Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems

Ortega, Romeo; Schaft, Arjan van der; Maschke, Bernhard; Escobar, Gerardo

Published in:
Automatica

PERGAMON

automatica

Automatica 38 (2002) 585–596

www.elsevier.com/locate/automatica

Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems ☆

Romeo Ortega[a,*], Arjan van der Schaft[b], Bernhard Maschke[c], Gerardo Escobar[d]

[a] *Lab. des Signaux et Systémes, Supelec, Plateau du Moulon, Gif-sur-Yvette 91192, France*
[b] *Faculty of Applied Mathematics, University of Twente, P.O. Box 217, AE Enschede 7500, The Netherlands*
[c] *Lab. d'Automatique, CNRS UMR 5007 CPE Lyon, Universite Claude Bernard, Lyon, Villeurbanne, France*
[d] *Department of Electrical Engineering, Northeastern University, 409 Dana Bldg., Boston, MA 02115, USA*

Received 22 February 1999; received in revised form 15 August 2001; accepted 3 September 2001

Abstract

Passivity-based control (PBC) is a well-established technique that has shown to be very powerful to design robust controllers for physical systems described by Euler–Lagrange (EL) equations of motion. For regulation problems of mechanical systems, which can be stabilized "shaping" only the *potential* energy, PBC preserves the EL structure and furthermore assigns a closed-loop energy function equal to the difference between the energy of the system and the energy supplied by the controller. Thus, we say that stabilization is achieved via energy balancing. Unfortunately, these nice properties of EL–PBC are lost when used in other applications which require shaping of the *total* energy, for instance, in electrical or electromechanical systems, or even some underactuated mechanical devices. Our main objective in this paper is to develop a new PBC theory which extends to a broader class of systems the aforementioned energy-balancing stabilization mechanism and the structure invariance. Towards this end, we depart from the EL description of the systems and consider instead port-controlled Hamiltonian models, which result from the network modelling of energy-conserving lumped-parameter physical systems with independent storage elements, and strictly contain the class of EL models. © 2002 Published by Elsevier Science Ltd.

Keywords: Nonlinear control; Passivity; Stabilization of nonlinear systems; Hamiltonian systems

1. Introduction

The term passivity-based control (PBC) was introduced in Ortega and Spong (1989) to define a controller design methodology which achieves *stabilization by passivation*. More precisely, the control objective is to passivize the system with a storage function which has a minimum at the desired equilibrium point. (A second requirement that ensures *asymptotic* stability is detectability of the passive output.) The idea has been very successful for simple mechanical systems that can be stabilized shaping only the potential energy. In this case, the closed-loop is still an Euler–Lagrange (EL) system with total energy being the difference between the stored and the supplied energies, hence stabilization can be explained in terms of *energy-balancing*, see Ortega, van der Schaft, Mareels, and Maschke (2001). These nice features—that simplify the controller commissioning—explain the practical success of PBC.

PBC has also been applied to physical systems described by EL equations of motion, which includes mechanical, electrical and electromechanical applications—for a complete set of references see Ortega, Loria, Nicklasson, and Sira-Ramirez (1998). Unfortunately, for applications that required the modification of the *kinetic* energy, the closed-loop—although still defining a passive operator—is no longer an EL system, and the storage function of the passive map (which is typically quadratic in the errors) does not have the interpretation of total energy. As explained in Section 10.3.1 of Ortega et al. (1998), this situation stems from the fact that these designs carry out an inversion of the system along the reference trajectories, which

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Carlos Canudas de Wit under the direction of Editor Hassan Khalil.

* Corresponding author. Tel.: 33-1-6985-1748; fax: 33-1-6941-3060.
E-mail addresses: rortega@lss.supelec.fr (R. Ortega), a.j.vanderschaft@math.utwente.nl (A. van der Schaft), maschke@lagep.univ-lyon1.fr (B. Maschke), gescobar@ece.neu.edu (G. Escobar).

PII: S0005-1098(01)00278-3

on one hand destroy the EL structure, and on the other hand impose an unnatural stable invertibility requirement to the system. In Section 7 we will elaborate further on these issues taking as case in point the example of a power converter.

Our *main objective* in this paper is to develop a new PBC theory—called *interconnection and damping assignment* (IDA) PBC—which extends, to a broader class of systems, e.g., to applications that require shaping of the total energy, the nice features of PBC of simple mechanical systems described above. Namely, that (i) the physical (Hamiltonian) structure is preserved in closed-loop, and (ii) the storage function of the passive map is precisely the total energy of the closed-loop system. We will, furthermore, give conditions on the dissipation such that this total energy is the difference between the stored and the supplied energies.

Towards this end, we depart from the EL description of the systems and consider instead *port-controlled Hamiltonian* (PCH) models, which encompass a very large class of physical nonlinear systems, strictly containing the class of EL models. They result from the network modelling of energy-conserving lumped-parameter physical systems with independent storage elements, and have been advocated in a series of recent papers, see Chapter 4 of van der Schaft (1999) and references therein, as an alternative to more classical EL (or standard Hamiltonian) models. Besides capturing the energy balance features of physical systems, as in EL models, PCH models provide a classification of the variables and the equations into those associated to phenomenological properties and those defining the interconnection structure related with the exchanges of energy. They are, therefore, well suited to carry out the basic steps of PBC of modifying the energy function and adding dissipation. Furthermore, the geometric structure of the state-space of PCH systems can be profitably used for PBC. For instance, the rank deficiency of the internal interconnection matrix reveals the existence of invariants of motion of the system dynamics which are *independent* of the energy function, the so-called Casimir functions (Marsden & Ratiu, 1994). The generation, through the action of the controller, of Casimir functions underlies the developments presented in this paper.

The main distinguishing feature between the "classical" PBC (e.g., as presented in Ortega et al., 1998) and IDA–PBC is that in the former we first *select* the storage function to be assigned and then design the controller that ensures this objective (by rendering the storage function non-increasing). On the other hand, in IDA–PBC the closed-loop energy function is obtained—via the solution of a partial differential equation (PDE)—*as a result* of our choice of desired subsystems interconnections and damping. It is well known that solving PDEs is, in general, not easy. However, the particular PDE that we have to solve in IDA–PBC is parameterized in terms of the interconnection and damping matrices, which can be judiciously chosen invoking physical considerations to solve it.[1] Even though we cannot provide explicit conditions for the existence of solutions of the PDE in general we prove, however, that the IDA–PBC methodology is "universally stabilizing", in the sense that it generates *all* asymptotically stabilizing controllers for PCH systems. One final advantage of the design is that it is rather systematic and amenable for symbolic computation.

The remaining of the paper is organized as follows. In Section 2 we briefly describe the class of PCH models studied in the paper. The main result of our work, namely a procedure to design a stabilizing PBC for PCH systems which preserves the Hamiltonian structure, is presented in Section 3. In Section 4 we prove the universal stabilization property of IDA–PBC. In Section 5 we show that, if the damping satisfies some structural constraints, then the energy function assigned by IDA–PBC is the difference between the stored and the supplied energies. In Section 6 we prove that in this case it is possible to invoke the method of Energy–Casimir functions (Marsden & Ratiu, 1994) to give an interconnection interpretation of IDA–PBC. In Section 7 we work out a simple power converter example, and wrap up the paper with some open problems and concluding remarks in Section 8.

2. Port controlled Hamiltonian systems

In this section we briefly describe the class of PCH models studied in the paper, derive some of its properties and formulate for them the, by now classical, passivation problem. The interested reader is referred to van der Schaft (1999), and references therein, for further details.

2.1. Systems model

Network modelling of lumped-parameter physical systems with independent storage elements leads to models of the form—called PCH systems van der Schaft (1999)

$$\Sigma: \begin{cases} \dot{x} = [J(x) - \mathscr{R}(x)]\frac{\partial H}{\partial x}(x) + g(x)u, \\ y = g^\top(x)\frac{\partial H}{\partial x}(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ are the energy variables, the smooth function $H(x): \mathbb{R}^n \to \mathbb{R}$ represents the total stored energy and $u, y \in \mathbb{R}^m$ are the port power variables.[2] The port variables u and y are conjugated variables, in the sense that their

[1] This point has been illustrated in several practical applications including mass–balance systems (Ortega, Astolfi, Bastin, & Rodriguez, 1999), electrical motors (Petrovic, Ortega, & Stankovic, 2000), magnetic levitation systems (Rodriguez, Ortega, & Mareels, 2000), power converters (Rodriguez, Ortega, Escobar, & Barabanov, 2000), underactuated mechanical systems (Ortega & Spong, 2000), design of power system stabilizers (Ortega, Galaz, Bazanella, & Stankovic, 2001) and underwater vehicles (Astolfi, Chhabra, & Ortega, 2001).

[2] We note that all vectors defined in the paper are *column* vectors, even the gradient of a scalar function.

duality product defines the power flows exchanged with the environment of the system, for instance currents and voltages in electrical circuits or forces and velocities in mechanical systems. The interconnection structure is captured in the $n \times n$ skew-symmetric matrix $J(x) = -J^{\top}(x)$ and the $n \times m$ matrix $g(x)$, while $\mathscr{R}(x) = \mathscr{R}^{\top}(x) \geqslant 0$ represents the dissipation, all these matrices depend smoothly on the state x.

We want to study also systems where the control acts through the *interconnection structure*. These are typically systems with switches where the controller commutes between different topologies. Assuming a sufficiently fast sampling and (for instance) a PWM implementation of the control action we can approximate the average behaviour of the switched system by a smooth system, where the control is now the PWM duty ratio. This situation, which is very common in power electronic devices (Ortega et al., 1998), leads us to consider systems of the form

$$\dot{x} = [J(x,u) - \mathscr{R}(x)]\frac{\partial H}{\partial x}(x) + g(x,u), \tag{2}$$

where $J(x,u) = -J^{\top}(x,u)$. The vector function $g(x,u)$ is introduced to capture two kind of interconnections, the standard $g(x)u$ and "constant source inputs", where u denotes the switching of the source input. See, for instance, the model of the Ć uk converter in Ortega et al. (1998).

2.2. Energy balance, passivity and stabilization

Evaluating the rate of change of the total energy we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}H = -\left[\frac{\partial H}{\partial x}(x)\right]^{\top} \mathscr{R}(x)\frac{\partial H}{\partial x}(x) + u^{\top} y, \tag{3}$$

where the first term on the right-hand side (which is non-positive) represents the dissipation due to the resistive (friction) elements in the system. Integrating (3), we obtain the *energy-balance* equation

$$\begin{aligned}&\underbrace{\int_0^t u^{\top}(s)y(s)\,\mathrm{d}s}_{supplied}\\ &\quad = \underbrace{H[x(t)] - H[x(0)]}_{stored}\\ &\qquad + \underbrace{\int_0^t \left[\frac{\partial H}{\partial x}[x(s)]\right]^{\top} \mathscr{R}[x(s)]\frac{\partial H}{\partial x}[x(s)]\,\mathrm{d}s}_{dissipated\ energy}\end{aligned} \tag{4}$$

which holds for all $t \geqslant 0$. If the total energy function $H(x)$ is bounded from below [3] PCH systems, analogously to EL systems, define a passive operator $\Sigma : u \mapsto y$ with storage function $H(x)$. In this case, (4) expresses the fact that a passive system cannot store more energy than it is supplied to it from the outside, with the difference being the dissipated energy. From (4) we also have that $-\int_0^t u^{\top}(s)y(s)\,\mathrm{d}s \leqslant H[x(0)]$, which shows that we can only extract a finite amount of energy from a passive system. (Notice that for the more general class of systems (2), since u is *not a port variable*, the passivity property is not established with respect to this signal, but between suitable elements of $\partial H/\partial x$ and $g(x,u)$. See, for instance, the example of Section 7.)

From (4) we easily see that the energy of the uncontrolled system (i.e., with $u(t) \equiv 0$) is non-increasing, that is, $H[x(t)] \leqslant H[x(0)]$, and it will actually decrease in the presence of dissipation. If the energy function is bounded from below, the system will eventually stop in a point of minimum energy. Also, the rate of convergence of the energy function is increased if we extract energy from the system, for instance setting $u = -K_v y$, with $K_v = K_v^{\top} > 0$, a so-called damping injection gain. The point where the open-loop energy is minimal is usually not the one of practical interest, and control is introduced to operate the system around some non-zero equilibrium point, say x_*.

Motivated by the discussion above, in PBC the stabilization problem is posed in terms of the following:

Passivation objective: Given the PCH system (1) (or (2)) and a desired (constant) equilibrium point x_*, find a control action [4] $u = \beta(x) + v$ so that the closed-loop dynamics is a PCH system satisfying the new energy-balancing equation

$$H_d[x(t)] - H_d[x(0)] = \int_0^t v^{\top}(s)y'(s)\,\mathrm{d}s - d_d(t), \tag{5}$$

where $H_d(x)$ is the desired total energy function, which has a strict (local) *minimum* at x_*, [5] y' (which may be equal to y) is the new passive output, and we have replaced the natural dissipation term by some function $d_d(t) \geqslant 0$ (which will in general be an increasing function) to increase the convergence rate.

It is clear that, with $v = 0$, the control that solves the passivation problem stabilizes x_* (with Lyapunov function $H_d(x)$). Stability will be asymptotic if some detectability-like conditions (to be specified later) are satisfied.

One of our main concerns in this paper is to understand the nature of the stabilization mechanism, and in particular what is the role of dissipation. Towards this end, we give conditions under which the energy function of the closed-loop PCH system is the difference between the stored and the supplied energy $H[x(t)] - \int_0^t u^{\top}(s)y(s)\,\mathrm{d}s$. In this case we will say that stabilization is achieved via *energy-balancing*.

[3] Since energy is only defined up to a constant, we can equivalently say that $H(x)$ is non-negative.

[4] We consider here *static state-feedback*. Also, we will assume the control is a function of the *full state* but, as will become clear below, all results apply as well to the partial state-feedback case. See (Rodriguez et al., 2000; Ortega et al., 1999) for application examples.

[5] That is, there exists an open neighbourhood $\mathscr{B}$ of x_* such that $H_d(x) > H_d(x_*)$ for all $x \in \mathscr{B}$.

3. Controller design

To solve the passivation problem we propose in this section the IDA–PBC design methodology. We present our derivations for the generalized PCH system (2), with slight notational modifications they can be adapted for the simpler case of (1).

3.1. Design procedure

The IDA–PBC design proceeds as follows: recalling that in PCH systems the internal energy exchanges are captured by the interconnection and damping matrices, we first *fix* the desired structure of these matrices—hence the name IDA. Then, we derive a PDE parameterized by the chosen matrices whose solutions characterize all the energy functions that can be assigned. Finally, from this family of solutions we choose one that satisfies the minimum requirement and compute the control. More precisely, the final objective of IDA–PBC is to find a static state-feedback control $u=\beta(x)$ such that the closed-loop dynamics is a PCH system with dissipation of the form

$$\dot{x} = [J_d(x) - \mathscr{R}_d(x)]\frac{\partial H_d}{\partial x}(x), \tag{6}$$

where the new energy function $H_d(x)$ has a strict local minimum at the desired equilibrium x_*, and $J_d(x) = -J_d^\top(x)$ and $\mathscr{R}_d(x) = \mathscr{R}_d^\top(x) \geqslant 0$ are some *desired* interconnection and damping matrices, respectively.

The following proposition presents our main result in its full generality, but in a rather conceptual form. More operational versions, which allows to effectively construct the stabilizing controllers, may be easily derived from it and are discussed in point 1 of Subsection 3.2.

Proposition 1. *Given* $J(x,u), \mathscr{R}(x), H(x), g(x,u)$ *and the desired equilibrium to be stabilized* $x_* \in \mathbb{R}^n$. *Assume we can find functions* $\beta(x), J_a(x), \mathscr{R}_a(x)$ *and a vector function* $K(x)$ *satisfying*

$$\begin{aligned}[J(x,\beta(x)) + J_a(x) - (\mathscr{R}(x) + \mathscr{R}_a(x))]K(x)\\ = -[J_a(x) - \mathscr{R}_a(x)]\frac{\partial H}{\partial x}(x) + g(x,\beta(x))\end{aligned} \tag{7}$$

and such that

(i) (Structure preservation)

$$J_d(x) := J(x,\beta(x)) + J_a(x) = -[J(x,\beta(x)) + J_a(x)]^\top,$$

$$\mathscr{R}_d(x) := \mathscr{R}(x) + \mathscr{R}_a(x) = [\mathscr{R}(x) + \mathscr{R}_a(x)]^\top \geqslant 0.$$

(ii) (Integrability) $K(x)$ *is the gradient of a scalar function. That is*,

$$\frac{\partial K}{\partial x}(x) = \left[\frac{\partial K}{\partial x}(x)\right]^\top. \tag{8}$$

(iii) (Equilibrium assignment) $K(x)$, *at* x_*, *verifies*

$$K(x_*) = -\frac{\partial H}{\partial x}(x_*). \tag{9}$$

(iv) (Lyapunov stability) *The Jacobian of* $K(x)$, *at* x_*, *satisfies the bound*

$$\frac{\partial K}{\partial x}(x_*) > -\frac{\partial^2 H}{\partial x^2}(x_*). \tag{10}$$

Under these conditions, the closed-loop system $u=\beta(x)$ *will be a PCH system with dissipation of the form* (6), *where*

$$H_d(x) := H(x) + H_a(x) \tag{11}$$

and

$$\frac{\partial H_a}{\partial x}(x) = K(x). \tag{12}$$

Furthermore, x_* *will be a* (*locally*) stable *equilibrium of the closed-loop. It will be* asymptotically *stable if, in addition, the largest invariant set under the closed-loop dynamics contained in*

$$\left\{ x \in \mathbb{R}^n \,\middle|\, \left[\frac{\partial H_d}{\partial x}(x)\right]^\top \mathscr{R}_d(x)\frac{\partial H_d}{\partial x}(x) = 0 \right\} \tag{13}$$

equals $\{x_*\}$. *An estimate of its* domain of attraction *is given by the largest* bounded *level set* $\{x \in \mathbb{R}^n \mid H_d(x) \leqslant c\}$.

Proof. For every given $\beta(x), J_a(x), \mathscr{R}_a(x)$, (and on any contractible neighbourhood of $\mathbb{R}^n$), the solution of Eq. (7) is a gradient of the form (12) if and only if the integrability condition (8) is satisfied. Using (11) it is easy to see that, in this case, the closed-loop is a PCH system of the form (6) and total energy (11).

We will now prove that, under (9), (10), the stability of the equilibrium is ensured. To this end, notice that the equilibrium assignment condition (9) ensures $H_d(x)$ has an extremum at x_*, while the Lyapunov stability condition (10) shows that it is actually an isolated minimum. On the other hand, from (5) (with $v=0$ and a suitably defined $d_d(t)$) we have that, along the trajectories of the closed-loop, $H_d(x(t))$ is non-increasing, hence it qualifies as a Lyapunov function, and we can conclude that x_* is a stable equilibrium. Asymptotic stability follows immediately invoking La Salle's invariance principle (Khalil, 1996) and the condition (13). Finally, to ensure the solutions remain bounded, we give the estimate of the domain of attraction as the largest bounded level set. □

3.2. Discussion

1. We have given in Proposition 1 a general—somehow "conceptual"—formulation of the design technique. More constructive procedures are as follows.

• For systems of the form (1) we can fix $J_a(x)$ and $\mathscr{R}_a(x)$ (in the first instance taken them equal to zero) and then look for a solution of the PDE

$$g^{\perp}(x)[J(x)+J_a(x)-(\mathscr{R}(x)+\mathscr{R}_a(x))]\frac{\partial H_a}{\partial x}(x) = -\,g^{\perp}(x)[J_a(x)-\mathscr{R}_a(x)]\frac{\partial H}{\partial x}(x)$$

in terms of $H_a(x)$, where $g^{\perp}(x)$ is a left annihilator of $g(x)$, i.e., $g^{\perp}(x)g(x)=0$. This is a *linear* PDE of the form $A(x)(\partial H_a/\partial x)(x)=b(x)$, for which powerful solution techniques, in particular the method of characteristics, are available. [6] The control can then be directly calculated using the formula

$$\beta(x)=[g^{\top}(x)g(x)]^{-1}g^{\top}(x)\left\{[J(x)+J_a(x)-(\mathscr{R}(x)+\mathscr{R}_a(x))]\frac{\partial H_a}{\partial x}(x)+[J_a(x)-\mathscr{R}_a(x)]\frac{\partial H}{\partial x}(x)\right\}.$$

- For the general case, we can fix again $J_a(x),\mathscr{R}_a(x)$, then solve the algebraic equation (7) for $K(x)$—which is trivial if the matrix $J(x,\beta(x))+J_a(x)-[\mathscr{R}(x)+\mathscr{R}_a(x)]$ is invertible. Finally, the integrability conditions (8) define a new PDE directly *for the control* $\beta(x)$.
- If there are no clear physical considerations for the choice of $J_a(x),\mathscr{R}_a(x)$ we can simply select them to simplify the solution of the PDE. This procedure is adopted in Ortega and Spong (2000) to design globally stabilizing IDA–PBCs for several underactuated mechanical systems. See also (Fujimoto & Sugie, 2001) for an alternative formulation.

2. Notice that in our construction we do not need to "guess" candidate energy–Lyapunov functions, moreover it does not even require its explicit derivation. (This can be obtained, though, as a by-product integrating $\partial H_a/\partial x(x)$.) This situation should be contrasted with classical Lyapunov-based designs (or EL–PBC, e.g., *à la* Ortega et al., 1998), where we fix a priori the Lyapunov (energy) function—typically a quadratic in the increments—, and then calculate the control law that makes its derivative negative definite.

3. Frobenius theorem (Nijmeijer & van der Schaft, 1990) provides us with a necessary and sufficient condition for the solvability of homogeneous equations of the form $A(x)\partial H_a/\partial x(x)=0$. Namely, that the dimension of the involutive closure of the distribution spanned by the vector fields formed by the rows of the matrix $A(x)$ is strictly less than n. We show in Ortega et al. (1999), that this simple test can help us in the choice of $J_a(x),\mathscr{R}_a(x)$. It is interesting to note that, for the case $\mathscr{R}(x)=\mathscr{R}_a(x)=0$, Frobenius' conditions are satisfied if $J(x,\beta(x))+J_a(x)$ satisfies the Jacobi identities (van der Schaft, 1999)

4. The IDA–PBC of Proposition 1 does not ensure, in general, passivity with respect to the natural output y, but with respect to $y'=g^{\top}(x)(\partial H_d/\partial x)(x)$. As pointed out in the introduction, to ensure robustness via à vis unmodelled dynamics which are wrapped around y—e.g., frictions and parasitic resistances—it is desirable to have y as the passive output. From (7) and (11) it follows immediately that this will be the case if $g^{\top}(x)K(x)\equiv 0$.

5. In some applications (Rodriguez et al., 2000; Ortega & Spong, 2000) physical considerations can be invoked to choose the desired interconnection and damping matrices. The choice of the latter should be done, however, cautiously. For instance, contrary to conventional wisdom, performance is not necessarily improved by adding positive damping. It is shown in Ortega et al. (1998) that, in some cases, increasing the damping to reduce the amplification factor of the energy of the input noise (in the spirit of the $\mathscr{H}_\infty$ approach) can actually degrade performance. Furthermore, we illustrate in the example of Section 7—see also (Ortega et al., 1999; Rodriguez et al., 2000)—that sometimes shuffling the damping between the channels can be beneficial for our control objective.

4. Universal stabilizing property of IDA–PBC

We have seen in the previous section that the success of our design hinges upon our ability to solve a PDE. We cannot provide explicit conditions for the existence of solutions of the PDE in general. Instead, we prove in this section that the IDA–PBC methodology is "universally stabilizing", in the sense that it generates *all* asymptotically stabilizing controllers for PCH systems of the form (1). Towards this end, we need the following.

Lemma 1. *If the system $\dot{x}=f(x)$, $f(x)\in\mathscr{C}^1$ has an asymptotically stable equilibrium point x_*, then there exist a $\mathscr{C}^1$ positive definite (w.r.t. x_*) function $H_d(x)$ and $\mathscr{C}^0$ matrix functions $J_d(x)=-J_d^{\top}(x)$, $\mathscr{R}_d(x)=\mathscr{R}_d^{\top}(x)\geqslant 0$ such that*

$$f(x)=[J_d(x)-\mathscr{R}_d(x)]\frac{\partial H_d}{\partial x}(x). \tag{14}$$

Proof. From the converse Lyapunov theorem (Khalil, 1996) we know the existence of a $\mathscr{C}^1$ positive definite (w.r.t. x_*) function $H_d(x)$ such that $[(\partial H_d/\partial x)(x)]^{\top}f(x)\leqslant 0$. Let us now define

$$\mathscr{R}_d(x):=-\frac{1}{|\partial H_d/\partial x(x)|^4}\frac{\partial H_d}{\partial x}(x)\left[\frac{\partial H_d}{\partial x}(x)\right]^{\top}f^{\top}(x)\frac{\partial H_d}{\partial x}(x),$$

$$J_d(x):=\frac{1}{|\partial H_d/\partial x(x)|^2}\left\{f(x)\left[\frac{\partial H_d}{\partial x}(x)\right]^{\top}-\frac{\partial H_d}{\partial x}(x)f^{\top}(x)\right\},$$

where $|\cdot|$ is the standard Euclidean norm. Clearly $J_d(x)=-J_d^{\top}(x)$, $\mathscr{R}_d(x)=\mathscr{R}_d^{\top}(x)$. Furthermore, the stability condition above also ensures $\mathscr{R}_d(x)\geqslant 0$, and direct substitution

[6] Notice that the solution of the PDE, which is parameterized in terms of free constants and function compositions, should satisfy the "boundary conditions" (9) and (10).

shows that (14) holds. Although these functions may not satisfy the required smoothness properties a standard regularization procedure will yield the desired result. [7] □

The proof of the proposition below follows immediately from Lemma 1.

Proposition 2. *If there exists a* $\mathscr{C}^1$ *function* $\beta(x)$ *such that* $\dot{x} = [J(x) - \mathscr{R}(x)](\partial H/\partial x)(x) + g(x)\beta(x)$ *is asymptotically stable, then there exist* $\mathscr{C}^0$ *matrix functions* $J_a(x)$, $\mathscr{R}_a(x)$ *and a* $\mathscr{C}^1$ *function* $H_a(x)$ *which satisfy the conditions of Proposition* 1. *In other words, IDA–PBC generates all asymptotically stabilizing controllers for PCH systems of the form* (1)

An alternative construction of the matrix functions $J_d(x), \mathscr{R}_d(x)$ proceeds as follows. First, given $H_d(x)$ and $f(x)$, solve for $\mathscr{R}_d(x)$ the equation

$$\left[\frac{\partial H_d}{\partial x}(x)\right]^\top f(x) + \left[\frac{\partial H_d}{\partial x}(x)\right]^\top \mathscr{R}_d(x)\frac{\partial H_d}{\partial x}(x) = 0.$$

Then, solve for $J_d(x)$ the linear equation

$$J_d(x)\frac{\partial H_d}{\partial x}(x) = \mathscr{R}_d(x)\frac{\partial H_d}{\partial x}(x) + f(x).$$

In this way we can avoid the regularization procedure required in Lemma 1.

5. Stabilization via energy-balancing

In this section we derive conditions on the systems natural damping such that stabilization is achieved via energy-balancing. More precisely, we investigate when the function $H_a(x)$ verifies

$$\begin{aligned} &H_a[\phi(x,t)] - H_a[\phi(x,s)] \\ &\quad = -\int_s^t \beta^\top[\phi(x,\tau)]g^\top[\phi(x,\tau)]\frac{\partial H}{\partial x}[\phi(x,\tau)]\,\mathrm{d}\tau \end{aligned}$$

for all x and all $t \geqslant s$, where $\phi(x,t)$ denotes the solution of (1) with $u = \beta(x)$ starting from the initial condition x at time t. With some abuse of notation we will say in this case that the energy function assigned to the closed-loop by the IDA–PBC satisfies

$$H_d(x(t)) = H(x(t)) - \int_0^t u^\top(s)y(s)\,\mathrm{d}s + \kappa \tag{15}$$

which is the difference between the total energy of the open-loop and the energy provided to the system from the controller, and κ is a constant determined by the initial conditions. We will give a necessary and a sufficient condition for energy-balancing stabilization.

[7] This procedure, being quite technical and not constructive, is omitted here. See also the derivations at the end of the section.

5.1. Necessary condition

Energy-balancing stabilization can be—in principle—applied to general *passive* (f,g,h) nonlinear systems of the form

$$\Sigma\colon \begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x). \end{cases} \tag{16}$$

We have the following simple proposition.

Proposition 3. *Consider the passive system* (16) *with differentiable storage function* $H(x)$ *and an admissible equilibrium* x_*. *If we can find a vector function* $\beta(x)$ *such that*

(i) *The partial differential equation*

$$[f(x) + g(x)\beta(x)]^\top \frac{\partial H_a}{\partial x}(x) = -[g(x)\beta(x)]^\top \frac{\partial H}{\partial x}(x) \tag{17}$$

can be solved for $H_a(x)$.

(ii) *The function* $H_d(x) = H(x) + H_a(x)$ *has a minimum at* x_*.

Then, $u = \beta(x)$ *is an energy-balancing stabilizer for the equilibrium* x_*. *That is, the Lyapunov function* $H_d(x)$ *satisfies* (15).

Proof. From the celebrated nonlinear version of the Kalman–Yakubovich–Popov lemma we know that for this class of system passivity is *equivalent* to the existence of a non-negative scalar function $H(x)$ such that

$$\left[\frac{\partial H}{\partial x}(x)\right]^\top f(x) \leqslant 0, \qquad h(x) = g^\top(x)\frac{\partial H}{\partial x}(x).$$

The proof then follows immediately noting that the left-hand side of (17) equals $\dot{H}_a$, while the right-hand side is $-y^\top u$, and then integrating from 0 to t. □

This result, although quite general, is of limited interest. First of all, (f,g,h) models do not reveal the role played by the energy function in the system dynamics. Hence, it is difficult to incorporate prior information to select a $\beta(x)$ to solve the PDE (17). Second, we will show now that, beyond the realm of mechanical systems, the applicability of energy-balancing stabilization is severely restricted by the systems natural dissipation. Indeed, the left-hand side of the PDE (17) is zero when evaluated at the equilibrium, hence the right-hand side, which is the *power* extracted from the controller, should also be zero at the equilibrium. This implies that the energy dissipated by the system is bounded, or equivalently that: the systems is stabilized *extracting a finite amount of energy* from the controller. This is possible in regulation of mechanical systems where the extracted power is the product of force and velocity and we want to drive the velocity to zero. Unfortunately, it is no longer the case for most electrical or electromechanical systems where power involves the product of voltages and currents, and the latter may be non-zero for non-zero equilibria. See Ortega et al. (2001) for further discussions on this issue.

5.2. Sufficient condition

Proposition 4. *Consider the IDA–PBC of Proposition* 1 *applied to the PCH system* (1) *without damping injection, i.e.*, $\mathscr{R}_a(x)=0$. *Assume the natural damping of the system verifies*

$$\mathscr{R}(x)\frac{\partial H_a}{\partial x}(x)=0. \tag{18}$$

Then, the IDA–PBC is an energy-balancing stabilizer.

Proof. The proof is easily established with the following simple calculations:

$$\dot H_d=\underbrace{u^\top y-\left[\frac{\partial H}{\partial x}(x)\right]^\top \mathscr{R}(x)\frac{\partial H}{\partial x}(x)}_{\dot H}+\dot H_a$$

$$=-\left[\frac{\partial H_d}{\partial x}(x)\right]^\top \mathscr{R}_d(x)\frac{\partial H_d}{\partial x}(x).$$

From the fact that $\mathscr{R}_d(x)=\mathscr{R}_a(x)+\mathscr{R}(x)$ we have that

$$\dot H_a=-u^\top y-\left[2\frac{\partial H}{\partial x}(x)+\frac{\partial H_a}{\partial x}(x)\right]^\top \mathscr{R}(x)\frac{\partial H_a}{\partial x}(x)$$

$$-\left[\frac{\partial H_d}{\partial x}(x)\right]^\top \mathscr{R}_a(x)\frac{\partial H_d}{\partial x}(x).$$

Consequently, if $\mathscr{R}_a(x)=0$ and the natural damping $\mathscr{R}(x)$ satisfies (18) we have that $\dot H_a=-u^\top y$ and, upon integration, we verify that $H_d[x(t)]$ satisfies (15). □

6. Energy–Casimir method

The IDA-PBC of Proposition 1 has been derived adopting a state-feedback viewpoint of the control action which somehow obscures how energy is exchanged between the controller and the plant. In order to unveil this fundamental feature of PBC it is necessary to adopt an alternative, and in many respects more natural, perspective of control-as-interconnection—where energy shaping is achieved adding up the energies of the plant and the controller. This approach has been proposed in Ortega, Loria, Kelly, and Praly (1995) to design EL controllers for potential energy shaping of mechanical systems, see also (Ortega et al., 1998). It has been discussed in Dalsmo and van der Schaft (1999) for static state-feedback control of mechanical systems in PCH form, and in Stramigioli, Maschke, and van der Schaft (1998) with a dynamic extension for output-feedback damping injection. In Maschke, Ortega, and van der Schaft (2000), which constitutes the analysis counterpart of the present paper, we introduced it for Lyapunov function generation for PCH systems with constant forcing inputs.

As pointed out in Maschke et al. (2000) the construction has some close connections with the Energy–Casimir method of mechanics (Marsden & Ratiu, 1994; van der Schaft, 1999), where we exploit the existence of dynamical invariants to generate Lyapunov functions. The purpose of this section is to elaborate upon these connections for the IDA-PBC proposed above. In particular, to show that the IDA–PBC reduces to the controller obtained via the Energy–Casimir method if and only if there is no damping injection and the natural damping of the plant satisfies the condition of Proposition 4. (Hence, the class of PCH systems stabilizable by IDA-PBC is strictly larger.) To prove this statement we view the IDA–PBC as a PCH *source* system in a power preserving *state-modulated* interconnection with the plant. To put our result in perspective we explain first the rationale of the Energy–Casimir method in its standard formulation.

6.1. PCH controllers: constant interconnection

We consider the system described by (1) in interconnection with a *PCH controller*

$$\Sigma_C:\ \begin{cases}\dot\zeta=u_C,\\ y_C=\dfrac{\partial H_C}{\partial\zeta}(\zeta)\end{cases} \tag{19}$$

with state $\zeta\in\mathscr{R}^m$, input u_C, output y_C, and $H_C(\zeta)$ the energy of the controller—which we assume bounded from below.[8] (See point 2 in the discussion below for an explanation for the choice of this structure of the PCH controller.)

The interconnection constraints are power-preserving of the form $u_C=y,\ u=-y_C$. The composed system is clearly still Hamiltonian and can be written as

$$\begin{bmatrix}\dot x\\ \dot\zeta\end{bmatrix}=\begin{bmatrix}J(x)-\mathscr{R}(x) & -g(x)\\ g^\top(x) & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H_{cl}}{\partial x}(x,\zeta)\\ \frac{\partial H_{cl}}{\partial \zeta}(x,\zeta)\end{bmatrix} \tag{20}$$

with $H_{cl}(x,\zeta)$ the *closed-loop* energy function (defined in an extended state space (x,ζ))

$$H_{cl}(x,\zeta):=H(x)+H_C(\zeta). \tag{21}$$

We can easily see that this energy function is non-increasing, and we would like to shape it to assign a minimum at the desired point. However, although $H_C(\zeta)$ can be freely assigned, the systems energy-function $H(x)$ is given, and its not clear how can we effectively shape the overall energy. One possibility is to restrict the motion of the closed-loop system to a certain subspace of the extended state space (x,ζ), say $\Omega\subset\mathbb{R}^{n+m}$, by rendering Ω *invariant*.[9] In this way, we can express the closed-loop total energy as a

[8] We have introduced the notation H_C here to highlight its interpretation as controller energy. We will see later, that this function play essentially the same role as H_a in Proposition 1.

[9] A set $\Omega\subset\mathbb{R}^{n+m}$ is invariant if the following implication holds: $(x(0),\zeta(0))\in\Omega\Rightarrow(x(t),\zeta(t))\in\Omega,\ \forall\, t\geqslant 0$.

function of x only. In the Energy–Casimir method (Dalsmo & van der Schaft, 1999; Marsden & Ratiu, 1994), we look for dynamical invariants which are *independent* of the Hamiltonian function. More precisely we look for functions $\mathscr{C}(x,\zeta)$—called Casimir functions—such that along the dynamics of the PCH system $(\mathrm{d}/\mathrm{d}t)\mathscr{C}(x,\zeta)=0$, *independent* of the energy function. Without loss of generality,[10] we consider Casimir functions of the form $\mathscr{C}(x,\zeta)=F(x)-\zeta$. Since we want these functions to remain constant along the trajectories of the closed-loop dynamics (20) *irrespective* of the precise form of $H_{cl}(x,\zeta)$, they should be solutions of the PDEs

$$\left[\left[\frac{\partial F}{\partial x}(x)\right]^\top \;\vdots\; -I_m\right]\begin{bmatrix} J(x)-\mathscr{R}(x) & -g(x) \\ g^\top(x) & 0\end{bmatrix}=0. \tag{22}$$

It is clear that the level sets $\Omega:=\{(x,\zeta)|\zeta=F(x)+\kappa\}$, $\kappa\in\mathbb{R}$, are invariant sets for the closed-loop system, hence the closed-loop total energy (in these level sets) becomes

$$H_d(x):=H(x)+H_C[F(x)+\kappa]. \tag{23}$$

This function can now be shaped with a suitable selection of the controller energy $H_C(\zeta)$. (Notice that, for obvious reasons, we have adopted the notation $H_d(x)$ of Proposition 1.)

We are in position to present the following proposition, whose proof may be found in van der Schaft (1999) and Ortega et al. (2001).

Proposition 5. *A necessary condition for the solution of the PDEs* (22), *hence for existence of Casimir functions for the closed-loop dynamics* (20), *is that the damping satisfies the energy-balancing constraint* (18).

6.2. State-modulated interconnection

In this subsection we view the IDA–PBC of Proposition 1 from the Energy–Casimir method perspective. Although we have already proven that energy-shaping can be achieved without generation of Casimir functions it is interesting to know under which conditions both methods yield the same controllers. We provide a definite answer to this question by establishing that the closed-loop admits Casimir functions *if and only if* $J_a(x)=\mathscr{R}_a(x)=0$ and the natural damping satisfies the condition (18).

To adopt an interconnection viewpoint for the IDA–PBC we introduce two key modifications. First, we consider the controller to be an (infinite energy) source, that is, a *scalar* system of the form (19) but with energy function

$$H_C(\zeta)=-\zeta. \tag{24}$$

(Notice that since $H_C(\zeta)$ is not bounded from below, the operator $u_C\mapsto y_C$ is not passive, hence we can extract an infinite amount of energy from the controller to stabilize systems with *infinite dissipation*.) Second, we interconnect the source system with the plant via a *state-modulated* interconnection of the form

$$\begin{bmatrix} u(s) \\ u_C(s)\end{bmatrix}=\begin{bmatrix} 0 & -\beta(x) \\ \beta(x) & 0\end{bmatrix}\begin{bmatrix} y(s) \\ y_C(s)\end{bmatrix}. \tag{25}$$

This interconnection is clearly power preserving, i.e. $u^\top y+u_C^\top y_C=0$, and the composed system is still Hamiltonian of the form

$$\begin{bmatrix} \dot x \\ \dot\zeta\end{bmatrix}=\begin{bmatrix} J(x)-\mathscr{R}(x) & -g(x)\beta(x) \\ \beta^\top(x)g^\top(x) & 0\end{bmatrix}\begin{bmatrix} \frac{\partial H_{cl}}{\partial x}(x,\zeta) \\ \frac{\partial H_{cl}}{\partial \zeta}(x,\zeta)\end{bmatrix} \tag{26}$$

with total energy $H_{cl}(x,\zeta)$ defined by (21) and (24).

The proposition below gives necessary and sufficient conditions for the Energy–Casimir method to apply. The proof is given in Ortega et al. (2001), hence is omitted.

Proposition 6. *Consider the PCH system* (1) *with a state-feedback control* $u=\beta(x)$. *The closed-loop, which can be represented as the augmented system* (26), (21), (24), *admits a Casimir function of the form*

$$\mathscr{C}(x,\zeta)=-H_a(x)-\zeta \tag{27}$$

if and only if

(i) *the integrability condition* (7), (8) *with* $J_a(x)=\mathscr{R}_a(x)=0$ *of Proposition* 1 *holds, that is*

$$[J(x)-\mathscr{R}(x)]\frac{\partial H_a}{\partial x}(x)=g(x)\beta(x)$$

(ii) *the natural damping matrix* $\mathscr{R}(x)$ *verifies the constraint* (18).

7. Power converter example

In this section we illustrate our main result with a power converter example. (Other practical applications of IDA–PBC include: underactuated mechanical systems (Ortega & Spong, 2000), mass–balance systems (Ortega et al., 1999), design of power system stabilizers (Ortega et al., 2001), synchronous generators (Petrovic et al., 2000), levitated systems (Rodriguez et al., 2000) and underwater vehicles (Astolfi et al., 2001). In all cases we derive *new controllers* that possess interesting implementation and robustness features.)

A. *Port-controlled Hamiltonian modelling*: We consider the well-known DC-to-DC boost power converter, whose (averaged) dynamics is described by a PCH model of the form (2) with $x=[x_1,x_2]^\top$, $g=[E,0]^\top$, and

$$J(u):=\begin{bmatrix} 0 & -u \\ u & 0\end{bmatrix},\quad \mathscr{R}:=\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R_L}\end{bmatrix},$$

[10] This is because we have assumed $g(x)$ and $J(x)-\mathscr{R}(x)$ full rank, hence this class generates (locally) all Casimir functions.

where

$$H(x)=\frac{1}{2L}x_1^2+\frac{1}{2C}x_2^2 \tag{28}$$

is the total energy of the circuit, x_1 is the inductance flux, x_2 is the charge in the capacitor, $u\in[0,1]$ represents the duty ratio of the PWM, R_L is the output load resistance and E is the DC voltage source.

It is important to remark that u is not a port variable. However, since the action of this signal is "workless", the circuit defines a passive operator $E\mapsto x_1/L$, independently of u. The control objective of the power converter of Fig. 1 is to drive the output capacitor voltage to some constant desired value $V_d>E$, maintaining internal stability. Notice that the *equilibrium point* for the desired output capacitor voltage V_d is given by $(x_{1*},x_{2*})=(LV_d^2/R_LE, CV_d)$, which is attained with the constant control $u_*=E/V_d$.

B. *Controller with natural interconnection and damping*: Following the method proposed here, with $J_a(x)=\mathscr{R}_a(x)=0$, we define the vector $K(x)$ as

$$K(x)=\begin{bmatrix}k_1(x)\\k_2(x)\end{bmatrix}=[J(\beta(x))-\mathscr{R}]^{-1}\begin{bmatrix}E\\0\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{E}{R_L\beta^2(x)}\\-\frac{E}{\beta(x)}\end{bmatrix}. \tag{29}$$

The integrability condition (8) reduces in this case to $(\partial k_2/\partial x_1)(x)=(\partial k_1/\partial x_2)(x)$, which yields the nonlinear PDE $(\partial k_2/\partial x_1)(x)+\frac{2}{R_LE}k_2(\partial k_2/\partial x_2)(x)=0$. A solution for this simple PDE may be obtained using standard computational tools as

$$k_2(x)=\frac{c_1x_2+c_3}{(2/R_LE)c_1x_1+c_2}, \tag{30}$$

where $c_i,\ i=1,2,3$ are arbitrary constants. Replacing (30) in (29) defines the control law

$$\beta(x)=-E\left(\frac{(2/R_LE)c_1x_1+c_2}{c_1x_2+c_3}\right). \tag{31}$$

In the sequel we will check the remaining conditions of Proposition 1 to define intervals for the constants $c_i,\ i=1,2,3$ so that the stabilization objective is achieved with $u=\beta(x)$. To enforce (9) we evaluate the control (31) at the equilibrium point. This gives the following linear function that relates the constants c_1, c_2 and c_3

$$c_2=-\left(\frac{2LV_d^2}{R_L^2E^2}+C\right)c_1-\frac{c_3}{V_d}. \tag{32}$$

It is interesting to note that, if we set $c_1=0$ in (31) and (32) we recover the open-loop control $u=u_*=E/V_d$. Hence, it is a member of the family of controllers that is generated by our method. After some simple calculations, we can verify that the Hessian condition (10) is satisfied provided $(R_L^2E^2/4LV_d^3)c_3<c_1<(1/CV_d)c_3$ if $c_3<0$, or $-(1/CV_d)c_3<c_1<-(R_L^2E^2/4LV_d^3)c_3$ if $c_3>0$.

Once again, we should underscore that we did not require the calculation of the Lyapunov function $H_d(x)$. It can be, however, easily computed as

$$H_d(x)=\frac{1}{2L}x_1^2+\frac{1}{2C}x_2^2+\frac{1}{2c_1}\frac{(c_1x_2+c_3)^2}{((2/R_LE)c_1x_1+c_2)}$$
$$-\frac{LV_d^4}{2R_L^2E^2}+\frac{V_dc_3}{2c_1},$$

where we have added the two last constant terms to enforce $H_d(x_*)=0$.

C. *Controller with damping assignment*: Even though the controller above ensures some stability properties it requires full state feedback and is sensitive to the load resistance, which might be unknown and/or time-varying. We will show now that changing the damping structure we can overcome these two drawbacks. Towards this end, we select the injected damping matrix as $\mathscr{R}_a=diag\{R_a,-1/R_L\}$, $R_a>0$, which yields the closed-loop damping $\mathscr{R}_d=diag\{R_a,0\}$. We thus obtain

$$K(x)=\frac{1}{\beta(x_2)}\begin{bmatrix}-\frac{1}{R_LC}x_2\\-\frac{1}{L}R_ax_1-E+\frac{R_a}{R_LC}\frac{x_2}{\beta(x_2)}\end{bmatrix}.$$

Taking into account that β is a function only of x_2, the integrability condition $(\partial k_2/\partial x_1)(x)=(\partial k_1/\partial x_2)(x)$ reduces to the simple ODE

$$\frac{d\beta}{dx_2}(x_2)=\frac{\alpha}{x_2}\beta(x_2),$$

where we have defined $\alpha\triangleq 1-R_aR_LC/L$. This ODE can be easily solved by the separation of variables method to get $\beta(x_2)=c_1x_2^\alpha$, where c_1 is a constant, which we choose as $c_1=u_*/x_{2*}^\alpha$, to assign the equilibrium. Now, to ensure that x_* is not just an extremum but a minimum of $H_d(x)$ we look at its Hessian (evaluated at x_*), and verify that it is positive definite if and only if $-1<\alpha<1$. Now, in the derivations above we have assumed that the matrix $J[\beta(x)]-\mathscr{R}_d$ is invertible, which is the case if $x_2\neq 0$. On the other hand, the model of the boost converter is physically meaningful only in the positive quadrant. It is easy to see that, restricting $0<\alpha<1$, $x_2(t)\equiv 0$ is a trajectory of the closed loop dynamics. Unfortunately, this does not mean that trajectories starting in the half-plane $x_2>0$ will remain there, because the closed-loop vector field is not continuous at $x_2=0$ and uniqueness of solutions is no more guaranteed! A more detailed analysis—carried out in Rodriguez et al., 2000—reveals that we can define a set of initial conditions contained in the domain of attraction of x_* and such that the trajectories of the closed-loop system remain in the positive quadrant. Furthermore, we prove that there exists a value of α such that control signal $u(t)$ ranges in the set $(0,1)$.

In summary, we have shown that the output feedback IDA-PBC

$$\beta(x_2)=u_*\left(\frac{x_2}{x_{2*}}\right)^\alpha,\quad 0<\alpha<1$$

asymptotically stabilizes x_* *for all* load resistances $R_L > 0$, while verifying the physical constraints.

D. *Comparison with the Euler–Lagrange approach*: In Ortega et al. (1998) the problem of stabilization of the boost converter is approached in the following form. First of all, the model of the system is written as $M\dot{z} + J(u)z + \mathscr{R}z = g$, where $z \in \mathbb{R}^2$ contains the input current and output capacitor voltage, and $M := diag\{L, C\}$. An implicit definition of the controller is derived from a copy of the system with additional damping as

$$M\dot{z}_d + J(u)z_d + \mathscr{R}z_d = g + \mathscr{R}_{di}\tilde{z}, \tag{33}$$

where $\mathscr{R}_{di} := diag\{R_1, 0\}$, $R_1 > 0$, $z_d \in \mathbb{R}^2$ is an auxiliary vector, $\tilde{z} := z - z_d$, and z_d will be defined later. The idea is that, for all u that verify (33), the error equation

$$M\dot{\tilde{z}} + [J(u) + \mathscr{R}_d]\tilde{z} = 0$$

with $\mathscr{R}_d := \mathscr{R} + \mathscr{R}_{di}$, is exponentially convergent, that is, $\tilde{z} \to 0$ (exp.). (This fact is easily established evaluating the derivative of the quadratic function $\tilde{z}^\top M\tilde{z}$.)

The next step is to find a control u such that $\tilde{z} \to 0 \Rightarrow z_2 \to V_d$. One is then tempted to set $z_{2d} = V_d$ in (33), solve for u and define a differential equation for z_{1d}. Unfortunately, as shown in Ortega et al., 1998, this will lead to an internally *unstable* system. This stems from the fact that the zero dynamics of the system associated to the output voltage is unstable, and the proposed controller is implementing an (asymptotic) inverse of the system. To go around this problem, which is by the way conspicuous by its absence in the PCH formulation elaborated here, we fix instead z_{1d} to its desired value V_d^2/RE in (33). This leads to the following asymptotically stabilizing control law

$$C\dot{z}_{2d} = -\frac{1}{R_L}z_{2d} + \frac{V_d^2}{R_L E z_{2d}}(E + R_1\tilde{z}_1),$$

$$u = \frac{1}{z_{2d}}(E + R_1\tilde{z}_1).$$

Notice that z_{2d} is the state of our dynamic controller. To complete the stability analysis we must show that z_{2d} remains bounded, which follows from the minimum phase properties of the system with output z_1.

Comparing the solution given above with the ones obtained by the new method we see that, besides the obvious complexity reduction, the latter is more "natural" for at least two reasons. First, in the EL approach we try to assign to the system a storage function which is quadratic in some error quantities. Although this is a suitable choice for linear systems it need not be a reasonable one in the nonlinear case (see Maschke et al., 2000). This point is corroborated by our new developments which yield rational or fractional polynomial Lyapunov functions! Second, as discussed in Ortega et al. (1998) and clearly illustrated here, an (asymptotic) inversion of the system is implicit in the EL design procedure. It is reasonable to expect that this will bring along some robustness problems inherent to the linearization ideas.

8. Concluding remarks and future research

We have presented in this paper a procedure to design stabilizing controllers for PCH models which effectively exploits the structural properties of the system. The main features of the proposed scheme may be summarized as follows:

(i) The Hamiltonian structure is preserved in closed-loop, which allows for an energy interpretation of the control action. (ii) Given the clear-cut definition of the interconnection structure and the damping (captured in the matrices $J(x)$ and $\mathscr{R}(x)$, respectively) the incorporation of the physical intuition is effectively enhanced. This aspect is very important, not only for the definition of the "desired dynamics", but also for the commissioning of the controller. (iii) Conditions on the damping are given to ensure that the closed-loop storage function is precisely the energy-balance function. (iv) In principle, there is no need to explicitly derive the Lyapunov function, only its existence need be ascertained. (v) The procedure is amenable for symbolic manipulation languages.

Proposition 1, is a generalization of the Energy–Casimir method, which has been developed for stability analysis, to the controller *synthesis* problem. Indeed, in the Energy–Casimir method one shapes the energy $H(x)$ to a desired function $H_d(x) = H(x) + H_a(C_1(x), \dots, C_m(x))$, with $C_i(x)$ the Casimirs. Note that for $J_a(x) = \mathscr{R}_a(x) = 0$ and no input, that is $g(x, \beta(x)) = 0$, (7) reduces to $[J(x) - \mathscr{R}(x)](\partial H_a/\partial x)(x) = 0$, which yields the Casimir functions.

The results reported here are restricted to the case of stabilization of fixed points. In some cases it is possible to adapt the procedure to treat the stabilization of periodic orbits. Current research is under way to extend our approach to stabilization of general periodic orbits, and eventually to handle the more challenging *tracking* problem. See Fujimoto and Sugie (2001) for some interesting results.

In IDA–PBC it is possible to modify the kinetic energy of mechanical systems by a suitable selection of the desired interconnection matrix $J_d(x)$. This feature has been used in Ortega and Spong (2000) to globally stabilize an inverted pendulum with an inertia disk and the ball-and-beam system. This possibility also allows us to establish some connections with the Controlled-Lagrangian controllers reported in Bloch, Leonhard, and Marsden (2000). In particular, it is possible to show that modifying the kinetic energy of a simple mechanical system without affecting the potential energy nor the damping (as done in Bloch et al. (2000)) is tantamount—in our formulation—to selecting the closed-loop interconnection matrix as

$$J_d(q,p) = \begin{bmatrix} 0 & M^{-1}(q)M_d(q) \\ -M_d(q)M^{-1}(q) & J_2(q,p) \end{bmatrix},$$

where $M_d(q), M(q)$ are the closed-loop and open-loop inertia matrices, respectively, and

$$J_2(q,p) = M_d(q)M^{-1}(q)\left\{\left[\frac{\partial}{\partial q}(M(q)M_d^{-1}(q)p)\right]^\top - \left[\frac{\partial}{\partial q}(M(q)M_d^{-1}(q)p)\right]\right\}M^{-1}(q)M_d(q).$$

See Blankenstein, Ortega and van der Schaft (2001) for a detailed comparison of both methods.

Acknowledgements

We would like to thank our collaborators A. Astolfi, N. Barabanov, I. Mareels and S. Stramigioli for many helpful discussions that improved the quality of this paper. B. Maschke acknowledges the financial support of the Foundation of Mathematical Research of the Netherlands (SWON). The research was partly funded by the TMR Nonlinear Control Network of the European Union and The Netherlands Organization for Scientific Research NWO (SWON Large Project).

References

Astolfi, A., Chhabra, D., & Ortega, R. (2001). Asymptotic stabilization of selected equilibria of the underactuated Kirchhoff's equations. *Systems and Control Letters*, to appear. See also: *ACC 2001*, Arlington, VA, USA, June 25–27.

Blankenstein, G., Ortega, R., & van der Schaft, A. J. (2001). The matching conditions of controlled Lagrangians and interconnection assignment passivity based control, *LSS Int. Report*, May. To appear in *International Journal of Control*.

Bloch, A., Leonhard, N., & Marsden, J. (2000). Controlled Lagrangians and the stabilization of mechanical systems. *IEEE Transactions of Automatic Control*, *AC-45*(12), 2253–2270.

Dalsmo, M., & van der Schaft, A. J. (1999). On representations and integrability of mathematical structures in energy-conserving physical systems. *SIAM Journal on Optimization and Control*, *37*(1).

Fujimoto, K., & Sugie, T. (2001). Canonical transformations and stabilization of generalized Hamiltonian systems. *Systems and Control Letters*, *42*(3), 217–227.

Khalil, H. (1996). *Nonlinear systems* (2nd ed.). New Jersey: Prentice-Hall.

Marsden, J., & Ratiu, T. (1994). *Introduction to mechanics and symmetry*. NY: Springer.

Maschke, B. M., Ortega, R., & van der Schaft, A. J. (2000). Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation. *IEEE Transactions on Automatic Control*, *45*(8), 1498–1502.

Nijmeijer, H., & van der Schaft, A. (1990). *Nonlinear dynamical control systems*. Berlin: Springer.

Ortega, R., Astolfi, A., Bastin, G., & Rodriguez, H. (1999). Output feedback stabilization of mass–balance systems. In: H. Nijmeijer & T. Fossen (Eds.), *Output-feedback stabilization of nonlinear systems*. Berlin: Springer (see also *Proceedings 2000 ACC*, Chicago, IL, June 28–30, 2000).

Ortega, R., Galaz, M., Bazanella, A., & Stankovic, A. (2001). An energy-shaping approach to excitation control of synchronous generators. *ACC 2001*, Arlington, VA, USA, June 25–27.

Ortega, R., & Spong, M. (1989). Adaptive motion control of rigid robots: A tutorial. *Automatica*, *25*(6), 877–888.

Ortega, R., & Spong, M. (2000). Stabilization of underactuated mechanical systems via interconnection and damping assignment. *IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Systems*. NJ: Princeton, March 16–18. To appear in *IEEE Transactions on Automatic Control*.

Ortega, R., van der Schaft, A., Mareels, I., & Maschke, B. (2001). Putting energy back in control. *IEEE Control Systems Magazine*, *21*(2), 18–33.

Ortega, R., Loria, A., Kelly, R., & Praly, L. (1995). On output feedback global stabilization of Euler–Lagrange systems. *International Journal of Robust and Nonlinear Cont.* In: H. Nijmeijer & A. van der Schaft (Eds.), Special Issue on Mechanical Systems. Vol. 5, No. 4 (pp. 313–324).

Ortega, R., Loria, A., Nicklasson, P. J., & Sira-Ramirez, H. (1998). *Passivity-based control of Euler–Lagrange systems*. Berlin: Springer. Communications and Control Engineering, September 1998.

Petrovic, V., Ortega, R., & Stankovic, A. (2000). Extending passivity-based control beyond mechanics: A synchronous motor example. *Automatisierungstechnik, Oldenbourg Verlag*, *48*(3), 106–115.

Rodriguez, H., Ortega, R., Escobar, G., & Barabanov, N. (2000). A robustly stable output feedback saturated controller for the boost DC-to-DC converter. *Systems and Control Letters*, *40*(1), 1–8.

Rodriguez, H., Ortega, R., & Mareels, I. (2000). Nonlinear control of magnetic levitation systems via energy-balancing. *ACC 2000*, Chicago, IL, June 28–30.

Stramigioli, S., Maschke, B. M., & van der Schaft, A. J. (1998). Passive output feedback and port interconnection. *Proceedings of 4th IFAC Symposium on Nonlinear Control Systems Design*, NOLCOS'98 (pp. 613–618). Enschede, NL, July 1–3.

van der Schaft, A. J. (1999). *L_2-gain and passivity techniques in nonlinear control*. Berlin: Springer.

Bernhard Maschke was graduated as engineer in telecommunication at the Ecole Nationale Supérieure des Télécommunications (Paris, France) in 1984. He received in 1990 his Ph. D. degree and in 1998 the Habilitation to Direct Researches both from the University of Paris-Sud (Orsay, France).

From 1987 until 1990 he has been working on the control of robots with flexible links at the Laboratory on Advanced Robotic Engineering at the Nuclear Research Center of Fontenay-aux-Roses, France.

From 1990 he has been associate professor at the Laboratory of Industrial Automation of the Conservatoire national des Arts et Métiers (Paris, France) and since 2000 he is professor in automatic control at the Laboratory of Control and Chemical Engineering of the University Claude Bernard of Lyon (Villeurbanne, France).

In 1996 he has spend a six month sabbatical with the group on "Differential geometry, dynamical systems and applications" (Prof. Ch.-M. Marle) of the Mathematical Institute of Jussieu of the University Pierre et Maris Curie (Paris, France). In 1990 he has spend six months, respectively, in 1998–2000 eighteen months, at the University of Twente (Enschede, The Netherlands) with the Control Laboratory (Prof. P.C. Breedveld) and with the Department of Systems, Signals and Control (Prof. A.J. van der Schaft).

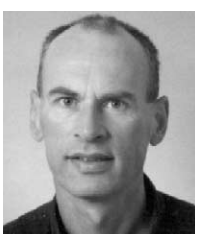
His research interests include the network modelling of physical systems using bond graphs, multibody systems, electro-mechanical systems and physico-chemical processes, Hamiltonian systems, irreversible thermodynamics, passivity-based control and control by interconnection, modelling and control of distributed parameter systems. He is co-author of the book *Dissipative Systems Analysis and Control*, Communication and Control Engineering Series, Springer, 2000.

Romeo Ortega was born in Mexico. He obtained his BSc in Electrical and Mechanical Engineering from the National University of Mexico, Master of Engineering from Polytechnical Institute of Leningrad, USSR, and the Docteur D'Etat from the Politechnical Institute of Grenoble, France in 1974, 1978 and 1984, respectively.

He then joined the National University of Mexico, where he worked until 1989. He was a Visiting Professor at the University of Illinois in 1987–1988 and at the McGill University in 1991–1992, and a Fellow of the Japan Society for Promotion of Science in 1990–1991. He has been a member of the French National Researcher Council (CNRS) since June 1992.

Currently, he is in the Laboratoire de Signaux et Systemes (SUPELEC) in Paris. His research interests are in the fields of nonlinear and adaptive control, with special emphasis on applications.

Dr. Ortega is a Fellow Member of the IEEE. He was the Chairman of the IEEE Working Group on Adaptive Control and Systems Identification, of the IFAC Technical Committee on Adaptive Control and Tuning and of the Automatica Paper Prize Award Committee. He is currently a member of the IFAC Technical Board and chairman of the IFAC Coordinating Committee on Systems and Signals. He is an Associate Editor of Systems and Control Letters, International Journal of Adaptive Control and Signal Processing, European Journal of Control and IEEE Transactions on Control Systems Technology.



A.J. (Arjan) van der Schaft was born in Vlaardingen, The Netherlands, in 1955. He received the undergraduate and Ph.D. degrees in Mathematics from the University of Groningen, The Netherlands, in 1979 and 1983, respectively. In 1982 he joined the Faculty of Mathematical Sciences, University of Twente, Enschede, The Netherlands, where he is presently a full professor in Mathematical Systems and Control Theory.

His research interests include the mathematical modelling of physical and engineering systems and the control of nonlinear and hybrid systems. He has served as Associate Editor for Systems & Control Letters, Journal of Nonlinear Science, and the IEEE Transactions on Automatic Control. Currently, he is Associate Editor for the SIAM Journal on Control and Optimization and Editor-at-Large for the European Journal of Control.

He is the (co-)author of the following books: System Theoretic Descriptions of Physical Systems (Amsterdam, The Netherlands: CWI, 1984), (with P.E. Crouch) Variational and Hamiltonian Control Systems (Berlin, Germany: Springer-Verlag, 1987), (with H. Nijmeijer) Nonlinear Dynamical Control Systems (Berlin, Germany: Springer-Verlag, 1990), L2-Gain and Passivity Techniques in Nonlinear Control (London,UK: Springer-Verlag, 1996, second enlarged edition, 2000), and (with J.M. Schumacher) An Introduction to Hybrid Dynamical Systems (London, UK: Springer-Verlag, LNCIS 251, 2000).

Gerardo Escobar Valderrama was born in Xochimilco, Mexico in 1967. He received his BSc degree in Electromechanics Engineering (speciality in Electronics) in 1991 and his MSc in Electrical Engineering (speciality in Automatic Control) in 1995, both from the Engineering Faculty of the National University of Mexico, and in May 1999 he received his Ph.D. from the Signals and Systems Lab. LSS—SUPELEC, Paris, France. He has worked as a Technical Assistant in the Automatic Control Lab. of the Graduate School of Engineering in the Nat. Univ. of Mexico from May 90 to April 91. From August 1991 to August 1995 he was an Assistant Professor in the Control Dep. of the Eng. Fac. in the Nat. Univ. of Mexico. He is currently a visiting scientist at the Northeastern University in Boston Massachusetts. His main research interest includes nonlinear control design, passivity based control, switching power converters, electrical drives and hybrid systems control.