij} T \right) \right] \right] .$$ (11)

The torsion two-form $T^a$ and the four-form field strength $H$ are defined by (our superspace conventions are those of Howe [12])

$$T^a = dE^a + E_\alpha E^\alpha, H = dB = \frac{1}{2} E^{DE} C E^A T^{CB} A, H_{ABC} = \frac{1}{3} E^{DE} C E^A T^{CB} A.$$

We now organize the terms in (11) according to the number of one-forms $E^a$ they contain. Those with three $E^a$ and two $E^a$ come only from the Wess-Zumino term. They must vanish separately, and this requires the constraints

$$H_{\alpha \beta \gamma} = H_{\alpha \beta \gamma} = 0 .$$ (13)

The cancellation of the terms linear in $E^a$ lead to the constraints

$$T^{a \alpha} = (\gamma^a)_{\alpha \beta} ,$$ (14)

$$H_{\alpha \beta \gamma} = - \frac{1}{3} (\gamma^a)_{\alpha \beta} ,$$ (15)

while the cancellation of the terms not containing $E^a$ require the constraint

$$\eta^{(a} T^{b)}_{\alpha} = \eta_{ab} A_{\alpha} ,$$ (16)

$$H_{\alpha \beta \gamma} = - \frac{1}{3} A_{\beta} (\gamma^a)_{\alpha \beta}.$$. (17)

Here $A_{\alpha}$ is an arbitrary spinor superfield which is vanishing in $d=11$ [10].

It important to realize that in obtaining (14)–(17) we have used the identity

$$\delta E^{\alpha} = \Gamma_{\alpha}^{\alpha} \delta E^{\beta} + (1 - \Gamma^2) \kappa^{\alpha} .$$ (18)

Using this identity in the variation of the kinetic term, the terms arising from $T^{\alpha \beta}$ in (18) can be shown to cancel similar terms coming from the variation of the Wess-Zumino term, modulo terms which cancel by an appropriate variation of $g_{ij}$. [Using the argument below (8) once.] In the remaining terms coming from $(1 - \Gamma^2)$, we use the argument given below (8) repeatedly to compute further contributions to $\delta g_{ij}$. Thus we find the result

$$X_{ij} = - \frac{1}{1} \epsilon^{ijkl} E_i^a E_j^b \delta E^a \delta E^b E_{ij} + \frac{1}{2} \kappa^{\alpha} E_{\alpha} (\gamma^d)_{\alpha \beta} E^{nd} g_{ij} (T^{k\ell} T^{\ell} + \delta_{ij} T^{k\ell} T^{\ell}) + i \neq j .$$ (19)

In summary, the action (3) is invariant under (6) provided that (13)–(17) hold, and $X^{ij}$ is given by (19). In addition, the following Bianchi identities must hold:

$$DT^A = - E^B \wedge R_{B} A, \quad DH = 0 .$$ (20)

The generalization of the results of this section to the general case of $n$-extended objects in $d$-dimensional supergravity is straightforward. The result is given in the appendix.

3. We observe that the superspace constraints of $d=11$ supergravity given by Cremmer and Ferrara [10] and Brink and Howe [10] do provide a solution to (13)–(17) and the Bianchi identities (20), with $A_{\alpha} = 0$.

In conclusion, we have shown that there exists a consistent coupling of a closed supermembrane to eleven-dimensional supergravity. (Note that it is natural to consider a closed supermembrane in eleven dimensions, since there are no matter multiplets in this dimension).

4. There are several directions in which the present work can be extended. We shall name a few.

Firstly, it is of interest to study the quantization of the supermembrane model in eleven dimensions. In particular, the question of whether massless gauge fields can possibly arise is a challenging one. Although usually one encounters difficulties in finding massless excitations of a membrane [13], it is encouraging that, here, we have a spacetime supersymmetric membrane action.

Secondly, it is natural to consider the dimensional reduction of our model from eleven- to ten-dimensional spacetime, and at the same time from three-dimensional world volume to a two-dimensional
world sheet. It would be interesting to see what kind of \( d=10 \) string theories could possibly emerge in an infinitely thin membrane limit.

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Appendix. In this appendix we construct the action for an \( n \)-extended object propagating in \( d \)-dimensional supergravity background. We also give the transformation rules, and the constraints on the background.

The action is

\[
S = \int d^2 \xi \left[ \frac{1}{2} \sqrt{-g} \right. \\
+ \epsilon^{i_1 \ldots i_{d+1}} E_{i_1} a_{i_1} \ldots E_{i_{d+1}} a_{i_{d+1}} B_{a_{d+1} \ldots a_{d+1}} - \frac{1}{2} (n-1) \sqrt{-g} \right] .
\]

The transformation rules are those in (6), where the matrix \( \Gamma^a \) is now given by

\[
\Gamma^a = \left[ \eta^a (n+1)! \frac{1}{\sqrt{-g}} \right. \\
\times \epsilon^{i_1 \ldots i_{d+1}} E_{i_1} a_{i_1} \ldots E_{i_{d+1}} a_{i_{d+1}} (\gamma_{a_{d+1} \ldots a_{d+1}})^{\alpha} \beta ,
\]

where \( \eta \) is given by

\[
\eta = (-1)^{(n+1)(n-2)/4} .
\]

Invariance of the action (A1) is ensured by imposing the following set of constraints:

\[
T^a_{\alpha \beta} = (\gamma^a)_{\alpha \beta} , \quad \eta \left( a T^a_{\beta} \right) = \eta_{ab} A^a ,
\]

\[
H_{a_{d+1} \ldots a_{d+1}} = \eta (n!)^2 (a_{d+1} \ldots a_{d+1})^\alpha ,
\]

\[
H_{a_{d+1} \ldots a_{d+1}} = \frac{1}{2} \delta E^a E^b \alpha ,
\]

\[
H_{a_{d+1} \ldots a_{d+1}} = 0 ,
\]

and by taking \( X_{ij} \) occurring in (6) to be

\[
X_{ij} = (-\eta/2n!)
\]

\[
\times \epsilon_{i_1 \ldots i_{d+1}} E_{i_1} a_{i_1} \ldots E_{i_{d+1}} a_{i_{d+1}} (\gamma_{a_{d+1} \ldots a_{d+1}})^{\alpha} \beta ,
\]

\[
+ \frac{1}{2} \kappa^a \left[ E^a \right. \\
\left. \delta E^a E^b \alpha + (n+1) A^a \right] ,
\]

\[
\times g_{ij} (T_{k_{d+1} \ldots k_{d+1}} - \delta_{k_{d+1} \ldots k_{d+1}} T_{k_{d+1} \ldots k_{d+1}} )
\]

\[
+ \delta_{k_{d+1} \ldots k_{d+1}} \delta \alpha \beta ,
\]

\[- \frac{1}{2} \delta E^a A^a g_{ij} + (i \leftrightarrow j) .
\]

References