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Networking improves robustness in flexible-joint multi-robot systems with only joint position measurements

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ABSTRACT

This paper studies the robustness in the coordination—via energy-shaping—of multiple nonidentical flexible-joint robots with only joint position measurements. The control objective is to drive all manipulators link positions to the same constant equilibrium. If the physical parameters are exactly known, then a classical decentralized energy-shaping controller solves the desired control objective. However, under parameter uncertainty, the globally asymptotically stable equilibrium point is shifted away from the desired value. The main contribution of the paper is to show that the steady-state performance is improved adding to the decentralized control policy information exchange between the agents. More precisely, it is proven that the equilibrium with the networked controller is *always closer* (in a suitable metric) to the desired one than that using the decentralized controller, provided the communication graph representing the network is *undirected* and *connected*. This result holds globally for sufficiently large interconnection gains and locally (in a suitably defined sense) for all values of the gains. An additional advantage of networking is that the asymptotic stabilization objective can be achieved injecting lower gains into the loop. The paper also provides simulation and experimental evidence, which illustrate the fact that networking improves robustness with respect to parameter uncertainty.

1. Introduction

The coordination of networks of multiple—linear and nonlinear—dynamical systems has been extensively studied in the past years [19,23,27,22,4,18]. The coordination main objective is that all systems reach a certain agreement point (consensus point). In many applications (*e.g.*, formation control, network consensus, flocking of agents, synchronization along a given trajectory) the agents of the network have to follow a given desired trajectory (leader) where either each agent controller has the complete knowledge of such trajectory [15,1,13,21], or it contains an internal model that captures the dynamics of the desired trajectory [26,5]. In the referenced results, a distributed control approach using a communication network allows all agents to reach a common control goal despite that the agents can be non-identical and that the communications may induce time-delays.

In a recent work [16] it has been shown that, for a class of fully actuated Euler–Lagrange (EL) systems with complete state feedback, there is an advantage in interconnecting the agents: in the presence of parameter uncertainty, the networked equilibrium is always closer to the desired value than the decentralized one. It should be underscored that, in [10], this robustness property has also been observed in the centroid formation control of multiple thrust propelled vehicles without a formal proof.

The purpose of this paper is to extend the results of [16,17] to a class of *under-actuated* EL-systems without full state measurements. More precisely, to consider flexible-joint robot manipulators and assume that the controller has access only to the joint (motor) positions. That is, the system has more degrees of freedom (DOF) than control actions and neither the link or joint velocities nor the links positions are available for measurement. Consensus problems for under-actuated EL-systems have been studied in [14,3], assuming full state measurement. In the former the Controlled-Lagrangian technique is employed to solve the network consensus problem, while in the latter the results of [15] are extended to robots with flexible joints.

Control of *single* flexible-joint robots with full state measurement was first solved in [25]. It was later extended to the case of joint position measurements in [2,6]. In [7,20] an interpretation of these controllers in terms of energy-shaping was given. It is well-known that, in the case of known parameters, these controllers

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generate a globally asymptotically stable (GAS) equilibrium at the desired constant value. However, this equilibrium is shifted away from the desired value in the face of parameter uncertainty. In a network, if the agents are identical, *i.e.*, their (uncertain) parameters are identical, then the shifted equilibrium is the same for the decentralized and networked approaches, and thus, exchanging information does not offer any advantage to the resulting steady-state error. On the other hand, if the non-identical case is considered, the papers' main result shows that adding information exchange between the agents to the decentralized controller improves the steady-state performance. More precisely, it is proven that if the agents exchange information through a network modeled by an *undirected* and *connected* communication graph, the equilibrium with the networked controller is *always closer* (in a suitable metric) to the desired one than that with the decentralized control policy. The result holds globally for sufficiently large gains and locally (in a suitably defined sense) for all values of the gains.

The robustness improvement of the networked controller is illustrated through numerical simulations, with ten 2-DOF flexible-joint robot manipulators, and with experiments using two 3-DOF robot manipulators.

Notation: Unless stated otherwise, throughout the paper the subindex i takes values in the set $\{1,\ldots,m\}$, where $m\in\mathbb{Z}_+$ is the number of flexible-joint robot manipulators. This clarification is omitted for brevity. Also, we define $\mathrm{col}(\mathbf{z}_i)$ as the mn-dimensional column vector $[\mathbf{z}_1^\top,\ldots,\mathbf{z}_m^\top]^\top$, where $\mathbf{z}_i\in\mathbb{R}^n$, and $\mathrm{diag}\{\mathbf{A}_i\}$ as the square block-diagonal matrix $\mathrm{diag}\{\mathbf{A}_1,\ldots,\mathbf{A}_m\}\in\mathbb{R}^{mn\times mn}$, where $\mathbf{A}_i\in\mathbb{R}^{n\times n}$. The notations $\mathbb{1}_m$ and $\mathbf{0}_m$ denote vectors of dimension m with all entries equal to 1 and equal to zero, respectively. The square of the Euclidean norm of a vector $\mathbf{x}\in\mathbb{R}^{mn}$ is denoted as $|\mathbf{x}|^2:=\mathbf{x}^\top\mathbf{x}$. For a positive definite matrix $\mathbf{W}\in\mathbb{R}^{mn\times mn}$, the square of the weighted Euclidean norm of $\mathbf{x}$ is defined by $\|\mathbf{x}\|_{\mathbf{W}}^2:=\mathbf{x}^\top\mathbf{W}\mathbf{x}$, the spectrum of $\mathbf{W}$ is denoted by $\sigma(\mathbf{W})$ while the minimum and the maximum of its spectrum are denoted by $\sigma_{\min}(\mathbf{W})$ and $\sigma_{\max}(\mathbf{W})$, respectively. Finally, the (transposed) gradient of a scalar function $V:\mathbb{R}^n\to\mathbb{R}$, is denoted by $\nabla V:=(\partial V/\partial\mathbf{x})^\top$ and its Hessian by $\nabla^2V:=\partial^2V/\partial\mathbf{x}^2$.

2. Background

In order to make the paper self-contained, this section briefly introduces the nonlinear model of the flexible-joint robot manipulators and the energy-shaping, position-feedback controller of [7]. This controller is used to illustrate the main result but, as will become clear later, the same result can be proven for any energy-shaping EL controller of the family given in Proposition 3.6 of [20].

2.1. Robots with joint flexibility

Let us consider a network of m non-identical, flexible-joint robot manipulators with n DOF. Directly actuated, revolute joints robots are assumed and the simplified model for flexibility of [24] is adopted. For every i, the nonlinear dynamics of the i-th manipulator is given by

$$\begin{aligned}\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i+\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i+\mathbf{g}_i(\mathbf{q}_i)+\mathbf{K}_i(\mathbf{q}_i-\boldsymbol{\theta}_i)&=\mathbf{0}_n\\ \mathbf{J}_i\ddot{\boldsymbol{\theta}}_i+\mathbf{K}_i(\boldsymbol{\theta}_i-\mathbf{q}_i)&=\boldsymbol{\tau}_i\end{aligned}\tag{1}$$

where $\mathbf{q}_i\in\mathbb{R}^n$ is the link angular position and $\boldsymbol{\theta}_i\in\mathbb{R}^n$ is the joint (motor) angular position. The matrix $\mathbf{M}_i(\mathbf{q}_i)\in\mathbb{R}^{n\times n}$ is the inertia matrix, the matrix $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\in\mathbb{R}^{n\times n}$ describes the Coriolis and centrifugal effects (defined via the Christoffel symbols of the first kind), the vector $\mathbf{g}_i(\mathbf{q}_i):=\nabla U_i(\mathbf{q}_i)$ is the gravity force with $U_i:\mathbb{R}^n\to\mathbb{R}$ the corresponding potential energy of the rigid dynamics, the matrix $\mathbf{J}_i\in\mathbb{R}^{n\times n}$ is the motor inertia at the joints, which is symmetric and positive definite, the matrix $\mathbf{K}_i\in\mathbb{R}^{n\times n}$ contains the joint stiffness coefficients, which is also symmetric and positive definite, and the vector $\boldsymbol{\tau}_i\in\mathbb{R}^n$ is the control input. Note that the overall potential energy of the system is given by

$$U_i(\mathbf{q}_i)+\frac{1}{2}\begin{bmatrix}\mathbf{q}_i\\ \boldsymbol{\theta}_i\end{bmatrix}^\top\begin{bmatrix}\mathbf{K}_i & -\mathbf{K}_i\\ -\mathbf{K}_i & \mathbf{K}_i\end{bmatrix}\begin{bmatrix}\mathbf{q}_i\\ \boldsymbol{\theta}_i\end{bmatrix},$$

that, following the principles of passivity-based control, is going to be shaped to assign a minimum at the desired equilibrium point.

For EL-systems described by (1), the following properties are well-known [24,8] and thus assumed throughout this paper.

(P1) $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric and there exists $\lambda_{mi},\lambda_{Mi}>0$ such that $0<\lambda_{mi}\mathbf{I}_n\le\mathbf{M}_i(\mathbf{q}_i)\le\lambda_{Mi}\mathbf{I}_n$ holds for all $\mathbf{q}_i\in\mathbb{R}^n$.
(P2) The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i)-2\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is skew-symmetric.
(P3) There exists $k_{gi}>0$ such that $|\partial\mathbf{g}_i(\mathbf{q}_i)/\partial\mathbf{q}_i|\le k_{gi}$. Hence, for all $\mathbf{q}_{i1},\mathbf{q}_{i2}\in\mathbb{R}^n$ the following inequality holds $|\mathbf{g}_i(\mathbf{q}_{i1})-\mathbf{g}_i(\mathbf{q}_{i2})|\le k_{gi}|\mathbf{q}_{i1}-\mathbf{q}_{i2}|$.

2.2. Stabilization with only joint position feedback

As described in Introduction, the control objective is to drive $\mathbf{q}_i(t)$ towards a constant desired value $\mathbf{q}_\star\in\mathbb{R}^n$ using only the measurement of the joint angular positions. For, this paper makes use of the output-feedback controller proposed in [7]. A relevant observation is that other recent controllers, as those in [12,11,9], cannot be employed here because they also require joint velocity and, the latter, joint torque measurements.

Thus, for every i, let

$$\boldsymbol{\theta}_{\star i}=\mathbf{q}_\star+\mathbf{K}_i^{-1}\mathbf{g}_i(\mathbf{q}_\star)\tag{2}$$

which is a function of $\mathbf{g}_i$, and consider the following output-feedback controller, proposed in [7]:

$$\left.\begin{aligned}\dot{\mathbf{y}}_i&=-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i\\ \boldsymbol{\tau}_i&=\mathbf{g}_i(\mathbf{q}_\star)-\mathbf{K}_{pi}(\boldsymbol{\theta}_i-\boldsymbol{\theta}_{\star i})-\mathbf{K}_{di}(-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i)\end{aligned}\right\}\tag{3}$$

where $\mathbf{y}_i\in\mathbb{R}^n$ is the controller state and $\mathbf{K}_{pi},\mathbf{A}_i,\mathbf{K}_{di}\in\mathbb{R}^{n\times n}$ are symmetric and positive definite matrices. As shown in [20], this controller assigns to the closed-loop system the desired potential energy function:

$$V_i(\mathbf{q}_i,\boldsymbol{\theta}_i):=U_i(\mathbf{q}_i)-\mathbf{q}_i^\top\mathbf{g}_i(\mathbf{q}_\star)+\frac{1}{2}\begin{bmatrix}\mathbf{q}_i-\mathbf{q}_\star\\ \boldsymbol{\theta}_i-\boldsymbol{\theta}_{\star i}\end{bmatrix}^\top\boldsymbol{\Phi}_i\begin{bmatrix}\mathbf{q}_i-\mathbf{q}_\star\\ \boldsymbol{\theta}_i-\boldsymbol{\theta}_{\star i}\end{bmatrix},\tag{4}$$

where

$$\boldsymbol{\Phi}_i:=\begin{bmatrix}\mathbf{K}_i & -\mathbf{K}_i\\ -\mathbf{K}_i & \mathbf{K}_i+\mathbf{K}_{pi}\end{bmatrix}.$$

Clearly, the gradient of V_i evaluated at $(\mathbf{q}_\star,\boldsymbol{\theta}_{\star i})$ satisfies $\nabla V_i(\mathbf{q}_\star,\boldsymbol{\theta}_{\star i})=\mathbf{0}_{2n}$. Moreover, $\nabla^2V_i(\mathbf{q}_i,\boldsymbol{\theta}_i)=\mathrm{diag}(\partial\mathbf{g}_i(\mathbf{q}_i)/\partial\mathbf{q}_i,\mathbf{0}_{n\times n})+\boldsymbol{\Phi}_i$. Hence, if $\sigma_{\min}(\boldsymbol{\Phi}_i)>k_{gi}$ then $\nabla^2V_i(\mathbf{q}_i,\boldsymbol{\theta}_i)>0$, achieving the desired energy-shaping objective.[1] Moreover, the controller injects a damping that back-propagates to ensure that $(\mathbf{q}_i,\boldsymbol{\theta}_i,\dot{\mathbf{q}}_i,\dot{\boldsymbol{\theta}}_i,\mathbf{y}_i)=(\mathbf{q}_\star,\boldsymbol{\theta}_{\star i},\mathbf{0}_n,\mathbf{0}_n,\mathbf{A}_i^{-1}\mathbf{K}_{di}\boldsymbol{\theta}_{\star i})$ is a GAS equilibrium. Indeed, the Lyapunov function:

$$W_i(\mathbf{q}_i,\boldsymbol{\theta}_i,\dot{\mathbf{q}}_i,\dot{\boldsymbol{\theta}}_i,\mathbf{y}_i)=\tfrac{1}{2}[\dot{\mathbf{q}}_i^\top\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i+\dot{\boldsymbol{\theta}}_i^\top\mathbf{J}_i\dot{\boldsymbol{\theta}}_i+|-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i|^2]+V_i(\mathbf{q}_i,\boldsymbol{\theta}_i)$$

is proper, *i.e.*, radially unbounded, and satisfies $\dot{W}_i(\mathbf{q}_i,\boldsymbol{\theta}_i,\ \dot{\mathbf{q}}_i,\dot{\boldsymbol{\theta}}_i,\mathbf{y}_i)=-\|-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i\|_{\mathbf{A}_i}^2\le0$. Since the closed-loop system with

[1] As shown in [20], a sufficient condition is $\min\{\mathbf{K}_i,\mathbf{K}_{pi}\}>\frac{1}{2}(3-\sqrt{5})k_{gi}\mathbf{I}_n$.

the output $-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i$ is observable (with respect to the equilibrium state), the GAS claim follows immediately. This GAS property has been established in [2].

3. Stabilization with parameter uncertainty

As stated in Introduction, this paper investigates the performance of the aforementioned output-feedback controller when the parameters appearing in the gravity term of the control law are not precisely known. It is assumed that the upper-bound constant k_{gi} is available and that the estimated gravity force $\hat{\mathbf{g}}_i$ satisfies $|\nabla\hat{\mathbf{g}}_i(\mathbf{q})|\le k_{gi}$ for all $\mathbf{q}$.

In what follows, the subscript D refers to the decentralized scheme while the subscript N corresponds to the networked scheme.

3.1. The decentralized controller

Due to the uncertainty in $\mathbf{g}_i$, let us define the desired motor joint position $\hat{\boldsymbol{\theta}}_{\star i}\in\mathbb{R}^n$ as

$$\hat{\boldsymbol{\theta}}_{\star i}:=\mathbf{q}_\star+\mathbf{K}_i^{-1}\hat{\mathbf{g}}_i(\mathbf{q}_\star). \tag{5}$$

In this case, the output-feedback decentralized controller (3) becomes

$$\left.\begin{aligned}\dot{\mathbf{y}}_i&=-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i\\ \boldsymbol{\tau}_{Di}&=\hat{\mathbf{g}}_i(\mathbf{q}_\star)-\mathbf{K}_{pi}(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i})-\mathbf{K}_{di}(-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i).\end{aligned}\right\} \tag{6}$$

The closed-loop system (1) and (6), using the fact that $\mathbf{K}_i(\hat{\boldsymbol{\theta}}_{\star i}-\mathbf{q}_\star)-\hat{\mathbf{g}}_i(\mathbf{q}_\star)=\mathbf{0}_n$, is given by

$$\left.\begin{aligned}&\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i+\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i+\mathbf{g}_i(\mathbf{q}_i)-\hat{\mathbf{g}}_i(\mathbf{q}_\star)+\mathbf{K}_i(\mathbf{q}_i-\mathbf{q}_\star)-\mathbf{K}_i(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i})=\mathbf{0}_n\\ &\mathbf{J}_i\ddot{\boldsymbol{\theta}}_i+(\mathbf{K}_i+\mathbf{K}_{pi})(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i})-\mathbf{K}_i(\mathbf{q}_i-\mathbf{q}_\star)+\mathbf{K}_{di}(-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i)=\mathbf{0}_n\\ &\dot{\mathbf{y}}_i+\mathbf{A}_i\mathbf{y}_i-\mathbf{K}_{di}\boldsymbol{\theta}_i=\mathbf{0}_n\end{aligned}\right\} \tag{7}$$

hence the EL-systems are endowed with the following potential energy:

$$V_{Di}(\mathbf{q}_i,\boldsymbol{\theta}_i):=U_i(\mathbf{q}_i)-\mathbf{q}_i^\top\hat{\mathbf{g}}_i(\mathbf{q}_\star)+\frac{1}{2}\begin{bmatrix}\mathbf{q}_i-\mathbf{q}_\star\\ \boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i}\end{bmatrix}^\top\boldsymbol{\Phi}_i\begin{bmatrix}\mathbf{q}_i-\mathbf{q}_\star\\ \boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i}\end{bmatrix}.$$

Remark that the extrema of this function are the solutions $(\overline{\mathbf{q}}_{Di},\overline{\boldsymbol{\theta}}_{Di})$ of

$$\begin{bmatrix}\mathbf{g}_i(\overline{\mathbf{q}}_{Di})-\hat{\mathbf{g}}_i(\mathbf{q}_\star)\\ \mathbf{0}_n\end{bmatrix}+\boldsymbol{\Phi}_i\begin{bmatrix}\overline{\mathbf{q}}_{Di}-\mathbf{q}_\star\\ \overline{\boldsymbol{\theta}}_{Di}-\hat{\boldsymbol{\theta}}_{\star i}\end{bmatrix}=\mathbf{0}_{2n}, \tag{8}$$

which clearly coincide with the desired values if $\mathbf{g}_i=\hat{\mathbf{g}}_i$, otherwise they are shifted. Another important property of V_{Di} is that $\nabla^2V_{Di}(\mathbf{q}_i,\boldsymbol{\theta}_i)=\nabla^2V_i(\mathbf{q}_i,\boldsymbol{\theta}_i)$ where V_i is as in (4). Therefore, similar to the known parameter case, if $\sigma_{\min}(\boldsymbol{\Phi}_i)>k_{gi}$ then $\nabla^2V_{Di}>0$ and thus V_{Di} is a convex function with an isolated global minimum at $(\overline{\mathbf{q}}_{Di},\overline{\boldsymbol{\theta}}_{Di})$. Similar to the previous subsection, it is straightforward to show that for every agent i the equilibrium point $(\mathbf{q}_i,\boldsymbol{\theta}_i,\dot{\mathbf{q}}_i,\dot{\boldsymbol{\theta}}_i,\mathbf{y}_i)=(\overline{\mathbf{q}}_{Di},\overline{\boldsymbol{\theta}}_{Di},\mathbf{0}_n,\mathbf{0}_n,\mathbf{A}_i^{-1}\mathbf{K}_{di}\overline{\boldsymbol{\theta}}_{Di})$ is GAS.

3.2. The networked controller

We will now consider the networked controller, where for every agent i, the output-feedback controller (6) is interconnected in a network as follows:

$$\left.\begin{aligned}\dot{\mathbf{y}}_i&=-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i\\ \boldsymbol{\tau}_{Ni}&=\boldsymbol{\tau}_{Di}-\mathbf{K}_I\sum_{j\in\mathcal{N}_i}(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i}-\boldsymbol{\theta}_j+\hat{\boldsymbol{\theta}}_{\star j}),\end{aligned}\right\} \tag{9}$$

where $\mathcal{N}_i$ is the set of agents transmitting information to the i-th agent, $\mathbf{K}_I=\mathbf{K}_I^\top\in\mathbb{R}^{n\times n}$ and $\mathbf{K}_I>0$. This corresponds to attaching linear springs between the joints of the "neighboring" manipulators. See Fig. 1 for the physical interpretation of the decentralized and the networked controllers using a couple of flexible-joint pendula.

In what follows, we will make use of the graph Laplacian matrix $\mathbf{L}\in\mathbb{R}^{m\times m}$, whose elements are defined by

$$\ell_{ij}=\begin{cases}\sum_{k=1}^{m}a_{ik}, & i=j\\ -a_{ij}, & i\neq j\end{cases} \tag{10}$$

where $a_{ij}=1$ if $j\in\mathcal{N}_i$ and $a_{ij}=0$ otherwise [19].

In order to ensure that the interconnection forces—*i.e.*, the second right hand terms in (9)—are generated from a potential energy function, the following assumption is needed:

Assumption 1. The undirected communication graph is connected.[2]

The closed-loop system (1) and (9) is given by

$$\left.\begin{aligned}&\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i+\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i+\mathbf{g}_i(\mathbf{q}_i)-\hat{\mathbf{g}}_i(\mathbf{q}_\star)+\mathbf{K}_i(\mathbf{q}_i-\mathbf{q}_\star)-\mathbf{K}_i(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i})=\mathbf{0}_n\\ &\mathbf{J}_i\ddot{\boldsymbol{\theta}}_i+(\mathbf{K}_i+\mathbf{K}_{pi})(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i})+\mathbf{K}_I\sum_{j\in\mathcal{N}_i}(\boldsymbol{\theta}_i-\hat{\boldsymbol{\theta}}_{\star i}-\boldsymbol{\theta}_j+\hat{\boldsymbol{\theta}}_{\star j})\\ &\quad-\mathbf{K}_i(\mathbf{q}_i-\mathbf{q}_\star)+\mathbf{K}_{di}(-\mathbf{A}_i\mathbf{y}_i+\mathbf{K}_{di}\boldsymbol{\theta}_i)=\mathbf{0}_n\\ &\dot{\mathbf{y}}_i+\mathbf{A}_i\mathbf{y}_i-\mathbf{K}_{di}\boldsymbol{\theta}_i=\mathbf{0}_n\end{aligned}\right\} \tag{11}$$

Defining $\boldsymbol{\theta}:=\mathrm{col}(\boldsymbol{\theta}_i)$, $\hat{\boldsymbol{\theta}}_\star:=\mathrm{col}(\hat{\boldsymbol{\theta}}_{\star i})$ and $\mathbf{q}:=\mathrm{col}(\mathbf{q}_i)$, the potential energy of the EL-system in (11) satisfies

$$V_N(\mathbf{q},\boldsymbol{\theta}):=\frac{1}{2}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_\star)^\top(\mathbf{L}\otimes\mathbf{K}_I)(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_\star)+\sum_{i=1}^{m}V_{Di}.$$

Furthermore, $\nabla V_N(\mathbf{q},\boldsymbol{\theta})$ is given by

$$\nabla V_N(\mathbf{q},\boldsymbol{\theta})=\begin{bmatrix}\mathbf{0}_{mn}\\ (\mathbf{L}\otimes\mathbf{K}_I)(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_\star)\end{bmatrix}+\begin{bmatrix}\mathbf{K} & -\mathbf{K}\\ -\mathbf{K} & \mathbf{K}+\mathbf{K}_p\end{bmatrix}\begin{bmatrix}\mathbf{q}-(\mathbb{1}_m\otimes\mathbf{q}_\star)\\ \boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_\star\end{bmatrix}+\begin{bmatrix}\mathbf{g}(\mathbf{q})-\hat{\mathbf{g}}(\mathbf{q}_\star)\\ \mathbf{0}_{mn}\end{bmatrix}$$

and

$$\nabla^2V_N(\mathbf{q},\boldsymbol{\theta})=\begin{bmatrix}\frac{\partial\mathbf{g}(\mathbf{q})}{\partial\mathbf{q}} & \mathbf{0}_{mn\times mn}\\ \mathbf{0}_{mn\times mn} & \mathbf{0}_{mn\times mn}\end{bmatrix}+\boldsymbol{\Psi},$$

where $\mathbf{K}:=\mathrm{diag}(\mathbf{K}_i)\in\mathbb{R}^{mn\times mn}$, $\mathbf{K}_p:=\mathrm{diag}(\mathbf{K}_{pi})\in\mathbb{R}^{mn\times mn}$, $\mathbf{g}(\mathbf{q}):=\mathrm{col}(\mathbf{g}_i(\mathbf{q}_i))$, $\hat{\mathbf{g}}(\mathbf{q}_\star):=\mathrm{col}(\hat{\mathbf{g}}_i(\mathbf{q}_\star))$ and

$$\boldsymbol{\Psi}:=\begin{bmatrix}\mathbf{K} & -\mathbf{K}\\ -\mathbf{K} & \mathbf{K}+\mathbf{K}_p+(\mathbf{L}\otimes\mathbf{K}_I)\end{bmatrix}. \tag{12}$$

Clearly, if $\sigma_{\min}(\boldsymbol{\Psi})>\max\{k_{gi}\}$ then V_N is a convex function with an isolated minimum at $(\overline{\mathbf{q}}_N,\overline{\boldsymbol{\theta}}_N)$ which are the unique solution to $\nabla V_N(\overline{\mathbf{q}}_N,\overline{\boldsymbol{\theta}}_N)=\mathbf{0}_{2mn}$. By denoting $\mathbf{y}:=\mathrm{col}(\mathbf{y}_i)$, $\mathbf{A}:=\mathrm{diag}(\mathbf{A}_i)$ and $\mathbf{K}_d:=\mathrm{diag}(\mathbf{K}_{di})$, it can be shown that the equilibrium point of the whole network $(\mathbf{q},\boldsymbol{\theta},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}},\mathbf{y})=(\overline{\mathbf{q}}_N,\overline{\boldsymbol{\theta}}_N,\mathbf{0}_n,\mathbf{0}_n,\mathbf{A}^{-1}\mathbf{K}_d\overline{\boldsymbol{\theta}}_N)$ is GAS.

[2] It is well known that Assumption 1 ensures that $\mathbf{L}=\mathbf{L}^\top$, $\mathbf{L}\mathbb{1}_m=\mathbf{0}$ and $\mathbb{1}_m^\top\mathbf{L}=\mathbf{0}$. Further, $\mathbf{L}$ has a single 0 eigenvalue and the rest of its spectrum is strictly positive. Moreover, for any $\mathbf{y}\in\mathbb{R}^{mn}$,

$$\mathbf{y}^\top(\mathbf{L}\otimes\mathbf{K}_I)\mathbf{y}=\frac{1}{2}\sum_{i=1}^{m}\sum_{j\in\mathcal{N}_i}(\mathbf{y}_i-\mathbf{y}_j)^\top\mathbf{K}_I(\mathbf{y}_i-\mathbf{y}_j)\ge 0,$$

where $\otimes$ is the standard Kronecker product.

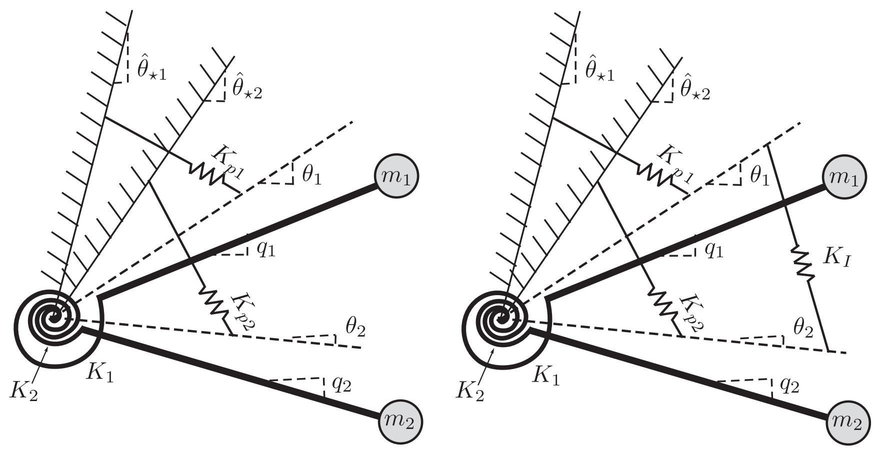

Fig. 1. Physical interpretation of the decentralized (left) and networked (right) controllers applied to a couple of flexible-joint pendula.

4. Networking improves robustness

In this section it is proved that the networked controller drives the equilibrium point closer (in a suitable defined metric) to the desired one than the decentralized controller. For, let us define the following equilibrium errors for the decentralized and the networked controllers:

$$\tilde{\mathbf{x}}_C := \begin{bmatrix} \tilde{\mathbf{q}}_C \\ \tilde{\boldsymbol{\theta}}_C \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{q}}_C - (\mathbb{1}_m \otimes \mathbf{q}_\star) \\ \overline{\boldsymbol{\theta}}_C - \hat{\boldsymbol{\theta}}_\star \end{bmatrix}, \tag{13}$$

where $\overline{\mathbf{q}}_C := \mathrm{col}(\overline{\mathbf{q}}_{Ci}) \in \mathbb{R}^{mn}$ and the subindex $C \in \{D, N\}$. Defining

$$\boldsymbol{\Upsilon} := \begin{bmatrix} \mathbf{K} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{K}+\mathbf{K}_p \end{bmatrix}, \quad \mathbf{G}(\mathbf{q}) := \begin{bmatrix} \mathbf{g}(\mathbf{q}) \\ \mathbf{0}_{mn} \end{bmatrix} \in \mathbb{R}^{2mn},$$

the decentralized equilibrium equations (8) can be written in matrix form as

$$\boldsymbol{\Upsilon}\tilde{\mathbf{x}}_D + \mathbf{G}(\overline{\mathbf{q}}_D) = \mathbf{G}(\mathbb{1}_m \otimes \mathbf{q}_\star). \tag{14}$$

Similarly, the networked equilibria satisfies

$$\boldsymbol{\Upsilon}\tilde{\mathbf{x}}_N + \boldsymbol{\Lambda}\tilde{\mathbf{x}}_N + \mathbf{G}(\overline{\mathbf{q}}_N) = \mathbf{G}(\mathbb{1}_m \otimes \mathbf{q}_\star), \tag{15}$$

where

$$\boldsymbol{\Lambda} := \begin{bmatrix} \mathbf{0}_{mn\times mn} & \mathbf{0}_{mn\times mn} \\ \mathbf{0}_{mn\times mn} & (\mathbf{L} \otimes \mathbf{K}_I) \end{bmatrix}.$$

Note, from (14) and (15) that the equilibria of the decentralized and the networked controller coincide if $\boldsymbol{\Lambda}\tilde{\mathbf{x}}_N = (\mathbf{L} \otimes \mathbf{K}_I)\tilde{\boldsymbol{\theta}}_N = \mathbf{0}_{mn}$ which implies that $\tilde{\boldsymbol{\theta}}_N = (\mathbf{1}_m \otimes \boldsymbol{\theta}_c)$ where $\boldsymbol{\theta}_c \in \mathbb{R}^n$. Hence, it is reasonable to assume the following:

Assumption 2. For all i and all $j \in \mathcal{N}_i$, $\tilde{\boldsymbol{\theta}}_{Ni} \neq \tilde{\boldsymbol{\theta}}_{Nj}$, *i.e.*, $\tilde{\boldsymbol{\theta}}_N$ is *not* in the identity set.

Assumption 2 is always satisfied when the agents are non-identical and the controllers have different proportional gains $\mathbf{K}_{pi}$ and different estimated parameters in the terms $\hat{\mathbf{g}}_i(\mathbf{q}_\star)$. This fact can be easily established from (8).

The following proposition states the main result of this work, that is (i) provided a minimum interconnection gain, the equilibrium error of the decentralized scheme is always greater than the equilibrium error of the networked scheme (in a suitable defined metric) and (ii) if both equilibria are close enough then, for all interconnection gains, the error of the decentralized scheme is strictly greater than the error of the networked scheme.

Proposition 1. *Consider a network of m, n-DOF, flexible-joint manipulators of the form (1) in closed loop with either the decentralized controller (3), verifying* $\sigma_{\min}(\boldsymbol{\Upsilon}) > \max\{k_{gi}\}$, *or the networked controller (9), verifying* $\sigma_{\min}(\boldsymbol{\Upsilon}+\boldsymbol{\Lambda}) > \max\{k_{gi}\}$. *Then, if Assumptions 1 and 2 hold, the following statements hold for all* $\hat{\mathbf{g}}_i(\cdot)$.

(i) *There exists* $K_I^{\min} > 0$ *such that for all* $\mathbf{K}_I$ *satisfying* $\sigma_{\min}(\mathbf{K}_I) \geq K_I^{\min}$, *the steady-state errors of the decentralized and the networked controllers in (13) satisfy*

$$\|\tilde{\mathbf{x}}_D\|^2_{\mathbf{W}_N} \geq \|\tilde{\mathbf{x}}_N\|^2_{\mathbf{W}_N} + \alpha \tag{16}$$

for some $\mathbf{W}_N > 0$ *and* $\alpha > 0$.

(ii) *If there exists* $\check{\mathbf{q}} \in \mathbb{R}^{mn}$ *such that* $|\overline{\mathbf{q}}_D - \check{\mathbf{q}}| \leq \varepsilon$ *and* $|\overline{\mathbf{q}}_N - \check{\mathbf{q}}| \leq \varepsilon$, *where* $\varepsilon > 0$ *is sufficiently small, then, for all* $\mathbf{K}_I > 0$,

$$\|\tilde{\mathbf{x}}_D\|^2_{\mathbf{W}} = \|\tilde{\mathbf{x}}_N\|^2_{\mathbf{W}} + \beta,$$

where $\beta := \|\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N\|^2_{\mathbf{W}} + \tilde{\boldsymbol{\theta}}_N^\top (\mathbf{L} \otimes \mathbf{K}_I)\tilde{\boldsymbol{\theta}}_N > 0$ *and* $\mathbf{W} := \boldsymbol{\Upsilon} + \nabla\mathbf{G}(\check{\mathbf{q}}) > 0$.

Proof. First note that, (14) and (15) imply the following:

$$\boldsymbol{\Upsilon}(\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N) = \boldsymbol{\Lambda}\tilde{\mathbf{x}}_N + \mathbf{G}(\overline{\mathbf{q}}_N) - \mathbf{G}(\overline{\mathbf{q}}_D). \tag{17}$$

Now, for ease of presentation, let us prove first claim (ii). By invoking the assumption that the equilibria are close to approximate the gravity forces as $\mathbf{g}(\mathbf{q}) \approx \mathbf{g}(\check{\mathbf{q}}) + \nabla\mathbf{g}(\check{\mathbf{q}})(\mathbf{q} - \check{\mathbf{q}})$, the expression (17) can be approximated by the linear equation:

$$\mathbf{W}(\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N) = \boldsymbol{\Lambda}\tilde{\mathbf{x}}_N. \tag{18}$$

Pre-multiplying (18) by $\tilde{\mathbf{x}}_N^\top$ and after some algebraic manipulations we get

$$\tilde{\mathbf{x}}_D^\top \mathbf{W}\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N^\top \mathbf{W}\tilde{\mathbf{x}}_N = (\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N)^\top \mathbf{W}(\tilde{\mathbf{x}}_D - \tilde{\mathbf{x}}_N) + 2\tilde{\mathbf{x}}_N^\top \boldsymbol{\Lambda}\tilde{\mathbf{x}}_N.$$

Assumptions 1 and 2 ensure that

$$2\tilde{\mathbf{x}}_N^\top \boldsymbol{\Lambda}\tilde{\mathbf{x}}_N = \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} (\tilde{\boldsymbol{\theta}}_{Ni} - \tilde{\boldsymbol{\theta}}_{Nj})^\top \mathbf{K}_I (\tilde{\boldsymbol{\theta}}_{Ni} - \tilde{\boldsymbol{\theta}}_{Nj}) > 0 \tag{19}$$

and if $\mathbf{K}_{pi}$ is set such that $\sigma_{\min}(\boldsymbol{\Upsilon}) > \max\{k_{gi}\}$ then $\mathbf{W} > 0$. This completes the proof of claim (ii).

Let us proceed now to establish claim (i). Using (13) we can write $\mathbf{g}(\overline{\mathbf{q}}_C) = \mathbf{g}((\mathbb{1}_m \otimes \mathbf{q}_\star) + \tilde{\mathbf{q}}_C)$. For every agent i, using the continuity of $\mathbf{g}_i$ and the mean-value theorem, we have that for every $k \in \{1, 2, \ldots, n\}$, there exist $\mathbf{a}_{ik} \in [\mathbf{q}_\star, \mathbf{q}_\star + \tilde{\mathbf{q}}_{Di}] \subset \mathbb{R}^n$ and $\mathbf{b}_{ik} \in [\mathbf{q}_\star, \mathbf{q}_\star + \tilde{\mathbf{q}}_{Ni}] \subset \mathbb{R}^n$, where $[\mathbf{q}_1, \mathbf{q}_2]$ denotes the straight line interval between

$\mathbf{q}_1$ and $\mathbf{q}_2$, such that

$$\mathbf{g}_{ik}(\mathbf{q}_\star+\tilde{\mathbf{q}}_{Di})=\mathbf{g}_{ik}(\mathbf{q}_\star)+\nabla\mathbf{g}_{ik}(\mathbf{a}_{ik})\tilde{\mathbf{q}}_{Di}$$
$$\mathbf{g}_{ik}(\mathbf{q}_\star+\tilde{\mathbf{q}}_{Ni})=\mathbf{g}_{ik}(\mathbf{q}_\star)+\nabla\mathbf{g}_{ik}(\mathbf{b}_{ik})\tilde{\mathbf{q}}_{Ni}$$

where $\mathbf{g}_{ik}$ denotes the k-th element of $\mathbf{g}_i$.

Now, denoting $\nabla\mathbf{G}_D{:=}\mathrm{diag}(\nabla\mathbf{G}_i(\mathbf{a}_{ik}))$ and $\nabla\mathbf{G}_N{:=}\mathrm{diag}(\nabla\mathbf{G}_i(\mathbf{b}_{ik}))$, where

$$\nabla\mathbf{G}_i(\mathbf{a}_{ik})=\begin{bmatrix}\nabla\mathbf{g}_{i1}^\top(\mathbf{a}_{i1})\\ \nabla\mathbf{g}_{i2}^\top(\mathbf{a}_{i2})\\ \vdots\\ \nabla\mathbf{g}_{in}^\top(\mathbf{a}_{in})\end{bmatrix},\quad \nabla\mathbf{G}_i(\mathbf{b}_{ik})=\begin{bmatrix}\nabla\mathbf{g}_{i1}^\top(\mathbf{b}_{i1})\\ \nabla\mathbf{g}_{i2}^\top(\mathbf{b}_{i2})\\ \vdots\\ \nabla\mathbf{g}_{in}^\top(\mathbf{b}_{in})\end{bmatrix},$$

then (17) can be compactly written as

$$\boldsymbol{\Upsilon}(\tilde{\mathbf{x}}_D-\tilde{\mathbf{x}}_N)=\boldsymbol{\Lambda}\tilde{\mathbf{x}}_N+\nabla\mathbf{G}_N\tilde{\mathbf{x}}_N-\nabla\mathbf{G}_D\tilde{\mathbf{x}}_D. \tag{20}$$

Due to the continuity of the potential energy $U_i(\mathbf{q}_i)$ and the fact that $\sigma_{\min}(\boldsymbol{\Upsilon})>\max\{k_{gi}\}$, we have that $\mathbf{W}_N{:=}\boldsymbol{\Upsilon}+\nabla\mathbf{G}_N$ and $\mathbf{W}_D{:=}\boldsymbol{\Upsilon}+\nabla\mathbf{G}_D$ are positive definite matrices. Using $\mathbf{W}_N$ and $\mathbf{W}_D$, (20) becomes $\mathbf{W}_D\tilde{\mathbf{x}}_D=\mathbf{W}_N\tilde{\mathbf{x}}_N+\boldsymbol{\Lambda}\tilde{\mathbf{x}}_N$, which using similar arguments to the proof of claim (ii), implies that

$$\tilde{\mathbf{x}}_D^\top\mathbf{W}_N\tilde{\mathbf{x}}_D-\tilde{\mathbf{x}}_N^\top\mathbf{W}_N\tilde{\mathbf{x}}_N=\tilde{\mathbf{x}}_D^\top\mathbf{W}_N\tilde{\mathbf{x}}_D-2\tilde{\mathbf{x}}_N^\top\mathbf{W}_D\tilde{\mathbf{x}}_D+\tilde{\mathbf{x}}_N^\top\mathbf{W}_N\tilde{\mathbf{x}}_N+2\tilde{\mathbf{x}}_N^\top\boldsymbol{\Lambda}\tilde{\mathbf{x}}_N.$$

Since $\tilde{\mathbf{x}}_N^\top\mathbf{W}_D\tilde{\mathbf{x}}_D=\tilde{\mathbf{x}}_D^\top\mathbf{W}_N\mathbf{W}_N^{-1}\mathbf{W}_D\tilde{\mathbf{x}}_N$ and adding and subtracting the term $\tilde{\mathbf{x}}_N^\top\mathbf{W}_D\mathbf{W}_N^{-1}\mathbf{W}_N\mathbf{W}_N^{-1}\mathbf{W}_D\tilde{\mathbf{x}}_N$ on the above equation, yields

$$\|\tilde{\mathbf{x}}_D\|^2_{\mathbf{W}_N}-\|\tilde{\mathbf{x}}_N\|^2_{\mathbf{W}_N}=\|\tilde{\mathbf{x}}_D-\mathbf{W}_N^{-1}\mathbf{W}_D\tilde{\mathbf{x}}_N\|^2_{\mathbf{W}_N}+\tilde{\mathbf{x}}_N^\top(2\boldsymbol{\Lambda}+\mathbf{W}_N-\mathbf{W}_D\mathbf{W}_N^{-1}\mathbf{W}_D)\tilde{\mathbf{x}}_N.$$

Fix $\mu>0$ and define

$$K_I^{\min}{:=}\frac{1}{\sum_{i=1}^m\sum_{j\in\mathcal{N}_i}|\tilde{\boldsymbol{\theta}}_{Ni}-\tilde{\boldsymbol{\theta}}_{Nj}|^2}[\mu+(\sigma_{\max}(\mathbf{W}_2\mathbf{W}_1^{-1}\mathbf{W}_2)-\sigma_{\min}(\mathbf{W}_N))|\tilde{\mathbf{x}}_N|^2].$$

Invoking (19) it is easy to see that, for all $\mathbf{K}_I$ satisfying $\sigma_{\min}(\mathbf{K}_I)\ge K_I^{\min}$, we have the bound

$$\tilde{\mathbf{x}}_N^\top(2\boldsymbol{\Lambda}+\mathbf{W}_N-\mathbf{W}_D\mathbf{W}_N^{-1}\mathbf{W}_D)\tilde{\mathbf{x}}_N\ge\mu,$$

and hence (16) holds with $\alpha=\|\tilde{\mathbf{x}}_D-\mathbf{W}_N^{-1}\mathbf{W}_D\tilde{\mathbf{x}}_N\|^2_{\mathbf{W}_N}+\mu$. This completes the proof. □

4.1. Remarks

(1) Claim (i) of Proposition 1 shows that networking reduces the steady-state error if the interconnection gain $\mathbf{K}_I$ is sufficiently large. In claim (ii) it is assumed that the equilibria are close, which is essential to obtain the linear equation (18) that approximates the link gravity forces in a neighborhood[3] of $\check{\mathbf{q}}$. The interest of (ii) is threefold. First, it allows to prove that the networked equilibrium point is strictly closer to the desired equilibria *for any* $\mathbf{K}_I>0$. Second, it replaces an estimate on the errors norms, with a sharp *identity*. Finally, an explicit expression for the norm weight, which is only known to exist for the general case, is given. It should be underscored that these results hold for any equilibria if the potential energy function is quadratic.

(2) The networked controller can potentially achieve the GAS objective using lower gains $\mathbf{K}_{pi}$ than the decentralized scheme. Indeed, the former condition is $\sigma_{\min}(\boldsymbol{\Upsilon}+\boldsymbol{\Lambda})>\max\{k_{gi}\}$, while the decentralized condition is $\sigma_{\min}(\boldsymbol{\Upsilon})>\max\{k_{gi}\}$. This shows that such condition is sufficient, but not necessary, for positivity of the Hessian $\nabla^2V_N(\mathbf{q},\boldsymbol{\theta})$. Hence a minimum can be assigned to the networked closed-loop potential energy with smaller controller gains $\mathbf{K}_{pi}$.

(3) It is important to remark that, if the stiffness coefficient of the linking spring $\mathbf{K}_I$ increases, the gap between the errors of the two approaches also increases, that is, α and β in Proposition 1 are larger. Clearly, the values of $\mathbf{K}_I$ required to increase the gap are inversely proportional to the distance of the equilibria to the unitary set. Moreover, for the global result (i), the value of $K_I^{\min}$ is unknown.

[3] It is important to underscore that this does not mean that the motion of the flexible joint manipulators are restricted to a small neighborhood, but only that their equilibria are close.

5. Numerical simulations

Some numerical simulations have been performed in order to illustrate the robustness improvement due to the interconnection. The simulations employ a network of ten 2-DoF revolute flexible-joint robot manipulators. Each manipulator nonlinear dynamics follows the EL-equations (1), whose inertia and Coriolis matrices are given, respectively, by

$$\mathbf{M}_i(\mathbf{q}_i)=\begin{bmatrix}\alpha_i+2\beta_i\mathrm{c}_{i_2} & \delta_i+\beta_i\mathrm{c}_{i_2}\\ \delta_i+\beta_i\mathrm{c}_{i_2} & \delta_i\end{bmatrix},\quad \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)=\begin{bmatrix}-2\beta_i\mathrm{s}_{i_2}\dot{q}_{i_2} & -\beta_i\mathrm{s}_{i_2}\dot{q}_{i_2}\\ \beta_i\mathrm{s}_{i_2}\dot{q}_{i_1} & 0\end{bmatrix}.$$

In these expressions, $\mathrm{c}_{i_k},\mathrm{s}_{i_k}$ are short notation for $\cos(q_{i_k})$ and $\sin(q_{i_k})$; q_{i_k} is the position of link k of manipulator i, with $k\in\{1,2\}$; $\alpha_i=l_{i_2}^2m_{i_2}+l_{i_1}^2(m_{i_1}+m_{i_2})$, $\beta_i=l_{i_1}l_{i_2}m_{i_2}$ and $\delta_i=l_{i_2}^2m_{i_2}$, where l_{i_k} and m_{i_k} are the respective lengths and masses of each link, respectively. The potential energy, due to gravity, of each manipulator is $U_i(\mathbf{q}_i)=gl_{i_1}(m_{i_1}+m_{i_2})(1+\mathrm{s}_{i_1})+gm_{i_2}l_{i_2}(1+\mathrm{s}_{i_{12}})$ where $\mathrm{s}_{i_{12}}$ stands for $\sin(q_{i_1}+q_{i_2})$ and g=9.81 m s^2 is the acceleration of gravity constant. In this case

$$\nabla^2U_i(\mathbf{q}_i)=-gm_{i_2}l_{i_2}\begin{bmatrix}\frac{l_{i_1}}{l_{i_2}}\left(\frac{m_{i_1}}{m_{i_2}}+1\right)\mathrm{s}_{i_1}+\mathrm{s}_{i_{12}} & \mathrm{s}_{i_{12}}\\ \mathrm{s}_{i_{12}} & \mathrm{s}_{i_{12}}\end{bmatrix}.$$

As stated in Section 3, the gravity term $\mathbf{g}_i(\mathbf{q}_\star)=\nabla U_i(\mathbf{q}_\star)$ is uncertain. In these simulations it is assumed that uncertainty appears due to the mass coefficients. The decentralized and networked controller employ only an estimated upper bound of the mass, denoted $\overline{m}_{i_1},\overline{m}_{i_2}$. Hence,

$$\hat{\mathbf{g}}_i^\top(\mathbf{q}_\star)=g[l_{i_1}(\overline{m}_{i_1}+\overline{m}_{i_2})\cos(q_{\star_1})+\overline{m}_{i_2}l_{i_2}\ \cos(q_{\star_1}+q_{\star_2}),\quad \overline{m}_{i_2}l_{i_2}\ \cos(q_{\star_1}+q_{\star_2})].$$

After some straightforward calculations, it can be shown that $|\nabla\hat{\mathbf{g}}_i(\mathbf{q}_\star)|\le(1/2)g\overline{m}_{i_2}l_{i_2}(b_i+2+\sqrt{b^2+4})$ where $b_i{:=}(l_{i_1}/l_{i_2})((\overline{m}_{i_1}/\overline{m}_{i_2})+1)$.

The flexible-joint manipulators network is composed of three different groups, with all members in each group equal. For simplicity, all m link lengths have been set to $l_{i_1}=l_{i_2}=0.25$ m. The rest of the physical parameters and the initial link positions for each manipulator are shown in Table 1. The initial joint (motor) positions are equal to the initial link positions, *i.e.*, $\boldsymbol{\theta}(0)=\mathbf{q}(0)$, and the initial velocities are all set to zero.

The mass upper bounds have all been set as $\overline{m}_{i_1}=3$ and $\overline{m}_{i_2}=1.9$. Thus, $|\nabla\hat{\mathbf{g}}_i(\mathbf{q}_\star)|\le18.2721$. In order to ensure the existence of a unique solution of the equilibria, the proportional gains $\mathbf{K}_{pi}$ for both controllers must be set such that $\sigma_{\min}(\boldsymbol{\Phi}_i)>|\nabla\hat{\mathbf{g}}_i(\mathbf{q}_\star)|$ for all manipulators. Setting $\mathbf{K}_{pi}=40\mathbf{I}_2$ ensures this condition since, for manipulators 1–4, $\sigma_{\min}(\boldsymbol{\Phi}_i)=19.2013$, for manipulators 5–7, $\sigma_{\min}(\boldsymbol{\Phi}_i)=19.0025$, and for manipulators 8–10, $\sigma_{\min}(\boldsymbol{\Phi}_i)=18.6725$. The filter gains are $\mathbf{K}_{di}=15\mathbf{I}_2$, $\mathbf{A}_i=25\mathbf{I}_2$ for all controllers. Finally, the desired position is $\mathbf{q}_\star=[0,0]^\top$ rad which is a point with maximum gravity torques.

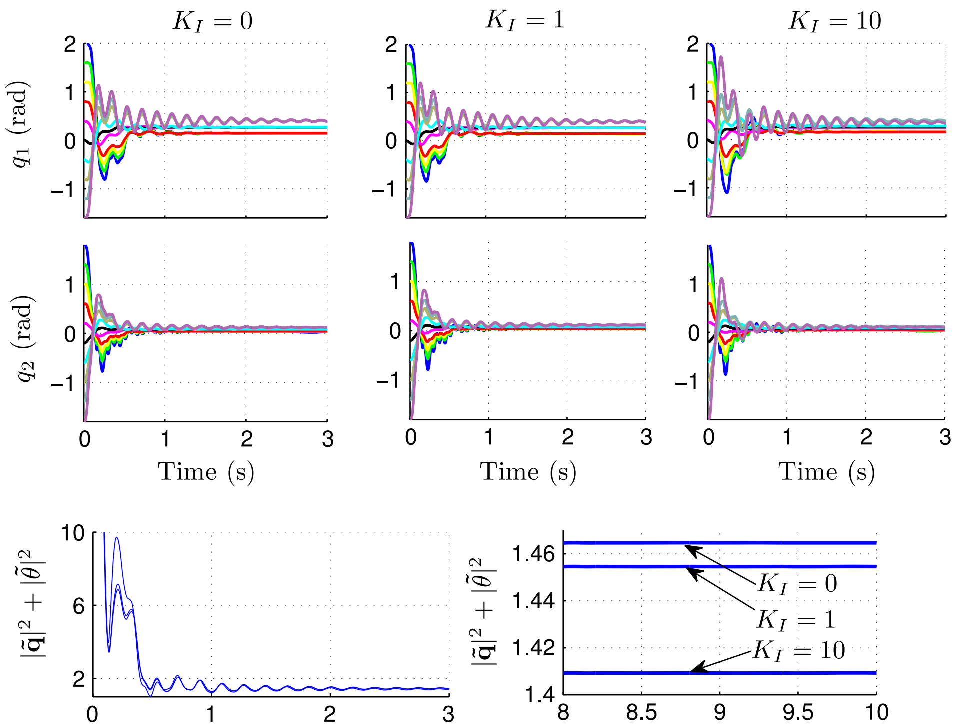

Fig. 2. Link angular positions and errors for different values of interconnection gains and for $\mathbf{q}_\star = [0, 0]^\top$.

The interconnecting Laplacian matrix is

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 3 \end{bmatrix}$$

which corresponds to a simply connected undirected graph. Note that $\mathbb{1}_{10}$ is the left and right eigenvector associated to the single zero eigenvalue of $\mathbf{L}$.

To verify the robustness improvement of the networked controller, different values of the interconnection gain have been simulated, namely $\mathbf{K}_I \in \{0.01\mathbf{I}_2, 0.1\mathbf{I}_2, \mathbf{I}_2, 10\mathbf{I}_2, 100\mathbf{I}_2, 1000\mathbf{I}_2\}$. Table 2 shows the numerical values of the norms of the decentralized and the networked equilibria for the different values of $\mathbf{K}_I$. The regulation results for the decentralized controller ($\mathbf{K}_I = \mathbf{0}_2$) can be seen in the first column of Fig. 2 (for sake of space only the link positions are shown). Since there are three different groups, each one with identical agents, it can be seen that there are three different equilibria that do not coincide with the desired link position $\mathbf{q}_\star$. On the other hand, the behavior of the system controlled with the networked approach is depicted, for some of the interconnection

Table 1
Physical parameters and initial positions for each EL-system in the network.

Index i	m_{i_1}, m_{i_2} (kg)	$\mathbf{J}_i$	$\mathbf{K}_i$	$\mathbf{q}_i^\top(0)$ (rad)
1	2, 1.2	$\mathbf{I}_2$	$250\mathbf{I}_2$	[2 1.8]
2				[1.6 1.4]
3				[1.2 1]
4				[0.8 0.6]
5	1.5, 0.8	$0.8\mathbf{I}_2$	$200\mathbf{I}_2$	[0.4 0.2]
6				[0 −0.2]
7				[−0.4 −0.6]
8	1, 0.4	$0.6\mathbf{I}_2$	$150\mathbf{I}_2$	[−0.8 −1]
9				[−1.2 −1.4]
10				[−1.6 −1.8]

Table 2
Numerical error norms of the decentralized and networked equilibria for $\mathbf{q}_\star = [0, 0]^\top$ rad for different values of the interconnection gain $\mathbf{K}_I$.

	$\mathbf{K}_I$						
Error Norm	$\mathbf{0}_2$	$0.01\mathbf{I}_2$	$0.1\mathbf{I}_2$	$\mathbf{I}_2$	$10\mathbf{I}_2$	$100\mathbf{I}_2$	$1000\mathbf{I}_2$
$\lvert\tilde{\mathbf{x}}_D\rvert^2$	1.4647						
$\lvert\tilde{\mathbf{x}}_N\rvert^2$		1.4646	1.4636	1.4546	1.4092	1.3461	1.3201

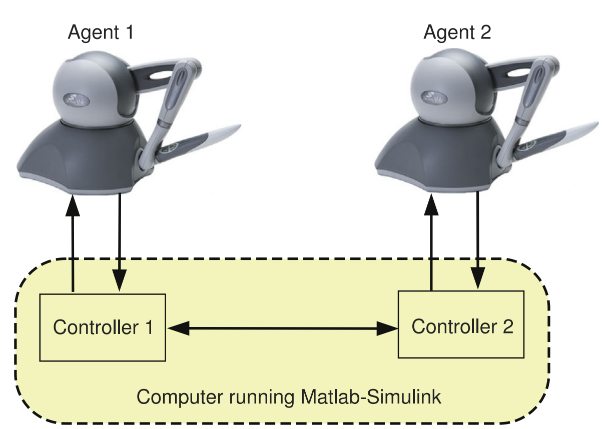

Fig. 3. Experimental setup, composed of two PHANToM Omni® devices.

gains, in the second and the third columns of Fig. 2. In all cases it has been confirmed that $|\tilde{\mathbf{x}}_D| > |\tilde{\mathbf{x}}_N|$.

6. Experiments

This section presents, in the same spirit as the previous one, some experimental evidence that supports the main thesis of the paper: *interconnection improves performance*. The experimental setup is composed of two 3-DOF mechanical systems. These devices (agents) are the PHANToM Omni®, from Sensable Technologies (http://sensable.com/). The devices run in the same computer connected through a Firewire 1934 port. Fig. 3 shows this experimental setup. The controller and all software are implemented in Matlab Simulink®.

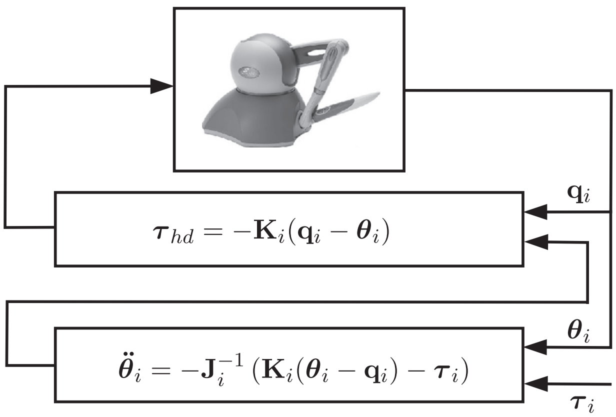

Fig. 4. Emulated flexible-joint manipulator.

Since the PHANToM Omni® devices are fully actuated, the flexible-joint (under-actuated) behavior is emulated using the control scheme in Fig. 4. The closed-loop behavior of such a system is the same as (1). The motor inertia has been set to $\mathbf{J}_i = \text{diag}(0.25, 0.15, 0.1)$ and the manipulator joint stiffness is set to $\mathbf{K}_i = 0.9\mathbf{I}_3$.

Both the decentralized and the networked controllers are the implementation of (6) and (9) with the gravity vector given by $\hat{\mathbf{g}}(\mathbf{q}) = \text{col}(0, \hat{g}_2, \hat{g}_3)$ where $\hat{g}_2 = \hat{p}_1 \sin(q_2+q_3) + \hat{p}_2 \cos(q_2)$ and $\hat{g}_3 = \hat{p}_1 \sin(q_2+q_3)$ for $\hat{p}_1 = \overline{m}_3 l_2$ and $\hat{p}_2 = \overline{m}_3 l_2 + \overline{m}_2 l_1$. In the experiments, the estimation of the physical parameters is $\hat{p}_1 = 0.1305$ K g m and $\hat{p}_2 = 0.2246$ K g m. It can be easily calculated that $\|\nabla \hat{\mathbf{g}}(\mathbf{q})\| \le 0.415$. Choosing $\mathbf{K}_{pi} = 1.5\mathbf{I}_3$ yields $\sigma_{\min}(\boldsymbol{\Phi}_i) = 0.4785$. The rest of the control gains is $\mathbf{K}_{di} = 0.8\mathbf{I}_3$ and $\mathbf{A}_i = 2.5\mathbf{I}_3$. The desired position is $\mathbf{q}_\star = [0, 0.6, 0.4]^\top$.

The experiments employ different values of interconnection gains, namely $\mathbf{K}_I \in \{0.1\mathbf{I}_3, \mathbf{I}_3, 2\mathbf{I}_3, 5\mathbf{I}_3, 10\mathbf{I}_3\}$. Table 3 shows the experimental values of the norms of the decentralized and networked equilibria errors for the different values of $\mathbf{K}_I$.

Fig. 5 shows the evolution of the link positions and the norm of the error for three different values of interconnection gains, from which it can be concluded that the experiments also confirm that interconnection improves robustness and hence $|\tilde{\mathbf{x}}_D| > |\tilde{\mathbf{x}}_N|$.

Table 3
Numerical error norms of the decentralized and networked equilibria for $\mathbf{q}_\star = [0, 0.6, 0.4]^\top$ rad for different values of the interconnection gain $\mathbf{K}_I$.

	$\mathbf{K}_I$					
Error Norm	$\mathbf{0}_3$	$0.1\mathbf{I}_3$	$\mathbf{I}_3$	$2\mathbf{I}_3$	$5\mathbf{I}_3$	$10\mathbf{I}_3$
$\lvert\tilde{\mathbf{x}}_D\rvert^2$	0.3009					
$\lvert\tilde{\mathbf{x}}_N\rvert^2$		0.3009	0.2406	0.2082	0.1760	0.1641

7. Conclusions

This paper shows that the robustness, of a class of (potential) energy-shaping controllers, for the coordination of multiple flexible-joint EL-systems is improved exchanging information between the agents, provided that the graph representing the

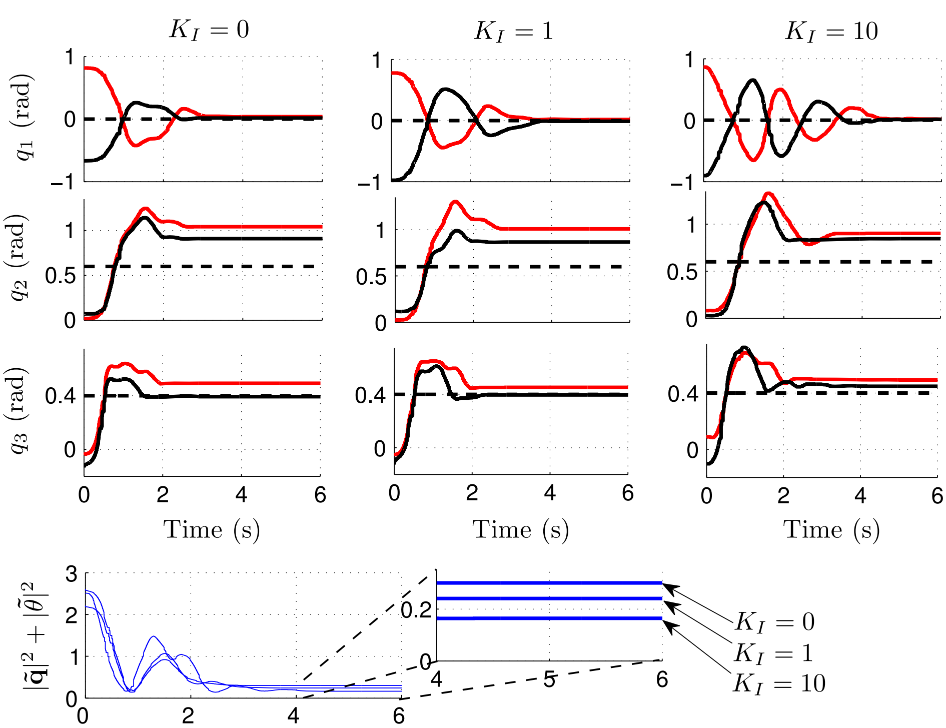

Fig. 5. Robot link positions and errors for different values of interconnection gains and for $\mathbf{q}_\star = [0, 0.6, 0.4]^\top$.

network is undirected and connected. It is proved that there exists a lower bound for the interconnection gain that ensures the improvement (in a suitable metric). This lower bound is inversely proportional to the distance of the joint (motor) equilibria to the unitary set. On the other hand, if the equilibria of the decentralized and networked controllers are close together, improvement is achieved for all interconnection gains.

In order to show the robustness effect, the paper presents some numerical simulations using a network composed of ten 2-DOF flexible-joint robot manipulators and experiments with two 3-DOF mechanical manipulators.

Two research avenues are currently being pursued. First, to analyze this robustifying feature when delays arise in the interconnection communications. Second, to extend the result to a larger class of underactuated EL-systems—that is, beyond robots with flexible-joints—and to other energy-shaping controllers, for instance, those including kinetic energy-shaping.

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