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Published in:
IEEE Transactions on Smart Grid

DOI:
[10.1109/TSG.2014.2318901](https://doi.org/10.1109/TSG.2014.2318901)

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Document Version
Final author's version (accepted by publisher, after peer review)

Publication date:
2014

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Larsen, G. K. H., van Foreest, N. D., & Scherpen, J. M. A. (2014). Distributed MPC Applied to a Network of Households With Micro-CHP and Heat Storage. *IEEE Transactions on Smart Grid*, 5(4), 2106-2114.
<https://doi.org/10.1109/TSG.2014.2318901>

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Distributed MPC applied to a Network of Households with micro-CHP and heat storage

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Abstract—This paper considers heat and power production from micro Combined Heat and Power systems and heat storage in a network of households. The goal is to balance the local heat demand and supply in combination with balancing the power supply and demand in the network. The on-off decisions of the local generators are done completely distributed based on local information and information exchange with a few neighbors in the network. This is achieved by using an information sharing model with a distributed model predictive control method based on dual-decomposition and sub-gradient iterations. Because of the binary nature of the decisions, a mixed integer quadratic problem is solved at each agent. The approach is tested with simulation, using realistic heat and power demand patterns. We conclude that the distributed control approach is suitable for embedding the distributed generation at a household level.

Index Terms—distributed model predictive control, smart energy management, distributed generation

I. INTRODUCTION

DISTRIBUTED power generation in the future smart grid, provides the potential for optimizing the efficiency of the energy chain. One advantage is that when the power is used in the local neighborhood of the generator, transmission losses in the power network are avoided. An interesting candidate for domestic power generation is the micro Combined Heat and Power (μ -CHP) system, which as the name states, produces heat and power at the same moment. Typically, μ -CHP systems run on gas, which makes them particularly interesting to install in households in countries like the Netherlands where the gas grid is dense [1]. The business potential is considered to be high, see e.g. [2] and [3], also for the μ -CHP technology considered in this paper.

In fact, in the north of The Netherlands, a demonstration project of a future energy-infrastructure called PowerMatching city is implemented with 25 households, see [4]. The connected households have smart appliances, including μ -CHP systems, that match their energy use in real time. Even though this demonstration project is generally perceived as a big success, it is also learning from some shortcomings, and thus improvements are considered. In particular, predictions of power and heat demand are not yet taken into account, as well as the coupling between heat and power, sometimes leading to a waste of heat production. This motivates to fully consider

the coupling between heat and power, and most importantly to consider an alternative method to match power supply and demand where predictions on the energy demand are included.

The energy output of the μ -CHP can easily be controlled, even though it is subject to several operational constraints, see e.g. [5]. In the literature, the μ -CHP unit is typically controlled according to heat-led or electricity-led strategies, meaning that the μ -CHP is turned on (off) according to the household's heat or power demand. However, in [6], it is stated that a combination of the two is the most economically efficient.

We add to the electricity-led literature with the following motivation: Our primary goal, is to cover the heat demand locally in the household. In this case, if the μ -CHP is controlled to follow the local heat demand, the electric power output fluctuates accordingly. Obviously, this may lead to a mismatch between local power supply and demand. However, by including storage in the network, we add flexibility to balance both heat and power. In particular, heat can be stored easily in water tanks in the household, which makes it easier to meet the heat demand. With the heat storage included, we can reverse the control problem in such a way that we control the power and store the heat. We then add the heat coverage problem as a constraint to the power control problem.

Electric power on the other hand, cannot be efficiently stored, but in the power network the power can easily be shared/traded amongst neighbors. This provides the opportunity to achieve a common goal, such as matching local power supply and demand real-time, at a network level. The problem we consider is therefore, how to coordinate the decisions made at household level regarding the μ -CHP in order to achieve such a power balance goal, and at the same time achieve heat balance?

Matching power in a large network, requires the control of a large number of agents. We therefore need a control strategy that scales well with the network. It is widely agreed that a centralized solution scheme for such an optimal control problem is too time consuming because of the computational complexity [7]. Therefore, efforts have been made suggesting scalable control methods in the Smart Grid setting. In [8], it is stated that common for the strategies used for device control at household level, is that there is one decision making agent present.

We however want to decide when to turn on or off a μ -CHP locally at a household level, only based on local information. To avoid a centralized structure, we propose an information network where each agent has local (imbalance) information about the system when they make their decisions. The agents in this virtual information network is an isolated subset of

This work is carried out as part of the Flexines project, financed by Koers Noord.

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the agents in the power grid. In a large network, the distance between suppliers and consumers is playing a role, and an agent only needs to exchange information directly with a subset of all agents in the information network according to the information structure. In our paper [9], this information sharing model was combined with a distributed Model Predictive Control (MPC) method [10], [11] to achieve a power balance. The method is based on dual-decomposition and sub-gradient iterations [12], [13]. MPC lets us include predictions [14], and the distributed method is expected to scale well because the computations are divided over the households. In fact, the complexity of the optimization problem per household is independent of the network size. However, due to the binary nature of the on-off decisions we must solve a Mixed Integer Quadratic Program [15].

In this paper, we combine our information sharing model with the adapted MPC algorithm as proposed in our paper [9]. However, a shortcoming in the previous work is that the heat output of the μ -CHP was not taken into consideration. Thus, when there is no domestic heat demand the heat is wasted and the μ -CHP is not operated optimally from an energy efficiency point of view. We assume that no heat can be dumped, and that we need to cover the local heat demand in the household in addition to optimize the on-off decisions regarding the μ -CHPs in order to cover the local electricity demand in the network. This means that the output of the μ -CHP is constrained by the heat demand. We use the model for a single μ -CHP and heat storage as presented in [5], and we focus on optimizing the integrated heat and power matching in a network of households with μ -CHP and heat storage. Therefore, we extend the analysis of [9] to include heat demand and dynamics from the heat storage at each household. This results in a completely local heat supply and demand balancing, and at the same time the power supply and demand is balanced in the network based on local decisions. We use realistic heat and power demand patterns generated by the method in [16], and we compare the total amount of resources used in the network in four different scenarios.

The rest of the paper is organized as follows: Section II models the device setup in a household and introduces the information network we propose for the network of households. Section III reviews the distributed MPC strategy to coordinate the power production in the network, while meeting local heat demand. Section IV presents simulation results of the network with realistic demand patterns. Finally, Section V concludes the paper.

II. SYSTEM DESCRIPTION

Here we first describe the agent model, being a household with μ -CHP and heat storage. This model specifies the technical constraints at each agent. Then we introduce the information sharing network of such households, which will help to achieve a balance of power in the network.

A. The Household

The setup of an agent is shown in Fig. 1. The agent has a μ -CHP fuelled on natural gas, and the power output is connected

to the power network while the heat output is stored in a hot water storage present in the house. The μ -CHP consists of a prime mover whose power and heat output is coupled, and an auxiliary burner which only has a heat output. Here we choose the prime mover to be a PEMFC¹, because of the high electric efficiency of this technology, see Table I and [5]. This means that we have high flexibility to control the power output and at the same time stay within given heat constraints. The auxiliary burner produces heat like a conventional boiler, and also stores the heat in the hot water storage. The auxiliary burner is part of the control problem because if the power demand is low and the heat demand is high, we prefer that the heat demand is covered by the auxiliary burner. In other words, we want to avoid a net power production in the network due to high heat demand. Further, the heat storage has to meet the households demand for hot water and central heating.

The μ -CHP can be modeled with different degrees of technical detail. Here we are interested in a model capturing the main features of the μ -CHP suitable for control of the electric power and heat output. Such a model of a prime mover was presented in [5] for demand response in one household. We use the same model for the μ -CHPs.

Each agent i has an electric power demand $d_i(k) \in \mathbb{R}_+$, and a heat demand $h_{d,i}(k) \in \mathbb{R}_+$ that can be measured at each discrete time-step k , where the heat demand is the aggregated space heating and domestic hot water needs. The prime mover produces electric power $p_i(k) \in \mathbb{R}_-$ with an efficiency $\eta_{p,i}$ and heat $h_{p,i}(k) \in \mathbb{R}_+$ with an efficiency $\eta_{h,i}$ when it is on. The efficiencies are approximated by a constant over the energy output, even if the efficiency can be slightly higher for a prime mover at low loads [5]. The power and heat output from the prime mover are therefore coupled to the power output by

$$h_{c,i}(k) = -\eta_i p_i(k), \quad (1)$$

where $\eta_i = \eta_{h,i}/\eta_{p,i}$, and the minus sign is due to our choice that the power production $p_i(k)$ is opposite in sign to the power demand $d_i(k)$. There is a minimum and a maximum power output $p_{\min,i}, p_{\max,i} \in \mathbb{R}_+$ that the μ -CHP can deliver. These constraints are given by

$$p_i(k) \in \{0\} \cup [-p_{\min,i}, -p_{\max,i}]. \quad (2)$$

The auxiliary burner provides additional heat $h_{a,i}(k) \in \mathbb{R}_+$ at times of high heat demand, and its range is given by

$$h_{a,i}(k) \in \{0\} \cup [h_{a,\min,i}, h_{a,\max,i}]. \quad (3)$$

This heat production has an efficiency $\eta_{a,i}$. The non-convexity due to the gap between 0 and $p_{\min,i}$, and 0 and $h_{a,\min,i}$, has consequences for the control algorithms that can be applied.

We assume that there are no thermal losses in the conversion and storage system, therefore, the dynamics of the heat storage level $h_{s,i}(k) \in \mathbb{R}_+$ is given by

$$h_{s,i}(k+1) = h_{s,i}(k) + h_{p,i}(k) + h_{a,i}(k) - h_{d,i}(k), \quad (4)$$

as in [5].

The maximum and minimum heat storage levels $h_{\min,i}, h_{\max,i} \in \mathbb{R}_+$ of the hot water tank depend on

¹Proton Exchange Membrane fuel cell

Fig. 1. This setup is taken from [3], and shows the energy flow in one household.

the mass m of the water, the specific heat constant c_p and the difference in minimum (maximum) temperature of the water and the room-temperature $\Delta T_{\min(\max),i}$, and is given by

$$h_{\min(\max),i} = m_i c_p \Delta T_{\min(\max),i}, \quad (5)$$

where $c_p = 4.18 \text{ kJ/(kg}\cdot\text{K)}$. We assume that no heat can be dumped, and the heat storage level needs to stay within the limits

$$h_{\min,i} \leq h_{s,i}(k) \leq h_{\max,i}, \quad (6)$$

at all times k .

At last, the prime mover has a relatively long start-up time $T_{\text{start},i}$ when it has been shut down. During this period an amount $g_{r,i}$ of gas is consumed, but no electricity and a neglectable amount of heat are produced. If the prime mover is frequently shut down, this could limit the opportunity to control the power output. A choice can therefore be to keep the prime mover at an idle on-state, in which state the prime mover can resume production at any time. However, in an idle on-state the prime mover consumes the minimum gas $g_{r,i}$ required to keep the reforming unit at the operating temperature, even if no power and heat are produced.

B. The Information Network

The agent's operation of the μ -CHP will influence the power balance in the power network. We propose an information sharing model in a network of n agents connected through an information network, which does not need to have the same structure as the corresponding power network. This will also help us to implement a completely distributed control strategy in the network. Here we repeat the network modeling considerations presented in [9], in order to be as self contained as possible.

We call the difference between power production $p_i(k) \in \mathbb{R}_-$ and demand $d_i(k) \in \mathbb{R}_+$ at an agent i , the real imbalance at agent i . Thus, we define the demand to be positive and the production to be negative.

In order for an agent to contribute to the local balancing of power by selling or buying power from neighbors, the agent requires some information about the overall power situation in the network. However, to avoid a centralized structure we introduce the state $x_i(k)$ which represents information about the imbalance of agent i , and depends also on information about imbalance of neighboring agents. This state is to be distinguished from the real imbalance at agent i . We introduce a virtual information network so that each agent has local information about the system when making the decision. The topology of the information network specifies which subset of

agents an agent i exchanges information with. Agent i 's set of information neighbors N_i is given by

$$N_i \subseteq \{1, \dots, n\} \setminus \{i\}, \quad (7)$$

where the agent itself is excluded.

We include the chosen information topology in our dynamic model of the information flow by adjusting information weights A_{ij} in the coupling between the agents' notion of imbalance in the system. By introducing the change in production

$$u_i(k) = p_i(k+1) - p_i(k), \quad (8)$$

the model for the *imbalance information* $x_i(k)$ at agent i is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \quad (9)$$

for all $i = 1, \dots, n$, where $w_i(k) = d_i(k) - d_i(k-1)$ represents the change in power demand at agent i , A_{ii} weighs the power imbalance information of agent i itself, and A_{ij} weighs the information received from neighbors $j \in N_i$. We choose the initial value of $x_i(0)$ to be the real physical power imbalance of agent i at the initial time, i.e. $x_i(0) = d_i(0) + p_i(0)$ and $w_i(0) = 0$. As time evolves, demand changes are spread through the network through the neighboring agents N_i . In this way close-by information-exchange agents can react faster to a change in demand $w_i(k)$ than information exchange agents further away. In Fig. 2, the solid lines represent one possible information network where each agent has one information neighbor $N_i = \{i-1\}$, and the self loops represent that agents take their own imbalance information into account as well.

In addition to the information network, all agents $i = 1, \dots, n$ are physically connected to the power grid, which is illustrated by the dashed lines in Fig. 2. This means that the power demand and production at each agent will affect the overall power imbalance in the system $\sum_{i=1}^n \tilde{x}_i(k)$. We relate the imbalance information to the physical power imbalance by proper choices of the weights in the *information matrix* $A \in \mathbb{R}^{n \times n}$, where the weights A_{ii}, A_{ij} in (9) are the elements of A . We impose four restrictions for how to choose the

Fig. 2. Three agents which are both connected in the power grid, represented by dashed lines, and in one possible information network, represented by solid lines.

weights in A :

- R1 $A_{ij} \neq 0$ if and only if information is exchanged from agent j to agent i .
- R2 All weights are non-negative: $A_{ij} \geq 0$, $i, j = 1, \dots, n$.
- R3 All columns sum up equal to one: $\sum_{i=1}^n A_{ij} = 1$, $j = 1, \dots, n$.
- R4 The graph corresponding to information matrix A is strongly connected [17]. Loosely said: there is a path in the communication graph from any agent to any agent.

See [9] for details. The current power network can be captured in the information matrix A , as information neighbors are physically virtual neighbors. We assume that the overall power shortage (excess) in the information network is imported from (exported to) an external network, and we will formulate our objective with the aim to minimize this exchange.

III. DISTRIBUTED DECISION MAKING

Here we first define the centralized MPC problem for the network, and then we review the distributed MPC method that we use to solve the network control problem completely distributed.

A. Model Predictive Control of the network of households

Due to the many operational constraints on the μ -CHP presented in Section II-A, we will solve the optimal control problem using MPC, which means that we optimize a control problem with finite horizon T_{pred} at each time step k , see e.g. [14]. The network goal will be expressed by means of an objective function, and local forecast about future energy demand can be taken into account when the control actions are determined. We also include prediction models based on the agents' and network dynamics and constraints presented in Section II-A and Section II-B. This means that we anticipate on how each agents' change in power demand and production affects the network power balance over the horizon, while explicitly taking into account the heat and operational constraints from the μ -CHP. New measurements of power demand, heat demand and heat storage levels are included as initial conditions at each MPC cycle. This corrects for inaccuracies in the models, and energy forecasts.

In order to include the non-convex constraints (2) and (3) in our MPC problem, we introduce binary decision variables $r_i(k)$ and $r_{a,i}(k)$, as was done for a single household in [18]. These variables are defined by

$$\begin{aligned} r_i(k) &= \begin{cases} 1 & \text{if the } \mu\text{-CHP is on,} \\ 0 & \text{otherwise,} \end{cases} \\ r_{a,i}(k) &= \begin{cases} 1 & \text{if the auxiliary burner is on,} \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (10)$$

and allow us to replace (2) and (3) by

$$\begin{aligned} -p_{\min,i} \cdot r_i(k) &\geq p_i(k) \geq -p_{\max,i} \cdot r_i(k), \\ h_{a,\min,i} \cdot r_{a,i}(k) &\leq h_{a,i}(k) \leq h_{a,\max,i} \cdot r_{a,i}(k). \end{aligned} \quad (11)$$

In that case that the μ -CHP is completely off before starting up again, as described in the end of Section II-A, we also need to include the start-up time $T_{\text{start},i}$ when the μ -CHP is on, but does not produce power or heat. We introduce two new binary variables to indicate whether the primary mover is still in the start-up phase, or is shut down at a given time-step k

$$\begin{aligned} a_{up,i}(k) &= \begin{cases} 1 & \text{if primary mover starts up at } k, \\ 0 & \text{otherwise,} \end{cases} \\ a_{down,i}(k) &= \begin{cases} 1 & \text{if primary mover shuts down at } k, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (12)$$

with relations

$$\begin{aligned} r_i(k) - r_i(k-1) &= a_{up,i}(k) + a_{down,i}(k), \\ a_{up,i}(k) + a_{down,i}(k) &\leq 1, \\ r_i(k+p) &\geq a_{up,i}(k), \quad p = 0, \dots, t_{\text{start},i}(k), \end{aligned} \quad (13)$$

where $t_{\text{start},i}(k)$ is a counter that is decreased by 1 each time-step k after the μ -CHP was turned on, and $t_{\text{start},i}(k)$ is reset to $T_{\text{start},i}$ when $a_{down,i}(k) = 0$. Relations (13) ensure proper behavior of the μ -CHP.

As we have seen, the agent has different choices for how to cover its energy need. It can get power from the network, or from its μ -CHP, and it can get heat from the primary mover or the auxiliary burner. We therefore specify the objective function, which will specify the control goal. The global control goal is to minimize the overall power imbalance in the network, while keeping the appropriate temperature in the local heat storage. For this purpose we define the local objective function

$$V_i(x_i(k), x_i(k), h_{a,i}(k)) = Q_{ii}x_i^2(k) + R_{ii}u_i^2(k) + H_{ii}\Delta h_{a,i}^2(k), \quad (14)$$

where Q_{ii}, R_{ii}, H_{ii} weigh the relative importance of the imbalance information, change in power production and change in auxiliary heat output respectively. In addition to minimizing the imbalance and change in power production, we also included the heat output of the auxiliary burner in the objective. This is done to make sure the auxiliary burner is only on when it is needed to meet the heat storage constraints, and we choose $H_{ii} \ll Q_{ii}$. We use hat-notation to indicate predictions of the variables presented in Section II, and minimize the following global objective

$$\sum_{\tau=k}^{k+T_{\text{pred}}} \sum_{i=1}^n V_i(\hat{x}_i(\tau), \hat{x}_i(\tau), \hat{h}_{a,i}(\tau)), \quad (15)$$

subject to the prediction models of (1), (4), (6), (9), (10), (11), (12), and (13) for all $i = 1, \dots, n$, and $\tau = k, \dots, k + T_{\text{pred}}$.

Solving (15) requires one central controller that has full information about states, measurements and forecasts of all agents in the network. However, the decision to turn on or off the primary and auxiliary burner in such a centralized way does not scale well [9]. The decisions will instead be taken completely distributed only based on local information, as presented next.

B. Distributed Model Predictive Control of the network

The minimization (15) can be approximated by dual decomposition and sub-gradient iterations, as explained in [10], [9]. The main idea is that the neighbor influence in (9), is replaced by a local guess $v_i(k)$ constrained by

$$v_i(k) = \sum_{j \in N_i} A_{ij}x_j(k), \quad (16)$$

which decomposes the state equation (9)

$$x_i(k+1) = A_{ii}x_i(k) + v_i(k) + u_i(k) + w_i(k). \quad (17)$$

Then the constraints (16) are moved to the objective function by standard Lagrangian relaxation, which introduces the Lagrangian multipliers $\lambda_i(k)$ related to the constraints (16). The

Result: Find $p_i(k), h_{p,i}(k), h_{a,i}(k)$ at each cycle k of the distributed MPC method

```

for  $k = k, \dots, T_{sim}$  do
  measure  $p_i(k), x_i(k), h_{s,i}(k), w_i(k)$ ;
  while  $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1}(\tau)| > \epsilon$  do
    for  $i = 1, \dots, n$  do
      minimize (18);
    end
    exchange  $\hat{x}_i(\tau)$ ;
    for  $i = 1, \dots, n$  do
      sub-gradient update (19);
    end
    exchange  $\hat{\lambda}_{i,r}(\tau)$ ;
  end
  implement  $p_i(k), h_{p,i}(k), h_{a,i}(k)$  to the system;
   $r = r + 1$ 
end

```

Algorithm 1: Distributed Model Predictive Control

complete decomposition is achieved by including sub-gradient iterations.

In Algorithm 1 the distributed MPC method is sketched. Given a sequence $\{\hat{\lambda}_i(\tau)\}_{\tau=k}^{k+T_{\text{pred}}}$, each agent $i = 1, \dots, n$ solves a local control problem

$$\text{minimize } V_{\text{agent}}^i, \quad (18)$$

subject to prediction models of (1), (4), (6), (17), (10), (11), (12), and (13) for all $\tau = k, \dots, k + T_{\text{pred}}$, and $V_{\text{agent}}^i = \sum_{\tau=k}^{k+T_{\text{pred}}} \left(V_i(\hat{x}_i(\tau), \hat{x}_i(\tau), \hat{h}_{a,i}(\tau)) + \hat{\lambda}_i(\tau) \hat{v}_i(\tau) - \sum_{j \in N_i} \hat{\lambda}_j(\tau) A_{ji} \hat{x}_i(\tau) \right)$ in order to find $\{\hat{p}_i(\tau)\}_{\tau=k}^{k+T_{\text{pred}}}$ and $\{\hat{h}_{a,i}(\tau)\}_{\tau=k}^{k+T_{\text{pred}}}$. The solution of (18) is found using a mixed integer quadratic program, see [5],[15]. Then $\{\hat{\lambda}_i(\tau)\}_{\tau=k}^{k+T_{\text{pred}}}$ is updated according to

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r} [\hat{v}_{i,r}(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_{j,r}(\tau)], \quad (19)$$

where r labels the sub-gradient iteration and $\gamma_{i,r}$ is the gradient step size, see e.g. [19]. The Lagrangian multipliers are often viewed as price signals. Agent i needs no information from neighbours in order to estimate $\hat{v}_i(\tau)$ while solving (18). However, the Lagrangian multipliers $\hat{\lambda}_{j,r}(\tau)$ that the agent receives from its neighbours, are inputs to the optimization problem and will influence the value of $\hat{v}_i(\tau)$ to converge to the true neighbour influence after some iterations r . The algorithm terminates when the update of the Lagrangian multipliers stays within a bound ϵ . Notice that a complete synchronization of the agents' clocks is an underlying assumption in Algorithm 1, i.e. an agent can only move to the next time-step when all agents have completed their computations. The physical implementation of the distributed MPC algorithm is discussed in detail in our paper [20].

For convex problems the solution of Algorithm 1 converges to the solution of the centralized problem (15). However, due to the non-convex constraint (2) and (3), modeled by including binary variables (10), such a convergence cannot

Fig. 3. The hot tap water $h_1(k)$, space heating $h_2(k)$, and electric power demand $d(k)$ of one household (right) and the sum of 25 households (left). In particular the tap water demand has large spikes distributed over the day. The aggregated plot shows that there is a peak in energy demand around $k = 250$ minutes.

be guaranteed. In fact, we expect situations to occur where the binary variables (10) oscillate between zero and one. This means that the update of the Lagrangian multiplier, at each iteration r , changes the solution of the optimization problem from $r_i(k) = 1$ to $r_i(k) = 0$ and back. In this situation, it is not clear what is the optimal state of the μ -CHP. We deal with this problem by fixing the binary variable as explained in Remark 1.

Remark 1. In order to use Algorithm 1 for our non-convex problem, we will fix the binary inputs (10) once a non-converging sequence of (18) and (19) is detected. The remaining overall optimization problem will then become convex, since the gap between zero and the minimum value in (2) or (3) are removed, and the values of the other variables will converge to values that are optimal given the fixed binary values.

IV. SIMULATIONS

Here we use the distributed MPC algorithm, as described in Section III, to control the network of households with μ -CHP and heat storage, described in Section II. We compare the solution of Algorithm 1 to the centralized solution (15).

A. Simulation Setup

First we describe the modeling parameters and simulation scenarios, then we turn to describe the parameters of the algorithm.

We model a circular information topology, where all agents weigh themselves by a weight 0.8, and two neighbors by a weight 0.1. Thus, the information matrix A is tri-diagonal with 0.8 on the diagonal.

We use realistic power demand and corresponding heat demand patterns [16]. Fig. 3 shows the demand patterns for a network of 25 households, which is the size of the PowerMatching city mentioned in Section I. The patterns are from a November month when the heat demand is high, and the resolution is one minute. Notice that in the simulations each household has a unique pattern based on five different household profiles. Fig. 3 also shows a zoom in on a single household with a low energy demand profile.

TABLE I
PARAMETERS FOR THE μ -CHP AND WATER STORAGE. THE VALUES ARE
BASED ON [5].

Parameter	Value	Unit
η_p, η_h	0.3, 0.7	-
$p_{\min}, p_{\max}$	0.1, 3.0	kW
p_{ramp}	0.15	kW/min
t_{start}	45	min
g_r	1	kW
η_a	1	-
$h_{a,\min}, h_{a,\max}$	4.0, 20.0	kW
m	200	1
$T_{\min}, T_{\max}$	55, 80	$^{\circ}\text{C}$

We see in Fig. 3 that the power demand per household has fluctuations of the order 1 kW, the space heating demand is smooth and of the order 2.5 kW for the large households, while the hot tap water demand has short spikes with demand as high as 20 kW. All patterns show a peak around $k = 250$ minutes, and for the heating patterns there is also a smaller peak around $k = 550$ minutes.

The parameters for the μ -CHP and heat storage model presented in Section II-A are given in Table I. The primary mover produces 70% heat and 30% power, and the auxiliary burner produce 100% heat. At the beginning of the simulation all μ -CHPs are turned off, $p_i(k) = h_{p,i}(k) = h_{a,i}(k) = 0$ kW, and the temperature of the heat storages are 60°C .

We compare the distributed and central solutions of the following four cases;

- 1) no μ -CHPs are installed and the electricity demand is imported from the network, while the heat demand is covered by boilers with 100% efficiency,
- 2) all households have a μ -CHP, but the auxiliary burner is not considered,
- 3) as Case 2, but only every third house has a μ -CHP and the households without a μ -CHP can get power from their neighbors and heat from a boiler,
- 4) all households have a μ -CHP and the auxiliary burners are considered in the control problem.

For each of these cases we report; 1: the total resources used for generating imported and local power, heating the reforming unit, and producing local heat, 2: total running time of the optimization per time-step k , 3: average number of sub-gradient iterations r per time-step k , and 4: the objective V given by

$$V = \sum_{k=1}^{T_{\text{sim}}} \sum_{i=1}^n [x_i^2(k) + u_i^2(k)]. \quad (20)$$

The gas consumption of the prime mover $g_{p,i}(k)$ and the auxiliary burner $g_{a,i}(k)$ are modeled by

$$g_{p,i}(k) = -p_i(k)/\eta_{p,i}, \quad (21)$$

$$g_{a,i}(k) = h_{a,i}(k)/\eta_{a,i}, \quad (22)$$

and in order to calculate the resources used to generate the imported power from external parties an efficiency of 45% is assumed.

As motivated in the Introduction, we aim for a real-time response of the μ -CHP system. Therefore, we include near-term

predictions in the distributed MPC algorithm. The resolution of the demand patterns and the algorithm are one minute, and the prediction horizon T_{pred} is five time-steps. A prediction horizon of five minutes is longer than the response time of the μ -CHP, since a PEMFC fuel cell has a very fast response (within one time-step). A longer prediction horizon will make the computations slower, while by simulation we verify that the on-off decision will not be influenced. We assume that each household can predict its demand patterns over the five time-steps. This means that $\hat{w}_i(\tau) = w_i(\tau)$, and $\hat{h}_{d,i}(\tau) = h_{d,i}(\tau)$ for $\tau = k, \dots, k+5$.

The implementation is done in Python with a mixed integer solver from Gurobi [21]. For the distributed algorithm we set the step size in (19) to be $\gamma_{i,r} = \frac{0.01}{\sqrt{r}}$, where r is the iteration number. These values are based on simulations. This might not be the optimal strategy of updating the step size in order to converge fast. However, it suits to prove the concept of the distributed implementation. The algorithm is terminated when all distances between the previous and current Lagrangian multiplier satisfy $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1}(\tau)| < 0.003$ for all $i = 1, \dots, n$ and $\tau = k, \dots, k+T_{\text{pred}}$. For a convex problem this distance can be decreased to get the distributed solution arbitrary close to the centralized solution. Other possible ways to terminate the algorithm could be to choose a fixed number of sub-gradient iterations r , or specify a criterion for the desired degree of sub-optimality of the solution compared to the centralized solution as presented in [10].

Since we use the algorithm which is valid for convex problems for a integer problem, we need to implement Remark 1 in order for the algorithm to always converge and terminate. Therefore, when we detect that a state $\hat{r}_i(\tau)$ switches six times from the on state to the off state during the sub-gradient iteration, the binary variables $\hat{r}_i(\tau)$ are fixed to its latest value. When an integer is fixed it means that the new problem is changed compared to the original problem.

B. Simulation Results

The simulation time is half a day, $T_{\text{sim}} = 720$ time steps. Table II shows the results for 10 households. In Case 1, the total amount of resources needed to cover the net energy demand is 351 kWh. In Case 2, the total amount of resources needed to cover the same energy demand is increased by 9%. The reason for this can be seen in Fig. 4. The network produces too much power, because the μ -CHP has to operate at full capacity due to the large heat demand. Notice that excess production is particularly high in the period $k = 200 - 400$ minutes, which is correlated with the peak in the heat demand in Fig. 3. In fact, the lower heat storage constraint is even violated around 90 minutes per household.

We see in Case 2, that the central and distributed algorithm performs similar with respect to the use of resources, i.e. the resource use is 384 kWh and 385 kWh. A disadvantage of the distributed algorithm compared to the centralized is that the objective (20) is twice as high, but an advantage is that it is about 10 times faster than the centralized solution. An important observation from Table III with respect to this advantage, is that when the network is increased from 10 till

TABLE II
SIMULATION RESULTS FOR HALF A DAY, $n = 10$.

Case	Method	V	Resources	Time	Itr
1	-	-	351 kWh	-	-
2	Cent	1.30 kWh ²	384 kWh	0.022 s	-
	Dist	2.34 kWh ²	385 kWh	0.002 s	14.7
3	Cent	0.09 kWh ²	320 kWh	0.013 s	-
	Dist	1.03 kWh ²	330 kWh	0.001 s	9.8
4	Cent	0.02 kWh ²	296 kWh	0.474 s	-
	Dist	0.22 kWh ²	328 kWh	0.012 s	22.2

TABLE III
SIMULATION RESULTS FOR HALF A DAY, $n = 25$.

Case	Method	V	Resources	Time	Itr
1	-	-	936 kWh	-	-
2	Cent	3.28 kWh ²	983 kWh	0.063 s	-
	Dist	6.04 kWh ²	975 kWh	0.002 s	31.8
3	Cent	0.38 kWh ²	842 kWh	0.031 s	-
	Dist	2.73 kWh ²	840 kWh	0.001 s	21.6
4	Cent	-	-	-	-
	Dist	0.74 kWh ²	857 kWh	0.003 s	27.1

25 households, the distributed algorithm is 30 times faster than the centralized solution for Case 2.

The same observations with respect to resources, objective and running time holds for Case 3. Here every third household has a μ -CHP, and the rest of the households can buy power from their neighbors and produce heat with a conventional boiler. Fig. 5 shows that the network can cover its power demand in this case, but the lower heat storage level is violated for the households with a μ -CHP during 40 minutes for each of these households. However, Table II shows that almost 9% is saved on the resources.

Fig. 6 shows that in Case 4, when all households have a μ -CHP with an auxiliary burner and heat storage, the power production can be well matched with the power demand in the network. At the same time, the heat demand is covered. Covering the tap water demand is only possible when using the auxiliary burner, even with the presence of heat storage. The distributed solution has a 7% saving compared to Case 1, while the centralized solution shows a 15% saving. We expect that the distributed solution can get closer to the centralized solution by tuning (decreasing the value of) the parameters $\gamma_{i,r}$ and ϵ in the sub-gradient iterations. However, due to the non-convexities of the problem we are not guaranteed that the distributed solution will converge to the centralized solution.

In Case 4, the number of binary variables are doubled, i.e. both $r_i(k)$ and $r_{a,i}(k)$. This has consequences for the scalability of the centralised implementation. Table III shows that for 25 households, no solution could be obtained within 1 minute for the centralized implementation, while the distributed implementation still finds a solution within 0.003 seconds for Case 4. This is promising for the scalability of the distributed algorithm.

V. DISCUSSION

In this paper, we modelled a network of households with μ -CHPs and heat storage. The model has the freedom to include different production capacities and heat storage sizes at each

Fig. 4. Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 2, where all households have a μ -CHP but the auxiliary burner is not included. There is a large excess of power production in the network, in particular between $k = 200$ and $k = 400$ minutes, when the heat storage levels hit its lower boundary and the μ -CHP is forced to produce at full capacity.

Fig. 5. Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 3, where the difference from Fig. 4 only every third house has a μ -CHP. We see that there is now enough generated power to cover the demand in the network. However, the μ -CHP can not provide enough heat for the local consumption. On average, the heat storage level lower boundary is violated 40 minutes per household with a μ -CHP.

Fig. 6. Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 4, where all households have a μ -CHP and the auxiliary burner is included. We see in the left plot that the network can balance the net power production and consumption, and at the same time the heat storage levels stay within its boundary at all times. By tuning the step-size and stopping criterion of the sub-gradient iterations, the distributed implementation shown in the right plot can also perform better.

household. By including information sharing dynamics, we achieve a balance between the power supply and demand in the network at the same time as the heat demand at household level is satisfied. The decision making is done by each household, and the decisions are coordinated by sub-gradient iterations as described in the distributed MPC approach presented in Section III-B. The algorithm is guaranteed to converge to

the centralised solution for convex problems. However, the output from the μ -CHP and auxiliary burner are non-convex, meaning that our control problem is not convex. Based on our simulation results, we see that when we implement Remark 1 in the algorithm, the distributed solution is comparable to the centralised solution. We compare the distributed and the centralized implementation, and study the influence on the resources needed to match supply and demand.

The distributed method can be implemented in a large network, since the number of optimisation variables per node is constant. Increasing the network, however, slowly increases the number of sub-gradient iterations needed to match the stopping criterion. The centralised problem can not be solved for large problems, in particular due to the large number of binary variables. Already with 25 households we could not solve Case 4.

Based on the simulation results, we conclude that when embedding the μ -CHPs and heat storage in the network, the network as a total uses less resources than when no μ -CHPs are present, and the auxiliary burner is needed to stay within the heat limits. The distributed control approach is suitable, as it scales well and helps balance the network.

We considered a circular information topology, while we could have considered any topology as long as the restrictions for the information matrix A given in Section II-B are met. How the steady state solution and the transient response of the system are influenced by the A matrix, is not considered in this paper. However, in the case that not all households have a μ -CHP, the households with a generator should be distributed evenly over the information network, since the information distance between two households determines how fast information travels from one node to the other. We also would like to note that the weights and connections in this A matrix can be updated from one MPC cycle to the next without any problem.

In the current market, several types of stakeholders are responsible for, or are exploring business opportunities at various levels in the power system. Two examples are the balance responsible parties, which commercially balances electricity supply and demand, and the Distribution System Operator (DSO), which is responsible for the low voltage and the medium voltage distribution grid.

One question is then, how do we organize the information sharing network, i.e. design the information matrix A in (9), with respect to the existing market parties? Should the information sharing network be formed by the free market, optimized with respect to technical performance, or should each energy company, who today contracts the small consumers, form independent information sharing networks? One possibility is to let the DSO organize the information network. This way, the flexibility can be coordinated to benefit the distribution network. The DSO can pay the end-users for the flexibility they provide, and it is free to set a target value for the power balance at which the network will operate.

Another option is presented in [22], where aggregators are introduced. The aggregator trades the flexibility services on behalf of the end-users connected in the grid. Each aggregator belongs to a balance responsible party, and the target value

for the objective function is a time varying reference signal instead of zero as was assumed in this paper.

The current study has examined distributed coordination of power production in a network with μ -CHPs and heat storage elements. However, when the price fluctuations are transparent for the end-user, we have reasons to believe that we will also see flexibility in the demand, see e.g. [23]. In [20] the modelling of the power demand side together with the distributed control approach is studied. It is interesting to study the combined problem of distributed power production and demand side control.

ACKNOWLEDGMENT

The authors would like to thank The Energy research Center of The Netherlands (ECN) for providing the heat and power demand patterns, and Auke-Dirk Pietersma for his help in finalizing this piece.

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