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ABSTRACT

We investigated here the influence of the lateral Casimir force on the dynamical actuation of devices with interacting materials covering a broad range of optical properties ranging from poor to good conductors, such as, for example, nitrogen doped SiC and Au, respectively. The conservative actuating system shows a central heteroclinic orbit surrounded by a finite number of homoclinic orbits, because at higher periods, an increased lateral Casimir force will be necessary to counterbalance the restoring force. As a result, the conservative system reaches stable operation sooner for the higher conductivity materials (Au–Au), indicating the significant impact of the material optical properties on the lateral Casimir force. Furthermore, for the non-conservative driven systems, the decrement of the Melnikov parameter α leads to a faster disappearance of the satellite homoclinic orbits in the Poincaré portraits, followed by a strong shrinkage of the central heteroclinic orbit toward unstable chaotic motion. The latter is more pronounced for the lower conductivity materials since comparison shows the Au–Au system to be significantly more stable than the SiC–SiC system. Therefore, in actuating systems where the lateral Casimir force could play a significant role, the higher conductivity materials appear to be a better choice to ensure stable operation against a chaotic motion.

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Nowadays, micro- and nanoelectromechanical systems are attracting great attention from scientific and technology points in diverse sensor technologies, accelerometers, microswitches, etc. Beyond the normal Casimir force, which is omnipresent and perpendicular to the surfaces of interacting bodies, when the latter are anisotropic or positioned asymmetrically, they can enable the generation of lateral Casimir forces. Our findings reveal that a conservative system reaches stable operation sooner for the higher conductivity materials, indicating the significant impact of the material optical properties on the lateral Casimir force. Furthermore, for the non-conservative driven systems, the lower conductivity materials lead to strong shrinkage of the central heteroclinic orbit and disappearance of any satellite homoclinic orbits toward chaotic motion. Therefore, we illustrate that in actuating systems where the lateral Casimir force could play a significant role, the higher conductivity materials appear to be a better choice to ensure stable operation against a chaotic motion and improve the long-term performance of devices.

I. INTRODUCTION

The significant role of the Casimir force in the design and actuation analysis of micro- and nanodevices has been attracting widespread attention from both the fundamental science and engineering points of view. By the advancement in microfabrication and consequent reduction of the scale of microdevices into the submicron range,^{1–13} the role of Casimir and electrostatic forces could become comparable. Although the electrostatic forces can be utilized to actuate micro/nano-electromechanical systems (MEMS/NEMS), the Casimir force is omnipresent and, in principle, it can always influence the actuation dynamics of operating devices. This is possible if the devices have surface areas large enough and separation gaps small enough for the Casimir force to play a significant role and, under certain conditions, to drag mechanical elements together, leading to permanent adhesion, which is a device malfunction known as stiction.^{2,9–13}

In any case, the Casimir force arises due to perturbation of quantum fluctuations of the EM field,³ as described by Casimir

in 1948 assuming two perfectly conducting parallel plates.¹⁴ Furthermore, Lifshitz and co-workers in the 1950s¹⁵ considered the general case of real dielectric plates by exploiting the fluctuation-dissipation theorem, which relates the dissipative properties of matter (via optical absorption by many microscopic dipoles) and the resulting EM fluctuations. This theory describes the attractive interaction due to quantum fluctuations for all separations covering both the Casimir (long-range) and van der Waals (short-range) regimes.^{1–5,14–17} Besides the optical properties^{18–30} and thermal contributions,^{31–35} by engineering the boundary surface between interacting components, it is feasible to tailor the direction and strength of the Casimir force and consequently drive the actuation dynamic of NEMS/MEMS in a multitude of different ways.

Beyond the normal Casimir force, which is perpendicular to the surfaces of interacting bodies, when the latter are anisotropic or positioned asymmetrically, they can enable the generation of the lateral Casimir force acting tangential to the surfaces.^{36–40} Both Casimir components originate from zero point quantum and thermal fluctuations of the EM field, but their influence on the dynamical behavior of actuating devices can be different. Because the Casimir force is omnipresent, it imposes a serious challenge for the design of MEMS. As a result of shrinking the size of devices, an unavoidable problem could be the occurrence of chaotic motion leading to device stiction or malfunction.^{41–44} So far, several works have shown the strong dependence of the normal Casimir force on the material optical properties and explored how these properties can be utilized to tune the actuation dynamics of devices that can exhibit transitions toward chaotic motion.^{27–30} However, still unexplored is the actuation dynamic of devices under the influence of the lateral Casimir force by taking into account significant contrast in the material optical properties and conductivity of the interacting surfaces.

Therefore, we will explore here how chaotic motion emerges under the influence of the lateral Casimir forces by taking into account the optical properties of the interacting materials and what abrupt changes it can generate in the dynamical behavior devices leading ultimately to malfunctions. Hence, for this purpose, we consider here a typical lateral system that is constructed from two components in close proximity that have sinusoidal corrugated surfaces both on a fixed plate and a sphere suspended by a mechanical spring. The latter is assumed sufficiently stiff to resist any bending along the normal to the surfaces, but it can allow lateral displacements, as shown in Fig. 1(a).⁴⁵

II. MATERIAL PROPERTIES AND DEVICE ACTUATION

In order to accomplish our goal with respect to the choice of material optical properties, Au was chosen as a good conductor for which the optical properties have been measured in a wide range of frequencies. On the other hand, as a poor conductor, we considered nitrogen doped silicon carbide (SiC), which is a promising material for device operating under harsh environments. Both materials were optically characterized in the past with the same equipment [J. A. Woollam Co., Inc. ellipsometers VUV-VASE (0.5–9.34 eV) and IR-VASE (0.03–0.5 eV)].^{24,25,46,47} To compare the static conductivity ratio ω_p^2/ω_τ in terms of the Drude model, with ω_p the plasma frequency and ω_τ the relaxation frequency, the calculation based on

FIG. 1. (a) The schematic of the lateral actuating device under consideration. (b) Dielectric functions at imaginary frequencies $\varepsilon(i\xi)$ for Au and SiC, which were used for the lateral Casimir force calculations.

the measured optical parameters yielded for these materials the values $\omega_p^2/\omega_\tau|_{\text{Au}} \approx 1600$ eV for Au^{24,25} and $\omega_p^2/\omega_\tau|_{\text{SiC}} = 0.4$ eV for SiC.^{46,47} These static conductivities differ by more than four orders in magnitude, and, in addition, the chosen materials show significant optical contrast for the dielectric function at imaginary frequencies $\varepsilon(i\xi)$ that is necessary input for the calculations of both the normal and lateral Casimir force [see Fig. 1(b)].

The equation of motion for the lateral micromechanical system [Fig. 1(a)], where both fixed (plate) and movable (sphere) components are coated with Au and/or SiC, is given by^{48–50}

$$M \frac{d^2 \Delta x}{dt^2} + \left(\frac{M\omega_0}{Q} \right) \frac{d\Delta x}{dt} = -F_{\text{res}} + F_{\text{Cas}}^{\text{lat}} + \varepsilon F_0 \cos(\omega t), \quad (1)$$

where M is the mass of the moving sphere, Δx is the lateral displacement between the components, and $(M\omega_0/Q)(d\Delta x/dt)$ is the intrinsic energy dissipation in the actuating system associated with a quality factor Q . The frequency ω_0 is assumed to be that of dynamic mode atomic force microscope (AFM) cantilevers or MEMS (typically $\omega = 300$ kHz).^{36,37,51,52} The parameter ε was introduced to distinguish between the conservative (frictionless) autonomous operation of the actuating system ($\varepsilon = 0$) and the non-conservative driven system by an external force ($\varepsilon = 1$) in the presence of dissipation (friction) having a finite quality factor Q . For a conservative system in practice, we consider devices with a high quality factor $Q > 10^4$ in order to neglect the energy dissipation.^{51,52}

FIG. 2. Lateral Casimir force vs λ ($= \Delta x / \Lambda$) for the Au–Au and SiC–SiC MEMS systems in this study. The inset shows for comparison also the normal Casimir force for these materials and the same sphere–plate geometry.

The vertical separation in the sphere–plate system is considered to be 200 nm, and the radius R of the sphere $100 \mu\text{m}$.^{36,37} The surface of both components is assumed to be sinusoidally corrugated with an amplitude $A_1 = 85 \text{ nm}$ and $A_2 = 14 \text{ nm}$ for the plate and the sphere, respectively.^{36,37} Also, the average period of the corrugation is assumed to be $\Lambda = 570 \text{ nm}$.^{36,37} In Eq. (1), the lateral Casimir force $F_{\text{Cas}}^{\text{lat}}$ was calculated by means of the Lifshitz formula using the proximity force approximation (PFA) (see Appendix) for bodies at room temperature ($T = 300 \text{ K}$).^{36,37} Furthermore, the lateral Casimir force is opposed by the elastic restoring force $F_{\text{res}} = -K \Delta x$ where K is the spring constant. According to Fig. 2, the normal force is stronger in magnitude than the lateral component with a typical ratio $F_{\text{lat}}/F_{\text{nor}} < 0.08$. The latter implies that the torsion stiffness K_b of the spring must be at least two orders of magnitude stronger than the normal horizontal stiffness K to prevent any torsion by the normal force. In practice, the normal stiffness, for example, of cantilevers, is significantly less than its torsional stiffness. Therefore, depending on the geometry under consideration, where the device operates, it is feasible to obtain springs with current fabrication technologies to sustain significant lateral motion as we describe in the current work.

III. RESULTS AND DISCUSSION

A. Conservative actuating system ($\varepsilon = 0$)

In order to investigate the effect of optical properties on the actuation of the lateral microsystem, we introduced the bifurcation parameter $\delta_{\text{Cas}} = F_{\text{res}}^{\text{M}}/F_{\text{Cas}}^{\text{lat(m)}}$ that represents the ratio of the maximum restoring force ($F_{\text{res}}^{\text{M}} = -k_b \Lambda$) to the minimum lateral Casimir force. A change of the parameter δ_{Cas} can lead to changes of the actuation of the device as it will be illustrated in Fig. 3.^{27,28} Therefore, it is useful to determine when there is a stable periodic solution for the system, which corresponds to sufficient restoring force to prevent instability in the system toward unperiodic motion.⁵³ Using

FIG. 3. Bifurcation diagrams δ'_{Cas} vs λ ($= \Delta x / \Lambda$) for the Au–Au and SiC–SiC systems.

δ_{Cas} , Eq. (1) assumes the more convenient form (for $\varepsilon = 0$)

$$\frac{d^2 \lambda}{dT^2} + \left(\frac{M \omega_0}{Q} \right) \frac{d\lambda}{dT} = -\lambda + \frac{1}{\delta'_{\text{Cas}}} \frac{F_{\text{Cas}}^{\text{lat}}}{F_{\text{Cas}}^{\text{lat(m)}}}, \quad (2)$$

where $\lambda = \Delta x / \Lambda$, $T = \omega_0 t$ and $\delta'_{\text{Cas}} = b \delta_{\text{Cas}}$. The equilibrium points for the conservative motion are obtained by the condition $F_{\text{total}} = -F_{\text{res}} + F_{\text{Cas}}^{\text{lat}} = 0$ or equivalently from Eq. (2) by the condition $-\lambda + (1/\delta'_{\text{Cas}})(F_{\text{Cas}}^{\text{lat}}/F_{\text{Cas}}^{\text{lat(m)}}) = 0$ that yields $\delta'_{\text{Cas}} = (1/\lambda)(F_{\text{Cas}}^{\text{lat}}/F_{\text{Cas}}^{\text{lat(m)}})$.

The bifurcation diagrams in Fig. 3 illustrate the influence of the optical properties on the stable and unstable regions. For a system under the influence of the lateral Casimir force, the corresponding bifurcation diagram has several maximums in every single period of lateral displacement ($n < \Delta x / \Lambda < n + 1$ with $|n| = 1, 2, 3, \dots$). Their magnitude strongly depends on the material conductivity, and it decreases with decreasing conductivity. In fact, when bifurcation parameter becomes weaker than the magnitude of the particular maximum in each period ($\delta_{\text{Cas}} < \delta_{\text{Cas}}^{\text{M,n}}$ with $|n| = 1, 2, \dots$, the number of periods), the counterbalance between the lateral Casimir force and the restoring force can lead the system to perform stable motion. By increasing the lateral displacement in higher periods, the restoring force becomes stronger, and it requires stronger lateral Casimir force in order to preserve stable motion for larger displacements. Consequently, at larger periods, the magnitude of maximums becomes weaker, and the system loses its stable motion sooner. The latter takes place faster in the system with lower conductivity due to weaker lateral Casimir forces.

Besides the bifurcation analysis, the sensitive dependence of the actuation dynamics on the optical properties is reflected by the Poincaré maps in Fig. 4. Both heteroclinic (central orbit) and homoclinic orbits separate the unstable (unperiodic) motion from the periodic closed orbits around the stable center points. From

FIG. 4. Contour plot of the transient time to instability for initial conditions in the $\lambda - d\lambda/dt$ phase plane for the Au–Au (left column) and SiC–SiC (right column) conservative systems for different value of δ'_{Cas} as they are indicated on the plots. For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dt)$. The red elliptical region contains initial conditions that lead to stable oscillations. The heteroclinic and homoclinic orbits separate sharply stable and unstable solutions reflecting the absence of chaotic behavior.

this figure, it is evident that actuating system reaches stable operation sooner for the higher conductivity materials. Nevertheless, the Poincaré portrait reveals the existence of a strong sensitivity on the decreasing value of δ_{Cas} . Therefore, so far, we can conclude

that the optical properties of the interacting materials can have significant effect on the conservative actuation dynamics of lateral micromechanical systems with an increase in magnitude of the lateral Casimir force.

B. Non-conservative actuating system MEMS ($\varepsilon = 1$)

Here, we performed calculations to investigate the existence of chaotic behavior in the dissipative lateral microsystem undergoing forced oscillation via an external driving force $F_0 \cos(\omega t)$.⁵⁴ The chaotic behavior is understood to occur if hetero- or homoclinic orbits of the conservative system split, leading to transitions in between them. The latter can be addressed by the so-called Melnikov function and Poincaré portrait analysis.^{54–58} In the driven system, the unstable equilibrium turns into an unstable periodic orbit. To calculate the Melnikov function, the heteroclinic and homoclinic solutions are necessary inputs. The latter are obtained by numerical solution of Eq. (2) for conservative motion, and they are denoted as $\lambda_{\text{het}}^C(T)$ and $\lambda_{\text{hom}}^C(T)$ for heteroclinic and homoclinic solutions, respectively. The Melnikov functions for the non-conservative system ($\varepsilon = 1$) are given by^{54–58}

$$M^{\text{het}}(T_0) = \frac{1}{Q} \int_{-\infty}^{+\infty} \left(\frac{d\lambda_{\text{het}}^C(T)}{dT} \right)^2 dT + \frac{F_0}{F_{\text{res}}^{\text{MAX}}} \int_{-\infty}^{+\infty} \frac{d\lambda_{\text{het}}^C(T)}{dT} \cos \left[\frac{\omega}{\omega_0} (T - T_0) \right] dT, \quad (3)$$

$$M^{\text{hom}}(T_0) = \frac{1}{Q} \int_{-\infty}^{+\infty} \left(\frac{d\lambda_{\text{hom}}^C(T)}{dT} \right)^2 dT + \frac{F_0}{F_{\text{res}}^{\text{MAX}}} \int_{-\infty}^{+\infty} \frac{d\lambda_{\text{hom}}^C(T)}{dT} \cos \left[\frac{\omega}{\omega_0} (T + T_0) \right] dT. \quad (4)$$

The separatrix splits if the Melnikov function has simple zeros so that $M(T_0) = 0$ and $M'(T_0) \neq 0$, while for $M(T_0) \neq 0$, the motion will not be chaotic. The conditions of non-simple zeros $M(T_0) = 0$ and $M'(T_0) = 0$ give the threshold condition for chaotic motion.^{54–58} If we define

$$\mu_{\text{het/hom}}^C = \int_{-\infty}^{+\infty} \left(\frac{d\lambda_{\text{het/hom}}^C(T)}{dT} \right)^2 dT$$

and

$$\beta(\omega) = \left| H \left[\text{Re} \left(F \left\{ \frac{d\lambda_{\text{het/hom}}^C(T)}{dT} \right\} \right) \right] \right|, \quad (5)$$

then the threshold condition for chaotic motion $\alpha = \beta(\omega)/\mu_{\text{het/hom}}^C$ with $\alpha = (1/Q)(F_0/F_{\text{res}}^{\text{MAX}})^{-1} = \gamma\omega_0 d/F_0$ obtains the final form

$$\alpha = \frac{\gamma\omega_0 d}{F_0} = \left| H \left[\text{Re} \left(F \left\{ \frac{d\lambda_{\text{het/hom}}^C(T)}{dT} \right\} \right) \right] \right| / \int_{-\infty}^{+\infty} \left(\frac{d\lambda_{\text{het/hom}}^C(T)}{dT} \right)^2 dT, \quad (6)$$

with $\gamma = I\omega_0/Q$, and $H[\dots]$ denoting the Hilbert transform.^{54–58} Figure 5 shows the threshold curves $\alpha = \gamma\omega_0 d/F_0$ vs the frequency ratio ω/ω_0 . For large values of α (above the curve), the dissipation dominates the driving force ($\alpha \sim \gamma/F_0$), leading to a regular periodic motion, while for values below the curve, the splitting of the separatrix leads possibly to chaotic motion.

Figure 5(a) demonstrates the threshold condition for possible chaotic behavior in the Au–Au system with $\delta'_{\text{Cas}} = 0.2$. The parameter μ (having integer values $\mu = 1, 2, 3, \dots$) denotes the homoclinic motion in the satellite twin intervals $-\mu - 1\Delta x/\Lambda < -\mu$ and $\mu < \Delta x/\Lambda < \mu + 1$ around the central heteroclinic orbit, where the system is under the constant influence of the restoring force. In fact, according to Fig. 4 in the Au–Au system for $\delta'_{\text{Cas}} = 0.2$, there are three satellite twin homoclinic orbits with $\mu = 1, 2, 3$. By

FIG. 5. Threshold curve $\alpha = \gamma\omega_0 d/F_0$ vs driving frequency ω/ω_0 (with ω_0 the natural frequency of the system). The area below the curve corresponds to parameters that can lead to chaotic motion with $\delta'_{\text{Cas}} = 0.2$: (a) Au–Au system and (b) both Au–Au and SiC–SiC systems.

increasing the lateral displacement, the possibility of chaotic motion increases as the area under the curve is growing with increasing μ . This behavior was also observed in the Poincaré portraits for the conservative system, where the stable actuating area became smaller by increasing distance from center. In any case, the most stable part for actuation in the phase portrait is the heteroclinic orbit, where the possibility for chaotic motion is minimum in this region.

Figure 5(b) provides an interesting comparison between the threshold conditions for chaotic behavior for the SiC–SiC and the Au–Au systems using the same bifurcation parameter $\delta'_{\text{Cas}} = 0.2$. According to the last row of Poincaré portraits for the conservative system in Fig. 4, for the SiC–SiC plot, there is only one pair of

satellite twins due to homoclinic motion corresponding to $\mu = 1$. The heteroclinic and homoclinic orbits of the Au system sustain more efficiently stable motion, while for the SiC chaotic motion is more likely to occur. This is because at higher periods, the increased restoring force will require an increased lateral Casimir force (which is the case for the Au–Au system) to preserve the force balance and stable motion. Since the lateral Casimir force is weaker for the SiC–SiC system, higher possibility of instability and subsequent chaotic motion is, therefore, expected.

Figures 6–8 show the transient times to instability for different values of the threshold parameter α . As it was shown in Fig. 4 for the conservative system, there is a sharp boundary separating stable (red area) and unstable solutions (blue area), while in the driven non-conservative system (Figs. 6 and 8), there is not a simple smooth boundary between the regions of stable and unstable solutions. Therefore, there are areas of intermediate cases, which are shown with different colors, and, according to the color bar, the instability could take place after several periods, affecting the long-term stability of the device. Figure 6 depicts in more detail how the decrement of the parameter α leads to faster disappearance of the homoclinic orbits followed also by shrinkage of the central heteroclinic orbit toward unstable chaotic behavior. In fact, Fig. 7 shows the area reduction of the heteroclinic orbit with decreasing α , which is more rapid for the Au–Au system where the lateral Casimir force is stronger. Figure 8 provides also a comparison between the two material systems and again confirms that the Au–Au system is more stable than the SiC–SiC in agreement with the Melnikov calculations in Fig. 5. Notably, for the Melnikov parameter $\alpha = 0.1$, where the homoclinic orbits for the SiC–SiC have been eliminated due to the chaotic motion, and for the Au–Au system, both the first and second order homoclinic twins are still present, manifesting the enhanced resilience of the system toward stable motion.

FIG. 6. Contour plot of the transient time to instability for initial conditions in the $\lambda - d\lambda/dT$ phase plane for the non-conservative Au–Au system with $\delta'_{\text{Cas}} = 0.2$ and $\omega/\omega_0 = 3$, and different values of α as they are indicated on the plots. For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dT)$. The red elliptical shape region shows the initial condition for which the microdevice shows still stable motion after 100 oscillations. With decreasing α , the chaotic behavior increases and the area of stable motion (red elliptical in shape region) shrinks rapidly until it disappears for completely unstable motion.

FIG. 7. The area S of the central heteroclinic orbit for both the Au–Au and SiC–SiC systems changes rapidly as a function of the Melnikov parameter α . The faster rise for the Au–Au system where the lateral Casimir force is stronger.

FIG. 8. Contour plot of the transient times to instability in the $\lambda - d\lambda/dT$ phase plane for the non-conservative Au-Au system (left column) and the SiC-SiC system (right column) with $\delta'_{Cas} = 0.2$, $\omega/\omega_0 = 4$, and different values of α as they are indicated on the plots. For the calculations, we used 150×150 initial conditions $(\lambda, d\lambda/dT)$. The red elliptical shape region shows the initial condition for which the microdevice shows still stable motion after 100 oscillations. With decreasing α , the chaotic behavior increases and the area of stable motion (red elliptical in shape region) shrinks more in the systems with less conductivity.

IV. CONCLUSIONS

In summary, we have investigated the influence of the lateral Casimir force on the dynamical actuation of devices with interacting materials covering a broad range of optical properties and electrical conductivities. For the conservative systems, bifurcation analysis and Poincaré portraits have been utilized to demonstrate the stable and unstable operating regimes. For conservative systems, the Poincaré portraits show one central heteroclinic orbit with symmetric satellite homoclinic orbits since the system is under the constant influence of the restoring force. A finite number of satellite homoclinic orbits occur because at higher periods, the increased restoring force will require an increased lateral Casimir force, which is the case, for example, for the Au-Au system, to preserve the force balance. Since the lateral Casimir force is weaker for the SiC-SiC system, a higher possibility of instability and chaotic motion is expected. Indeed, the conservative actuating system reaches stable operation sooner for the higher conductivity material system, indicating the significant impact on the conservative dynamics with increasing magnitude of the lateral Casimir force. Finally, for the driven non-conservative system, instability could take place after several periods prohibiting the long-term stability of devices. The Poincaré portraits depict with detail that the decrement of the Melnikov parameter α leads to faster disappearance of the satellite homoclinic orbits followed by a strong shrinkage of the central heteroclinic orbit towards chaotic behavior. The latter is more pronounced for the lower conductivity materials since comparison shows that the Au-Au system is more stable than the SiC-SiC system in agreement with the Melnikov calculations. Therefore, in actuating systems, where the lateral Casimir forces could play

a prominent role, the choice of higher conductivity interacting materials appears to be a better choice to sustain long-term stable operation.

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APPENDIX: BRIEF THEORY OF THE LATERAL CASIMIR FORCE AND DIELECTRIC FUNCTION OF MATERIALS WITH EXTRAPOLATIONS

The lateral Casimir force $F_{Cas}(d, \varphi)$ in Eq. (1) for the plate-sphere system with sinusoidal corrugated surfaces, using the proximity force approximation (PFA), is given by²⁷

$$F_{Cas}^{lat}(d, \varphi) = \alpha \frac{\sin \varphi}{\beta} \sum_{n=1}^{\infty} \frac{1}{n} \sum_{l=0}^{\infty} \int_{\zeta_1}^{\infty} dy [r_{TM}^{2n}(i\zeta_1, y) + r_{TE}^{2n}(i\zeta_1, y)] \exp(-ny) I_1(nby), \quad (A1)$$

where

$$\alpha = \frac{\pi k_B T R A_1 A_2}{2d^3 \Lambda} \quad \text{and} \quad \beta = \frac{\sqrt{A_1^2 + A_2^2 - 2A_1 A_2 \cos(\varphi)}}{d}. \quad (A2)$$

In both equations, d is the vertical separation distance and $\varphi = 2\pi \Delta x / \Lambda$. The prime in the first summation of Eq. (A1) indicates that the term corresponding to $l = 0$ should be

multiplied with a factor $1/2$. Also, we have $y = 2dq_l$ with $q_l = \sqrt{k_{\perp}^2 + \xi_l^2/c^2}$ and ξ_l the imaginary frequency. The Fresnel reflection coefficients are given by $r_{TE}^{(i)} = (k_0 - k_i)/(k_0 + k_i)$ and $r_{TM}^{(i)} = (\epsilon_i k_0 - \epsilon_0 k_i)/(\epsilon_i k_0 + \epsilon_0 k_i)$ for the transvers electric (TE) and magnetic (TM) field polarizations, respectively. $\xi_1 = 2d\xi_l/c$ and $k_i = \sqrt{\epsilon_i(i\xi_l) + k_{\perp}^2}$ ($i = 0, 1, 2$) represent the out-of-plane wave vector in the gap between the interacting plates (k_0) and in each of the interacting plates ($k_{i=(1,2)}$). $k_{\perp}$ is the in-plane wave vector.

Furthermore, $\epsilon(i\xi)$ is the dielectric function evaluated at imaginary frequencies, which is the necessary input for calculating the Casimir force between real materials using the Lifshitz theory. The latter is given by¹¹

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^{\infty} \frac{\omega \epsilon''(\omega)}{\omega^2 + \xi^2} d\omega. \quad (A3)$$

For the calculation of the integral in Eq. (A2), one must use the measured data for the imaginary part $\epsilon''(\omega)$ of the frequency dependent dielectric function $\epsilon(\omega)$. The materials were optically characterized by ellipsometry over a wide range of frequencies at J. A. Woollam Co. with the VUV-VASE (0.5–9.34 eV) and IR-VASE (0.03–0.5 eV) ellipsometers.^{19,30} The experimental data for the imaginary part of the dielectric function cover only a limited range of frequencies $\omega_1 (= 0.03 \text{ eV}) < \omega < \omega_2 (= 8.9 \text{ eV})$. Therefore, for the low optical frequencies ($\omega < \omega_1$), we extrapolated using the Drude model³⁰

$$\epsilon''_L(\omega) = \frac{\omega_p^2 \omega_{\tau}}{\omega(\omega^2 + \omega_{\tau}^2)}, \quad (A4)$$

where ω_p is the plasma frequency and ω_{τ} is the relaxation frequency. Furthermore, for the high optical frequencies ($\omega > \omega_2$), we extrapolated using³⁰

$$\epsilon''_H(\omega) = \frac{A}{\omega^3}. \quad (A5)$$

Finally, using Eqs. (A3)–(A5), the function $\epsilon(i\xi)$ is given by

$$\epsilon(i\xi) = 1 + \frac{2}{\pi} \int_{\omega_1}^{\omega_2} \frac{\omega \epsilon''_{\text{exp}}(\omega)}{\omega^2 + \xi^2} d\omega + \Delta_L \epsilon(i\xi) + \Delta_H \epsilon(i\xi), \quad (A6)$$

with

$$\Delta_L \epsilon(i\xi) = \frac{2}{\pi} \int_0^{\omega_1} \frac{\omega \epsilon''_L(\omega)}{\omega^2 + \xi^2} d\omega$$

and

$$\Delta_H \epsilon(i\xi) = \frac{2}{\pi} \int_{\omega_2}^{\infty} \frac{\omega \epsilon''_H(\omega)}{\omega^2 + \xi^2} d\omega. \quad (A7)$$

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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