University of Groningen

Review of S. Shapiro's Varieties of Logic

Geerdink, L. M.; Dutilh Novaes, C.

Published in:
History and Philosophy of Logic

DOI:
[10.1080/01445340.2015.1064956](https://doi.org/10.1080/01445340.2015.1064956)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2015

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):
Geerdink, L. M., & Dutilh Novaes, C. (2015). Review of S. Shapiro's Varieties of Logic. *History and Philosophy of Logic*, 37(2), 194-196. <https://doi.org/10.1080/01445340.2015.1064956>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Varieties of Logic

L.M. Geerdink & C. Dutilh Novaes

To cite this article: L.M. Geerdink & C. Dutilh Novaes (2016) Varieties of Logic, History and Philosophy of Logic, 37:2, 194-196, DOI: [10.1080/01445340.2015.1064956](https://doi.org/10.1080/01445340.2015.1064956)

To link to this article: <https://doi.org/10.1080/01445340.2015.1064956>

Published online: 29 Jul 2015.

Submit your article to this journal [↗](#)

Article views: 964

View related articles [↗](#)

View Crossmark data [↗](#)

Book Reviews

S. SHAPIRO, *Varieties of Logic*. Oxford: Oxford University Press, 2014. i + 226 pp.
\$55.49, €50.93. ISBN 978-0-19-969652-9.

Reviewed by L.M. GEERDINK and C. DUTILH NOVAES, *Faculty of Philosophy, University of Groningen, Oude Boteringestraat 52, Groningen, GL 9712, The Netherlands*,
l.m.geerdink@rug.nl; c.dutilh.novaes@rug.nl¹

© 2015 L.M. Geerdink

Logical pluralism is the view according to which there is more than one correct (or adequate) logic. Its main competitor in the literature is logical monism – the view that there can be only One True Logic – but it is also opposed to logical nihilism – the view that there is *no* correct or adequate logic. (The latter view has so far not enjoyed much popularity, but it is starting to be considered more carefully, even if not yet endorsed, by philosophers such as Aaron Cortnoir and Gillian Russell.²) In effect, for much of the history of logic and its philosophy, logical monism has been the dominant position. In the early 2000s, J.C. Beall and Greg Restall launched a sustained defense of logical pluralism, in particular their 2006. Beall and Restall's offensive has sparked lively debates, so much so that logical pluralism has become one of the main topics within the philosophy of logic of the last decade. However, much of this literature has focused on coming to grips with what the position in fact amounts to, as obviously either a defense or a critique of logical pluralism would presuppose a sufficiently precise formulation of the thesis itself. And yet, it has proven surprisingly difficult to formulate the thesis adequately (as argued, *inter alia*, in Keefe 2014).

In his recent book *Varieties of Logic*, Stewart Shapiro's goals are to provide a much-needed deeper analysis of the thesis of logical pluralism – which (as he points out) can receive a number of equally legitimate formulations – and to defend a version of logical pluralism that he calls *eclectic pluralism*. To this end, he resorts to a range of key recent developments in the philosophy of language, in particular pertaining to the concepts of contextualism, relativism, and vagueness. Indeed, Shapiro claims that, at least in its most promising variants, logical pluralism amounts to a form of *relativism* (a view Beall and Restall had explicitly rejected, despite their reliance on the notion of cases): we say that a logic is correct or adequate only *relative* to suitably defined parameters. Once Shapiro establishes a more precise formulation of the thesis itself, he then goes on to argue in its favor, on the basis of the variety of logics used in mathematical practice (or so he claims). His main exhibits are: Heyting Arithmetic with Church's Thesis; Intuitionistic Analysis;

¹ We thank Rohan French for a detailed discussion of this review. We also wish to reciprocally thank Shawn Standefer for detailed discussions about the book.

² In presentations.

and Smooth Infinitesimal Analysis. These are all bona fide, viable mathematical theories which are rendered inconsistent (and thus trivial) with the addition of excluded middle, and are thus incompatible with classical logic. In all three cases, the underlying logic is intuitionistic.

Indeed, taking mathematical practice seriously for the philosophy of mathematics has been one of Shapiro's main concerns for decades (see *Shapiro 1985*). The main point of the book seems to be that adopting logical monism in favor of classical logic would entail dismissing as inconsistent and trivial a group of mathematical theories which nevertheless constitute valuable, legitimate mathematics; in other words, adopting logical monism would entail a significant departure from actual mathematical practice, which for Shapiro would be a highly undesirable result.

We now present a brief sketch of the contents of the book per chapter. After some preliminary remarks in the first chapter, Shapiro shows in the second chapter that there are several ways in which the thesis of logical pluralism could be formulated. He thereby disentangles different varieties of pluralism that are often conflated, adding much-needed clarity to the debates.

Shapiro's own position only becomes clear in the third chapter, where he argues that what counts as a correct logic is *relative to a mathematical structure or theory* – hence his conception of logical pluralism as a form of relativism. Taking his cue from Hilbert, Shapiro argues that any non-trivial axiomatization implicitly defines a mathematical theory. One of his main aims is 'to show how a wide variety of theories, studied by mathematicians whose credentials can hardly be challenged, are legitimate' (p. 38). Besides the three theories whose underlying logic is intuitionistic mentioned above, Shapiro briefly considers other non-classical logics, but recognizes that there may not be (for now at least) robust, sophisticated mathematical theories relying on these other logics.

In Chapters 4 and 5, Shapiro takes up the question whether 'logical consequence' has the same meaning across mathematical theories with different background logics. To address this question, Shapiro has to engage with an issue made famous by Quine; do logical constants have the same meaning across these theories, or do different theories simply talk past each other when purportedly disagreeing on the correct properties of logical constants?

Discussing this issue can be done through (at least) two routes. First, there is the *prima facie* acceptable idea that the *same* statement is, or is not, a theorem of two or more theories with different background logics. For instance, it seems to be the case that the law of the excluded middle is a theorem of classical logic but not of intuitionistic logic, that the theorems of Heyting Arithmetic are a proper subset of the theorems of Peano Arithmetic, and that the Fundamental Theorem of Calculus is provable in Smooth Infinitesimal Analysis. In these cases, one can think of the theories as sharing the same vocabulary, with the consequence relation being indexed relative to a theory. Thus, in this case, one has pluralism with respect to the consequence relation itself.

Second, if the meanings of logical constants change across logics, as argued by Quine, the various logics arise from the different meanings for the logical constants rather than from the consequence relation. In this case, one ends up with a monism with respect to the consequence relation, although Shapiro notes that the result is a 'toothless' monism because one has to give up the universal applicability of logic (p. 122).

Whether 'logical consequence' has the same meaning across theories therefore depends on which of these two *prima facie* acceptable viewpoints is adopted. Shapiro argues that whether the logical vocabulary has the same meaning across different theories is relative because he thinks the relation 'x has the same meaning as y' is itself vague and context-sensitive (as argued in Chapter 5). But if this relation is vague (and this might be disputed), whether this vagueness also gives rise to pluralism depends, of course, on one's theory of

vagueness. And so, to obtain the version of pluralism he is after, Shapiro needs to wheel in some kind of contextualist theory of vagueness, such as the one he himself developed in Shapiro 2006. This is perhaps the most contentious part of the book, and we suspect that critics will focus on this point.

In the last two chapters, Shapiro investigates the ideal of a unified foundation for mathematics and issues pertaining to meta-theoretic results. Although Shapiro agrees in Chapter 6 with Maddy 2007 that we may *want* to have a single unified (meta-)theory in which we can compare all mathematical theories, in practice, meta-theories used to compare mathematical theories are chosen on a case by case basis. We therefore do not need what we want, according to Shapiro.

Shapiro further discusses whether it matters which background logic our meta-theory uses when proving meta-theoretical results. He shows that it sometimes does, and it sometimes does not, and views this as a further argument in favor of his own view. It is indeed a problem for logical monists if they cannot prove important meta-theoretical results about what they take to be the One True Logic without invoking another logic. But on Shapiro's pluralist view, meta-theoretical results hold even if a different logic is required. Sometimes, we simply need to shift our perspective in order to see a result.

We could not possibly do justice to the sophistication and philosophical richness of this book within the confines of this review. Among other things, it offers sorely needed clarification of key concepts, or at least provides further tools to start making sense of them. We share Shapiro's worry that all too often 'the deep, interesting, and important matters get lost or confused in the terminology' (p. 6). If only because of this, we expect that *Varieties of Logic* will become the standard text on logical pluralism, and will likely set the agenda for debates on the topic for years to come. The main take-home methodological message seems to be: before entering a debate, make sure you know what its key terms mean, but in most (interesting) cases there is no unique meaning one could attribute to them. Indeed, philosophical concepts such as 'consequence', 'meaning', etc. are 'open texture' (a term Shapiro borrows from Waismann in Chapter 5); they do not have a fixed, unique meaning.

As for his more substantive positions, the book certainly contains enough contentious claims to generate further debate. One may wonder for example how to capture both the pluralism with respect to the consequence relation and the pluralism with respect to the logical vocabulary without resorting to Shapiro's (contentious?) theory of vagueness. In effect, given that he is fighting a number of battles on different fronts, at times treatments of certain issues are rather brief, perhaps too brief. (For example, the historical considerations offered in Chapter 1 are very sketchy, despite Shapiro's endorsement of the motto 'let's start from the very beginning, a very good place to start'.) Such brief treatments call for further discussion, and thus we can all look forward to the continuation of these debates on the basis of the helpful conceptual framework laid out by Shapiro.

References

- Beall, J. C. and G. Restall. 2006. *Logical Pluralism*, Oxford: Oxford University Press.
 Keefe, R. 2014. 'What logical pluralism cannot be', *Synthese*, **191**, 1375–90.
 Maddy, P. 2007. *Second Philosophy*, Oxford: Oxford University Press.
 Shapiro, S. 1985. 'Second-order languages and mathematical practice', *Journal of Symbolic Logic*, **50**, 714–42.
 Shapiro, S. 2006. *Vagueness in Context*, Oxford: Oxford University Press.

<http://dx.doi.org/10.1080/01445340.2015.1064956>