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Stable, nonsingular bouncing universe with only a scalar mode

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We study a class of higher derivative, nonlocal gravity with a cosmological constant which admits homogeneous and isotropic nonsingular bouncing universes in the absence of matter. We show that, at the linearized level around the bounce, it is possible to constrain the nonlocal form factor such that there is only a scalar propagating degree of freedom, and no vector or tensor modes. The scalar mode can be made free from perturbative ghost instabilities and has oscillatory and bounded evolution across the bounce.

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I. INTRODUCTION

Bouncing cosmologies have been an attractive paradigm for resolving the big bang singularity, which is inevitable in general relativity (GR) under reasonable assumptions [1]. They have been studied in various approaches to quantum gravity, like string theory [2,3] and loop quantum gravity [4]. Bouncing solutions are typically found by modifying the matter sector by adding fields which violate the null energy condition (NEC) and/or by modifying GR, for instance, by adding higher derivative terms to the Einstein-Hilbert action [1,5–10]. NEC violation and higher derivative actions generally imply sicknesses in the form of ghost (fields with wrong-sign kinetic terms) and gradient instabilities. Another problem with bouncing cosmologies is the growth of vector (metric and matter) modes with time during the contraction phase, thereby invalidating perturbation theory [7]. In some bouncing models [11], and even in power-law inflation [12], scalar modes can also grow. Of central importance is then the question of finding a healthy, nonsingular bouncing scenario with bounded amplitudes of perturbations.

In the context of higher derivative modified gravity, quadratic scalar curvature ($R + R^2$) is an interesting model to consider when hunting for nonsingular solutions [13,14]. After adding a cosmological constant to $R + R^2$ theory, one can indeed obtain bouncing solutions, which nevertheless are plagued by the presence of negative radiation energy density $\rho < 0$. Extending the action by adding an arbitrary number of higher derivative terms typically leads to ghost degrees of freedom around, for instance, flat space and (anti-)de Sitter (A)dS. On the other hand, an infinite covariant derivative, nonlocal extension can potentially resolve the problem of ghosts by choosing a suitable function that captures all order derivative terms in the action. In this nonlocal quadratic scalar curvature model, bouncing solutions were found with $\rho > 0$ around spatially flat, homogeneous, and isotropic backgrounds [15–20]. The stability of scalar perturbations around these solutions

in the presence of radiation was studied in [16,21]. One can deduce that the presence of radiation implies growth of vector perturbations during the contraction phase [22].

Thus far, infinite derivative gravity theories have been constructed to be ghost-free around flat space [18,23–25] and (A)dS [26,27]. In an arbitrary background, inevitable mixing of scalar-vector-tensor (SVT) modes makes the task of removing ghosts challenging, but one can still remove them around specific nonmaximally symmetric backgrounds [28].

In this paper, we find a class of nonlocal $R + R\mathcal{F}R$ theory with a cosmological constant which is ghost-free around homogeneous and isotropic bouncing cosmological backgrounds. $\mathcal{F}$ is the nonlocal term (discussed below) essential for avoiding ghosts. To the best of our knowledge, we show for the first time that a class of higher derivative gravity can give stable, nonsingular bouncing scenarios, in the absence of matter and without any vector or tensor modes at the linearized level around a time dependent background. Only one scalar mode survives which can be made ghost-free and exhibits bounded, oscillatory evolution across the bounce.

The paper is organized as follows: In Sec. II, we provide the nonlocal action used throughout the paper, discuss the equations of motion, and obtain conditions on the background geometry following from assuming a simplifying ansatz for the background Ricci scalar. In Sec. III, we study perturbations both at the linearized level of the equations of motion and at the quadratic level of the action around the background chosen in the previous section. In particular, we show how the theory propagates only 1 scalar degree of freedom and no vector or tensor modes, and how the kinetic operator can be chosen such that there are no extra/ghost modes. In Sec. IV, we choose a homogeneous and isotropic cosmological bouncing background and study the time evolution of the lone scalar mode, showing its bounded and oscillatory behavior. We conclude in Sec. V. The metric signature is $(-+++)$, overbars on quantities like $\bar{R}$ indicate their background value, and $\hbar = c = 1$.

II. ACTION

Nonlocal generalization of $R + R^2$ by adding an infinite series of covariant derivatives reads [15]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + R \mathcal{F}(\square) R - \Lambda \right], \quad (1)$$

where Λ is the cosmological constant and the form factor $\mathcal{F}(\square)$ is an arbitrary analytic function of the d'Alembertian, $\square = g_{\mu\nu} \nabla^\mu \nabla^\nu$, containing all orders of $\square$. It has the power series expansion $\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square^n$, where f_n are dimensionless coefficients, dimensionless $\square_s = \square/M_s^2$, and $M_s (< M_p \sim 10^{18} \text{ GeV})$ is a new high energy scale of nonlocality below the Planck scale M_p . The equations of motion (EOMs) in Eq. (1) are [29]

$$\begin{aligned} E^\mu{}_\nu \equiv & -[M_p^2 + 4\mathcal{F}(\square)R]G^\mu{}_\nu - R\mathcal{F}(\square)R\delta^\mu{}_\nu \\ & + 4(\nabla^\mu \partial_\nu - \delta^\mu{}_\nu \square)\mathcal{F}(\square)R + 2\mathcal{K}^\mu{}_\nu \\ & - \delta^\mu{}_\nu (\mathcal{K}^\sigma{}_\sigma + \tilde{\mathcal{K}}) - \Lambda \delta^\mu{}_\nu = 0, \end{aligned} \quad (2)$$

where $\mathcal{K}^\mu{}_\nu = \frac{1}{M_s^2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\partial^\mu \square_s^l R)(\partial_\nu \square_s^{n-l-1} R)$ and $\tilde{\mathcal{K}} = \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\square_s^l R)(\square_s^{n-l} R)$. The trace of the EOM is

$$(M_p^2 - 12\square\mathcal{F}(\square))R - 2\mathcal{K}^\mu{}_\mu - 4\tilde{\mathcal{K}} - 4\Lambda = 0. \quad (3)$$

The structure of the form factor $\mathcal{F}(\square)$ is determined by finding the second order action around a fixed background and demanding that all perturbations be ghost-free. Note that most of the upcoming computations are carried out covariantly. Solving the EOM in Eq. (2) in full generality is nontrivial but the following ansatz for the background Ricci scalar helps in making headway [15,16,21]:

$$\bar{\square} \bar{R} = r_1 \bar{R} + r_2, \quad (4)$$

where r_1 and r_2 are dimensionful constant parameters. The ansatz equation (4) is essentially the trace EOM of local

$R + R^2$ theory with a cosmological constant given by $\mathcal{L} = \frac{M_p^2}{2} R + f_0 R^2 - \Lambda$ [15]. The ansatz is solved by bouncing backgrounds with cosine hyperbolic and exponential type scale factors, as we will soon see. Straightforwardly, we then obtain

$$\bar{\square}^n \bar{R} = r_1^n \left(\bar{R} + \frac{r_2}{r_1} \right) \Rightarrow \mathcal{F}(\bar{\square}) \bar{R} = \mathcal{F}_1 \bar{R} + \mathcal{F}_2, \quad (5)$$

where

$$\mathcal{F}_1 = \mathcal{F}(r_1) \quad \text{and} \quad \mathcal{F}_2 = \frac{r_2}{r_1} (\mathcal{F}_1 - f_0) \quad (6)$$

are constants. Substituting the ansatz equation (4) into the EOM in Eq. (2) and the trace EOM in Eq. (3) gives us the following unique conditions on the form factor $\mathcal{F}(\bar{\square})$:

$$\mathcal{F}_1 = \mathcal{F}'(r_1) = 0, \quad \mathcal{F}_2 = -\frac{M_p^2}{4}, \quad \Lambda = -\frac{M_p^2}{16f_0}, \quad (7)$$

where $\mathcal{F}'(r_1)$ denotes the derivative of $\mathcal{F}(\bar{\square})$ with respect to $\bar{\square}_s$, evaluated at $\bar{\square} = r_1$. From Eqs. (6) and (7), we can deduce that

$$f_0 < 0 \quad \text{for } \Lambda > 0, \quad f_0 > 0 \quad \text{for } \Lambda < 0. \quad (8)$$

Note that in Refs. [16,21], bouncing solutions and linear perturbations were studied in the presence of fluid radiation, which corresponds to the case when $\mathcal{F}_1 < 0$. In this paper, we emphasize that a bounce can be achieved in the theory satisfying the conditions in Eq. (7) without requiring any radiation/matter.

III. PERTURBATIONS

We now study perturbations and their stability around backgrounds satisfying the ansatz equation (4) in two equivalent ways—both at the linearized level of the EOM and at the quadratic level of the action. Linearizing the full EOM in Eq. (2) and the trace EOM in Eq. (3) around the background given by Eq. (4) and substituting the conditions into Eq. (7), we obtain, respectively,

$$\begin{aligned} \delta E^\mu{}_\nu &= [(4\bar{G}^\mu{}_\nu + 2\delta^\mu{}_\nu \bar{R} - 4(\bar{\nabla}^\mu \bar{\nabla}_\nu - \delta^\mu{}_\nu \bar{\square}))(\bar{\square} - r_1) - 2(\partial_\nu \bar{R} \partial^\mu + \partial^\mu \bar{R} \partial_\nu) + 2\delta^\mu{}_\nu (\partial_\sigma \bar{R} \partial^\sigma + \bar{\square} \bar{R})] \mathcal{Z}(\bar{\square}) \zeta = 0, \\ \delta E &= [\partial^\mu \bar{R} \partial_\mu + 2(r_1 \bar{R} + r_2) + 3(\bar{\square} - r_1)^2 + (\bar{R} + 3r_1)(\bar{\square} - r_1)] \mathcal{Z}(\bar{\square}) \zeta = 0, \end{aligned} \quad (9)$$

where

$$\zeta = \delta(\square) \bar{R} + (\bar{\square} - r_1) \delta R \quad (10)$$

and

$$\mathcal{Z}(\bar{\square}) = \frac{\mathcal{F}(\bar{\square})}{(\bar{\square} - r_1)^2}. \quad (11)$$

Note that the perturbed EOM in Eq. (9) has only one perturbation variable ζ , and we will shortly see that this leads to interesting physics. One may wonder about the absence in Eq. (9) of the perturbation of the Einstein tensor from GR. This can be easily understood since the following coefficient of the Einstein tensor in Eq. (2) vanishes upon using the conditions in Eq. (7),

$$[M_p^2 + 4\mathcal{F}(\bar{\square})\bar{R}] = 0. \quad (12)$$

From the $i \neq j$ perturbed EOM in Eq. (9), we ultimately obtain the simple EOM for ζ

$$\mathcal{Z}(\bar{\square})\zeta = 0. \quad (13)$$

$$\begin{aligned} \delta^2 S = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{M_p^2}{4} (\delta_{GR} - \delta^{(2)}R) - \Lambda \left(\frac{h^2}{8} - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} \right) + \frac{h}{2} \bar{R} \mathcal{F}(\bar{\square}) \delta R + \frac{h}{2} \bar{R} \delta \mathcal{F}(\bar{\square}) \bar{R} + \delta R \delta \mathcal{F}(\bar{\square}) \bar{R} \right. \\ \left. + \bar{R} \delta \mathcal{F}(\bar{\square}) \delta R + \bar{R} \delta^{(2)} \mathcal{F}(\bar{\square}) \bar{R} \right\}, \end{aligned} \quad (14)$$

where $\sqrt{-\bar{g}} \delta_{GR} \equiv \delta^{(2)}(\sqrt{-\bar{g}}R)$ and $\delta^{(2)}R$ is the second variation of R . After considerable manipulation of terms in Eq. (14) (see the Appendix for some useful formulas), the quadratic action remarkably simplifies to

$$\delta^2 S = \int d^4x \sqrt{-\bar{g}} \zeta \mathcal{Z}(\bar{\square}) \zeta. \quad (15)$$

As expected, at the quadratic level of the action, the presence of the sole variable, ζ , is consistent with our earlier result obtained from the linearized EOM in Eq. (13)¹—by varying $\delta^2 S$ with respect to ζ , we exactly recover the linearized EOM for ζ .

To clearly expose the physical degrees of freedom in our theory at the level of the quadratic action, we study SVT modes in the context of (1 + 3) cosmological perturbation theory [31]. We fix the background $\bar{g}_{\mu\nu}$ to be spatially flat Friedmann-Lemaître-Robertson-Walker given by the line element $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, with $a(t)$ being the scale factor, satisfying the ansatz equation (4). We write the full metric as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is the perturbation. In longitudinal gauge $h_{\mu\nu}$ has components

$$h_{00} = a^2(-2\phi), \quad h_{0i} = a^2(\hat{B}_i), \quad h_{ij} = a^2(-2\psi\delta_{ij} + 2\hat{h}_{ij}) \quad (16)$$

so that $h_{\mu\nu}$ has 6 degrees of freedom (2S + 2V + 2T) after gauge fixing² [22]. Note that $\partial^i \hat{B}_i = \partial^i \hat{h}_{ij} = \hat{h}_i{}^i = 0$. Given this decomposition, it is straightforward to see that ζ [defined in Eq. (10)] is a linear combination of ϕ and ψ and therefore is effectively a scalar under $SO(3)$. The quadratic action (15) then only has 1 scalar propagating degree of freedom ζ . It is easy to understand why the vector $\hat{B}_i$ and tensor mode $\hat{h}_{ij}$ are absent. The background

¹Our result [Eq. (15)] turns out to be the same as the nonlocal part of the quadratic action around an inflationary background satisfying $\bar{\square} \bar{R} = r_1 \bar{R}$ [30].

²The longitudinal gauge removes 2 scalar and 2 vector degrees of freedom from the original total of 10 in $h_{\mu\nu}$.

To deduce the kinetic structure of ζ and, therefore, the (background dependent) ghost-free form factor $\mathcal{F}(\bar{\square})$, we compute the second variation of the action Eq. (1) around Eq. (4), further imposing the conditions $\mathcal{F}_1 = 0$, $\mathcal{F}_2 = -\frac{M_p^2}{4}$,

conditions we imposed [see Eq. (7)] lead to the removal of vector and tensor modes that could potentially come from the term proportional to M_p^2 in Eq. (14). All other terms in $\delta^2 S$ generate only scalars in terms of ϕ and ψ . This confirms the result we had obtained earlier from the linearized EOM.

For the theory to have no extra degrees of freedom or ghosts at the quadratic level of the action in Eq. (15), the kinetic operator $\mathcal{Z}(\bar{\square})$ can have at most a single zero, according to the Weierstrass product theorem. For the case of $\Lambda > 0$ [see Eq. (8)], we therefore make the following minimal choice for $\mathcal{F}(\bar{\square})$ [see Eq. (11)] while satisfying all of the necessary conditions in Eq. (7):

$$\mathcal{F}(\bar{\square}) = \frac{1}{M_s^6} (\bar{\square} - m^2)(\bar{\square} - r_1)^2 e^{\gamma(\bar{\square})}, \quad (17)$$

where $m^2 \geq 0$ is some arbitrary mass scale and γ is an arbitrary entire function of $\bar{\square}/M_s^2$. This choice ensures that the kinetic term for ζ has only one zero at $\bar{\square} = m^2$. Note that the kinetic term for ζ has the correct sign to avoid ghosts. Similarly, for the case of $\Lambda < 0$, the minimal choice for $\mathcal{F}(\bar{\square})$ would be one without any zeros for $\mathcal{Z}(\bar{\square})$,

$$\mathcal{F}(\bar{\square}) = \frac{1}{M_s^4} (\bar{\square} - r_1)^2 e^{\gamma(\bar{\square})}. \quad (18)$$

In this case, ζ contains no zeros in the kinetic operator and acts like a p -adic scalar [32].

IV. EXAMPLE: COSMOLOGICAL BOUNCING BACKGROUND

We now specialize to known bouncing solutions [15,16,33]. They all satisfy the ansatz equation (4), are vacuum solutions of the nonlocal theory equation (1), and possess only a single ghost-free scalar mode (and no vector or tensor modes) around the bouncing background at the linearized level when the nonlocal form factor $\mathcal{F}(\bar{\square})$ is chosen appropriately as just discussed:

- (1) Cosine hyperbolic bounce with $a(t) = a_0 \times \cosh(\sqrt{r_1/2}t)$. In this case, $\Lambda > 0$.
- (2) Exponential bounce with $a(t) = a_0 e^{\frac{\lambda}{2}t^2}$. In this case, $\Lambda > 0$ for $\lambda > 0$ and $\Lambda < 0$ for $\lambda < 0$. For $\lambda > 0$, we have contraction followed by expansion as in the usual bounce. For $\lambda < 0$, the Universe reaches a maximum scale factor at $t = 0$ and then starts to decrease in size. This is also a possible way the big bang singularity could be resolved.
- (iii) Cyclic universe. This scenario was numerically studied in [16]. It was found that in the case of $\Lambda < 0$ there exists a cyclic scenario where the Universe undergoes multiple bounces.

Let us restrict to the case of $\Lambda > 0$ and a cosine hyperbolic bounce, in which case $\mathcal{F}(\bar{\square})$ is given by Eq. (17), and the corresponding quadratic action for ζ reads

$$\delta^2 S = \frac{1}{M_s^6} \int d^4 x \sqrt{-g} \zeta e^{\gamma(\bar{\square})} (\bar{\square} - m^2) \zeta. \quad (19)$$

For a stable bounce, we would require the perturbation ζ to be well behaved in time. All solutions of the nonlocal EOM for ζ in Eq. (13) for $\mathcal{F}(\bar{\square})$ given in Eq. (17) are captured by solutions of the local equation

$$(\bar{\square} - m^2)\zeta = 0 \quad (20)$$

because the nonlocal factor $e^{\gamma(\bar{\square})}$ does not introduce any new zeros in the kinetic operator [15,32]. Setting $a_0 = 1$, $r_1 = m^2 = M_s^2$ for the cosine hyperbolic bounce, the generic solution of Eq. (20) in Fourier space looks like

$$\zeta_k(z) = c_1 H_k^{(+)}(z) + c_2 H_k^{(-)}(z), \quad (21)$$

where

$$H_k^{\pm} = \frac{\sqrt{e^{\sqrt{2}z}}}{1 + e^{\sqrt{2}z}} \left[\frac{(1 \pm i\sqrt{e^{\sqrt{2}z}})^2}{1 + e^{\sqrt{2}z}} \right]^{\sqrt{\frac{2k^2}{M_s^2} + 1}}, \quad (22)$$

FIG. 1. Evolution in cosmic time $z = M_s t$ of a Fourier component of the scalar mode ζ_k for a cosine hyperbolic bounce. We have set $a_0 = 1$, $r_1 = m^2 = M_s^2$, $c_1 = c_2 = 0.1M_s$, and $k = 20M_s$ (blue curve) and $k = 0$ (red curve).

where $z = M_s t$ and $c_{1,2}$ are constants. We can easily deduce that ζ_k is real for $c_1 = c_2$. Here, we do not quantize ζ and merely wish to observe how it evolves classically through the bouncing phase. We have shown the solution ζ_k in Fig. 1, where we can see that it is oscillatory and bounded for the chosen values of $c_{1,2}$ and k . This behavior persists to hold for any value of k with an increasing number of bounded oscillations as $k \rightarrow \infty$. We do not present here the solutions for ζ around other bouncing backgrounds mentioned earlier since the nature of perturbations remains very similar to that of the cosine hyperbolic bounce.

For the case of the cosine hyperbolic bounce, we will have a de Sitter (inflationary) evolution as a late time attractor [15,17,21,34]. We can observe from Fig. 1 that $\zeta \rightarrow 0$ in the late time leading to $(\bar{\square} - r_1)\delta R \rightarrow 0$. This observation confirms that we exactly recover the late time behavior of scalar fluctuations found in Ref. [21].

There is also the intriguing possibility of a phase of superinflation after the cosine hyperbolic bounce and before cosmic inflation. This makes this type of bounce an interesting framework for explaining low multipoles in the observed data for cosmic microwave background radiation [35]. It is important to note that the late time de Sitter stage is significantly different in our case than in [21] because of the absence of any tensor modes due to our background conditions in Eq. (7). Note that one can take the GR limit at the level of the action in Eq. (1), but not at the level of our background solution satisfying the EOM in Eq. (2) with the uniquely tuned conditions in Eq. (7).³

V. CONCLUSIONS

To summarize, we have found a simple model of nonsingular, homogeneous, and isotropic bouncing cosmology (with a cosmological constant) in vacuum with spatially flat geometry. Moreover, for the background satisfying certain conditions[see Eq. (7)], there exists only

³It is possible to have special points in the theory space of a gravity model where the physical spectrum differs from the usual expectations—for example, pure R^2 theory propagates a massless scalar and tensor modes around the (A)dS background, which can be seen by conformally transforming pure R^2 to the Einstein frame, where the action turns out to be GR with a massless scalar and cosmological constant [36]. However, around flat space, the spectrum contains only a massless scalar and no tensor modes (here, the conformal transformation into the Einstein frame is singular). Another example would be critical gravity described by $\mathcal{L} = \frac{M_P^2}{2}(R - 2\Lambda + \alpha R^{\mu\nu} R_{\mu\nu} + \beta R^2)$ —here, the tunings $\alpha = -3\beta$ and $\beta = -1/2\Lambda$ remove the massive scalar mode and make the massive spin-2 mode massless [37]. In the context of nonlocal theories, the theory described by the action in Eq. (1) permits special classes of (A)dS and nonmaximally symmetric covariantly constant curvature backgrounds around which the physical spectrum consists of only a scalar mode and no tensor or vector modes [28]. In this paper, we find a special class of dynamical backgrounds around which we can have such a physical spectrum with only a scalar mode as a result of Eq. (7).

a scalar perturbation at the linearized level, and no propagating vector or tensor modes. This is contrary to the case in GR and infinite derivative gravity in general, which propagate tensor modes. Finding the curious ghost-free form factor $\mathcal{F}(\bar{\square})$ around a dynamical background that leads to only a scalar propagating mode is nontrivial, and it is the main highlight of this paper. To our knowledge, this is the first such study in infinite derivative theories of gravity. Moreover, in a cosine hyperbolic bouncing scenario, we have explicitly shown that the lone scalar propagating degree of freedom is stable and that its evolution is oscillatory and bounded. Of course, these results are valid at the linearized level in perturbation theory and it would be interesting to see what happens at higher orders, which we leave for future discussions.

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APPENDIX: DERIVATION OF $\delta^2 S$ IN EQ. (15)

Simplifying the quadratic action (14) and the linearized EOM in Eq. (9) involves using the formulas

$$\delta\mathcal{F}(\square) = \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \square_s^l \delta(\square_s) \square_s^{n-l-1} \quad (\text{A1})$$

and

$$\begin{aligned} \delta^{(2)}\mathcal{F}(\square) &= \sum_{n=1}^{\infty} \sum_{l=0}^{n-1} f_n \square_s^l \delta^{(2)}(\square_s) \square_s^{n-l-1} \\ &+ 2 \sum_{n=2}^{\infty} \sum_{l=0}^{n-2} \sum_{k=0}^{n-l-2} f_n \square_s^l \delta(\square_s) \square_s^k \delta(\square_s) \square_s^{n-k-l-2}. \end{aligned} \quad (\text{A2})$$

Using these, the background ansatz equation (4), and the conditions in Eq. (7), we can simplify the last five terms in the quadratic action by using the following relations

$$\delta(\mathcal{F}(\square)R) = (\bar{\square} - r_1)\mathcal{Z}(\bar{\square})\zeta, \quad (\text{A3})$$

$$\begin{aligned} \int d^4x \sqrt{-\bar{g}} \bar{R} \delta\mathcal{F}(\square) \delta R &= \int d^4x \sqrt{-\bar{g}} \left[\left(\bar{R} + \frac{r_2}{r_1} \right) \delta(\square) \mathcal{Z}(\bar{\square}) (\bar{\square} - r_1) \delta R + \frac{h r_2}{2 r_1} (\mathcal{F}(\bar{\square}) - f_0) \delta R \right], \\ \int d^4x \sqrt{-\bar{g}} \bar{R} \delta^{(2)}\mathcal{F}(\square) \bar{R} &= \int d^4x \sqrt{-\bar{g}} \left[\left(\bar{R} + \frac{r_2}{r_1} \right) \delta(\square) \mathcal{Z}(\bar{\square}) \delta(\square) \bar{R} \right. \\ &\quad \left. + \frac{h r_2}{2 r_1} (\bar{\square} - r_1) \mathcal{Z}(\bar{\square}) \delta(\square) \bar{R} + \frac{M_p^2 h}{4 r_1 2} (\delta(\square) \bar{R} + \delta^{(2)}(\square) \bar{R}) \right]. \end{aligned} \quad (\text{A4})$$

Substituting Eqs. (A3) and (A4) into Eq. (14), using the following identity for some scalars X and Y , performing numerous integration by parts, and omitting total derivatives, we finally obtain the result in Eq. (15),

$$\int d^4x \sqrt{-\bar{g}} Y \delta(\square) X = \int d^4x \sqrt{-\bar{g}} \left[X \delta(\square) Y - \frac{h}{2} (Y \bar{\square} X - X \bar{\square} Y) \right]. \quad (\text{A5})$$

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