The Galactic halo: 
formation history and dynamics

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# Contents

1 Introduction 1
   1.1 The Universe and $\Lambda$CDM ........................................... 1
   1.2 Galaxies ................................................................. 2
   1.3 Gaia and other surveys ............................................... 6
   1.4 The Milky Way ......................................................... 8
   1.5 The Galactic halo - structure and formation ....................... 13
   1.6 Outline of Thesis ..................................................... 15
   1.7 Where to go from here? ............................................... 18

Part I Formation history 25

2 One large blob and many streams frosting the nearby stellar halo
   in Gaia DR2 27
   2.1 Introduction ........................................................... 27
   2.2 Data and Methods .................................................... 29
   2.3 Analysis ............................................................... 29
   2.4 Discussion ............................................................ 35

3 The merger that led to the formation of the Milky Way's inner stellar halo
   and thick disc 39
   3.1 Main section ............................................................ 40
   Appendix 3.A Naming ..................................................... 47
   Appendix 3.B Dataset, selection criteria and the effect of systematics ................................................... 47
   Appendix 3.C Random sets and significance of features ............... 51
   Appendix 3.D Context and link to other substructures ................. 52

4 Characterisation and history of the Helmi streams with Gaia DR2 55
   4.1 Introduction ............................................................. 55
   4.2 Data ............................................................... 57
   4.3 Finding members .............................................................. 59
   4.4 Analysis of the streams .................................................. 65
   4.5 Simulating the streams ................................................. 71
5 Multiple retrograde substructures in the Galactic halo:  
A shattered view of Galactic history  
5.1 Introduction 85  
5.2 Data 87  
5.3 Results 88  
5.4 Discussion and Conclusions 92  

6 Origin of the system of globular clusters in the Milky Way 99  
6.1 Introduction 99  
6.2 The dataset: dynamics, ages, and metallicities 100  
6.3 Assignment of clusters 101  
6.4 Summary and Conclusions 108  
Appendix 6.A 112  

7 A massive mess:  
When a large dwarf and a Milky Way-like galaxy merge 117  
7.1 Introduction 117  
7.2 Methods 119  
7.3 Results 120  
7.4 Conclusions 124  
Appendix 7.A Observational datasets used 127  
Appendix 7.B Computation of the orbital parameters for the simulations 128  

Part II Dynamics 129  

8 The Reduced Proper Motion selected halo:  
methods and description of the catalogue 131  
8.1 Introduction 132  
8.2 Methods 133  
8.3 Data selection and calibration 135  
8.4 Spatial distribution of the RPM sample 143  
8.5 Velocity content of the RPM sample 149  
8.6 The velocity distribution of the local halo 155  
8.7 Discussion and Conclusions 162  
Appendix 8.A Velocity maps without MSTO stars 167  
Appendix 8.B Selection Effects 167  
Appendix 8.C RVS sample 170
The Sun, all the stars that are visible in the night sky, and hundreds of billions of other stars are all part of a single galaxy: the Milky Way. The study of the Galaxy and its contents is known as Galactic Astronomy. In this introduction, we describe the status of the field and some of the tools used, and outline some important questions. How far back can we trace the genealogy of the Milky Way? What are its structural components? How massive is the Galaxy, and how is this mass distributed? Moreover, we aim to place these questions and their possible solutions in the bigger picture of galaxy formation and the formation of the large-scale structure of the Universe.

The introduction of this Thesis starts with a brief and basic description of the formation of the Universe (Sec. 1.1), and of the formation and properties of galaxies (Sec. 1.2). These topics are intertwined and also lay out the basic principles that form the foundations of the following sections. For example, we will see that the first galaxies to form are small and have evolved in terms of their size and chemical composition to give rise to systems such as the Milky Way. Moreover, by placing studies of the Milky Way in a bigger context, we can develop our understanding of fundamental physics such as the properties of the first galaxies and the nature of the dark matter particle.

The Milky Way provides unique insights into the physics of galaxy formation because it is one of the few galaxies in which we can resolve individual stars. Ironically, it notoriously difficult to study because we are embedded inside it. The Gaia mission, described in Sec. 1.3, is providing a map of the Milky Way that is unprecedented in size and detail. In Sec. 1.4 we will discuss the general properties of the Milky Way and the Galactic halo is fleshed out in Sec. 1.5. The outline of the Thesis is described in Sec. 1.6, and in Sec. 1.7 we hypothesise about the direction in which the field might move next.

## 1.1 The Universe and $\Lambda$CDM

The earliest account that we have of the Universe is from circa 400 000 years after the Big Bang: it is the image of the cosmic microwave background (CMB). This image evidences what is known as the cosmic principle: the Universe is homogeneous and isotropic - on the largest scales. The CMB displays only microscopic anisotropies isotropic up to one part in a 100 000. However, it is these microscopic density fluctuations that form the seeds of the galaxies and large-scale structure that we see in the present-day Universe.

The CMB marks the time of ‘recombination’, which is when the first atoms form. The primordial baryonic matter comprises $\sim$75% Hydrogen, $\sim$25% Helium and a trace amount of Lithium. Regions in the Universe with a slight over-density start grow under the influence of Gravity - and mostly driven by dark matter, which makes up most of the mass in the Universe (e.g. Hinshaw et al., 2013). In contrast with this dark matter,
baryons do interact with the electromagnetic field. They radiate away their energy, cool down, and condense at the centres of the dark structures. It is here where they form the first population of stars. Figure 1.1 summarises our current understanding of the formation and evolution of the Universe, the last \( \sim 13.7 \) billion years of which are described in the next section.

The model that fits best with the observations of the CMB, primordial nucleosynthesis, and the resulting large-scale structure, is one where the dark matter particle is cold (CDM). Considering also the expansion of the Universe, this model is known as \( \Lambda \)CDM. This model is not fully without problems (e.g. Bullock & Boylan-Kolchin, 2017) so other types of dark matter are also being considered (such as warm, fuzzy, and self-interacting dark matter, Spergel & Steinhardt 2000; Hu et al. 2000; Bode et al. 2001; Hui et al. 2017).

## 1.2 Galaxies

We will describe here the basics of the theory of galaxy formation and evolution. This will help in placing the Milky Way in a more general context. We also describe several of the tools used in this Thesis.

### 1.2.1 Galaxy Formation

In the currently widely accepted \( \Lambda \)CDM model, structure forms hierarchically (White & Rees, 1978; Searle & Zinn, 1978; White & Frenk, 1991). The proto-galaxies that form shortly after the Big Bang enter a cascade of mergers that results in the galaxies that are present nowadays in the Universe. Figure 1.2 shows the formation of a disc galaxy in the
EAGLE simulations (Schaye et al., 2014) as traced by its gas (where the colour red is the densest). Gas brought in by mergers, as well as inflow from the intergalactic medium, enables galaxies to form their stars. Many galaxies are still forming stars today but the star-formation density peaks at a redshift of $z = 2$ (Madau & Dickinson, 2014)$^1$. About half of the baryonic mass of Milky Way-like galaxies is accreted from the intergalactic medium and the other half is obtained from mergers (e.g. Grand et al., 2019, although only up to $\sim 10\%$ of the stars are accreted, Rodriguez-Gomez et al. 2016). On the other hand, most of the dark material comes from mergers (e.g. Wang et al., 2011).

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$^1$ a redshift of $z=2$ the Universe is $\sim 3.3$ Gyr old and stars formed then are now $\sim 10$ Gyr old. (Wright, 2006)
1.2.2 Galaxy Types

The canonical picture of galaxy formation described above leaves room for variations between galaxies through environmental dependencies (e.g. Dressler, 1980) and stochastic processes such as galaxy mergers (e.g. Toomre & Toomre, 1972; Barnes, 1988, 1992). Nevertheless, most galaxies seem to adhere to several remarkably narrow scaling relations. By placing the Milky Way on these relations we can study the formation paths of galaxies similar to it.

Despite the many similarities between galaxies on average, there exists a large variety of galaxy morphologies (the classification of which is among the oldest disciplines in modern astronomy, e.g. Hubble 1926). The two most general types are spiral galaxies and spheroidal galaxies. The former have a strong disc-like component and are typically less massive. The latter are spheroidal, often more massive, and typically have stopped forming stars at $z = 0$. Spiral galaxies make up about 66% of all galaxies. They are mostly found in low-density regions, which hints at an environmentally dependent formation path. Spheroidal galaxies are mostly found in galaxy clusters.

1.2.3 Chemical evolution

One of the ‘fingerprints’ of a star is its chemical composition. Recall that the primordial gas mostly contains Hydrogen and Helium. All elements apart from these two are known as ‘metals’ in the context of Astronomy, they are formed in stars. Some metals are formed in every star, others require specific circumstances such as neutron-star mergers and supernova explosions (e.g. Burbidge et al., 1957; Kasen et al., 2017). The stochastic nature of some of these events leads to a large difference in the chemical evolution between galaxies (e.g. from very evolved to enriched by only a single event, Ji et al. 2016).

Metals are often subdivided based on their (dominant) formation channel. For example, some of the lightest metals are known as the $\alpha$-elements. These elements form from the nuclear fusion of $\alpha$-particles (Helium), and thus only have even proton numbers (e.g. Magnesium and Silicon). Lighter elements like Carbon and Oxygen may also be included in the $\alpha$-elements, but they form a complex chemical cycle together with Nitrogen (the CNO cycle). The next set of elements that is useful to define are the iron-peak elements. Nuclear fusion is energy efficient (exothermic) until the formation of iron (Fe), beyond which it is energy costly. Elements heavier than Zinc are formed through neutron-capture. It is useful to divide these heavy elements into two sets: one formed by slow neutron-capture (or s-process) and the other by rapid neutron-capture (r-process). The s-process produces elements like Yttrium, Barium, and a typical r-process element is Europium. The r- and s-processes only occur in rare events like supernovas and binary neutron-star mergers.

The various sets of elements are created on different time-scales and sometimes require rare conditions. Therefore, by measuring the relative fractions of elements in the constituents of a galaxy, we can determine quite precisely when and under which conditions
it formed its stars. For the Milky Way, and nearby galaxies, we typically measure the chemical compositions in individual stars and compare the chemical sequences of the ensemble (e.g. Tolstoy et al., 2009). Perhaps the most often used chemical indicator, as such, is \([\alpha/\text{Fe}]^2\), because it is relatively simple to measure. Supernova Type II (high-mass stars) create relatively more \(\alpha\) elements than supernova Type Ia (low-mass stars, through mass transfer in binary systems). However, the latter create relatively more iron. Because the Type Ia take longer to evolve, they start to contribute later to the chemical evolution of a galaxy. The time-scale difference between the two processes acts as a chemical ‘clock’ that is visible as a ‘knee’ in the \([\text{Fe}/\text{H}]\) versus \([\alpha/\text{H}]\) diagram when plotting individual stars in a galaxy.

### 1.2.4 Star formation and stellar evolution

Stars are not all born the same, a newly formed stellar population typically shows a range in masses. This range is described as the initial mass function (IMF) of a population. Most stars that are being formed have a low mass and, depending on the IMF, the most likely mass is below a solar mass.

The evolution of stars and stellar populations can be mapped by the Hertzsprung-Russel diagram (HRD). There exist several versions of this diagram, the one that we use in this thesis is the ‘colour-magnitude’ diagram, which is commonly used for observational data. It is far beyond the scope of this Thesis to describe all details of stellar evolution and their location in the HRD, so we will provide only a summary (parts of which are based on Binney & Merrifield, 1998).

Stars move along a predetermined track in the HRD that varies with the stars mass and chemical composition. They spend most of their lifetime on a part that is called the main sequence (MS). As stars move away from the MS, their radius and surface temperature (or colour and magnitude) will vary, causing them to move upwards (except for stars with masses \( M \gtrsim 8 M_\odot \)). Stars on the MS are known as ‘dwarfs’ and the phase after the MS is known as the ‘giant’ phase. The first part of the giant-track is the red-giant branch (RGB), then comes the horizontal branch (HB), and finally the asymptotic giant branch (AGB). The fate of a star after the AGB is strongly mass-dependent.

The HRD provides a powerful tool in recovering the star formation history of a galaxy. For simple systems it is often used to determine their relative ages (e.g. for globular clusters, Marín-Franch et al., 2009).

### 1.2.5 Galactic dynamics

It is notoriously difficult to calculate, by hand, the trajectories of stars in a gravitational system. The motion of one- and two-body systems are relatively simple. However, already three-body systems are cumbersome and contain configurations that are chaotic (and a fraction of them are fundamentally unpredictable, e.g. Boekholt et al., 2020). Therefore, the orbits of stars are commonly calculated with numerical integration algorithms.

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\(^2\) The ratio of elements is typically measured in logarithmic units and relative to the solar value

\[
[\alpha/\text{Fe}] = \log(N_\alpha / N_{\text{Fe}}) - \log(N_\alpha / N_{\text{Fe}})_\odot
\]
It quickly becomes difficult to keep track of the relative locations of stars in many-body systems. For this reason, astronomers often default to simple, smooth, static, and symmetric descriptions of galaxies. These approximations work well because encounters between individual stars are rare and because the time over which galaxies change is much longer than the typical orbital time-scale. The theory and the applications of these kinds of models are comprehensively described by Binney & Tremaine (2008, also the 1987 edition).

In this Thesis, we most often approximate the Milky Way with an axisymmetric model, because that resonates well with the predominance of the Galactic disc (in many cases we use the potential described by McMillan (2017)). Axisymmetric models are often expressed in cylindrical coordinates \( (R, z) \) where \( z = 0 \) in the plane of symmetry. It is simple to show that the circular velocity \( (v_c) \) in the plane of symmetry is

\[
\frac{v_c^2(R)}{R} = |F_G| = \frac{d\Phi(R)}{dR}, \tag{1.1}
\]

where \( |F_G| \) is the gravitational force at radius \( R \), and \( \Phi(R) \) is the potential energy of a star. Another useful property is the escape velocity, which is the velocity that is necessary to escape from a galaxy

\[
v_{esc}(R, z) = \sqrt{2|\Phi(R, z)|}. \tag{1.2}
\]

Trajectories in axisymmetric potentials often can be defined by two classical integrals of motion (IOM), and sometimes a third ‘non-classical’ integral (for which there, in general, is no analytic expression). The two classical integrals are the Hamiltonian and \( L_z \). Here \( L_z \) is the component of the angular-momentum vector \( r \times p \) in the \( z \)-direction: \( L_z = r v_\phi \). The integrals of motion, and other orbital properties such as the eccentricity and circularity, are often used to characterise the orbits of stars.

1.3 Gaia and other surveys

Observing the entirety of the Milky Way is notoriously difficult. Even the latest surveys map only parts of the Galaxy. We will here focus on surveys that have been used in this Thesis.

1.3.1 The Gaia mission

The space-based Gaia mission has been invaluable for the field of Galactic Astronomy. The satellite was launched in December 2013 and reached L2 about 2 months later. Since then, it has been mapping the Milky Way. The nominal time (5 years) for the mission has passed, but it has been extended because of its success and is still running at the time of writing this.

Gaia scans the sky by spinning around its axis, as illustrated in Fig. 1.3. By slowly varying the axis of rotation the full sky is mapped, one entire cycle taking 63 days. During this scanning, the satellite is storing the positions and photometry of all objects it detects.
(these include stars, quasars, and asteroids). The mission is expected to be complete for all objects brighter than $\sim 20$ mag, depending on the length of the extension. *Gaia* carries three instruments: an astrometric, a photometric, and a spectroscopic instrument.

The astrometric instrument provides astrometric parameters down to the $\mu$as level. The motions and distances of stars are obtained by mapping small variations in the location of a star over the period of several years. *Gaia* will map each star on average $> 70$ times. Variations in a star’s location are a result of the combination of its finite distance (i.e. the parallax measurement) and its intrinsic motion (proper motion). This combined motion can be decomposed by combining multiple observations taken over an extended period of time. The astrometric instrument also provides broad-band photometry in the G-band. Colour-information is provided by the photometric instrument, which takes low-resolution spectrophotometric observations of the blue and red part of the G-band ($G_{BP}$ and $G_{RP}$). The photometric data is used to map stellar variability, providing the classification of variable stars (Cepheids, RR-Lyrae, e.g. Clementini et al., 2019), and the BP and RP bands are aimed to provide astrophysical parameters (extinction, temperature, surface-gravity).
The quality of the astrometric parameters (location on the sky, proper motion, and parallax) increases significantly as stars are mapped more often. And especially the motions benefit from having long time-scales between mappings. The currently available data, which is the second release (DR2), only includes the observations of the first two years of operations. This period is long enough to calculate proper motions for 1.3 billion of its total 1.7 billion mapped stars (Gaia Collaboration, Brown et al., 2018; Lindegren et al., 2018). However, the uncertainty in the astrometric parameters will significantly decrease with the coming data releases, comprising observations over a longer time span. Further data releases are expected soon, at the end of this year (2020) an early data release (EDR3) will be made public. This EDR3 will provide updated astrometric parameters (most significantly the parallax and proper motion). For more information on the satellite and more, the reader could consult the website\(^3\) and the science-performance website\(^4\) (on which some of the information in this section is based).

### 1.3.2 Spectroscopic surveys

The spectroscopic instrument of *Gaia* is known as the Radial-Velocity Spectrometer (RVS) and has a brightness limit of \(G_{\text{RVS}} \approx 16\) mag. With a resolution of \(\sim 11,500\), the RVS spectra (will) provide radial velocities, astrophysical parameters, and elemental abundances for bright stars (e.g. abundances only for stars brighter than \(G_{\text{RVS}} \approx 11\) mag). The set of stars with all 5 astrometric parameters plus radial velocities available is known as the RVS or 6D sample. Currently, only one in every \(\sim 200\) stars in *Gaia* DR2 is part of this subset (Katz et al., 2019) - and the first metallicity indicators have yet to be released. Sometimes it is feasible to do spectroscopic follow up of (a couple of) individual objects, but it is not very efficient. It has proven to be more fruitful to cross-match the *Gaia* data with spectroscopic surveys.

In this thesis we have made use of data from the following spectroscopic surveys: LAMOST (Cui et al., 2012), APOGEE DR14 and DR16 (Abolfathi et al., 2018; Ahumada et al., 2020), RAVE DR5 (Kunder et al., 2017). These surveys all provide accurate radial velocities but provide (accurate) abundance information only for a handful of elements. In the near future we will see a surge of these cross-matching possibilities as the next generation (multi-fibre) spectroscopic surveys like WEAVE (Dalton et al., 2012), 4MOST (de Jong et al., 2012), SDSS-V (Kollmeier et al., 2017), DESI (DESI Collaboration et al., 2016) will come online.

### 1.4 The Milky Way

Here we will provide a quick overview of the various large components of the Milky Way, most of which are shown in Fig. 1.4. Galactic taxonomy is a task that is far from trivial. The components seamlessly blend together and contain sub-components that,
most of the times, also have no clear boundaries. In the following, I will discuss the main components (central region, the disc(s), and the halo). This Thesis focuses mostly on the Galactic Halo, the luminous part of which is annotated in the figure. Therefore, we will describe the other components only briefly, mainly highlighting their (tentative) connection to the halo.

### The Milky Way in numbers

- The Milky Way owes most of its mass to the dark halo, which weighs $\sim 10^{12} \, M_\odot$ (e.g. Callingham et al., 2019).
- The total stellar mass of the Galaxy is $M_* \approx 5 \pm 1 \cdot 10^{10} \, M_\odot$ (Bland-Hawthorn & Gerhard, 2016)
- And the stellar mass of the halo $M^\text{halo}_* \sim 1.3 \cdot 10^9 \, M_\odot$ (Deason et al., 2019; Mackereth & Bovy, 2020).

From which can be derived that the mass-to-light ratio of the Milky Way is roughly 20:1 and only one in every hundred (1:100) stars belongs to the halo.

### 1.4.1 The central part

The central region is known as the bulge/bar area and is one of the most elusive parts of our Galaxy. It is heavily obscured by stars, gas, and dust that are located in the plane of the disc. Mainly the dust impedes observations as it absorbs the blue part of the spectrum,
causing stars to appear dim and reddened (i.e. ‘reddening’ or dust extinction). The other issue is that the central regions of the halo and the disc (in form of the bar) come together in the Galactic Centre, as well as other components such as a ‘classical bulge’, ‘nuclear star cluster’ (e.g. Bland-Hawthorn & Gerhard, 2016, and references therein).

The total stellar content of the bulge weighs $\sim 2\times 10^{10} \, M_\odot$, making up about 40% of the total stellar mass of the Milky Way (Valenti et al., 2016). In general, there is not expected to be a lot of dark matter in the central region, as the mass-to-light ration is $M/L \sim 1$ requiring only a dark fraction of 10–25% (Zoccali et al., 2000; Bland-Hawthorn & Gerhard, 2016). The very centre of it all hosts a supermassive black hole with a mass of $4.1\times 10^6 \, M_\odot$ (GRAVITY Collaboration et al., 2018).

After years of debate, the existence of the bar has been settled. However, its size, orientation, and pattern-speed are still under scrutiny (e.g. Wegg et al., 2015; Momany et al., 2006; Portail et al., 2017; Bovy et al., 2019). The time-dependence of the rotating bar induces resonances in the orbits of stars (e.g. Dehnen, 2000; Monari et al., 2017, 2019) and can play an important role for stellar streams and other halo substructures that come near it (Bonaca et al., 2020b; Pearson et al., 2017). Carefully pinning down the properties of the bar can, therefore, enhance dynamical models of the Milky Way.

There is ample evidence that the central region of the Milky Way comprises several stellar populations that are distinct in chemistry and kinematics (e.g. Ness et al., 2013; Rojas-Arriagada et al., 2014). The most metal-rich stars show a strong rotational signal but the populations become increasingly more pressure-supported with decreasing metallicity (e.g. Arentsen et al., 2020). The pressure-supported and metal-poor population(s) could belong to the halo.

1.4.2 The disc(s)

The most conspicuous component of the Milky Way is its disc, as it contains most of the stars. Besides stars, there is also dust and gas. The latter accumulates in the mid-plane and, when sufficiently compressed, turns into new stars. The current rate of star formation is roughly $1 \, M_\odot/\text{yr}$ (Licquia & Newman, 2015). Most of the material is found inside the solar radius ($r \lesssim 8 \, \text{kpc}$), even though the disc extends out to $\sim 16 \, \text{kpc}$. The stars and gas in the disc follow roughly an exponential profile both in the radial and vertical direction, although with different characteristic parameters.

Moreover, depending on the classification criteria, the disc is a superposition of several sub-discs. For example, we often differentiate between the ‘thin’ and ‘thick’ disc. The thin disc comprises mainly young stars and is flat, with a scale height of $\sim 300 \, \text{pc}$. On the other hand, the stars in the thick disc are old and extend further away from the mid-plane. The thick disc has a scale height of $\sim 1 \, \text{kpc}$ and stellar mass of $M^{\text{thick}}_* \approx 6\times 10^9 \, M_\odot$. The thin disc is more massive, with a stellar mass of $M^{\text{thin}}_* \approx 3.5\times 10^{10} \, M_\odot$ (Bland-Hawthorn & Gerhard, 2016).

Classically, another classification of the discs is made based on the chemistry of the stars. Stars in the disc are known to be distributed in two distinct chemical components, separated by the $\alpha$-elements. These two populations are known as the high-
Fig. 1.5: Comparison of the structures in the outer regions of galaxies seen in observations and simulations. These structures originate in merger events and are typically difficult to observe because of their low surface-brightness. Image credit, Left: CFHT-OmegaCam, Coelum (Duc et al., 2012), Right: Cooper et al. (2010)

and low-\(\alpha\) discs. In this thesis (and often in the literature), these two populations are conflated with the thin and thick disc (i.e. the thin disc being the low-\(\alpha\) disc and the thick disc corresponding to the high-\(\alpha\) disc). This link is supported by the typical stellar ages of the two: the majority of the low-\(\alpha\) disc stars are younger than 6 Gyr and the high-\(\alpha\) disc stars are typically older than 8 Gyr (e.g. Silva Aguirre et al., 2018).

1.4.3 The Halo - general properties

When the halo is mentioned in this Thesis, we are most often referring to the stellar halo. Unlike the other components of the Milky Way, the stellar halo is dominated by its dark counterpart. The dark halo is roughly a thousand times more massive than the stellar halo. In fact, most of the mass of the Milky Way is stored in its dark halo.

One of the reasons for studying the stellar halo is that trajectories in the outer halo have very long time-scales. The stars belonging to accreted systems remain spatially coherent for several Gyr. Another reason for studying the halo is that its stars are among the oldest and most metal-poor in the Galaxy. Halo stars are often called the ‘fossil records’ of the Milky Way, making the study of which known as ‘Galactic Archaeology’.

In the stellar halo, we expect to find the remainders of the galaxies that have merged with the Milky Way (e.g. Johnston et al., 1996; Helmi & White, 1999; Bullock et al., 2001) possibly together with a component that formed \textit{in situ}. Figure 1.5 shows an impression of the kind of accretion debris that might be found in the outer regions of halos. We will describe the properties and formation history of the halo in more detail in Sec. 1.5.
1.4.4 The satellite system

The Milky Way’s gravitational pull has trapped around 200 luminous satellite systems, together with the tentative hundreds of thousands of dark satellites (see Sec. 1.5.1). The luminous satellites are typically divided into two classes: globular clusters and dwarf galaxies. Gaia DR2 has also been advantageous for determining, or improving our estimates of, the motions of satellites (Gaia Collaboration, Helmi et al., 2018; Fritz et al., 2018; Massari & Helmi, 2018; Vasiliev, 2019).

Globular Clusters

Globular clusters (GC) are among the densest stellar systems in the Universe, with $\sim 10^5$ stars packed into a few pc. They are generally thought to be free of dark matter and their stars have very similar ages and metallicities. The halo of the Milky Way is known to host over 150 globular clusters (Harris, 1996), with new ones being found on a sporadic basis (often in obscured regions like in the bulge, e.g. Minniti et al. 2017; Palma et al. 2019).

The GC system of a galaxy holds clues to its formation history (West et al., 2004). The GC’s of the Milky Way can be separated into two distinct populations based on their age and metallicity. The populations are thought to have a different origin, with one being accreted and the other formed in situ (e.g. Forbes & Bridges, 2010; Leaman et al., 2013; Renaud et al., 2017). Accreted GC’s once likely belonged to dwarf galaxies, they decoupled when the latter was accreted by the Milky Way. One clear example of this is the Sagittarius galaxy, which tentatively brought in several globular clusters (e.g. Law & Majewski, 2010). Several advances have recently been made in this field (e.g. Myeong et al., 2018d; Massari et al., 2019; Kruijssen et al., 2019, 2020; Pfeffer et al., 2020).

Dwarf galaxies

As the name suggests, dwarf galaxies are comparable to regular galaxies but they are smaller. The Milky Way hosts $\sim 30$ dwarf galaxies (Mcconnachie, 2012), although also here new candidates are still being found (e.g. Torrealba et al., 2016, 2019).

The ‘building blocks’ of the Milky Way probably had a similar mass and size as the surviving dwarf galaxies, according to our current understanding of the cosmological model. However, the chemical compositions of halo stars are different from that of the surviving dwarf galaxies (e.g. Venn et al., 2004; Tolstoy et al., 2003), suggesting that the Milky Way’s ‘building blocks’ had different properties (Robertson et al., 2005; Font et al., 2006). This hypothesis finds support in recent cosmological simulations (De Lucia & Helmi, 2008; Deason et al., 2016; Fattahi et al., 2020). Basically, the surviving dwarfs have had several Gyr longer to form stars and evolve both structurally and chemically.
1.5 The Galactic halo - structure and formation

1.5.1 Inspiration from simulations

Cosmological simulations have been vital in building our understanding of the formation history of Milky Way-like galaxies (and of galaxy formation in general, e.g. Vogelsberger et al., 2020). The most state-of-the-art cosmological simulations reproduce the observed distribution of galaxies, albeit with the use of (tweaked) sub-grid physics. They successfully account for the effects of dark matter, dark energy, and for the baryonic physics governing the evolution of gas, and star-formation. They can be useful in exploring the differences between CDM and more exotic forms and their influences on small-scale structure formation (Vogelsberger et al., 2012; Macciò et al., 2019; Vogelsberger et al., 2016; Bozek et al., 2016).

One of the vexing problems with galaxy formation in ΛCDM is the apparent lack of substructure in the dark halos of galaxies like our own. This problem is known as the ‘missing satellites problem’ and became apparent with the first generations of high-resolution dark-matter-only simulations (Klypin et al., 1999; Moore et al., 1999) but is also present in the later generations of dark-matter-only simulations (Diemand et al., 2008; Springel et al., 2008). Besides tweaking the nature of the dark matter particle, a straight forward solution might be the inclusion of baryonic components and baryonic physics (D’Onghia et al., 2010; Governato et al., 2010; Zolotov et al., 2012; Brooks et al., 2013; Zhu et al., 2016; Sawala et al., 2017). The additional baryonic physics suppresses the amount of dark substructure but does not really solve the problem.

Other types of simulations, such as zoom-in, semi-analytical, and fully hydrodynamical simulations have been tailored to make predictions specific for the formation history of the Milky Way (Brook et al., 2003; Helmi et al., 2003; Bullock & Johnston, 2005; Johnston et al., 2008; Springel et al., 2008; Cooper et al., 2010; Wang et al., 2011; Pillepich et al., 2014). We will summarise some relevant insights that emerge from these simulations. Even though the Galaxy likely merged with hundreds of small systems, most of the accreted mass comes from a handful of very large systems. The outer halo is dominated by debris originating from smaller systems and is still forming (e.g. Helmi, 2008). Most of these systems were accreted > 8 Gyr ago, leading to a relatively quiescent assembly history. The most massive mergers dominate the inner halo and may have affected on the (ancient) Galactic disc (Brook et al., 2004; Villalobos & Helmi, 2008, 2009; Purcell et al., 2009; Moster et al., 2010).

1.5.2 Observations of substructure

Stellar streams and large structures

The Milky Way is known to host a range of different substructures, from cold streams to large overdensities (see Grillmair & Carlin, 2016, for a compilation). Deep photometric surveys have been very successful in mapping spatially coherent structures like streams and large overdensities (Belokurov et al., 2006; Bonaca et al., 2012a; Bernard et al.,
2016; Shipp et al., 2018), but also Gaia has contributed to this field (Malhan et al., 2018; Ibata et al., 2019). Figure 1.6 illustrates some of the structures that have been detected with these surveys.

The first discovery of a Galactic merger event (that is still ongoing) is that of the Sagittarius stream and dwarf galaxy (Ibata et al., 1994, 1995), which has recently been mapped in full-sky (Antoja et al., 2020; Ibata et al., 2020; Ramos et al., 2020). Some of the other dominant structures in the halo are the Virgo stellar stream (VSS) and Virgo overdensity (VOD) (Newberg et al., 2002; Duffau et al., 2006; Juric et al., 2008; Bonaca et al., 2012b) and the Hercules-Aquila cloud (HAC) (Belokurov et al., 2007; Simion et al., 2014) which shows the characteristics of a phase-mixed structure (Simion et al., 2018). The VOD and HAC structures might be related to each other (Simion et al., 2019).

The halo also contains substructures that are not thought to have been accreted, like Tri-And, A13, Monoceros, and the Anti-centre stream. For these structures there exists ample evidence - from chemical information and dynamical modelling - that they once belonged to the disc (Rocha-Pinto et al., 2004; Newberg et al., 2002; Morganson et al., 2016; Price-Whelan et al., 2015; Bergemann et al., 2018; Sheffield et al., 2018; Laporte et al., 2019). These structures might have been created during a merger event.

**Local fragments of streams**

A complementary approach of finding signs of accretion events is to look for clustering of stars in velocities or IOM, a technique that was pioneered by Helmi & White (1999) (but see also Helmi & de Zeeuw, 2000). Helmi et al. (1999) report on the first evidence of accretion in the local halo: the Helmi Streams. These streams have been discovered owing to their characteristic clustering in angular momentum space.

After the discovery of the Helmi Streams, several other structures have been discovered using similar techniques (e.g. Helmi et al., 2006, 2017; Chiba & Beers, 2000; Klement et al., 2008, 2009; Williams et al., 2011; Beers et al., 2017; Myeong et al., 2018a,b,c). The technique of looking for clustering in the IOM has culminated in the discovery of
Gaia-Enceladus, a relatively massive dwarf galaxy (Helmi et al., 2018; Belokurov et al., 2018) that had a similar size as the LMC (a stellar mass of $5 \cdot 10^8 - 5 \cdot 10^9$, e.g. Helmi et al. 2018; Mackereth et al. 2019; Fattahi et al. 2019; Zinn et al. 2019; Vincenzo et al. 2019) that merged with the Milky Way roughly 10 Gyr ago (Di Matteo et al., 2019; Gallart et al., 2019; Chaplin et al., 2020).

Dual halo?
One of the puzzles that might have been solved with Gaia data and the discovery of Gaia-Enceladus, is the dual-halo phenomenon (e.g. Helmi, 2020). Historically, there has been a debate on the existence of a dual halo, where one component is flattened and rotation supported and the other is more spherical, pressure supported, and in general more metal-poor (Chiba & Beers, 2000; Morrison et al., 2009; Carollo et al., 2007; Sesar et al., 2011). The local stellar halo ($r < 15$ kpc) has a metallicity of $[\text{Fe/H}] \sim -1.5$, which shifts to $\lesssim -2.0$ in the outer halo (Carollo et al., 2007; De Jong et al., 2010; Lee et al., 2017, 2019; Dietz et al., 2020), although such a metallicity gradient is not observed by all surveys (e.g. Sesar et al., 2011; Conroy et al., 2019) In this context, it is also interesting to follow the discussion on the high and low-Mg populations in the local halo (Nissen & Schuster, 2010; Schuster et al., 2012; Hawkins et al., 2014; Hayes et al., 2018).

The dichotomy of the stellar halo was fleshed out by the Gaia, which shows two clear populations in the HRD (Gaia Collaboration, Babusiaux et al., 2018) - one of which corresponds to debris from Gaia-Enceladus (Koppelman et al., 2018; Haywood et al., 2018). The other sequence corresponds to stars from the ancient disc that were heated and put on more halo-like orbits (Gallart et al., 2019; Di Matteo et al., 2019; Bonaca et al., 2017, 2020a; Belokurov et al., 2020). This second population has a net prograde rotation and is fairly flat, it does not correspond to the classical in situ halo formed, for example, from monolithic collapse (e.g. Di Matteo et al., 2019; Helmi, 2020).

1.6 Outline of Thesis
The contents of this Thesis are bundled into two parts: I) formation history and II) dynamics. These parts are very intertwined and are split mainly for readability. Part I draws from a range of dynamical tools to tackle Galactic Archaeology questions. It has a clear narrative that describes some of the major events in the formation history. On the other hand, Part II contributes to our understanding of the dynamical properties of the stellar halo, some of which might be influenced by its history.

1.6.1 Part I: formation history
One of, if not the most exciting day in my PhD must have been on the 25th of April in 2018. On this day, the second data release of Gaia was made available to everyone around the globe at the same time: 12:00 CEST. I remember frantically refreshing the Gaia website while a news reporter stood behind me asking whether I had already ‘discovered something exciting’... The reporter was too early.
However, 5 days and 3 hours and $\sim 53$ minutes later we submitted the contents of Chapter 2. In this chapter, we use simple analysis techniques to identify halo stars in RVS sample of Gaia. These techniques virtually entail manually drawing lines based, however, on experience from simulations. After identifying $\sim 6000$ stars in the local stellar halo we set out to identify sub-structures in the form of groups of stars with similar orbits - that as a collective stand out from the rest of the halo. The nearby halo contains about five of such groups that are clearly identifiable, most of which are in the retrograde part of the halo. However, much to our surprise, most of the halo is concentrated in a large disperse structure (‘blob’) of stars. Interestingly, this ‘blob’ coincides with one of the two stellar populations that had previously been identified in the Hertzsprung-Russel Diagram (HRD) of the local stellar halo (see Gaia Collaboration, Babusiaux et al., 2018). We tentatively link this ‘blob’ to the remainders of the last relatively massive merger, that possibly triggered the formation of (parts of) the thick disc.

Intrigued by this ‘blob’ of stars, we set out to analyse its properties in Chapter 3. Here we use similar techniques as in Chapter 2 to identify members of this structure. Based on a comparison with the simulations of Villalobos & Helmi (2008), we hypothesise that this ‘blob’ corresponds to the debris of a relatively massive dwarf galaxy that merged with the Milky Way at a redshift of $z \approx 1.8$. We find evidence for this link in the chemical compositions of the stars, which are obtained from a cross-match with APOGEE data. We conclude that the ‘blob’ originates in a single progenitor, which we estimate had a stellar mass of $6 \cdot 10^8$ M$_\odot$. The progenitor was named Gaia-Enceladus, after the satellite that made its discovery possible and Enceladus, the offspring of Gaia who was a Giant and was said to be buried underneath Mount Etna.

Using similar techniques of cross-matching data and comparing the current dynamical state of the system to simulations, we characterise in Chapter 4 one of the smaller structures identified in Chapter 2. This structure corresponds to the Helmi streams (see Sec. 1.5). In Chapter 4 we show that the progenitor of this structure likely was a dwarf galaxy of $10^8$ M$_\odot$ which merged 5 – 8 Gyr ago.

In Chapter 5 we build yet further on the previous chapters by investigating the collective of structures in the retrograde part of the halo. We find evidence for at least one new structure, which we label Thamnos. And in Chapter 6 we aim to look for the globular clusters that came in together with the merged dwarf galaxies. Globular clusters are more resilient to tidal disruption and thus could still be orbiting the Milky Way - on orbits close to those of the stars of these dwarfs. We assign groups of globular clusters to several merger events and find little evidence for yet undiscovered events, except for perhaps in the central region of the Milky Way.

We investigate the link between (some of) the retrograde structures and Gaia-Enceladus in Chapter 7. To this end, we study in detail the best matching simulation of the suite of Villalobos & Helmi (2008). The simulations were not designed to reproduce the debris of Gaia-Enceladus but still have been very useful in interpreting its debris. For example, after merging, the debris of a (relatively) massive merger can display a range in orbital parameters such as eccentricities and angular momenta. Especially discy progenitors show a complex gradient in the orbital parameters, giving rise to halo substructures that
seem dynamically uncorrelated. These substructures stem from regions of the dwarf stripped at different times, starting from the outer regions. Typically these outer regions of a (dwarf) galaxy are less chemically enriched. Therefore, the dynamically different substructures can also have different chemical compositions. We conclude with the warning that the merger of a relatively massive satellite can result in multiple halo structures that appear both dynamically and chemically distinct from the main body - despite once belonging to a single dwarf galaxy.

1.6.2 Part II: dynamics

In Part II we move away from the clear theme that outlines Part I into a narrative where dynamics is more at the centre, rather than being a tool. We explore the dynamical properties of substructures in the local halo and link them to the underlying mass-profile of the Milky Way.

In Chapter 8 we identify halo stars based on a combination of their photometry and proper motion, known as a reduced proper motion (RPM). This selection method has the advantage of not needing the line-of-sight velocity nor the parallax, which are either often missing in the Gaia data or have too large uncertainties to be able to use reliably. The RPM selection is extremely powerful in identifying halo stars, which otherwise would be hidden among the much more numerous disc stars. Moreover, because the selected stars are all MS stars we can use their approximately linear colour - absolute magnitude relation to calculate photometric distances (they typically have an accuracy of $\sim 7\%$). Using this sample of RPM selected halo stars, we deploy a range of techniques to surmount the missing velocity component and to interpret the data. The amount of structure that is present in the data sample appears to be fully consistent with the amount of structure in the 6D subsample.

In Chapter 9 we determine a lower limit for the local escape velocity using the RPM halo sample from the previous chapter. The escape velocity depends on the mass of the Galaxy, as we discussed in Sec. 1.2.5. We determine the escape velocity using a commonly used method, which is to fit the tail of the velocity distribution. One of the main issues with this method is that the velocity distribution not necessarily reaches the escape velocity. So, by fitting this tail we can only obtain a lower-limit for the true escape velocity. Our estimate of the lower limit is $v_{\text{esc}}(r_\odot) = 497^{+40}_{-24}$ km/s which results in a lower limit on the mass of the Milky Way’s dark halo of $M_{200} = 6.7^{+3}_{-1.5} \cdot 10^{11}$ M$_\odot$.

Finally, in Chapter 10, we conclude the thesis with a study of the evolution of gaps in stellar streams. Streams are put forwards as tentative probes of dark halo substructures. If a dark subhalo interacts with a thin, cold stellar stream it will create a gap in the otherwise nearly smooth distribution of the latter. There exist some models that predict the evolution of the gap, but they all have their own limitations. Our unique, fully analytical approach, using the action-angle variables, allow us to express the properties of the gap (size, central density) in terms of the subhalo’s properties and the parameters describing the configuration of the collision. With such expressions, and soon to be available observations of gaps in streams, one could potentially determine if and what kind of dark subhalos are orbiting the Milky Way.
1.7 Where to go from here?

At the start of my PhD I identified the following open questions

- What is the origin of the duality/multi-modality of the halo?
- What is the balance of dissipative vs. dissipationless formation?
- Are Milky Way satellites survivors of building blocks?
- What constraints can we infer from streams in the halo of the Milky Way?

Markedly, many if not all of these questions have been answered from the collective efforts of the field including this Thesis, and thanks to the data coming from Gaia and additional spectroscopic surveys. So what remains? What are the new open questions? In what direction will the field move?

The field of Galactic Archaeology and Galactic Dynamics are at a unique time. New data will (very) soon be available in the form of significant updates from the Gaia mission, multi-fibre and high-resolution spectroscopic surveys, and asteroseismic observations. On the other hand, the flood of results stemming from the already released Gaia data has given many interesting insights. They require the development of new theory, stepping away from the assumption of equilibrium, and new generations of (tailored) hydrodynamic simulations. There are many different directions in which the field could move, only some of which I will discuss here. What is clear is that it is imperative to update our understanding of the Galaxy on all fronts - observations, theory, and simulations.

One of the important developments will be to model time-dependent effects in the field of dynamics. Most of the current models of the Milky Way are time-invariant and assume equilibrium, but there is mounting evidence that these assumptions might not be valid. One example of this is the infall of the LMC, which clearly affects the dynamics of the rest of the Galactic halo (Vera-Ciro & Helmi, 2013; Petersen & Peñarrubia, 2020; Erkal et al., 2020). Moreover, we will have to quantify what effects the (ongoing) mergers have on the dynamics and star-formation rate of the Galactic disc (e.g. the recent studies of Ruiz-Lara et al., 2020; Mor et al., 2019; Fantin et al., 2020). Also in this context, it has been shown that the local disc contains wave-like features that might be triggered by (ongoing) merger events (Antoja et al., 2018; Kawata et al., 2018). All of these results raise the question of which types of dynamical models can be run in isolation and which require to model the Milky Way holistically. When is the approximation of the Galaxy being in equilibrium acceptable and when is it not?

In conjunction with testing the assumption of equilibrium, we will have to verify the current sets of simulations, which often are dark-matter-only. We have already discussed that the inclusion of baryons can drastically alter the dark substructure of a galaxy. Also, based on the recent findings on the Galaxy's assembly history, it will be possible to more precisely identify Milky Way analogues in large cosmological simulations. A venture that is already showing interesting comparisons, indicating that there might be less Milky Way-like galaxies than expected (Bignone et al., 2019; Bose et al., 2020; Fattahi et al., 2019; Evans et al., 2020; Elias et al., 2020; Brook et al., 2020) - although any emergent structure is unique if one includes enough details. Some of the candidates identified
in these studies might soon be followed-up by zoom-in simulations. Therefore, we can expect that a new generation of cosmologically-motivated, tailored, fully-hydrodynamical simulations are necessary to interpret the upcoming data releases from Gaia and other surveys. Have past and ongoing mergers induced significant star-formation? Locally, or everywhere in the Galaxy? What happened to the gas of Gaia-Enceladus? But also: what are the signatures of group infall of satellites? Can we determine with confidence whether Gaia-Enceladus came in alone, or did it have its own dwarf satellites?

Another interesting development will be to expand ‘chemical tagging’ by adding multi-element information from high-resolution spectroscopic observations and to complement it with stellar ages (which are notoriously difficult to observe, e.g. Soderblom 2010). Complementary to this will be the forthcoming data releases from Gaia, which will include significantly more accurate astrometry, and yet unavailable data such as light-curves of transient objects, elemental abundances, and the radial velocities for many more stars than currently are available.

And finally, we will work towards a complete inventory of all of the different structures in the stellar halo and their connection (along the lines of Naidu et al., 2020; Bonaca et al., 2020a). However, perhaps even more excitingly we will see the rise of studies looking into the detailed properties of these structures. One intriguing application is to use the orbital frequencies to identify exactly when objects were accreted time (McMillan & Binney, 2008; Gómez & Helmi, 2010). Moreover, hopefully, we will be able to probe the population of dark substructures from their effects on the luminous substructures. Perhaps from careful analysis of dynamical anomalies, such as gaps in streams, we will be able to put firm constraints on the nature of the dark matter particle.

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Part I

Formation history
One large blob and many streams frosting the nearby stellar halo in *Gaia* DR2


**Abstract**

We explore the phase-space structure of nearby halo stars identified kinematically from *Gaia* DR2 data. We focus on their distribution in velocity and in “integrals of motion” space as well as on their photometric properties. Our sample of stars selected to be moving at a relative velocity of at least 210 km/s with respect to the Local Standard of Rest, contains an important contribution from the low rotational velocity tail of the disk(s). The $V_R$-distribution of these stars depicts a small asymmetry similar to that seen for the faster rotating thin disk stars near the Sun. We also identify a prominent, slightly retrograde “blob”, which traces the metal-poor halo main sequence reported by Gaia Collaboration, Babusiaux et al. (2018). We also find many small clumps especially noticeable in the tails of the velocity distribution of the stars in our sample. Their HR diagrams disclose narrow sequences characteristic of simple stellar populations. This stream-frosting confirms predictions from cosmological simulations, namely that substructure is most apparent amongst the fastest moving stars, typically reflecting more recent accretion events.

**2.1 Introduction**

The *Gaia* 2nd data release (Gaia Collaboration, Brown et al., 2018) has just become available and has surpassed all expectations as evidenced by the science verification publications accompanying its release (e.g. Gaia Collaboration, Helmi et al., 2018; Gaia Collaboration, Katz et al., 2018; Gaia Collaboration, Babusiaux et al., 2018). It will take many years to fully exploit the vastness of the dataset and especially the fantastic increase in accuracy. On the other hand, also because of the same reasons, already a simple first exploration of the dataset yields exciting new insights.

We report here the results of the analysis of the *Gaia* DR2 set of 7 million stars with full phase-space information (derived from the astrometric and from the radial velocity spectrometer data, Lindegren et al., 2018; Katz et al., 2018), with the aim of identifying substructure in the nearby Galactic halo. This Galactic component is particularly important for understanding the assembly of the Milky Way in a cosmological
context. It is here where we expect to find merger debris and some of the most pristine stars (Bullock & Johnston, 2005; Starkenburg et al., 2017). Large photometric surveys such as SDSS and PanSTARRS have uncovered large overdensities and stellar streams in the outer halo (Belokurov et al., 2006; Bernard et al., 2016), indicative of relatively recent accretion activity. However, the inner regions of the stellar halo, despite containing most of the mass, have remained more of a mystery so far partly because of the shorter dynamical timescales. Yet it is these inner halo stars that tell us about the early assembly history of the Milky Way.

In this Chapter we analyse a sample of halo stars selected from a Toomre diagram constructed from Gaia DR2 data (Sec. 2.2). We have inspected their kinematics and dynamics as well as their distribution in the Hertzsprung-Russell diagram (Sec. 2.3). As reported in Gaia Collaboration, Babusiaux et al. (2018) two clear main sequences are evident amongst halo stars in the Solar vicinity. Here we are able to associate at least in part the older and more metal-poor sequence to a prominent slightly retrograde “blob”, hinted at in previous datasets (Carollo et al., 2007; Morrison et al., 2009; Helmi et al., 2017) but never so easily discernible. We also report the presence of several small clumps of stars that populate the tails of the kinematic distribution, including the Helmi stream (Helmi et al., 1999). Cosmological simulations of halo build-up (Helmi et al., 2003) have long predicted that substructure due to accretion should be more apparent at high-velocities (see also Re Fiorentin et al., 2015, for first hints). Thus it is very plausible that these substructures are in fact the remnants of the more recent accretion of small dwarf galaxies contributing stars to the Solar neighbourhood.
2.2 Data and Methods

Halo stars are relatively rare (less than 1% of the stars in the Solar neighbourhood, e.g. Helmi, 2008, and references therein), hence it is paramount to have a good method of identification (see Veljanoski et al., 2018). Traditionally halo stars have been selected on the basis of their metallicity or their large velocities with respect to the disk (see Posti et al., 2017, for a recent comparison of selection methods). Here we follow the second approach, and use the traditional Toomre diagram selection to isolate halo stars (see e.g. Bonaca et al., 2017). Although subject to biases (against halo stars that have similar motions as the disk(s)), it allows removing the large number of nearby disk stars that dominate the counts (Brown et al., 2005).

The Toomre diagram plots the velocity in the direction of rotation $V_y$ against the other two components: $\sqrt{V_x^2 + V_z^2}$. We use the Gaia DR2 sample with 6D phase-space information and consider only those stars with relative parallax error $\varpi/\sigma_\varpi > 5$, which allows us to compute distances as $d = 1/\varpi$ with relative errors of 20% at most. This sample contains 6,366,744 stars.

Fig. 2.1 shows the Toomre diagram for the stars in this sample and located within 1 kpc from the Sun. Our reference system is oriented such that X is positive towards Galactic longitude $l = 0$, Y in the direction of rotation, and Z for Galactic latitudes $b > 0$, with the Sun located at $R_{\odot} = 8.2$ kpc on the negative X-axis. The velocities are also oriented in these directions and have been corrected assuming a Local Standard of Rest velocity $V_{\text{LSR}} = 232$ km/s (McMillan, 2017) and a peculiar motion for the Sun of $(U_{\odot}, V_{\odot}, W_{\odot}) = (11.1, 12.24, 7.25)$ km/s (Schönrich et al., 2010).

We isolate a set of halo stars using the criterion $|V - V_{\text{LSR}}| > V_{\text{cut}}$, where we take $V_{\text{cut}} = 210$ km/s, (i.e. slightly stricter than Nissen & Schuster, 2010). This kinematically selected halo sample contains a total of 5,980 stars, with typical velocity errors of 10 km/s. Of these, 3,040 are main sequence stars. The coloured solid circles in Fig. 2.1 correspond to various easily discernible overdensities identified and discussed in more detail in the next section.

2.3 Analysis

2.3.1 Velocities

Fig. 2.2 shows the velocity distribution of the kinematically selected halo stars. This distribution is rather complex, particularly in the $V_x - V_y$ projection (top panel of Fig. 2.2). This is in part due to the sharpness of the selection criterion applied on the Toomre diagram. This figure shows also that relaxing slightly the value of $V_{\text{cut}}$ would lead to the inclusion of more stars from the tail of the velocity distribution of the disk(s) (Schönrich & Binney, 2009; Bonaca et al., 2017). Their imprint is a noticeable asymmetry in the $V_x$-distribution, which is characteristic of the perturbation induced by the Galactic bar (e.g. Antoja et al., 2015). In our sample, this asymmetry has a smaller amplitude than
Fig. 2.2: Velocity distribution of halo stars (black dots and coloured circles) selected according to the Toomre diagram shown in Fig. 2.1. The blue density maps show the velocity distributions of all stars within 1 kpc from the Sun and reveal the contribution of the disk(s). The top panel exposes particularly clearly the effect of our kinematic selection criterion. The coloured stars mark the location of tight clumps that are easy to discern because of their large velocities.

for the thin disk, but it is nonetheless clearly, and possibly unexpectedly, present at $V_y \gtrsim 100$ km/s, where the number of stars with $V_x < 0$ is 389, whereas for $V_x > 0$ there are 322 stars ($\sim 3\sigma$ excess). To properly understand the dynamical properties of this transition region of velocity space, would likely require a multi-dimensional
probabilistic analysis to assign stars to different physical components using also chemical and age information (Binney et al., 2014; Posti et al., 2017), which is beyond the scope of this Chapter.

The top panel of Fig. 2.2 shows a broad overdensity of stars for positive $V_x$ at $V_y \sim 50$ km/s, just where the contribution from the low velocity tail of the disks would be expected to die away. Although to understand its nature requires an in-depth analysis, the location of this overdensity seems unlikely to be related to the Toomre diagram-based selection criterion.

Fig. 2.2 shows the presence of a prominent component with a large dispersion in $V_x$ (equivalent to $V_R$ near the Sun), that has a slightly retrograde mean motion of a few tens of km/s. Although this could be considered as the “traditional” halo, it is slightly too retrograde and asymmetric towards more negative $V_y$. Fig. 2.2 also reveals that the high-velocity tails of the distribution of the stars in our halo sample are populated by several cold clumps. The structure at $V_y \sim 150$ km/s and $V_z \sim -250$ km/s (in green, 25 stars), can be associated to one of the streams found by Helmi et al. (1999), and reported also in Gaia Collaboration, Helmi et al. (2018) using only proper motion information (their Fig. 25). The second stream found by Helmi et al. (1999) is less conspicuous with 12 stars, but present at $V_y \sim 150$ km/s and $V_z \sim 230$ km/s. The asymmetry in the number of stars in each of the streams implies that the accretion event from which these streams originate must have happened 6 to 9 Gyr ago according to the models presented in Kepley et al. (2007).

There are also other clumps in Fig. 2.2 and these are marked with different colours. Some appear to overlap with previously reported hints of substructure (e.g. Re Fiorentin et al., 2015). For example, the orange circles are probably related to the “retrograde outlier stars” of Kepley et al. (2007), and those with blue colour overlap with the structure Ve1He1–4 from Helmi et al. (2017).

### 2.3.2 “Integrals of motion”-space

We explore now the distribution of stars in the space of “integrals of motion” defined by the $z$-component of the angular momentum $L_z$, the perpendicular component $L_\perp = \sqrt{L_x^2 + L_y^2}$, and the energy $E$. For the stars in our sample we compute their total energy $E$ as the sum of a kinetic and a potential term, where the amplitude of the latter is estimated using a suitable Galactic potential (see Helmi et al., 2017, for details).

Note that $L_\perp$ is not really conserved in an axisymmetric potential like that of our Galaxy, but it has proven to be nonetheless a useful proxy for a third integral, and to help in discriminating substructures with different orbital properties (Helmi et al., 1999; Helmi & de Zeeuw, 2000)

Figure 2.3 shows the distribution of the halo stars in our sample in the $E - L_z$ space. Although this figure is reminiscent of that obtained using a TGAS x RAVE sample (Helmi et al., 2017).
Fig. 2.3: Distribution of the stars in our kinematically selected halo sample in the “integrals of motion” space defined by their energy $E$ and $z$-component of their angular momentum $L_z$. To make the structure visually more apparent we have used here a stricter value of $V_{\text{cut}} = V_{\text{LSR}}$, which reduces the contrast between the tails of the disks and the rest of the halo.

et al., 2017) or perhaps even TGAS×SDSS (Myeong et al., 2018), it is much more spectacular.

Figure 2.3 shows a clear prominent “blob” or “plume” that is slightly retrograde, the counterpart of the structure seen in velocity space (Fig. 2.2) and also in the Toomre diagram (Fig. 2.1). This region has been previously associated to where a possible progenitor of OmegaCen would deposit debris (Dinescu, 2002; Bekki & Freeman, 2003; Majewski et al., 2012; Helmi et al., 2017). Whether this is the only progenitor populating this region of phase-space remains to be seen. Because of its large extent in energy, this structure contributes stars to the outer halo, and so may well be at least partly responsible for the “retrograde” component previously reported by Carollo et al. (2007, and subsequent work). If a single event, its size in “integrals of motion” space suggests it was very significant.

Fig. 2.4 shows a scatter plot of the distribution of kinematically selected stars in the $E - L_z$ (top) and $L_{\perp} - L_z$ (bottom) spaces. We have plotted here as a density map the contribution of all the stars in the Gaia DR2 6D sample located within 1 kpc from the Sun and with distance errors smaller than 20%. These figures clearly show where the disks fall and the sharpness of our kinematic selection criterion in the transition region between the disks and the (traditional) halo.

Several tight overdensities also apparent in the panels of Fig. 2.4. There is a close correspondence between these overdensities and those identified in velocity space. The boxes shown here mark the clumps easy to identify in the “integrals of motion” space, while the remaining coloured clumps have been identified in velocity space. All structures have been highlighted using the same colour-coding as in Fig. 2.2.
To establish the significance level of these structures we randomise the whole halo dataset by reshuffling the velocities of the stars, while keeping their spatial distribution. We make 10 000 such random realisations and recompute for each, the distribution in “integrals of motion” space. We find that in none of these realisations, a substructure is apparent that has a similar extent and location as any of the overdensities identified in the various figures.

2.3.3 HR diagram
The stars that are part of the large “blob/plume” identified in Fig. 2.3 (inside the grey box in the top panel of Fig. 2.4) define a blue sequence in the HR diagram of kinematically selected halo stars, as indicated by the red dots in the large panel in Fig. 2.5. The absolute magnitude given here is calculated using the parallax from Gaia and has not
Fig. 2.5: HR diagrams of the stars in our kinematically selected halo sample (in gray). The location of the stars in the retrograde “blob” seen in Fig. 2.3 are shown in red in the large panel. The subpanels show the sequences defined by the other structures identified in Fig. 2.2 and 2.4 using the same colour schemes. To guide the eye we include here two isochrones having [M/H] ∼ –1.3 dex and 13 Gyr old (blue), and [M/H] ∼ –0.5 dex and 11 Gyr old (black), following Gaia Collaboration, Babusiaux et al. (2018).
been corrected for extinction. This sequence coincides with that reported by Gaia Collaboration, Babusiaux et al. (2018). These authors estimated an age of 13 Gyr and a metallicity $[\text{M}/\text{H}] \sim -1.3$ dex for this population, obtained by comparison to the blue isochrone shown here and obtained from Marigo et al. (2017), considering an $\alpha$-enhancement of 0.23 (Salaris et al., 1993). The redder stars are fitted better by an isochrone with $[\text{M}/\text{H}] \sim -0.5$ dex and age of 11 Gyr (in black in Fig. 2.5), as already shown in Gaia Collaboration, Babusiaux et al. (2018).

The stars associated to the blue sequence (and the corresponding isochrone) are more metal-rich than the “outer halo” component of Carollo et al. (2007), which has $[\text{Fe/H}] \sim -2.2$ dex. This could imply that this region of phase-space is more complex than anticipated (see also Nissen & Schuster, 2010). On the other hand, if this overdensity is due to the merger of a single large progenitor system, this could have had a metallicity gradient. This would imply that stars at larger distances could be more metal-poor on average (as seen also for the streams of the Sagittarius dwarf, Bellazzini et al., 2006), thus explaining the findings of Carollo et al. (2007).

The HR diagrams for the stars belonging to the other substructures are plotted as small panels in Fig. 2.5. We do not attempt to fit isochrones to their distribution because of the relative low number of member stars (ranging from 12 to 37 stars). Note however, that their HR diagrams are very coherent, and suggest low metallicities and old ages, similar to those of the large “plume” shown in the large panel of the figure. Exploration of the full Gaia DR2 should disclose additional members. These could be either more distant giants stars or fainter nearby dwarfs. These dwarf stars would not have full phase-space information in Gaia DR2 (because of the magnitude cut of the spectroscopic sample, Katz et al., 2018), but should have accurate proper motions and parallaxes, and be present in large numbers.

2.4 Discussion

Gaia DR2 has revealed that the phase-space structure of halo stars selected using a kinematic selection, namely in the Toomre diagram, is rather complex. It includes large overdensities and several tight kinematic streams or clumps.

The distributions of stars in velocity and in “integrals of motion” space both indicate the presence of what appears to be the low rotational velocity extension of the Galactic disks, well into the region traditionally associated to the halo. Surprisingly, this distribution is asymmetric in $V_x \ (V_R)$ and follows the thin disk’s characteristic shape believed to be due to the effect of the Galactic bar. This is presumably the metal-rich $\sim 11$ Gyr halo component reported in the HR diagram of Gaia Collaboration, Babusiaux et al. (2018).

We find a prominent slightly retrograde component, which we interpret to be at least in part merger debris from one or more large objects. The distribution of stars in the Toomre diagram shown in Fig. 2.1 resembles very closely Fig. 7 of Villalobos & Helmi (2009) obtained from a simulation of the merger of a relatively massive object with a pre-existing disk (merger ratio 1 : 5), that subsequently gave rise to the formation
of a thick disk. This large “blob” could thus have been responsible for the puffing up of an ancient Galactic disk. An alternative explanation is that this “blob” is an ancient non-rotating halo, and that the assumed Local Standard of Rest velocity should be shifted by a few tens of km/s (from the 232 km/s we assume here), which at face value seems somewhat unlikely, especially given the asymmetric shape of the “blob” towards more negative rotational velocities.

The velocity distribution of stars in our halo sample also reveals the presence of streams located in the high-velocity tails, as predicted by cosmological simulations of the build-up of galactic halos (Helmi et al., 2003). This implies that accretion has played a role in the assembly of the halo near the Sun. How important this process has been remains to be established with more sophisticated analysis. Further explorations of Gaia DR2 in other regions of the Galaxy, using different tracers, and only proper motion and parallax information are necessary to fully grasp the complexity of the stellar halo. Given the superb quality of the data, there is no doubt that Gaia holds many surprises for those in the quest to unravel the assembly history of the Galaxy.

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Bibliography

The merger that led to the formation of the Milky Way’s inner stellar halo and thick disc

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Abstract

The assembly process of our Galaxy can be retrieved using the motions and chemistry of individual stars (Freeman & Bland-Hawthorn, 2002; Helmi et al., 2003). Chemo-dynamical studies of the nearby halo have long hinted at the presence of multiple components such as streams (Helmi et al., 1999), clumps (Morrison et al., 2009), duality (Carollo et al., 2007) and correlations between the stars’ chemical abundances and orbital parameters (Chiba & Beers, 2000; Nissen & Schuster, 2010; Beers et al., 2017). More recently, the analysis of two large stellar surveys (Abolfathi et al., 2018; Gaia Collaboration, Prusti et al., 2016) have revealed the presence of a well-populated chemical elemental abundance sequence (Nissen & Schuster, 2010; Hayes et al., 2018), of two distinct sequences in the colour-magnitude diagram (Gaia Collaboration, Babusiaux et al., 2018), and of a prominent slightly retrograde kinematic structure (Belokurov et al., 2018; Koppelman et al., 2018) all in the nearby halo, which may trace an important accretion event experienced by the Galaxy (Haywood et al., 2018). Here report an analysis of the kinematics, chemistry, age and spatial distribution of stars in a relatively large volume around the Sun that are mainly linked to two major Galactic components, the thick disc and the stellar halo. We demonstrate that the inner halo is dominated by debris from an object which at infall was slightly more massive than the Small Magellanic Cloud, and which we refer to as Gaia-Enceladus. The stars originating in Gaia-Enceladus cover nearly the full sky, their motions reveal the presence of streams and slightly retrograde and elongated trajectories. Hundreds of RR Lyrae stars and thirteen globular clusters following a consistent age-metallicity relation can be associated to Gaia-Enceladus on the basis of their orbits. With an estimated 4 : 1 mass-ratio, the merger with Gaia-Enceladus must have led to the dynamical heating of the precursor of the Galactic thick disc and therefore contributed to the formation of this component approximately 10 Gyr ago. These findings are in line with simulations of galaxy formation, which predict that the inner stellar halo should be dominated by debris from just a few massive progenitors (Helmi et al., 2003; Cooper et al., 2010).
### 3.1 Main section

The sharp view provided by the second data release (DR2) of the *Gaia* mission (Gaia Collaboration et al., 2018), has recently revealed (Koppelman et al., 2018) that, besides a few tight streams, a significant fraction of the halo stars near the Sun are associated with a single large kinematic structure that has slightly retrograde mean motion and which dominates the Hertzsprung-Russell diagram’s (HRD) blue sequence revealed in the *Gaia* data (Gaia Collaboration, Babusiaux et al., 2018). This large structure is readily apparent (in blue) in Fig. 3.1a, which shows the velocity distribution of stars (presumably belonging to the halo) in the Solar vicinity inside a volume of 2.5 kpc radius from *Gaia* data (see Methods for details). Figure 3.1b shows the velocity distribution from a simulation of the formation of a thick disc via a 20% mass-ratio merger (Villalobos & Helmi, 2008). The similarity between the panels suggests that the retrograde structure could be largely made up of stars originating in an external galaxy that merged with the Milky Way in the past.

![Fig. 3.1: Velocity distribution of stars in the Solar vicinity in comparison to a merger simulation.](image)

In the left panel, the velocities of stars in the disc are plotted with grey density contours (because of the large number of stars), while the halo stars (selected as those with $|v-v_{\text{LSR}}| > 210$ km/s, where $v_{\text{LSR}}$ is the velocity of the Local Standard of Rest) are shown as points. The blue points are part of a prominent structure with slightly retrograde mean rotational motion, and have been selected here as those having $-1500 < L_z < 150$ kpc km/s and energy $E > -1.8 \times 10^5$ km$^2$/s$^2$ (see Methods for details). The panel on the right shows the distribution of star particles in a small volume extracted from a simulation (Villalobos & Helmi, 2008) of the formation of a thick disc via a $5:1$ merger between a satellite (in blue) and a pre-existing disc (in black). The overall morphology and the presence of an arch (from $V_y \sim -450$ km/s and $V_\perp = \sqrt{V_x^2 + V_z^2} \sim 50$ km/s to $V_y \sim -150$ km/s and $V_\perp \sim 300$ km/s seen in the left panel) can be reproduced qualitatively after appropriately scaling the velocities (see Methods), in a simulation where the satellite is discy (rather than spherical, as the arch-like feature is sharper), and on a retrograde orbit inclined by $\sim 30^\circ$ to $60^\circ$. 

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40 Chapter 3 The Gaia-Enceladus merger
Support for this hypothesis comes from the chemical abundances of stars provided by the APOGEE survey (Abolfathi et al., 2018). In Fig. 3.2a we plot the [$\alpha$/Fe] vs [Fe/H] abundances for a sample of stars cross-matched to Gaia DR2 (see Methods for details). $\alpha$-elements are produced by massive stars that die fast as supernovae (SNII), while iron, Fe, is also produced in SNI explosions of binary stars. Therefore in a galaxy, [$\alpha$/Fe] decreases with time (as [Fe/H] increases). Fig. 3.2a shows the well-known sequences defined by the thin and thick discs. The vast majority of the retrograde structure’s stars (in blue), follow a well-defined separate sequence that extends from low to relatively high [Fe/H]. (The presence of low-$\alpha$ stars with retrograde motions in the nearby halo has in fact been reported before (Nissen & Schuster, 2010, 2011) but for a small sample. The existence of a well-populated sequence with lower [$\alpha$/Fe] was demonstrated very recently using also APOGEE data (Hayes et al., 2018)). An independent analysis (Haywood et al., 2018) has confirmed the relation between Gaia’s HRD blue sequence and the kinematic structure shown in Fig. 3.1a, and established firmly the link to the low-$\alpha$ stars using both earlier data (Nissen & Schuster, 2010) as well as APOGEE, thereby putting the accretion hypothesis on more secure ground.

The large metallicity spread of the retrograde structure stars depicted in Fig. 3.2b, implies that they did not form in a single burst in a low mass system. Furthermore because the more metal-rich stars have lower [$\alpha$/Fe] at the characteristic metallicity of the thick disc ([Fe/H] $\sim$ –0.6), this means that they were born in a system with a lower star formation rate than the thick disc. The star formation rate required to match the $\alpha$-poor sequence of the APOGEE data has recently been calculated using a chemical evolution model and including different elemental abundances (Fernández-Alvar et al., 2018), and found to be 0.3 M$_{\odot}$/yr lasting for about 2 Gyr. This implies a stellar mass for the progenitor system of $\sim$ 6 $\times$ 10$^8$ M$_{\odot}$, a value that is consistent with the large fraction of nearby halo stars being associated with the structure given estimates of the local halo density (Helmi, 2008), and which is comparable to the present-day mass of the Small Magellanic Cloud (van der Marel et al., 2009). Interestingly, previous work (Hayes et al., 2018) has shown that the trends in the abundances of low metallicity stars in the Large Magellanic Cloud actually overlap quite well with the sequence, implying that the structure was comparable to the Large Magellanic Cloud in its early years. Furthermore and perhaps even more importantly, because [$\alpha$/Fe] must decrease as [Fe/H] increases, the stars in the structure could not have formed in the same system as the vast majority of stars in the Galactic thick disc. They must have formed, as previously suggested (Nissen & Schuster, 2010; Koppelman et al., 2018; Haywood et al., 2018), in a separate galaxy, which we refer to as Gaia-Enceladus hereafter (see Methods for the motivation behind the naming).

We now explore whether the Gaia-Enceladus galaxy could have been responsible at least partly for the formation of the thick disc (Belokurov et al., 2018; Koppelman et al., 2018; Haywood et al., 2018), as the comparison between the data and the simulation shown in Fig. 3.1 would suggest. In that case, a pre-existing disc must have been in place at the time of the merger. Figure 3.2c plots the HRD of the halo stars in
Fig. 3.2: Astrophysical properties of stars in Gaia-Enceladus. Panel a) shows the chemical abundances for a sample of stars located within 5 kpc from the Sun resulting from the cross-match between Gaia and APOGEE. The blue circles correspond to 590 stars that have $-1500 < L_z < 150$ km/s kpc and $E > -1.8 \times 10^5$ km$^2$/s$^2$ (as in Fig. 3.1a, but now for a larger volume to increase the sample size, see Methods). Note the clear separation between the thick disc and the sequence defined by the majority of the stars in the retrograde structure, except for a small amount of contamination (17%) by thick disc stars (i.e. on the $\alpha$-rich sequence) that share a similar phase-space distribution as the structure. The error bar in the lower left corner shows the median error for the sample. The solid (dotted) histogram in panel b) shows the metallicity distribution of the structure without (with) the subset of $\alpha$-rich stars. Their distribution peaking at $[\text{Fe/H}] \sim -1.6$, is very reminiscent of that of the stellar halo (Helmi, 2008). Panel c) is the HRD for halo stars (black points, selected as in Fig. 3.1a with the additional photometric quality cuts (Gaia Collaboration, Babusiaux et al., 2018): $E(B-V) < 0.015$ to limit the impact in the magnitudes and colours to less than 0.05 mag, and phot-bp-rp-excess-factor < 1.3 + 0.06(G$G_{\text{BP}}$ − G$G_{\text{RP}}$)$^2$) and shows Gaia’s blue and red sequences. Gaia-Enceladus stars are plotted with dark blue symbols, with those in APOGEE within 5 kpc and with $[\alpha/\text{Fe}] < -0.14 - 0.35$ [Fe/H], in light blue. The superimposed isochrones (Marigo et al., 2017) based on previous work (Hawkins et al., 2014) show that an age range from 10 to 13 Gyr is compatible with the HRD of Gaia-Enceladus.

Fig. 3.1a showing the Gaia-Enceladus stars (in blue) populating Gaia’s blue sequence (Gaia Collaboration, Babusiaux et al., 2018; Koppelman et al., 2018; Haywood et al., 2018). The thinness of this sequence is compatible with an age range from $\sim 10$ to 13 Gyr given the stars’ abundance sequence, as indicated by the plotted isochrones (Marigo et al., 2017). Previous studies (Schuster et al., 2012; Hawkins et al., 2014), on which this age range is based, have shown that the stars on the $\alpha$-poor sequence are younger than those on the $\alpha$-rich sequence for $-1 < [\text{Fe/H}] < -0.5$. This implies that the progenitor of the Galactic thick disc was in place when Gaia-Enceladus fell in, which based on the ages of its youngest stars, would suggest that the merger took place around 10 Gyr ago, i.e. at redshift $z \sim 1.8$. 

Chapter 3  The Gaia-Enceladus merger
Fig. 3.3: Sky distribution of tentative Gaia-Enceladus members from a Gaia subsample of stars with full phase-space information. These stars have $\varpi > 0.1$ mas, relative parallax error of 20%, and are colour-coded by their distance from the Sun (from near in dark red to far in light yellow). They satisfy the condition $-1500 < L_z < 150$ kpc km/s. Because of the larger volume explored, we do not include additional selection criteria based on energy, as done for Fig. 3.2 (since energy depends on the Galactic potential whose spatial variation across the volume explored is less well-constrained than its local value), nor on velocity as for Fig. 3.1a (because of spatial gradients). We thus expect some amount of contamination by thick disc stars, especially towards the inner Galaxy (see Methods). The starry symbols are Gaia RR Lyrae stars potentially associated to this structure. To identify these, we bin the sky in $128 \times 128$ elements, and log $\varpi$ in bins of 0.2 width (mimicking the relative parallax error), and measure the average proper motion of Gaia-Enceladus stars in each 3D bin. We then require that the RR Lyrae have the same proper motion (within 25 km/s in each component at their distance), which for example corresponds to 1 mas/yr for those with $\varpi \sim 0.2$ mas. Globular clusters with $L_z < 250$ kpc km/s, located between 5 and 15 kpc from the Sun, and $40^\circ$ away from the Galactic Centre, are indicated with solid circles.

Such a prominent merger must have left debris over a large volume of the Galaxy. To explore where we may find other tentative members of Gaia-Enceladus beyond the solar neighbourhood, we consider stars in the Gaia 6D sample with 20% relative parallax error, with $\varpi > 0.1$ mas and having $-1500 < L_z < 150$ kpc km/s. Fig. 3.3 shows that nearby tentative Gaia-Enceladus stars (with $\varpi > 0.25$ mas, darker points) are distributed over the whole sky, this subset being more than 90% complete. More distant stars are preferentially found in specific regions of the sky, and although for such small Gaia parallaxes ($\varpi = 0.1 – 0.25$ mas) the zero-point offset ($\sim -0.03$ mas) is significant and this affects the selection in $L_z$, it does not to the extent that it can produce the observed asymmetry on the sky. At least in part this asymmetry is due to the 20% relative parallax error cut, as highlighted in Fig. 3.4 (see Methods for more details and also for
possible links to known overdensities). In Figure 3.3 we have also overplotted (with starry symbols) a subset of 200 Gaia RR Lyrae stars (Clementini et al., 2019). These have proper motions similar to the mean of the candidate Gaia-Enceladus stars with full phase-space information, at their sky position and parallax. Thirteen globular clusters can also be associated to Gaia-Enceladus on the basis of their angular momenta (Gaia Collaboration et al., 2018) (Fig. 3.3). All these clusters show a consistent age-metallicity relation (VandenBerg et al., 2013).

Fig. 3.4: Kinematic properties of Gaia-Enceladus tentative members on the sky. The plotted stars are a subset of those in Fig. 3.3 with 0.1 < \( \varpi \) < 0.2 mas, and are colour coded by their radial velocity with the arrows indicating their direction of motion. To avoid cluttering the panels correspond to different proper motion ranges, and we have removed stars close to the bulge (within 30° in longitude and 20° in latitude). All velocities have been corrected for the Solar and for the Local Standard of Rest motions. The grey contours encompass 90% of the stars in the 6D Gaia set with 0.1 < \( \varpi \) < 0.2 mas and having 20% relative parallax error, and clearly demonstrate this selection criterion impacts our ability to identify distant Gaia-Enceladus stars in certain regions of the sky. Notice the large-scale pattern in the radial velocity, as well as its correlation with the proper motion component \( \mu_l \): stars with \( \mu_l > 0 \) (top panels) have \( v_{\mathrm{GSR}} > 0 \) for \( l \gtrsim 75^\circ \) and \( v_{\mathrm{GSR}} < 0 \) for \( l \lesssim -75^\circ \), while the opposite occurs for \( \mu_l < 0 \). Such a global pattern (and its reversal for \(-75^\circ \lesssim l \lesssim 75^\circ\)) arises because of the coherent retrograde sense of rotation of the stars in their orbits (i.e. they have mostly \( L_z \lesssim 0 \)), but the correlation with \( \mu_l \) is a result of their elongated orbits, e.g. we see that if \( \mu_l > 0 \) and \( l \gtrsim 75^\circ \) stars are typically moving outwards with high speed and away from the Solar radius (\( v_{\mathrm{GSR}} \gtrsim 100 \text{ km/s} \)).

Fig. 3.4 shows the velocity field of the more distant stars associated to Gaia-Enceladus. Notice the large-scale gradient in the radial velocity across the full sky. Such a coherent pattern can only be obtained if stars are moving in the same (retrograde) direction on elongated orbits. The proper motions, depicted by the arrows, reveal a rather complex velocity field. This is expected, given the large mass of the progenitor object and the
short mixing timescales in the inner Galaxy (Helmi et al., 2003). Nonetheless, in this complexity we see streams: close stars often move in the same direction. This is a very significant effect as established by comparing to mock sets constructed assuming a multivariate Gaussian for the velocities (see Methods for details).

We conclude that the halo near the Sun is strongly dominated by a single structure of accreted origin, as hinted also by other work (Belokurov et al., 2018; Koppelman et al., 2018), and leaving little room for an in-situ contribution (Haywood et al., 2018). It is however, not necessarily representative of the whole stellar halo, as debris from other accreted large objects (with e.g. different chemical abundance patterns) might dominate elsewhere in the Galaxy. We also conclude that the Milky Way disc experienced a significant merger in its history. We estimate the mass-ratio of this merger at the time it took place as

$$\frac{M_{\text{vir}}^{\text{GE}}}{M_{\text{vir}}^{\text{MW}}} = \frac{f^{\text{MW}}}{f^{\text{GE}}} \times \frac{M_{*}^{\text{GE}}}{M_{*}^{\text{MW}}},$$

where $f$ is the ratio of the luminous-to-halo mass of the object. At the present-time, $f^{\text{MW},0} \sim 0.04$ for the Milky Way (McMillan, 2017), and if we assume that Gaia-Enceladus would be similar to the Large Magellanic Cloud had it evolved in isolation, then $f^{\text{GE},0} \sim 0.01$ (van der Marel et al., 2009). It has been shown (Behroozi et al., 2013) that the redshift evolution of $f$ between $z = 2$ and $z = 0$ for objects of the Magellanic Cloud scale and the Milky Way is similar, implying that $f^{\text{MW}}/f^{\text{GE}} = f^{\text{MW},0}/f^{\text{GE},0} \sim 4$. Therefore, taking $M_{*}^{\text{MW}}$ at the time of the merger to be the mass of the thick disc (McMillan, 2017), i.e. $\sim 10^{10} \text{M}_\odot$, we obtain a mass-ratio for the merger of $\sim 0.24$. This implies that the merging of Gaia-Enceladus must have led to significant heating and to the formation of a thick(er) disc.

**Bibliography**

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Appendix 3.A Naming

We describe here the motivation behind the name Gaia-Enceladus In Greek mythology Enceladus was one of the Giants (Titans), and the offspring of Gaia (which represents the Earth), and Uranus (representing the Sky). Enceladus was said to be buried under Mount Etna and responsible for earthquakes in the region. The analogies to the accreted galaxy reported and characterised in this Chapter are many, and they include: i) being offspring of Gaia and the sky, ii) having been a “giant” compared to other past and present satellite galaxies of the Milky Way, iii) being buried (in reality first disrupted by the Milky Way and then buried, also in the Gaia data as it were), and iv) being responsible for seismic activity (i.e. shaking the Milky Way and thereby leading to the formation of its thick disc). We refer to the accreted galaxy as Gaia-Enceladus to avoid confusion with one of Saturn’s moons, also named Enceladus.

Appendix 3.B Dataset, selection criteria and the effect of systematics

For the work presented in the main section of the Chapter, we selected stars from the Gaia 6D-dataset (Gaia Collaboration et al., 2018) with small relative parallax error $\varpi/\sigma_{\varpi} > 5$, which allows us to compute their distance as $d = 1/\varpi$. For Figure 3.1, we consider only stars with $\varpi > 0.4$ mas (i.e. within 2.5 kpc from the Sun) to limit the impact of velocity gradients. The velocities were obtained using the appropriate matrix transformations form the observables $\alpha, \delta, \mu_{\alpha*}, \mu_\delta, \nu_{\text{los}}$ and distances $d$. These velocities have then been corrected for the peculiar motion of the Sun (Schönrich et al., 2010) and the Local Standard of Rest velocity, assuming a value (McMillan, 2017) of $V_{\text{LSR}} = 232$ km/s.
Fig. B.1: Slices of phase-space used to isolate Gaia-Enceladus stars. Panel a): Energy $E$ vs $L_z$ for stars in the 6D Gaia dataset, satisfying the quality criteria described in the text, with $\varpi > 0.2$ mas (5 kpc from the Sun) and with $|v-v_{\text{LSR}}| > 210$ km/s. The straight lines indicate the criteria used to select Gaia-Enceladus stars, namely $-1500 < L_z < 150$ kpc km/s and $E > -1.8 \times 10^5$ km$^2$/s$^2$. These criteria follow roughly the structure’s shape (see for comparison Fig. C.1b), but are slightly conservative for the upper limit of $L_z$ to prevent too much contamination by the thick disc. However, small shifts such as considering an upper limit of 250 kpc km/s or a lower limit of $-750$ kpc km/s for $L_z$, or $E > -2 \times 10^5$ km$^2$/s$^2$ do not result in drastic changes to the results presented in the Chapter. Panel b): $L_z$ vs Galactocentric distance $R$ for all stars in the 6D Gaia with $\varpi > 0.2$ mas. The black points are the halo sample shown in panel a). Panel c): same as panel b) for star particles in the merger simulation (Villalobos & Helmi, 2008) shown in Fig. 3.1b, where blue correspond to the stars from the satellite, and grey to the host disc, and the positions and velocities have been scaled as described in the text. In this figure, the energy has been scaled by $E_{\text{sun}}$ (which is $-1.63 \times 10^5$ km$^2$/s$^2$ in the Galactic potential used), $L_z$ by $L_{z,\text{sun}} = 1902.4$ kpc km/s, and $R$ by the solar distance $R_{\text{sun}} = 8.2$ kpc.

We select halo stars (such as the black points in Figure 3.1a) as those that satisfy $|v-v_{\text{LSR}}| > 210$ km/s (Koppelman et al., 2018). This condition is an attempt to remove the contribution of the disc(s), although towards the inner Galaxy, this is less effective because of the increasing velocity dispersion of disc stars (Gaia Collaboration et al., 2018). To select members of the retrograde structure (such as the blue points in Figure 3.1a), we inspect the energy vs $L_z$ distribution of the stars in our dataset. The energy is computed assuming a Galactic potential including a thin disc, bulge and halo components (Helmi et al., 2017). For example, the left panel of Fig. B.1 shows the energy vs $L_z$ distribution for all halo stars within 5 kpc from the Sun ($\varpi > 0.2$ mas). We have here removed stars with phot-bp-rp-excess-factor $> 1.27$ (this is enough to remove some not so well-behaved globular cluster stars so we do not apply a colour-dependent correction (Arenou et al., 2018)). This figure shows that the regions occupied by the retrograde structure and by the disc are relatively well-separated. There is however some amount of overlap, particularly for higher binding energies and lower angular momenta. Therefore even the selection criteria of $L_z < 150$ kpc km/s and $E > -1.8 \times 10^5$ km$^2$/s$^2$, indicated by the straight lines, will not yield a pure (thick disc free) sample of stars in the structure. This figure reveals also the large range of energies in the structure, indicating that member stars are expected over a large range of distances.
Because the energies of stars depend on the gravitational potential of the Galaxy, and its form and amplitude are not so well-constrained beyond the Solar neighbourhood, we use a criterion based only on $L_z$ to find additional members of the structure/Gaia-Enceladus beyond the immediate vicinity of the Sun (as for Fig. 3.3 of the main section). The central panel of Fig. B.1 shows the $L_z$ vs Galactocentric distance in the disc plane $R$, for all stars in the Gaia 6D-dataset with $\varpi/\sigma_\varpi > 5$, and including stars with parallaxes $\varpi > 0.2$ mas. This plot shows that a selection based only on $L_z$ works relatively well to isolate Gaia-Enceladus stars near the Sun and also farther out in the Galaxy. However for the inner regions there is much more overlap and hence the distinction between the thick disc and Gaia-Enceladus is less straightforward, and the amount of contamination by thick disc stars is likely to be much higher. Furthermore, we expect the orbits of some stars in the progenitor of the thick disc to have been perturbed so significantly during the merger (Jean-Baptiste et al., 2017) that they will “mingle” with those from Gaia-Enceladus.

The rightmost panel of Fig. B.1 shows the $z$-angular momentum as function of cylindrical radius of stellar particles in a simulation of the merger of a pre-existing disc and a massive satellite (Villalobos & Helmi, 2008, 2009) (the same of Fig. 3.1b). The example here corresponds to the redshift $z = 1$ simulation of a disc with $M_* = 1.2 \times 10^{10} M_\odot$ and a satellite with $M_{*,\text{sat}} = 2.4 \times 10^9 M_\odot$. Because of the lower host mass used in this simulation (compared to the present-day mass of the Milky Way), the spatial scales and velocities are typically smaller compared to the data. Therefore in the simulations, we consider as solar vicinity a volume centered at $R_{\text{sun}}^{\text{sim}} = 2.4 \times R_{\text{final thick}}^{\text{thick}}$, where $R_{\text{final thick}}^{\text{thick}} = 2.26$ kpc (Villalobos & Helmi, 2008). We also scale the positions by $R_{\text{sun}}/R_{\text{sun}}^{\text{sim}} = 1.5$ and the velocities by $v_{\text{thick,sun}}/v_{\text{thick,sim}}^{\text{final}}$, where $v_{\text{thick,sun}} = 173$ km/s is the rotational velocity of the thick disc near the Sun (Morrison et al., 1990) and $v_{\text{thick,sim}}^{\text{final}}$ is that of the thick disc in the simulation at $R_{\text{sun}}^{\text{sim}}$. Figure B.1c shows that like for the data, the separation between accreted and host disc stars is less effective for small radii.

For Fig. 3.2a, we have cross-matched the catalogues from Gaia DR2 and APOGEE (Abolfathi et al., 2018; Majewski et al., 2017) DR14 and retained only stars with estimated distances from both these catalogues (i.e. spectrophotometric and trigonometric parallaxes) consistent with each other at the 2$\sigma$ level. We also impose a relative parallax error of 20%. More than 100 000 stars within 5 kpc from the Sun satisfy these conditions. The abundances shown in Fig. 3.2 stem from the ASCAP pipeline (García Pérez et al., 2016).

The presence of a parallax zero-point offset in the Gaia data (Lindegren et al., 2018) has been established thoroughly, and is partly (if not only) due to a degeneracy between the parallax and the basic-angle variation of the Gaia satellite (Butkevich et al., 2017). Its amplitude varies with location on the sky (Arenou et al., 2018; Lindegren et al., 2018), and is on average $-0.029$ mas and has an RMS of $\sim 0.03$ mas (Gaia Collaboration et al., 2018). Such variations make it very difficult to perform a correction a posteriori for the full Gaia DR2 dataset (although the expectation is that its effect will be smaller for Gaia DR3). The discovery and characterisation of Gaia-Enceladus was done using stars
Fig. B.2: Effect of a zero-point offset in the parallax on $L_z$. Panel a) shows the distribution of the difference between the initial and “measured” (after error convolution) $L_z$ for GUMS stars with “measured” distances between 5 and 10 kpc and with $l = (-60^\circ, -20^\circ)$. Panel b) shows the mean value of the difference over the full sky.

with parallaxes $\varpi > 0.4$ mas for Fig. 3.1 of the main section, and in Fig. 3.2 for stars with $\varpi > 0.2$ mas from the cross-match of $Gaia$ and APOGEE. We therefore expect the derived kinematic and dynamical quantities for these subsets to be largely unaffected by the systematic parallax error. However, for Figs. 3.3 and 3.4 of the main section of the Chapter, we selected stars on the basis of their $L_z$ although we focused on properties which are independent of the parallax, such as position on the sky and proper motions. Nonetheless, to establish how important the parallax zero-point offset is on the selection via $L_z$ we perform the following test.

We use the $Gaia$ Universe Model Snapshot GUMS v18.0.0 (Robin et al., 2012), and select stars according to the following criteria: $6 \leq G \leq 13.0$, $0.2 \leq \log g \leq 5$ and $3000 \leq T_{\text{eff}} \leq 9000$ K. This selection leads to a total of 7403454 stars distributed across all Galactic components. For these stars we compute error-free velocities and $L_z$. We convolve their true parallax with a Gaussian with a dispersion of depending on the magnitude of the star\textsuperscript{1}. The parallax is reconvolved with a Gaussian with a mean of $-0.029$ mas and a dispersion of 0.030 mas. Using these observed parallaxes, we compute “observed” velocities and $L_z$.

We find that for measured distances smaller than 5 kpc, there is no shift in the derived $L_z$, while for a shell between 5 and 7.5 kpc the median amplitude of the shift is $\sim -50$ kpc km/s, making the observed $L_z$ more retrograde. For a shell between 9 and 10 kpc, the median shift is small and has an amplitude of 20 kpc km/s, presumably reflecting that at such large distances, the random errors on the individual stars’ measurements dominate. The results are shown in the left panel of Fig. B.2 where we plot the difference between the true (initial) and “measured” distributions of $L_z$ for stars “observed” to be located at distances between 5 and 10 kpc, for $l = (-60^\circ, -20^\circ)$. The panel on the

\textsuperscript{1}See https://www.cosmos.esa.int/web/gaia/science-performance. These end of the mission uncertainties have been scaled to account for DR2 shorter timespan.
Fig. C.1: Distribution of stars’ dynamical properties for a smooth dataset. Panel a) shows the velocity distribution and panel b) the $E$ vs $L_z$ distribution for a dataset obtained by reshuffling the velocities of the stars plotted in Fig. 3.1a and in Fig. B.1a, respectively. The visual comparison to those figures shows that these random sets are less clumped than the observed distributions of the Gaia halo stars.

right shows the distribution of the mean value of the difference over the whole sky, and although it reveals certain patterns, these are different from those seen in Fig. 3.3. As stated in the main section of the Chapter, the lack of distant stars in the regions outside of the contours plotted in Fig. 3.4, is the result of a quality cut in the relative parallax error of 20%. This selection criterion allows for parallax errors in the range 0.02 to 0.04 mas for the most distant stars (with $0.1 < \varpi < 0.2$ mas), and these are only reached in those regions of the sky that have been surveyed more frequently by Gaia, such as around the ecliptic poles. The Gaia RR Lyrae stars associated to Gaia-Enceladus suffer of course also from this effect, as a lower number of visits leads to more difficult identification and hence to lower levels of completeness (Clementini et al., 2019).

Appendix 3.C Random sets and significance of features

To understand how different the dynamical properties of the Gaia 6D dataset are in comparison to a smooth distribution, we plot the distribution of velocities in Fig. C.1a and of $E$ vs $L_z$ in Fig. C.1b for randomised datasets. These smooth datasets have been obtained from the Gaia data shown in Fig. 3.1a and in Fig. B.1a, respectively, by re-shuffling the velocities. That is, for each star, we assign randomly a $v_y$ and $v_z$ velocity from two other stars in the sample. This results in distributions with the same 1D velocity distributions as the original data, but without any correlations or lumpiness. The comparison of Fig. 3.1a to Fig. C.1a shows that the distribution in the random dataset is indeed much smoother than the data, and that the overall velocity dispersion in the $y$-direction has increased because there no longer is a clear separation between the region occupied by
Gaia-Enceladus and by the thick disc. The comparison of Figs. 3.1b and Fig. C.1b is even more revealing and clearly shows that the structure defined in $E$ vs $L_z$ by Gaia-Enceladus stars has effectively disappeared in the randomised dataset. Similar conclusions are reached when, instead of using a reshuffled dataset, we compare the distributions to those in the GUMS model.

Fig. 4 of the main section of the Chapter shows the radial velocities and proper motions (corrected for the Solar and for the Local Standard of Rest motions) for stars with $0.1 < \varpi < 0.2$ mas and $-1500 < L_z < 150$ kpc km/s. These stars are tentative members of Gaia-Enceladus, although as discussed earlier towards the inner Galaxy contamination by thick disc stars becomes more important for large distances. The arrows depicting the proper motions suggest that stars that are close by on the sky move in similar directions. We establish here whether this is significant by comparing to a mock dataset.

The mock dataset uses the measured positions of the stars that are plotted in Fig. 4, but their velocities are generated randomly according to a multivariate Gaussian distribution with dispersions in $v_R$, $v_\phi$ and $v_z$ of 141, 78 and 94 km/s respectively (Posti et al., 2018). During the process of generation, we only keep stars’ velocities that satisfy $-1500 < L_z < 150$ kpc km/s, as in the real data. To quantify the degree of coherence in the proper motions of neighbouring stars on the sky, we perform the following test. For each star, we find its nearest neighbour on the sky, and then determine the angle between their proper motion vectors. We then count the number of such pairs having a given angle. Figure C.2 shows the distribution of these pairs for the Gaia subsample (in blue) and for the mock (in red). There is a clear excess of pairs of stars with similar directions of motion in the data in comparison to the mock.

Appendix 3.D Context and link to other substructures

Hints of the presence of a population like Gaia-Enceladus have been reported in the literature in the last two decades, and were typically based on small samples of stars. These hints were of chemo-dynamical nature (Chiba & Beers, 2000; Beers et al., 2017; Carollo et al., 2013) and sometimes attributed to accretion (Morrison et al., 2009; Brook et al., 2003), but also based purely on chemical signatures, such as the $\alpha$-poor sequence (Nissen & Schuster, 2010, 2011). More recently, cross-matches to the first data release of the Gaia mission (Gaia Collaboration et al., 2016) also revealed the contrast between the metal-rich population supported by prograde rotation and associated to the tail of the thick disc (Bonaca et al., 2017), and the metal-poor halo, i.e. what we have just identified as Gaia-Enceladus. Furthermore, in one study (Belokurov et al., 2018) the difference in the kinematics of these two populations, and the measurement of a very radially biased velocity ellipsoid for halo stars with $[\text{Fe/H}] > -1.7$, led to the proposal that this population (which was termed “Gaia sausage”) could be the result of a significant merger. Although this could be also attributed to an in-situ formation via a radial collapse, this
Fig. C.2: Distribution of angles between the proper motion vectors for neighbouring stars on the sky. The blue and red histograms correspond respectively, to Gaia-Enceladus and to a mock dataset. This mock dataset uses the positions of the stars in Gaia-Enceladus, but velocities generated randomly according to a multivariate Gaussian distribution (Posti et al., 2018), after which only stars’ velocities that satisfy \(-1500 < L_z < 150\) kpc km/s are kept, as in the real data. For each star, we find its nearest neighbour on the sky, and then determine the angle \(\Delta \theta\) between their proper motion vectors for the data and for the mock. We then count the number of such pairs having a given angle \(\Delta \theta\).

The hypothesis gained further supported by their orbits leading to the break in the halo density profile at \(\sim 20\) kpc (Deason et al., 2018). All of these pieces together outline the case for the discovery and detailed characterisation of Gaia-Enceladus reported here.

The more distant Gaia-Enceladus debris occupies large portions of the sky not extensively covered by other existing surveys. There is however, a recent detection of an overdensity identified in PanSTARRS and WISE with the help of Gaia proper motions (Conroy et al., 2018), which overlaps with the northern part of the more distant Gaia-Enceladus stars for \(-2 < \mu_\alpha < -1\) mas/yr and \(-1 < \mu_\delta < 0\) mas/yr, and partly (but not fully because of the PanSTARRS footprint) with the southern part, for \(0 < \mu_\alpha < 1\) mas/yr and \(-3 < \mu_\delta < -1\) mas/yr. There could potentially be also a relation to the Hercules Aquila Cloud (Belokurov et al., 2007) identified in SDSS, although this appears to be offset both in the northern and southern hemispheres and located at a larger distance. The location on the sky of intermediate distance Gaia-Enceladus stars would seem to overlap with the Hercules thick disc cloud (Larsen et al., 2011), especially in the fourth Galactic quadrant below the Galactic plane.

Appendix 3.D  Context and link to other substructures
Characterisation and history of the Helmi streams with *Gaia* DR2

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Abstract

The halo of the Milky Way has long been hypothesised to harbour significant amounts of merger debris. For more than a decade this view has been supported by wide-field photometric surveys which have revealed the outer halo to be lumpy. The recent release of *Gaia* DR2 is allowing us to establish that mergers also have been important and possibly built up the majority of the inner halo. In this Chapter we focus on the Helmi streams, a group of streams crossing the solar vicinity and known for almost two decades. We characterise their properties and relevance for the build-up of the Milky Way’s halo. We identify new members of the Helmi streams in an unprecedented dataset with full phase-space information combining *Gaia* DR2, and the APOGEE DR2, RAVE DR5 and LAMOST DR4 spectroscopic surveys. Based on the orbital properties of the stars, we find new stream members up to a distance of 5 kpc from the Sun, which we characterised using photometry and metallicity information. We also perform N-body experiments to constrain the time of accretion and properties of the progenitor of the streams. We find nearly 600 new members of the Helmi streams. Their HR diagram reveals a broad age range, from approximately 11 to 13 Gyr, while their metallicity distribution goes from $-2.3$ to $-1.0$, and peaks at $[\text{Fe/H}] \sim -1.5$. These findings confirm that the streams originate in a dwarf galaxy. Furthermore, we find seven globular clusters to be likely associated, and which follow a well-defined age-metallicity sequence whose properties suggest a relatively massive progenitor object. Our N-body simulations favour a system with a stellar mass of $\sim 10^8 \, \text{M}_\odot$ accreted 5 – 8 Gyr ago. The debris from the Helmi streams is an important donor to the Milky Way halo, contributing approximately 15% of its mass in field stars and 10% of its globular clusters.

4.1 Introduction

According to the concordance cosmological model ΛCDM, galaxies grow by mass through mergers. Typically a galaxy’s halo is formed through a handful of major mergers accompanied by a plethora of minor mergers. This model’s predictions stem both from dark-matter-only simulations (Helmi et al., 2003) combined with semi-analytic models of galaxy formation (e.g. Bullock & Johnston, 2005; Cooper et al., 2010) and hydrodynamical simulations (e.g. Pillepich et al., 2014).
When satellites merge with a galaxy like the Milky Way they get stripped of their stars by the tidal forces (e.g. Johnston et al., 1996). These stars follow approximately the mean orbit of their progenitor and this leads to the formation of streams and shells. Wide-field photometric surveys have already discovered many cold streams likely due to globular clusters, for example Pal 5 (Odenkirchen et al., 2001); GD-1 (Grillmair & Dionatos, 2006), and more disperse streams caused by dwarf galaxies, for example Sagittarius (Ibata et al., 1994), whose total extent and importance can only be appreciated in full-sky maps (Belokurov et al., 2006; Bernard et al., 2016; Shipp et al., 2018). Many of these streams are distant and have become apparent only after meticulous filtering (e.g. Rockosi et al. 2002; Grillmair 2009; Malhan et al. 2017).

Tidal debris in the vicinity of the Sun is predicted to be very phase-mixed (Helmi & White, 1999; Helmi et al., 2003). Typically one can expect to find many stream wraps originating in the same object, that is groups of stars with different orbital phase sharing a common origin. Rather than to search for substructure in spatial coordinates, it is more productive to study tidal debris in spaces where the degree of clustering is either constant, such as in integrals of motion or action space (Helmi & de Zeeuw, 2000; Mcmillan & Binney, 2008), or even increases with time (Helmi & White, 1999). Until recently, a few studies on the nearby stellar halo identified different small groups of stars that likely were accreted together (Helmi et al., 1999; Chiba & Beers, 2000; Helmi et al., 2006; Klement et al., 2008, 2009; Williams et al., 2011; Majewski et al., 2012; Beers et al., 2017; Helmi et al., 2017), see Newberg & Carlin (2016) for a comprehensive review.

A new era is dawning now that the Gaia mission is delivering full phase-space information for a billion stars. As it is clear from the above discussion, this is crucial to unravel the merger history of the Milky Way and to characterise the properties of the satellites with which it merged. The strength of Gaia, and especially of the 6D sample, is that it can identify stream members based on the measured kinematics (e.g. Koppelman et al., 2018; Price-Whelan & Bonaca, 2018). In fact, the second data release of the Gaia mission (Gaia Collaboration, Brown et al., 2018) is already transforming the field of galactic archaeology. One example is the recent spectacular discovery that the inner halo was built largely via the accretion of a single object, as first hinted from the kinematics (Belokurov et al., 2018; Koppelman et al., 2018), and the stellar populations (Gaia Collaboration, Babusiaux et al., 2018; Haywood et al., 2018), all pieces put together in Helmi et al. (2018). This accreted system, known as Gaia-Enceladus, was discy and similar in mass to the Small Magellanic Cloud of today, and hence led to the heating of the Galactic proto-disc some 10 Gyr ago (Helmi et al., 2018).

Gaia-Enceladus debris, however, is not the only substructure present in the vicinity of the Sun. Detected about 20 years ago, the Helmi streams (Helmi et al., 1999, H99 hereafter) are known to cross the solar neighbourhood. Their existence has been confirmed by Chiba & Beers (2000) and Smith et al. (2009), among other studies. In the original work, 13 stars were detected based on their clumped angular momenta which clearly differ from other local halo stars. Follow-up work by Kepley et al. (2007) estimated that the streams were part of the tidal debris of a dwarf galaxy that was accreted 6 – 9 Gyr
ago, based on the bimodality of the $z$-velocity distribution. This bimodal distribution is the distinctive feature of multiple wraps of tidal debris crossing the solar neighbourhood. Since the discovery in 1999, a handful of new tentative members have been found, increasing the total number of members to $\sim 30$ (e.g. Kepley et al., 2007; Re Fiorentin et al., 2005; Klement et al., 2009; Beers et al., 2017), while several tens more were reported in Gaia Collaboration, Helmi et al. (2018). Also structure S2 from Myeong et al. (2018a), consisting of $\sim 60$ stars, has been recognised to be related to the Helmi streams (W. Evans priv. comm.).

Originally, the Helmi streams were found using Hipparcos proper motions (Perryman et al., 1997) combined with ground-based radial velocities (Beers & Sommer-Larsen, 1995; Chiba & Yoshii, 1998). In this Chapter, we aim to find new members and to characterise better its progenitor in terms of the time of accretion, initial mass and star formation history. To this end, we focus on the dynamics, metallicity distribution and colour-magnitude diagram of its members. Furthermore, we also identify globular clusters that could have potentially been accreted with the object (Leaman et al., 2013; Kruijssen et al., 2018), as for example seen for the Sagittarius dwarf galaxy (Law & Majewski, 2010; Massari et al., 2017; Sohn et al., 2018), and also for Gaia-Enceladus (Myeong et al., 2018b; Helmi et al., 2018).

This Chapter is structured as follows: in Sect. 4.2 we present the data and samples used, while in Sect. 4.3 we define a core selection of streams members that serves as the basis to identify more members. In Sect. 4.4 we analyse the spatial distribution of the debris. In Sect. 4.5 we supplement the observations with N-body simulations. We discuss possible associations of the Helmi streams with globular clusters in Sect. 4.6. Finally, we present our conclusions in Sect. 4.7.

4.2 Data

4.2.1 Brief description of the data

The recently published second data release (DR2) of the Gaia space mission contains the on-sky positions, parallaxes, proper motions, and the $G$, $G_{BP}$ and $G_{RP}$ optical magnitudes for over 1.3 billion stellar sources in the Milky Way (Gaia Collaboration, Brown et al., 2018). For 7 224 631 stars with $G_{RVS} < 12$, known as the 6D subsample, line-of-sight velocity information measured by the Gaia satellite is available (Gaia Collaboration, Katz et al., 2018). The precision of the observables in this dataset is unprecedented: the median proper motions uncertainties of the stars with full phase-space information is 1.5 mas/yr which translates to a tangential velocity error of $\sim 7$ km/s for a star at 1 kpc, while their median radial velocity uncertainties are 3.3 km/s. This makes the Gaia DR2 both the highest-quality and the largest-size single survey ever available for studying the kinematics and dynamics of the nearby stellar halo and disc.
4.2.2 Cross-matching with APOGEE, RAVE and LAMOST

To supplement the 6D Gaia subsample, we add the radial velocities from the cross-matched catalogues APOGEE (Wilson et al., 2010; Abolfathi et al., 2018) and RAVE DR5 (Kunder et al., 2017), see Marrese et al. (2018) for details. We also add radial velocities from our own cross-match of Gaia DR2 with LAMOST DR4 (Cui et al., 2012).

For the cross-match with LAMOST we first transform the stars to the same reference frame using the Gaia positions and proper motions, and then we match stars within a radius of 10 arcsec with TOPCAT/STILTS (Taylor, 2005, 2006). We find that over 95% of the stars have a matching radius smaller than 0.5 arcsec. In total, we find 2 868 425 matches between Gaia and LAMOST DR4, with a subset of 8 404 overlapping also with RAVE, and 50 650 with APOGEE. Because the LAMOST radial velocities are known to be offset by +4.5 km/s with respect to APOGEE (Anguiano et al., 2018), we correct for this effect.

Since the radial velocities of RAVE and APOGEE have been shown to be very consistent with those of Gaia (Sartoretti et al., 2018), for our final catalogue we use first the radial velocities from APOGEE if available, then those from RAVE, and finally from LAMOST for the stars for which there is no overlap with either two of the other surveys. After imposing a quality cut of parallax_over_error > 5, this yields a sample of 2 361 519 stars with radial velocities. We note that all these surveys also provide additional metallicity information for a subset of the stars.

When combined with the Gaia 6D sample, this results in a total of 8 738 322 stars with 6D information and parallax_error > 5. The median line-of-sight velocity error of the stars from the ground-based spectroscopic surveys is 5.8 km/s, while that of the pure Gaia sample is 1 km/s for the same parallax quality cut.

4.2.3 Quality cuts and halo selection

To isolate halo stars, we follow a kinematic selection, meaning that stars are selected on the basis of their very different velocity from local disc stars. By cutting in velocity we introduce a clear bias: halo stars with disc-like kinematics are excluded from this sample (Nissen & Schuster, 2010; Bonaca et al., 2017; Posti et al., 2018; Koppelman et al., 2018). Nevertheless, the amplitude of the Z-velocities of the Helmi streams stars is > 200 km/s, that is very different from the disc, so our selection will not impact our ability to find more members. Such a selection reduces significantly the sample size (since by far most stars are in the thin disc) and helps in making the Helmi streams more apparent.

We start from our extended 6D data sample (obtained as described in the previous section). Because of the zero-point offset of ~ –0.03 mas known to affect the Gaia parallaxes (Arenou et al., 2018; Gaia Collaboration, Brown et al., 2018; Lindegren et al., 2018) we also discard stars with parallaxes < 0.2 mas. Distances for this reduced sample are obtained by inverting the parallaxes. Following Koppelman et al. (2018) we select stars that have |V – V_{LSR}| > 210 km/s, where V_{LSR} is the velocity vector of the local standard of rest (LSR). A selection like this removes all thin-disc stars, assuming those
move at the LSR velocity. Thick disc stars moving at \(~ 170\) km/s (Morrison et al., 1990), will be mostly removed except for those that rotate slowly and/or have an exceptionally large vertical velocity. We apply the velocity selection after correcting for the motion of the Sun using the values, \((U_{\odot}, V_{\odot}, W_{\odot}) = (11.1, 12.24, 7.25)\) km/s (Schönrich et al., 2010) and that of the LSR, \(V_{\text{LSR}} = 232.8\) km/s (Mcmillan, 2017). We do not take the uncertainties of the velocities into account, as these are typically very small with a median value of \(~ 4\) km/s.

Our Cartesian reference frame is pointed such that \(X\) is positive towards the Galactic Centre, \(Y\) points in the direction of the motion of the disc, and \(Z\) is positive for Galactic latitude \(b > 0\). In this frame the Sun is located at \(X = -8.2\) kpc from the Galactic Centre (Mcmillan, 2017). This final sample contains 79 318 tentative halo stars, with 12 472 located within 1 kpc from the Sun. Slightly more than half of these stars stem from the \textit{Gaia}-only 6D sample.

### 4.3 Finding members

#### 4.3.1 Core selection

Using the halo sample described above, we select ‘core members’ by considering only those stars within 1 kpc from the Sun from the \textit{Gaia}-only sample. The parallaxes for this subsample are very good, with a median \textit{parallax\_over\_error} of 46. In such a local sample, streams are very clustered in velocity-space because the gradients caused by the orbital motion are minimised. In the next section we will use the orbits of these core members to find members at distances beyond 1 kpc.

Figure 4.1 shows our selection of the streams’ core members in velocity space with green boxes. The boxes are placed on top of the positions of the original H99 members of...
Tab. 4.1: Gaia DR2 source_id of the Helmi streams’ core members.

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<tr>
<td>6615661065172699776</td>
<td>6914409197757803008</td>
</tr>
</tbody>
</table>

the Helmi streams. The boundaries in \((V_Z, V_Y)\) for the left box are: \([-290, -200] \ [90, 190]\) km/s, and for the right box: \([200, 270] \ [110, 190]\) km/s. Using the SIMBAD database we find that ten of the original 13 members have Gaia DR2 distances smaller than 1 kpc. Of these ten stars, nine have radial velocities in our extended data sample. Only one star with updated radial velocity information has very different velocities, leaving the original sample with eight reliable members with full Gaia 6D parameters within 1 kpc.

One of the key characteristics of the Helmi streams are the two groups in the \(V_Y – V_Z\) plane, one moving with positive \(V_Z\) and one with negative \(V_Z\). Using the selection described above we find 40 core members in the Gaia-only sample, of which 26 with \(V_Z < 0\). The Gaia DR2 source_ids are given in Table 4.1. While we use the kinematically biased halo sample to find these core members, there are no stars that did not make it to the core sample because of this. The halo sample is used here to illustrate how conspicuous the streams are with respect to the background halo in velocity space. The asymmetry in the number of stars in the two groups can be used as an indicator of the time of accretion and/or mass of the progenitor since tidal streams will only produce multiple wraps locally after having evolved for a sufficiently long time. In Sect. 4.5.1 we explore what the observed asymmetry implies for the properties of the progenitor of the Helmi streams.
4.3.2 Beyond the core selection

The original 13 H99 members are located within 2.5 kpc from the Sun. Beyond this distance, we expect that other members of the streams will have different kinematic properties because of velocity gradients along their orbit. Therefore the best way of finding more members beyond the local volume is to use integrals of motion (IOM) such as angular momenta and energy or the action integrals (Helmi & de Zeeuw, 2000). From this section onward, we use the extended sample described in Section 4.2.2 and which includes radial velocities from APOGEE/RAVE/LAMOST.

We will mainly base the membership selection on the angular momentum of the stars, namely the $L_z$-component $L_z$ and the perpendicular component: $L_{\perp}^2 = L_x^2 + L_y^2$, although the latter is not fully conserved in general. Since the energy $E$ depends on the assumed model for the Galactic potential, we use it only to check for outliers.

To calculate the energy of the stars we model the Milky Way with a potential that is similar to that used by Helmi et al. (2017): it includes a Miyamoto-Nagai disc with parameters $M_d = 9.3 \cdot 10^{10} M_\odot$, $(a_d, b_d) = (6.5, 0.26)$ kpc, an NFW halo with parameters $M_h = 10^{12} M_\odot$, $r_{\rm s,h} = 21.5$ kpc, $c_h = 12$, and a Hernquist bulge with parameters $M_b = 3 \cdot 10^{10} M_\odot$, $c_b = 0.7$ kpc. The circular velocity curve of this potential is similar to that of the Milky Way. Since this potential is axisymmetric $L_z$ is a true IOM. For convenience, in what follows we flip the sign of $L_z$ such that it is positive for the Sun.

In Fig. 4.2 we show the distribution of $L_z$ versus $L_{\perp}$ (left), and $L_z$ versus $E$ (right). A grey density map of all the stars in the Gaia 6D sample with 20% relative parallax error and with parallax $> 0.2$ marks the location of the disc. The black dots correspond to all
kinematically selected halo stars within 2.5 kpc from the Sun. The transition of the halo into the disc is smooth, and only appears sharp because of this particular visualisation. The streams are clumped around \((L_\perp, L_z) \sim (2000, 1250)\) kpc km/s, and are highlighted by overlaying the core members from Sect. 4.3.1 with green dots. In the right panel, we see that some of the core members appear to be outliers with too high or low energy (but the analyses carried out in the next sections show they are indistinguishable in their other properties, except for their large \(v_R\) velocities, see Section 4.5.1).

Red, dashed lines indicate two boxes, labelled A & B, that we use to select tentative additional stream members. The limits of box A are: 1750 < \(L_\perp\) < 2600 kpc km/s and 1000 < \(L_z\) < 1500 kpc km/s, and those of box B are: 1600 < \(L_\perp\) < 3200 kpc km/s and 750 < \(L_z\) < 1700 kpc km/s. The boxes are placed on top of the core members, their sizes are chosen somewhat arbitrarily. The selection is tight where we expect to find a large amount of contamination, for example on the lower boundary (towards the disc), and looser for the upper boundary. The exact footprint of the entire stream in this diagram could even be larger than box B depending on the size of the progenitor galaxy (as for example can be seen from the numerical simulations shown in Fig. 4.15). Our decision to explore the two boxes A&B allows us to check how the level of contamination changes with box size. 

The number of stars located within 5 kpc that fall in boxes A&B are 235 and 523, respectively. At most 20 stars that we identify as members of the Helmi streams in the IOM space (grey points inside the selection boxes), do not satisfy our halo selection (meaning that they have \(|V - V_{LSR}| < 210\) km/s). In the following sections, we focus on the streams members that fall in selection B unless mentioned otherwise. We remind the reader that selection B includes stars from the full extended sample comprising radial velocities from \(Gaia\) and from APOGEE/RAVE/LAMOST. For a table with the ids of all the sources in selection B see the published version of the article.

### 4.3.3 Members without radial velocities

Most of the stars in the \(Gaia\) DR2 dataset lack radial velocities, which makes the search for additional tentative members of the Helmi streams less straightforward. In the work of Gaia Collaboration, Helmi et al. (2018) new members were identified using locations on the sky where the radial velocity does not enter in the equations for the angular momentum, namely towards the Galactic Centre and anti-centre. At those locations, the radial velocity is aligned with the cylindrical \(v_R\) component, therefore, it does not contribute to \(L_z = rv_\phi\), and \(L_y = -xv_z\). The degree to which the radial velocity contributes to the angular momenta increases with angular distance from these two locations on the sky. Based on a simulation of a halo formed through mergers created by Helmi & de Zeeuw (2000), we estimate that within 15 degrees from the (anti-)centre, the maximum difference between the true angular momenta of stars and that computed assuming a zero radial velocity, is \(\sim 1000\) kpc km/s. Since the size of Box B is \(\sim 1000\) kpc km/s, we consider 15 degrees as the maximum tolerable search.

---

\(^1\)We have explored an even larger box size but noticed that the contamination increased significantly when inspecting for example, the metallicity distribution presented in Sect. 4.4.4.
Fig. 4.3: Distribution of stars from the 5D subset of Gaia, located within 15 degrees from the Galactic Centre or anti-centre, in (pseudo)angular momentum space. The angular momenta are calculated here by assuming that the line-of-sight velocities are zero. The grey density map reveals the location of all of the stars in these windows, most of which are disc stars. The black dashed lines show the 2.5% and 97.5% quantiles of the \( \tilde{L}_y \)-distribution. The two boxes indicated with red dashed lines are used to identify candidate members of the Helmi streams, here shown with blue symbols.

radius. We denote the angular momenta computed assuming zero radial velocity as \( \tilde{L}_y \) and \( \tilde{L}_z \), where we change the sign of \( \tilde{L}_z \) such that it is positive in the (prograde) direction of rotation of the disc.

We select stars from the full Gaia DR2 5D-dataset within 15 degrees from the (anti-)centre and with \texttt{parallax\_over\_error} > 5. Moreover, we apply the following photometric quality cuts described in Sect. 2.1 of Gaia Collaboration, Babusiaux et al. (2018):

\[
\begin{align*}
\text{phot\_g\_mean\_flux\_over\_error} & > 50, \\
\text{phot\_rp\_mean\_flux\_over\_error} & > 20, \\
\text{phot\_bp\_mean\_flux\_over\_error} & > 20, \\
1.0 + 0.015 \times \text{power(bp\_rp, 2)} & < \text{phot\_bp\_rp\_excess\_factor}, \\
1.3 + 0.06 \times \text{power(bp\_rp, 2)} & > \text{phot\_bp\_rp\_excess\_factor}.
\end{align*}
\]

Figure 4.3 shows the distribution of all the stars in this subsample with a grey density map in \( \tilde{L}_y \) vs \( \tilde{L}_z \) space. The two boxes marked with red dashed lines show the criteria we apply to identify additional members of the Helmi streams. The size and location of the boxes are based on those in Fig. 4.2. They are limited by \( 750 < \tilde{L}_z < 1700 \) kpc km/s, while we use a tighter constraint on \( |\tilde{L}_y| \) to prevent contamination from the disc. The black dashed lines in Fig. 4.3 indicate upper and lower quantiles of the full \( \tilde{L}_y \)-distribution.
such that 95% of the stars in the 5D subsample are located between these dashed lines. The lower limits of selection boxes in the $\tilde{L}_y$ direction are offset by 500 kpc km/s from the dashed lines, and are located at $\tilde{L}_y$ at 1782 and –1613 kpc km/s, respectively.

The blue symbols in Fig. 4.3 correspond to the 105 tentative members that fall inside the boxes. Most of these stars are within 2.5 kpc from the Sun. The two clumps in $\tilde{L}_y$ have a direct correspondence to the two streams seen in the $V_Z$ component for the 6D sample. The clumps have 24 and 81 stars each, implying a $\sim 1 : 3$ asymmetry which is quite different from that seen in the number of core member stars associated with each of the two velocity streams. The difference could be caused in part by incompleteness and crowding effects together with an anisotropic distribution of the stars in the streams (see e.g. Fig. 4.7). We use the 5D members in this Chapter only for the photometric analysis of the Helmi streams carried out in Section 4.4.4.

![Fig. 4.4: Spatial distribution of members of the Helmi streams for the two selection boxes A (top) and B (bottom). The 6D stream members are indicated with green circles, local halo stars are shown in the background with grey symbols. The total number of 6D stream members is indicated in the top left of each panel. Tentative members from the 5D data set are shown with blue symbols.](image-url)
4.4 Analysis of the streams

4.4.1 Spatial distribution

Figure 4.4 shows the distribution of the stream members in the $XZ$-plane (left) and $XY$-plane (right), for the selection box A (top) and for B (bottom). Those identified with 6D information are indicated with green circles, a local sample of halo stars is shown in grey in the background. There is a lack of stars close to the plane of the disc, likely due to extinction (Gaia Collaboration, Katz et al., 2018). This gap is filled with tentative members from the 5D sample (in blue) which have, by construction, low galactic latitude. Figure 4.4 reveals the streams stars to be extended along the $Z$-axis, as perhaps expected from their high $V_Z$ velocities, but there is also a clear decrease in the number of members with distance from the Sun.

To establish whether the spatial distribution of the streams differs from that of the background, we proceed as follows. We compare our sample of streams stars to $10^4$ samples randomly drawn from the background. The random samples contain the same number of stars as the streams, and the background comprises all of the stars in the halo sample, described in Sect. 4.2, excluding the streams members. In this way, we account for selection effects associated with the different footprints of the APOGEE/RAVE/LAMOST surveys as well as with the 20% relative parallax error cut (since the astrometric quality of the Gaia data is not uniform across the sky), and which are likely the same for the streams and the background.

Figure 4.5 shows a comparison of the distribution of heliocentric $R$ (left) and $Z$ (right) coordinates of the stars in our sample (in green) and in the random samples (black).
The sizes of the grey and black markers indicate the $1\sigma$ and $3\sigma$ levels respectively, of the random samples. The $Z$-distribution of the streams members shows minimal differences with respect to the background, except near the plane $Z \sim 0$. On the other hand, their distribution in $R$ shows very significant differences with respect to the background, depicting a very steeply declining distribution. This was already hinted at in Fig. 4.4, and would suggest that the streams near the Sun have a cross section of $\sim 500$ pc.

4.4.2 Flows: Velocity and spatial structure

Fig. 4.6: Distribution of the stars in the Helmi streams in the $XZ$ plane (i.e. perpendicular to the disc of the Milky Way), with the arrows illustrating their motions (amplitude and direction) in this plane. The symbols are colour-coded according to the amplitude of their $V_Y$ velocity (i.e. perpendicular to the projected plane). The top and bottom rows show stream members with $V_Z < 0$ and $V_Z > 0$, respectively. The left and right columns show stars with $V_X < 0$ and $V_X > 0$, respectively. In all the four panels streaming motions and substructures are clearly apparent.

The main characteristic of stars in streams is that they move together through space as in a flow. Figure 4.6 illustrates this by showing the spatial distribution of the members (according to selection B), in the $XZ$-plane. The arrows indicate the direction and amplitude of the velocities of the stars, with stars with $V_Z < 0$ shown in the top row and $V_Z > 0$ shown in the bottom row. The left and right columns show stars with $V_X < 0$ and $V_X > 0$, respectively. In all the four panels streaming motions and substructures are clearly apparent.

2defined as the distance at which the counts of stars has dropped by a factor two.
those with $V_Z > 0$ in the bottom row of the figure. Every star is colour-coded according to its velocity component in and out of the plane of projection (i.e. its $V_Y$). The left column shows stars with $V_X < 0$, while the right column shows stars with $V_X > 0$.

The flows seen in Fig. 4.6 reveal that the two characteristic clumps in $V_Z$ (i.e. those shown in the left panel of Fig. 4.1) actually consist of several smaller streams. For example, the top and bottom right panels of Fig. 4.6 both clearly show two flows: one with $V_X \sim 0$, the other with a large $V_X$.

![Fig. 4.7: Compilation of orbits based on the 6D positions of the members of the Helmi streams. These orbits have been integrated for 100 Myr backwards and forward in time. The position of the Sun is illustrated with a red star, the Galactic Centre is at $(X, Z) = (0, 0)$ in this frame. The trajectories of stars that currently have $V_Z > 0$ are coloured black, while those with $V_Z < 0$ are shown in blue. Close to the solar position, the majority of the Helmi streams' members move perpendicular to the plane of the disc, and are close to pericentre.]

To enhance the visibility of the flows we integrate the orbits of the streams stars forward and backwards in time. The potential in which the orbits are calculated is the same as the one described in Section 4.3.2. The trajectories of all the stars are integrated for $\pm 100$ Myr in time and are shown in Fig. 4.7 projected onto the $XZ$ plane. With a red star we indicate the solar position. The trajectories of stars that belong to the group with $V_Z > 0$ are coloured black, while those with $V_Z < 0$ are given in blue.

Clearly, the stars found in the solar vicinity are close to an orbital turning point and on trajectories elongated in the $Z$-direction, as expected from their large vertical velocities. Figure 4.7 serves to understand the observed spatial distribution of the member stars (i.e. narrower in $X$ (or $R$) and elongated in $Z$) seen in Fig. 4.4. Finally, we also note the presence of groups of orbits tracing the different flows just discussed, such as for
example the group of stars moving towards the upper left corner of the figure (and which corresponds to some of the stars shown in the bottom left panel of Fig. 4.6).

### 4.4.3 Ratio of the number of stars in the two $V_Z$ groups

As mentioned in the introduction, the ratio of the number of stars in the two $V_Z$ groups was used in Kepley et al. (2007) to estimate the time of accretion of the object. The ratio these authors used was 1 : 2, in good agreement with the ratio found here in Sect. 4.3.1 for the core members. Using numerical simulations, this implied an accretion time of 6 – 9 Gyr for an object of total dynamical mass of $\sim 4 \times 10^8 M_\odot$. Now with a sample of up to 523 members, we will analyse how this ratio varies when exploring beyond the immediate solar vicinity.

Figure 4.8 shows the ratio of the number of stars in the two streams in $V_Z$ for selection B, as a function of the extent of the volume considered. Blue indicates the ratio of all the stars and green is for the 6D Gaia-only sample. The central lines plot the measured ratio and the shaded areas correspond to the 1$\sigma$ Poissonian error, showing that there is good agreement between the samples. The dashed lines at 1 : 2, 3 : 5 and 2 : 5 encompass roughly the mean and the scatter in the ratio. Evidently, the ratio drops slightly beyond the 1 kpc volume around the Sun, however, overall it stays rather constant and takes a value of approximately 1 : 2 for stars with $V_Z > 0$ relative to those with $V_Z < 0$. 

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**Fig. 4.8:** Ratio of the number of stars in the two clumps in $V_Z$ using selection box B, for different volumes and for the two samples of stars as indicated. The shaded area corresponds to the Poisson error on the measured ratio.
4.4.4 HR diagram and metallicity information

![HR diagram](image)

**Fig. 4.9:** Hertzsprung-Russel diagram of members of the Helmi streams. Those identified in the 6D sample are shown in green, while those without radial velocities are indicated with blue symbols. Members that are likely highly reddened are indicated with black dots, see Appendix 4.A. Superimposed are single population isochrones taken from Marigo et al. (2017). They serve to illustrate that the Helmi streams include a range of old, metal-poor stellar populations which did not form in a single event.

Photometry from *Gaia* combined with auxiliary metallicity information from the APOGEE/RAVE/LAMOST surveys can give us insights into the stellar populations of the Helmi streams. By using the *Gaia* parallaxes we construct the Hertzsprung-Russell (HR) diagram shown in Fig. 4.9. We have used here a sample of high photometric quality by applying the selection criteria from Arenou et al. (2018) and described in Sect. 4.3.3. Because the photometry in the BP passband is subject to some systematic effects especially for stars in crowded regions (see Gaia Collaboration, Brown et al., 2018), we use \((G – G_{RP})\) colour.

In Fig. 4.9 the streams’ members from the 6D sample are plotted with green symbols, and in blue if they are from the 5D dataset. On the basis of a colour-colour diagram we have identified stars that are likely reddened by extinction, see Appendix 4.A, and these are indicated with black dots. Since most stars follow a well-defined sequence in the \([(G – G_{RP}), (G – G_{BP})]\) space, outliers can be picked out easily. We consider as outliers...
those stars with a \((G - G_{\text{BP}})\) offset greater than 0.017 from the sequence (i.e. \(5 \times\) the mean error in the colours used). We find that especially the members found in the 5D dataset appear to be reddened. This is expected as all of these stars are located at low Galactic latitude (within 15 degrees from the Galactic Centre or anti-centre). Figure 4.9 shows also that we are biased towards finding relatively more intrinsically bright than fainter stars, and this is due to the quality cuts applied and to the magnitude limits of the samples used. We expect however that there should be many more fainter, lower main sequence stars that also belong to the Helmi streams, hidden in the local stellar halo.

The HR diagram that is shown in Fig. 4.9 does not resemble that of a single stellar population, but rather favours a wide stellar age distribution of \(~ 2\) Gyr spread, based on the width of the main sequence turn-off. To illustrate this we have overlayed two isochrones from Marigo et al. (2017) for single stellar populations of 11 and 13 Gyr old age and with metallicities \([\text{Fe/H}] = -1.0\) and \(-2.3\) respectively. To take into account the difference between the theoretical and actual \(Gaia\) passbands (Weiler, 2018), we have recalibrated the isochrones on globular clusters with similar age and metallicity (NGC104, NGC6121, NGC7099, see Harris, 1996), which led to a shift in \((G - G_{\text{RP}})\) colour of 0.04 mag.

Fig. 4.10: Histogram of the metallicities of the Helmi streams stars that are in the APOGEE/RAVE/LAMOST datasets. The distributions are very similar for selections A & B, peaking at \([\text{Fe/H}] \sim -1.5\) and revealing a broad range of metallicities for the Helmi streams stars. The metal-rich tail \((\text{[Fe/H]} \sim -0.5)\) is likely due to contamination from the thick disc but it is minimal for both selection boxes.
The spread in metallicities used for the isochrones is motivated by the metallicity distribution shown in Fig. 4.10, and derived using the stream members found in the APOGEE/RAVE/LAMOST datasets. We have used the following metallicity estimates for the different surveys: \(\text{Met}_K\) for RAVE, \(\text{FE}_\text{H}\) for APOGEE and \(\text{feh}\) for LAMOST. The median errors for these quantities are: 0.17 dex for RAVE, 0.12 dex for LAMOST, and 0.02 for APOGEE. The distribution plotted in Fig. 4.10 shows a range of metallicities \([\text{Fe/H}] = [-2.3, -1.0]\), with a peak at \([\text{Fe/H}] = -1.5\). The small tail seen towards the metal-rich end, that is at \([\text{Fe/H}] \sim -0.5\) is likely caused by contamination from the thick disc. The shape of the distribution shown in Fig. 4.10 is reminiscent of that reported by Klement et al. (2009) and by Smith et al. (2009) for much smaller samples of members of the Helmi streams. Roederer et al. (2010) have carried out detailed abundance analysis of the original streams' members which confirm the range of \(\sim 1\) dex in \([\text{Fe/H}]\) found here. All this evidence bolsters our claim that the streams originate in an object that had an extended star formation history.

### 4.5 Simulating the streams

We focus here on N-body experiments which we carried out to reproduce some of the properties of the Helmi streams. To this end, we used a modified version of GADGET 2 (Springel et al., 2005) that includes the rigid, static host potential described in Sect. 4.3.2 to model the Milky Way.

The analysis presented in previous sections, and in particular the HR diagram and metallicity distribution of member stars, supports the hypothesis that the Helmi streams stem from a disrupted (dwarf) galaxy. We therefore model the progenitor of the streams as a dwarf galaxy with a stellar and a dark matter component. We consider four possible progenitors whose characteristics are listed in Table 4.2.

For the stellar component, we use \(10^5\) particles distributed following a Hernquist profile, whose structural properties are motivated by the scaling relations observed for dwarf spheroidal galaxies (Tolstoy et al., 2009). For the dark matter halo we use \(6 \times 10^5\) particles following a truncated NFW profile (similar to the model introduced in Springel & White, 1999, but where the truncation radius \(r_{\text{c, trunc}}\) and the decay radius \(r_{\text{d, trunc}}\) are specified independently), with characteristic parameters taken from Correa et al. (2015). We truncate the NFW halo at a radius where its average density is three times that of the host (at the orbital pericentre). After setting the system up using the methods described in Hernquist (1993), we let it relax for 5 Gyr in isolation. We then place it on an orbit around the Milky Way. This orbit is defined by the mean position and velocity of the stars that were identified as core members of the stream with \(V_Z < 0\).

Figure 4.11 illustrates the evolution of a high-mass dwarf galaxy in the top two rows, and a low-mass dwarf galaxy in the bottom two rows. In each panel, we show the star particles in galactocentric Cartesian coordinates for different times up to 8 Gyr after infall. This comparison shows that increasing the mass results in more diffuse debris. On the other hand, the time since accretion has an impact on the length of the streams and on how many times the debris wraps around the Milky Way.

\(^3\)We take the \(V_Z < 0\) clump as this has the largest number of members.
Tab. 4.2: Structural parameters of the simulated dwarf galaxies. We quote both the dark halo’s original and truncated mass. This truncation depends on two parameters: \( r_{c,\text{trunc}} \) and \( r_{d,\text{trunc}} \), the cut-off and the decay radii, respectively.

<table>
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<td>1.62</td>
<td>2.13</td>
</tr>
</tbody>
</table>

Fig. 4.11: Spatial evolution of the Helmi streams at four different snapshots in our simulations. The top two rows show the evolution of a progenitor with a stellar mass of \( 10^8 M_\odot \), while the bottom two rows correspond to a system with a stellar mass of \( 5 \times 10^6 M_\odot \). In both cases, the orbit of the progenitor is the same. The appearance of the debris is seen to depend on the time since accretion as well as on the mass of the progenitor.
4.5.1 Estimation of the mass and time of accretion

To constrain the history of the progenitor of the Helmi streams we use the ratio of the number of stars in the two clumps in $V_Z$ as well as their velocity dispersion. Typically, the simulated streams do not have a uniform spatial distribution and they show variations on small scales. Furthermore, the azimuthal location of the Sun in the simulations is arbitrary. Therefore, and also to even out some of the small-scale variations, we measure the ratio of the number of stars in the streams and their velocity dispersion in 25 volumes of 1 kpc radius located at 8.2 kpc distance from the centre, and distributed uniformly in the azimuthal angle $\phi$.

Figure 4.12 shows the mean of the ratio for the different volumes as a function of time with solid curves, with the colours marking the different progenitors listed in Table 4.2. The shaded areas correspond to the mean Poissonian error in the measured ratio for the different volumes. The horizontal dashed lines are included for guidance and correspond to the lines shown in Fig. 4.8 for the actual data.

The mean ratio is clearly correlated with the properties of the progenitor, the most massive one being first to produce multiple streams in a given volume. Massive satellites have a larger size and velocity dispersion, which causes them to phase mix more quickly because of the large range of orbital properties (energies, frequencies). Based solely on Fig. 4.12, and taking a ratio between 0.55 and 0.7 as found using the Gaia-only sample in a 1 kpc sphere, we would claim that there is a range of possible ages of the stream, with the youngest being $\sim 4.5$ Gyr for the most massive progenitor, while for the lowest mass object the age would have to be at least 8 Gyr.
Fig. 4.13: Velocity dispersions of the simulated streams 8 Gyr after accretion. These have been computed using the principal axis of the velocity ellipsoid of stars in a given $V_Z$ clump, open markers are used for the smallest clump and closed markers for the largest. We show the results for volumes satisfying the ratio in the number of stars in the clumps as observed for the data. The green markers indicate the measured velocity dispersion from the data, the error bars illustrate the scatter in 1000 randomly down sampled sets of the streams members. These measured dispersions are deconvolved of their errors, and a 1σ clip in $V_R$ is applied.

A different way of probing the properties of the progenitor is to use the velocity dispersion of the streams. In Fig. 4.13 we show the dispersion along two principal axes of the velocity ellipsoid for the 1 kpc volumes that satisfy the ratio-constraint on the number of stars in the two $V_Z$ clumps, with open/closed markers used for the least/most populated clump. Green markers indicate the measured velocity dispersions. These dispersions are deconvolved of their errors, and calculated after clipping 1σ outliers in $V_R$ (see next paragraph). Typically the simulations have fewer particles in such 1 kpc volumes than observed in the debris. Therefore, we down-sample the data to only 15 stars per stream, which corresponds to the average number of particles in the simulations. The error bars indicate the maximum scatter in the velocity dispersion for 1000 such random down sampled sets.

The velocity ellipsoid of the streams' members is roughly aligned in cylindrical coordinates with $V_1$, $V_2$, and $V_3$ corresponding respectively to $V_\phi$, $V_Z$, $V_R$. We note that only for the most massive progenitor (and for times > 5 Gyr) the velocity dispersions are in good agreement with the observed values for $\sigma_{V_1}$ and $\sigma_{V_2}$, and that in all the remaining simulations, the dispersions are too small compared to the data. We should point out however, that the largest velocity dispersion (i.e. that along $V_R$) is less well reproduced in our simulations, possibly indicating that the range of energies of the debris is larger. However, if we clip 1σ outliers in $V_R$ for the data, and recompute $\sigma_{V_3}$, we find much better agreement with the simulations (while the values of $\sigma_{V_1}$ and $\sigma_{V_2}$ remain largely the same). The clipped members of the Helmi streams are all in the high energy, $E$, tail (see Fig.4.2), suggesting perhaps that the object has suffered some amount of dynamical friction in its evolution.
Combining the information of the ratio (Fig. 4.12) and the corresponding velocity dispersion measurements (Fig. 4.13) we therefore conclude that the most likely progenitor of the Helmi streams was a massive dwarf galaxy with a stellar mass of $\sim 10^8 \, M_\odot$. In general, the simulations suggest a range of plausible accretion times from $5 – 8$ Gyr. Although these estimates of the time of accretion are different than those obtained by Kepley et al. (2007), our results are consistent when a similar progenitor mass is used (i.e. of models 1 and 2), which however, does not reproduce well the observed kinematical properties of the streams.

### 4.5.2 Where to find new members across the Milky Way

To investigate where to find new members of the Helmi streams beyond the solar neighbourhood we turn to the simulations.

In Fig. 4.14 we show a sky map of stars from the N-body simulation corresponding to the dwarf galaxy with $M_* = 10^8 \, M_\odot$, $8.0$ Gyr after accretion (same as shown in the top right panel of Fig.4.11). The coordinates shown here are galactic $l$ and $b$ plotted in a Mollweide projection with the Galactic Centre located in the middle (at $l = 0$). Nearby stars (in blue) are mainly distributed along a ‘polar ring-like’ structure between longitudes $\pm 60^\circ$ (see for comparison Fig. 4.6). The most distant members are found behind the bulge (in red), but because of their location they may be difficult to observe.
4.6 Association with globular clusters

If the progenitor of the Helmi streams was truly a large dwarf galaxy, it likely had its own population of globular clusters (see Leaman et al., 2013; Kruijssen et al., 2018). To this end we look at the distribution of the debris in IOM-space for the simulation of the progenitor with $M_\ast = 10^8 \, M_\odot$ and overlay the data for the globular clusters from Gaia Collaboration, Helmi et al. (2018).

Figure 4.15 shows the energy $E$ vs $L_z$ (left) and the $L_\perp$ vs $L_z$ (right) of star particles in the simulation (black) and the stream members (green) together with the globular clusters (white open circles). For easy comparison we have overplotted the same red selection boxes as those in Fig.4.2. We have labelled the globular clusters that show overlap with the streams members in this space. Those that could tentatively be associated on the basis of their orbital properties are: NGC 4590, NGC 5024, NGC 5053, NGC 5272, NGC 5634, NGC 5904, and NGC 6981.

This set of globular clusters shows a moderate range in age and metallicity: they are all old with ages $\sim 11$ – $12$ Gyr and metal-poor with metallicities [Fe/H] = $[-2.3, -1.5]$. These age estimates are from Vandenberg et al. (2013), while for NGC 5634 we set it to 12 Gyr from comparison to NGC 4590 based on the zero-age HB magnitude (Bellazzini et al., 2002), and assume an uncertainty of 0.5 Gyr. Figure 4.16 shows the age-metallicity relation of all globular clusters that have reliable ages with those that we have associated with the Helmi streams coloured green. Interestingly the Helmi streams’ clusters follow a relatively tight age-metallicity relation, and which is similar to that expected if they originate in a progenitor galaxy of $M_\ast \sim 10^7$ – $10^8 \, M_\odot$ (see Leaman et al., 2013, Fig. 4).
Fig. 4.16: Age-metallicity distribution of Milky Way globular clusters based on Vandenberg et al. (2013). The green symbols mark the clusters associated with the Helmi streams on the basis of their orbital properties. They follow a well-defined age-metallicity relation.

Fig. 4.17 shows the Gaia colour-magnitude diagrams (CMD) of the globular clusters tentatively associated with the Helmi streams. Although not all CMDs are well-populated because of limitations of the Gaia DR2 data, their properties do seem to be quite similar, increasing even further the likelihood of their association to the progenitor of the Helmi streams.

Some of the associated globular clusters, namely NGC 5272, NGC 5904, and NGC 6981 have in fact, been suggested to have an accretion origin (of a yet unknown progenitor, see Kruijssen et al., 2018, and references therein). Two other clusters, NGC 5024 and NGC 5053, have at some point been linked to Sagittarius, although recent proper motion measurements have demonstrated this association is unlikely (see Law & Majewski, 2010; Sohn et al., 2018; Gaia Collaboration, Helmi et al., 2018). Also NGC 5634 has been related to Sagittarius (Law & Majewski, 2010; Carretta et al., 2017) based on its position and radial velocity. However, the proper motion of the system measured by Gaia Collaboration, Helmi et al. (2018) is very different from the prediction by for example, the Law & Majewski (2010) model of the Sagittarius streams.

### 4.7 Conclusions

Using the latest data from Gaia DR2 combined with the APOGEE/RAVE/LAMOST surveys we find hundreds of new tentative members of the Helmi streams. In the 6D sample that we built, we identified 523 members on the basis of their orbital properties, in particular
Fig. 4.17: Gaia colour-magnitude diagrams for the globular clusters that are likely associated with the Helmi streams on the basis of their orbital properties. All the CMDs correspond to stellar populations that are old and metal-poor. The ages and metallicities shown are taken from Vandenberg et al. (2013), except for the age of NGC 4590, which is based on the zero-age HB magnitude (Bellazzini et al., 2002).
their energy and angular momenta. On the other hand, we found 105 stars in the full Gaia 5D dataset in two $15\degree$-radius fields around the Galactic Centre and anticentre, using only the tangential velocities of the stars (which translate directly into two components of their angular momenta). Despite the large number of newly identified members we expect that many, especially faint stars are still hiding, even within a volume of 1 kpc around the Sun.

Having such an unprecedented sample of members of the streams allows us to characterise the streams and the nature of their progenitor. The HR diagram of the members suggests an age range of $\sim 11 – 13$ Gyr, while their metallicity distribution goes from $[\text{Fe/H}] \sim -2.3$ to $-1.0$, with a peak at $[\text{Fe/H}] \sim -1.5$. We are also able to associate to the streams seven globular clusters on the basis of their dynamical properties. These clusters have similar ages and metallicities as the stars in the streams. Remarkably they follow a well-defined age-metallicity relation, and similar to that expected for clusters originating in a progenitor galaxy of $M \sim 10^7 – 10^8 M_\odot$ (Leaman et al., 2013).

This relatively high value of the stellar mass is also what results from N-body simulations that aim to recreate the observed dynamical properties of the streams. From the ratio of the number of stars in the two clumps in $V_Z$ and their velocity dispersion we estimate the time of accretion to be in the range 5–8 Gyr and a stellar mass for the dwarf galaxy of $\sim 10^8 M_\odot$.

Although 5 Gyr ago would imply a relatively recent accretion event, one might argue that the object was probably on a less bound orbit and sunk in via dynamical friction (thanks to its large mass) and started to get disrupted then. This could explain the mismatch between the age of the youngest stars in the streams (approximately 11 Gyr old), and the time derived dynamically.

Despite the fact that the simulations are able to recreate the observations reasonably well, they fail to reproduce fully the observed velocity distribution in particular in the radial direction. This could be due to the lack of dynamical friction, but also by the limited exploration of models for the potential of the Milky Way. Other important improvements will be to consider the inclusion of gas particles and star formation in the simulations, as well as different initial morphologies for the progenitor systems (not only spherical, but also disc-like).

Originally, H99 determined that 10% of the stellar halo mass beyond the solar radius could belong to the progenitor of the Helmi streams. The lack of a significant increase in the number of members subsequently discovered by other groups (Chiba & Beers, 2000), led to the suggestion that the fraction may be lower. Our best estimate of the stellar mass of the progenitor of the Helmi streams is $\sim 10^8 M_\odot$, implying that it does significantly contribute to the stellar halo. For example Bell et al. (2008) estimate a stellar mass for the halo of $(3.7 \pm 1.2) \times 10^8 M_\odot$ between galactocentric radii of 1 to 40 kpc, and hence being a lower limit to the total stellar halo mass. Other estimates, based on the local density of halo stars give $7 – 10 \times 10^8 M_\odot$ (see Morrison, 1993; Bland-Hawthorn & Gerhard, 2016). This implies that the Helmi streams may have contributed $\sim 10 – 14\%$ of the stars in the Galactic halo.
Acknowledgements

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We have also made use of data from: (1) the APOGEE survey, which is part of Sloan Digital Sky Survey IV. SDSS-IV is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS Collaboration (http://www.sdss.org). (2) the RAVE survey (http://www.rave-survey.org), whose funding has been provided by institutions of the RAVE participants and by their national funding agencies. (3) the LAMOST DR4 dataset, funded by the National Development and Reform Commission. LAMOST is operated and managed by the National Astronomical Observatories, Chinese Academy of Sciences.

For the analysis, the following software packages have been used: vaex (Breddels & Veljanoski, 2018), numpy (Van Der Walt, 2011), matplotlib (Hunter, 2007), jupyter notebooks (Kluyver et al., 2016).

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Fig. A.1: Colour-colour diagram used to identify sources that are likely affected by extinction. Sources that offset from the main population by more than $5 \times$ the mean error in the used colours, i.e. 0.017 in $(G - G_{BP})$, are considered to be reddened. The main population is fit with a second-degree polynomial (dashed line). The dashed line is offset by 0.017 in $(G - G_{BP})$ to indicate which sources are reddened, i.e. those above the line. The members identified in the full 6D sample are shown with green markers, those without radial velocities are blue.

Appendix 4.A  Colour colour selection

On the basis of the colour-colour diagram shown in Fig. A.1 we select possible sources that are heavily affected by extinction. The full sample of members is fit with a second-degree polynomial (dashed line). As explained Sect. 4.4.4, we consider as outliers those stars with a $(G - G_{BP})$ offset greater than 0.017 from the sequence (i.e. $5 \times$ the mean error in the colours used). The dashed line shown in the figure is offset by this 0.017 in $(G - G_{BP})$ to illustrate clearly which stars are considered to be reddened. All sources above the dashed line are marked as reddened sources.
Multiple retrograde substructures in the Galactic halo: A shattered view of Galactic history

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Abstract
Several kinematic and chemical substructures have been recently found amongst Milky Way halo stars with retrograde motions. It is currently unclear how these various structures are related to each other. This Chapter aims to shed light on this issue. We explore the retrograde halo with an augmented version of the Gaia DR2 RVS sample, extended with data from three large spectroscopic surveys, namely RAVE, APOGEE, and LAMOST. In this dataset, we identify several structures using the HDBSCAN clustering algorithm. We discuss their properties and possible links using all the available chemical and dynamical information. In concordance with previous work, we find that stars with [Fe/H] < −1 have more retrograde motions than those with [Fe/H] > −1. The retrograde halo contains a mixture of debris from objects like Gaia-Enceladus, Sequoia, and even the chemically defined thick disc. We find that the Sequoia has a smaller range in orbital energies than previously suggested and is confined to high energy. Sequoia could be a small galaxy in itself, but since it overlaps both in integrals-of-motion space and chemical abundance space with the less bound debris of Gaia-Enceladus, its nature cannot yet be fully settled. In the low-energy part of the halo, we find evidence for at least one more distinct structure: Thamnos. Stars in Thamnos are on low-inclination, mildly eccentric retrograde orbits, moving at \( v_\phi \approx −150 \text{ km/s} \), and are chemically distinct from the other structures. Even with the excellent Gaia DR2 data, piecing together all the fragments found in the retrograde halo remains challenging. At this point, we are very much in need of large datasets with high-quality high-resolution spectra and tailored high-resolution hydrodynamical simulations of galaxy mergers.

5.1 Introduction
A wide variety of cosmological simulations, typically performed in a ΛCDM setting, have shown that the stellar halo of the Milky Way is an excellent testbed for galaxy formation models (Helmi et al., 2003; Bullock & Johnston, 2005; Johnston et al., 2008; Cooper et al., 2010; Pillepich et al., 2014; Grand et al., 2017). In ΛCDM, the halos of galaxies like the Milky Way grow in size by merging with other galaxies, mostly through minor mergers.
mergers. Galaxies that merge leave behind debris in the form of a trail of stars, and at the solar position this debris typically is very phase-mixed (Helmi & White, 1999). Disentangling the superimposed trails of different mergers is in principle possible with the help of detailed dynamical information like the integrals of motion (Helmi & de Zeeuw, 2000), or the actions (McMillan & Binney, 2008). In a local volume, each stream (a portion of a trail with stars with similar orbital phase) has typically a very low density, and has been estimated to contain on average 0.25% and at maximum 5% of the total number of local halo stars (Gould, 2003).

The outer stellar halo of the Milky Way is consistent with being completely built up through mergers (e.g. Belokurov et al., 2006; Bell et al., 2008; Helmi et al., 2011). With Gaia (Gaia Collaboration, Prusti et al., 2016; Gaia Collaboration, Brown et al., 2018) it has become possible to map the kinematics of the local stellar halo in great detail (e.g. Helmi et al., 2017; Myeong et al., 2018a,b; Koppelman et al., 2018). An impressive finding in the field of Galactic archaeology since the release of Gaia DR2 is the debris of Gaia-Enceladus-Sausage (Belokurov et al., 2018; Helmi et al., 2018): a massive dwarf galaxy that contributed a large fraction of the local stellar halo. The initial stellar mass of this object was $5 \cdot 10^8$–$5 \cdot 10^9$ M$_\odot$ (Belokurov et al., 2018; Helmi et al., 2018; Mackereth et al., 2019; Vincenzo et al., 2019) and it was accreted $\sim$ 10 Gyr ago (Helmi et al., 2018; Di Matteo et al., 2018; Gallart et al., 2019). We refer to this dwarf as Gaia-Enceladus, but the kinematic footprint of this dwarf galaxy is sometimes also referred to as Gaia-Sausage.

Besides Gaia-Enceladus, the Helmi streams (Helmi et al., 1999) are located in the prograde part of the halo. These streams originate in a dwarf galaxy of $M_\star \sim 10^8$ M$_\odot$ that was accreted 5-8 Gyr ago (Koppelman et al., 2019, see also Kepley et al. 2007). While a large fraction of stars with retrograde motions appears to be debris from Gaia-Enceladus, especially for high-eccentricity (Helmi et al., 2018, see also Belokurov et al. 2018), for very retrograde motions ($v_\phi < -100$ km/s) the situation is less clear. This portion of the halo contains several small structures (e.g. Myeong et al., 2018b; Koppelman et al., 2018; Matsuno et al., 2019), and plausibly also debris of Gaia-Enceladus. Also, Mackereth et al. (2019) postulate that the low-eccentricity region had a more complex formation history and would be composed by a mixture of stars formed in situ, debris from Gaia-Enceladus, and debris from other structures. One such structure would be the Sequoia (Myeong et al., 2019), whose existence builds on the discovery of a large globular cluster with very retrograde halo-like motion, FSR-1758 (Barba et al., 2019).

In this Chapter we quantify the degree of clustering in a local sample of halo stars using both dynamical and metallicity information. This allows us to discover debris from another small object, which we term Thamnos, as well as to establish on firmer grounds the reality and relationship between the different structures reported thus far in the literature in this rapidly evolving field.
5.2 Data

We use here an augmented version of the Gaia RVS sample, extended with radial velocities from APOGEE DR14 (Abolfathi et al., 2018), LAMOST (Cui et al., 2012), and RAVE DR5 (Kunder et al., 2017); see Sect. 2.1 and 2.2 of Koppelman et al. (2019) for more details. Because the metallicity scales of the three different surveys are not necessarily the same, we use the LAMOST values, unless stated otherwise. The results do not depend on this choice, except that the cross-matches with APOGEE and RAVE have considerably fewer stars. In total, our sample comprises 8,738,322 stars with full 6D phase-space information and high-quality parallaxes (parallax_over_error > 5) of which 3,404,432 have additional [Fe/H] information and 189,444 have chemical abundances from APOGEE. To calculate the distance we invert the parallaxes. Even when using high-quality parallaxes, biases in the distances could be introduced by inverting the parallaxes. However, we find that the structures identified in this Chapter are robust to using other distance estimates such as those provided by McMillan (2018) or Schönrich et al. (2019). Because of the systematic parallax offset in Gaia DR2 (Arenou et al., 2018; Gaia Collaboration, Brown et al., 2018; Lindegren et al., 2018), which for the RVS sample might even be more significant (Schönrich et al., 2019), we restrict our analysis to stars within 3 kpc of the Sun. When inspecting velocities we use a selection of stars in an even smaller volume to optimise the amount of clumpiness (by avoiding possible velocity gradients).

The velocities of the stars are corrected for the solar motion assuming \((U, V, W) = (11.1, 12.24, 7.25)\) km/s (Schönrich et al., 2010), and for the motion of the local standard of rest (LSR) using \(v_{\text{LSR}} = 232.8\) km/s (McMillan, 2017). Cartesian coordinates are calculated such that \(X\) points towards the Galactic Centre, and \(Y\) points in the direction of the motion of the disc. Cylindrical coordinates are derived in a right-handed system, although we flip the sign of \(v_\phi\) such that it coincides with the \(Y\)-axis at the solar position. In this system, the Sun is located at \(X = -8.2\) kpc. We use the implementation of the McMillan (2017) potential in AGAMA (Vasiliev, 2019) to calculate orbital parameters such as the total energy \((E_n)\), eccentricity \((e)\), circularity \((c)\), apocentre \((apo)\), and pericentre \((peri)\). The circularity is calculated as \(c = L_z/L_{z,circ}\), where \(L_{z,circ}\) is the vertical component of the angular momentum for a circular orbit with the same \(E_n\).

In this Chapter, we identify halo stars by their kinematics, a selection mostly used for illustrative purposes. As we are mainly interested in the retrograde halo we impose a relatively conservative cut by removing stars with \(|V - V_{\text{LSR}}| < 230\) km/s.

In this Chapter, we identify halo stars by their kinematics, a selection mostly used for illustrative purposes. As we are mainly interested in the retrograde halo we impose a relatively conservative cut by removing stars with \(|V - V_{\text{LSR}}| < 230\) km/s.
5.3 Results

5.3.1 The metal-poor, retrograde halo

Figure 5.1 shows a velocity diagram of the local stellar halo (distance < 1 kpc) split in a metal-poor (top) and a metal-rich (bottom) sample. Two-dimensional histograms show the distribution of all the stars in the given \([\text{Fe/H}]\) selection, while halo stars are highlighted with small black dots. The vertical dashed line indicates the very retrograde limit and highlights the large amount of small-scale substructure present for low metallicity. This is consistent with previous work reporting that the retrograde halo is more metal-poor (e.g. Carollo et al., 2007; Matsuno et al., 2019; Myeong et al., 2019). One of the structures seen is the arch reaching from \((v_\phi, (v_z^2 + v_R^2)^{1/2}) = (-100, 300)\) to \((-450, 0)\) km/s, which overlaps with the retrograde structures of Myeong et al. (2018b) and with the red and purple structures in Koppelman et al. (2018). The arch was associated to Gaia-Enceladus (Helmi et al., 2018) on the basis of resemblance to the simulations of Villalobos & Helmi (2008).

Besides the arch, there is another retrograde structure apparent in the metal-poor halo, at \(v_\phi = -150\) km/s and with \((v_z^2 + v_R^2)^{1/2} < 150\) km/s, that is with counter-rotating thick-disc-like kinematics. A subset of this retrograde component was picked up as the VelHel-4 structure Helmi et al. (2017) and as the blue and orange structures reported in Koppelman et al. (2018).

The debris of Gaia-Enceladus, which we identify here as the dominant contributor to the halo in the range \(-100 < v_\phi < 50\) km/s, has more stars with \([\text{Fe/H}] < -1\) (top panel) but also contributes to the metal-rich (bottom) panel. The only structure that is more abundant in the metal-rich part of the halo is the extension of the thick-disc, identified as the slow-rotating tail of the thick-disc (e.g. Koppelman et al., 2018; Haywood et al., 2018; Di Matteo et al., 2018). Stars with thin-disc-like motions appear to also exist with \([\text{Fe/H}] < -1\).

5.3.2 Selecting distinct substructures

Figure 5.1 on its own does not clear up if and how the retrograde structures are related. To study this in more detail we apply the clustering algorithm HDBSCAN\(^1\) (McInnes et al., 2017). We use the default parameters of the algorithm, after setting \(\text{min}\_\text{samples} = 3\), \(\text{min}\_\text{cluster}\_\text{size} = 15\), and \(\text{cluster}\_\text{selection}\_\text{method} = \text{‘leaf’}\). These settings, especially the leaf mode, tune the algorithm to find fine-grained structure instead of large overdensities. In our experience, no clustering algorithm is capable of picking out each of the halo overdensities given the large amount of overlap, the measurement errors, and the lack of metallicities for most sources. Therefore, we aim to break up the halo into small, robust groups that can be used to trace the large structures. Based on these groups we then place selection boxes to select the larger structures.

\(^1\)Hierarchical Density-Based Spatial Clustering of Applications with Noise, a clustering algorithm that excels over the better known DBSCAN both because it is less sensitive to the parameter selection and because it can find clusters of varying densities. See also https://hdbscan.readthedocs.io
Fig. 5.1: Velocity diagram of the local ($d < 1$ kpc) stellar halo split in a metal-poor (top) and a metal-rich sample (bottom). The colour-coding of the 2D histogram scales with the logarithm of the number of stars in each bin. All stars outside of the red dashed line are tentatively labelled as halo stars and are shown as black dots. We note that most if not all of the halo left of the dashed vertical line ($v_\phi < -100$ km/s) is more metal-poor than [Fe/H] = −1.
Fig. 5.2: Top row: Distribution of the stars in the groups identified by HDBSCAN in $E_n$, $L_z$, $ecc$, and [Fe/H] space, and colour-coded by [Fe/H], with the rest shown with black dots. In the bottom-right panel, we over-plotted lines of constant circularity and used coloured boxes to indicate our selection of substructures. We note that the Helmi Streams (HStr) are selected in $L_z - L_{\perp}$ space as described in the text. The bottom-left panel shows the kinematic properties of the stars in these substructures. The stars are coloured based on the boxes in the bottom-right panel (black dots mark stars that are not part of a box).

As input parameters for the algorithm we use $E_n$, $L_z$, $ecc$, and [Fe/H], which are all often used to find substructure in the stellar halo (e.g. Helmi & de Zeeuw, 2000; Helmi et al., 2017; Koppelman et al., 2018; Mackereth et al., 2019). The space that is defined by these parameters is scaled with RobustScaler implemented in scikit-learn (Pedregosa et al., 2011) using the default settings of the code. We select all stars within 3 kpc of the Sun and $|V - V_{LSR}| > 180$ km/s, because we are mainly interested in picking up structure in the halo. This selection includes a significant amount of thick-disc stars that should be identified as a distinct component if the algorithm works properly. There are no thin-disc stars in this selection.

Figure 5.2 shows the stars associated with substructures according to HDBSCAN, colour-coded by [Fe/H], while the remaining stars are shown with black dots. The top-left panel is similar to Fig. 5.1 and shows a very clear gradient of metallicity with $v_\phi$. Both the arch
and the low \((v_x^2 + v_R^2)^{1/2}\) structures are picked up as (metal-poor) groups (in yellow), while the thick-disc is also apparent (in purple). When varying the HDBSCAN parameters the individual groups change slightly, but the large structures which they trace persist. The results are also unaffected by changes in the limiting distance of the stars, at least up to 5 kpc from the Sun.

The top-right panel of Fig. 5.2 shows the distribution of the clusters in \(E_n-L_z\) space. In this space, it becomes clear that the arch structure strongly overlaps with the retrograde group (i.e. Sequoia) identified by Myeong et al. (2018b). As a reference we add the globular clusters FSR 1758 and \(\omega\)-Cen to this diagram, both of which have tentatively been assigned to the Sequoia by Myeong et al. (2019, although Massari et al. (2019) argues that the latter is more likely associated with Gaia-Enceladus). In the bottom-right panel of Fig. 5.2 we have overlaid lines of constant circularity (\(\text{circ} = -0.2, -0.4, -0.6\)), with solid lines corresponding to circular orbits in the Galactic plane. We select here regions occupied predominantly by the various structures as follows:

- **Gaia-Enceladus:** \(-1.5 < E_n/[10^5 \text{ km}^2/\text{s}^2] < -1.1\) and \(-0.20 < \text{circ} < 0.13\);
- **Sequoia:** \(-1.35 < E_n/[10^5 \text{ km}^2/\text{s}^2] < -1.0\) and \(-0.65 < \text{circ} < -0.4\);
- The low \((v_x^2 + v_R^2)^{1/2}\) structure is split in two based on the different metallicities:
  - **i):** with \(v_\phi \sim -200 \text{ km/s}, \text{circ} < -0.75\) and \(-1.65 < E_n/[10^5 \text{ km}^2/\text{s}^2] < -1.45\); and
  - **ii):** with \(v_\phi \sim -150 \text{ km/s}, -0.75 < \text{circ} < -0.4\) and
  \(-1.8 < E_n/[10^5 \text{ km}^2/\text{s}^2] < -1.6\).

We use different colours to show how the stars in these selections are distributed in velocity space in the bottom-left panel of Fig. 5.2. As a reference, we also plot the Helmi streams (HStr), selected as all stars with \(1600 < L_\perp/[\text{kpc km/s}] < 3200\) and \(1000 < L_z/[\text{kpc km/s}] < 1500\) (c.f. Koppelman et al., 2019). We note that in this figure, no globular clusters are found on the region occupied by the stars colour-coded dark blue (c.f. Massari et al., 2019). On the other hand, the cyan stars are located near \(\omega\)-Cen and therefore if we follow the argument of Myeong et al. (2018b), they could belong to the Sequoia. It is possible however that these cyan stars are tracing a new structure, are associated with those in the dark blue selection, or are a mixture of several structures.

For completeness, in Fig. 5.3 we plot the structures in other projections of velocity space. Their distribution is highly reminiscent of the simulated substructures in Helmi & de Zeeuw (2000), suggesting that they could indeed belong to different dwarf-galaxy progenitors.

### 5.3.3 Chemical analysis

Figure 5.4 shows the distribution of stars in our sample with abundances from APOGEE (with \textsc{aspcapflag} == 0), colour coded according to our selections. Particularly for \([\text{Fe/H}] > -1.5\), we see that the stars in cyan reveal contamination from the chemically defined thick disc as indicated by the annotation (despite their very retrograde motion) and Gaia-Enceladus. For low \([\text{Fe/H}]\), these stars typically have higher \([\text{Mg/Fe}]\) than Sequoia and Gaia-Enceladus, indicating a different origin. We tentatively refer to the
structure defined by the cyan and dark blue stars as Thamnos, that is ‘shrubs’, because these stars stand at the foot of a Greek giant and a tall tree in both velocity and $E_n - L_z$ spaces. We keep for now the distinction between the stars with $v_\phi \sim -200$ km/s and those with $v_\phi \sim -150$ km/s and refer to them as Thamnos 1 and 2, respectively.

The Sequoia stars on the other hand overlap with the metal-poor tail of Gaia-Enceladus, making it difficult to argue that they truly originate in a different system. It should be noted that much of this analysis is tentative as it is only based on a small sample of stars and at low [Fe/H] the errors are significant. Furthermore, the various other (independent) elements in APOGEE also have errors that are too large to be of help. With the amounts of data coming in the next few years this analysis will be much improved.

5.4 Discussion and Conclusions

We used Gaia DR2 data supplemented with line-of-sight velocities and chemical abundances from RAVE, APOGEE, and LAMOST to shed more light on the nearby stellar halo and its substructures. The stars in the retrograde halo are predominantly metal-poor. In fact, as the motions of the stars become more retrograde, the stars are even more metal-poor (i.e. a ‘gradient’ in $v_\phi$ with [Fe/H]). This gradient is reminiscent of the dual halo reported in Carollo et al. (2007), but its nature is more complex. Our analysis seems to suggest that the outer halo is more retrograde because it is dominated by debris from (the outskirts of) Gaia-Enceladus and Sequoia. This was already hinted at by Helmi et al. (2017), who have shown that at high energies the halo is retrograde. On the other hand, stars on very retrograde motions with orbits in the inner halo belong to a newly identified (but previously reported in part in Helmi et al., 2017; Koppelman et al., 2018; Mackereth et al., 2019) substructure which we have named Thamnos.
Fig. 5.4: Detailed chemical abundances (from APOGEE) for stars in the different substructures. The bars at the bottom of the panel indicate the mean error at that [Fe/H]. In the background we show a 2D histogram of all of the stars in our dataset, colour-coded by the logarithm of the number of stars per bin.
5.4.1 Notes on Sequoia

Using a mixture of spectroscopic data, the Stellar Abundances for Galactic Archeology (SAGA) database (Suda et al., 2008), Matsuno et al. (2019) have shown that the trend defined by Sequoia members is slightly offset from that of Gaia-Enceladus. Despite the fact that we only have three stars with full abundance information provided by APOGEE, we derive a similar conclusion.

In spite of using the same APOGEE dataset as Myeong et al. (2019) we reach different conclusions on the nature of Sequoia. One of the reasons driving this is that we find that it is more natural to separate the very retrograde halo into a high-\( En \) (Sequoia) and a low-\( En \) (Thamnos) substructures. Sequoia is not only chemically different from Thamnos, but it seems also difficult to reconcile its metallicity with the large extent in \( En \) proposed by Myeong et al. (2019, starting from the more unbound Gaia-Enceladus debris down to the energy of \( \omega \)-Cen). Such a large range in energy can only be produced by a very massive object, as illustrated in Fig. 5.5 where we have overlaid contours on the \( En - L_z \) diagram using the extent of mock dwarf galaxies. The outer contour corresponds to a mock dwarf galaxy of \( M_* = 10^8 \, M_\odot \) and the inner to \( M_* = 5 \cdot 10^6 \, M_\odot \) (see for details...
on the mock dwarfs Sect. 5 of Koppelman et al., 2019). The mocks are centred on an orbit that is chosen to be roughly in the centre of the debris of Sequoia and Thamnos in this diagram (only slight shifts are found when a different central orbit is chosen). The contours encompass 80% of the stars in the mock dwarfs. Therefore the extent of an object in $E_n - L_z$ space reflects - to some degree - the initial mass and size of the progenitor. Figure 5.5 shows that the contours for the Sequoia and Thamnos do not overlap, confirming our assessment that these are likely distinct systems.

On the other hand, having lower binding energy and more retrograde motion than the bulk of the debris of Gaia-Enceladus and overlapping with its metal-poor tail, Sequoia could well be, at least in part, debris from the outer regions of Gaia-Enceladus (see Fig.1 of Helmi et al., 2018), lost at early times. This analysis suggests that at best we are dealing with a bonsai Sequoia.

5.4.2 Thamnos
We find evidence for one or two more distinct components in the local retrograde halo: Thamnos 1 & 2. The debris of these objects is characterised by strong retrograde rotation and high binding energy. Especially the values of $E_n$ suggests that these structures may have been accreted a very long time ago. The distribution of these structures in $E_n - L_z$ space is compatible with them originating in the same dwarf galaxy; see Fig. 5.5. Figure 5.2 (bottom, right) shows that neither $\omega$-Cen nor FSR 1758 fall inside the selection boxes for Thamnos. When comparing to the full catalogue of Massari et al. (2019), we find no globular clusters to fall inside the selection for Thamnos. Compared to Gaia-Enceladus, the chemical composition of the stars of Thamnos are more metal-poor and significantly more $\alpha$-enhanced. As far as we can judge and given their similar abundances, Thamnos 1 & 2 share the same progenitor whose stellar mass $M_* \lesssim 5 \times 10^6 M_\odot$.

5.4.3 The chemically defined thick-disc
Our analysis reveals the presence of stars from the thick-disc with retrograde motions, identified chemically because they are metal-rich and $\alpha$-enhanced. It will be interesting to study these stars detailed chemical composition: they are amongst the oldest stars that formed in the in situ disc of the Milky Way. Since they were present at the time of the merging of Gaia-Enceladus (which explains their hot orbits) such a study would allow the characterisation of the disc at $z \gtrsim 2$. Early attempts of such studies have dated the merger event of Gaia-Enceladus (Di Matteo et al., 2018) and found an ultra metal-poor disc component (Sestito et al., 2019).

5.4.4 Final note
The main conclusion of this Chapter is that even with the excellent Gaia DR2 data, putting the shattered pieces together to reconstruct history, as in true Galactic archaeology, remains challenging at present. The different substructures identified in dynamical space show significant overlap. Chemical tagging helps with disentangling, but the current sample of high-quality and reliable abundances is too small to lead to firm conclusions. At
this point we are in desperate need for high-quality spectroscopic observations of the halo stars to supplement the *Gaia* data, as fortunately planned for WEAVE (Dalton et al., 2012) and 4MOST (de Jong et al., 2012). Furthermore, significant progress could be made by comparing the detailed properties of the substructures to tailored high-resolution hydrodynamical simulations of mergers of satellites with Milky Way-like galaxies.

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Origin of the system of globular clusters in the Milky Way

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Abstract

The assembly history experienced by the Milky Way is currently being unveiled thanks to the data provided by the Gaia mission. It is likely that the globular cluster system of our Galaxy has followed a similarly intricate formation path. To constrain this formation path, we explore the link between the globular clusters and the known merging events that the Milky Way has experienced. To this end, we combined the kinematic information provided by Gaia for almost all Galactic clusters, with the largest sample of cluster ages available after carefully correcting for systematic errors. To identify clusters with a common origin we analysed their dynamical properties, particularly in the space of integrals of motion. We find that about 40% of the clusters likely formed in situ. A similarly large fraction, 35%, appear to be possibly associated to known merger events, in particular to Gaia-Enceladus (19%), the Sagittarius dwarf galaxy (5%), the progenitor of the Helmi streams (6%), and to the Sequoia galaxy (5%), although some uncertainty remains due to the degree of overlap in their dynamical characteristics. Of the remaining clusters, 16% are tentatively associated to a group with high binding energy, while the rest are all on loosely bound orbits and likely have a more heterogeneous origin. The resulting age–metallicity relations are remarkably tight and differ in their detailed properties depending on the progenitor, providing further confidence on the associations made. We provide a table listing the likely associations. Improved kinematic data by future Gaia data releases and especially a larger, systematic error-free sample of cluster ages would help to further solidify our conclusions.

6.1 Introduction

According to the ΛCDM cosmological paradigm, structure formation proceeds bottom-up, as small structures merge together to build up the larger galaxies we observe today. The Milky Way is a prime example of this formation mechanism, as first demonstrated by the discovery of the Sagittarius dwarf spheroidal galaxy in the process of disruption (Ibata et al., 1994), then in halo stellar streams crossing the solar neighbourhood (Helmi et al., 1999), and more recently by the discovery of stellar debris from Gaia-Enceladus, revealing the last significant merger experienced by our Galaxy (Helmi et al. 2018, see also Belokurov et al. 2018).
As a natural result of such events, not only field stars but also globular clusters (GCs) may have been accreted (Peñarrubia et al. 2009). Starting with Searle & Zinn (1978) there has been a quest to understand which of the approximately 150 GCs hosted by the Galaxy actually formed in situ and which formed in different progenitors that were only later accreted. Recently, the availability of precise relative ages (with formal errors of \( \lesssim 1 \text{ Gyr} \); e.g. Marín-Franch et al., 2009; VandenBerg et al., 2013) and homogeneous metallicity measurements (Carretta et al. 2009) led to the discovery that the age–metallicity relation (AMR) of Galactic GCs is bifurcated (Marín-Franch et al. 2009; Forbes & Bridges 2010; Leaman et al. 2013). Although limited, kinematic information (e.g. Dinescu et al. 1997, 1999; Massari et al. 2013) nonetheless helped to reveal that the metal-poor branch of young GCs have halo-like kinematics (and are therefore more likely to be accreted), whereas GCs in the young and metal-rich branch have disc-like kinematics, and are consistent with having formed in situ (see also Recio-Blanco 2018).

With the advent of the second data release (DR2) of the Gaia mission (Gaia Collaboration, Brown et al. 2018), we have, for the first time, full six-dimensional phase space information for almost all of the Galactic GCs (Gaia Collaboration, Helmi et al., 2018; Vasiliev, 2019a). Therefore, it is timely to revisit the origin of the Galactic GC system. The goal of this Chapter is to use this information to provide a more complete picture of which GCs formed outside our Galaxy and in which progenitor (among those currently known or yet to be discovered). The main result of this analysis is given in Table A.1, which lists all the Galactic GCs and the progenitors they have likely been associated with. These associations may be seen to reflect the best of our current understanding, although some are only tentative. We may expect to be able to draw firmer conclusions with improved GC age datasets and larger samples of field stars with full-phase space kinematics.

6.2 The dataset: dynamics, ages, and metallicities

We put together a dataset of 151 clusters with full 6D phase-space information of Galactic GCs known based on the compilations by Gaia Collaboration, Helmi et al. (2018) and Vasiliev (2019a) (for more details, see Appendix A.1). We used the AGAMA package (Vasiliev 2019b) with the McMillan (2017) potential to compute GCs orbital parameters like the apocenter (apo), maximum height from the disc \( Z_{\text{max}} \), and eccentricity (ecc). We also computed the orbital circularity as \( \text{circ} = L_Z/L_{Z,\text{circ}} \), where \( L_{Z,\text{circ}} \) is the angular momentum of a circular orbit with the cluster energy, which thus takes extreme values +1 or –1 for co-planar circular prograde or retrograde orbits, respectively.

In this Chapter we study the AMR for a subsample of GCs that has a homogeneous set of ages and metallicities. This sample includes the catalogue by VandenBerg et al. (2013), who provide absolute ages and uncertainty for 55 GCs, and objects from the compilation by Forbes & Bridges (2010) who gathered relative age estimates from Salaris & Weiss (1998), De Angeli et al. (2005), Marín-Franch et al. (2009), which we add after checking
Fig. 6.1: Age–metallicity relations for the sample of 69 GCs, colour-coded according to their dynamical properties. We note that clusters on the young metal-rich branch share similar dynamical properties (in red), and that clusters with these characteristics are also present for low metallicities and are typically older.

6.3 Assignment of clusters

Figure 6.1 shows the AMR for the clusters in our sample, colour-coded according to various dynamical properties, namely apo, circ, $Z_{\text{max}}$ and ecc. It is immediately clear that the clusters located on the young and metal-rich branch of the AMR are dynamically different from those on the young and metal-poor branch. Young and metal-rich GCs typically do not reach high altitudes above the Galactic plane ($Z_{\text{max}}$), have smaller apocentres, and tend to have lower eccentricities. As already recognised in the literature (e.g. Leaman et al. 2013), these are the clusters formed in situ, either in the disc or in the bulge, in what we hereafter refer to as the Main Progenitor. For the first time we can recognise from Fig. 6.1 some old metal-poor GCs with orbital properties characteristic of the young metal-rich branch, which thus would also belong to the Main Progenitor.
6.3.1 **In situ clusters**

We therefore use the dynamical properties of the GCs that populate the young and metal-rich branch of the AMR of Fig. 6.1, and which were likely born in the Milky Way, to define simple criteria to identify Main Progenitor clusters:

- **Bulge clusters**: those placed on highly bound orbits (with apo < 3.5 kpc). There are 36 GCs selected in this way.

- **Disc clusters** satisfy: i) \( Z_{\text{max}} < 5 \) kpc and ii) \( \text{circ} > 0.5 \). While this does not guarantee a “pure” disc sample, and there may be a small amount of contamination, the effectiveness of these criteria is supported by the fact that the vast majority of the selected clusters tend to describe an AMR qualitatively similar to that found by Leaman et al. (2013), except for two clusters located on the young and metal-poor branch (NGC 6235 and NGC 6254). The dynamical parameters of these clusters are in the extremes of those characteristic of the Main Progenitor, and therefore we exclude them. This leaves a total of 26 Disc clusters.

We note that for \([\text{Fe/H}] < -1.5\), Main Progenitor clusters are older than the average. The 62 *in situ* clusters are listed in Table A.1 as Main-Disc (M-D) or Main-Bulge (M-B).

6.3.2 **Accreted clusters**

We now analyse the remaining clusters looking for a common association with the progenitors of known merger events experienced by the Milky Way. To do so, we investigate the integrals of motion (IOM) space defined by \( E \), \( L_z \), and \( L_{\text{perp}} \). Here \( L_{\text{perp}} = \sqrt{L_X^2 + L_Y^2} \) is the angular momentum component perpendicular to \( L_Z \), which, despite not being fully conserved in an axisymmetric potential like that of the Milky Way, still helps in discriminating groups of stars (or clusters) with similar origin (Helmi & de Zeeuw 2000), also if the potential has varied with time (see Peñarrubia et al., 2006; Gómez et al., 2013). In particular, we use the (known) extent of each progenitor stellar debris in the IOM space to provisionally identify associated GCs.

**The Sagittarius dwarf spheroidal galaxy**

The Sagittarius dwarf spheroidal galaxy constituted the first discovery of a merger with the Galaxy (Ibata et al., 1994). By exploiting numerical models that accurately reproduce the position and radial velocity of stars belonging to the Sagittarius streams, Law & Majewski (2010) provided a list of candidate GCs that could have been associated to the dwarf. This list has been recently refined by adding the information on their proper motions as measured from HST and *Gaia* observations (Massari et al. 2017; Sohn et al. 2018), and currently includes 6 GCs, namely M 54, Arp 2, Pal 12, Terzan 7, Terzan 8, and NGC 2419.

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1 Nuclear clusters of dwarf galaxies accreted long ago could also end up on bulge-like orbits due to dynamical friction. The current uncertainty on the proper motions of bulge GCs (they are typically highly extincted and only few stars are detected by *Gaia*), and the lack of an age estimate for most of them, prevents us from investigating this possibility.
Fig. 6.2: Two projections of IOM space for the 151 GCs in our sample, colour-coded according to their associations with different progenitors (blue symbols mark the Main Progenitor, red is for Gaia-Enceladus, green for Sagittarius, orange for the progenitor of the Helmi streams, brown for Sequoia, pink for the low-energy group, and cyan for the high-energy group). For visualisation purposes, two clusters (Crater and E1) with extremely negative $L_Z$ have not been plotted. Empty symbols correspond to tentative associations. The two star symbols mark the young and metal-poor clusters excluded by the Disc selection.

These six clusters describe a well-defined subgroup in IOM space: i) $3700 < L_{\text{perp}} < 6200$ km/s kpc and ii) $0 < L_Z < 3000$ km/s kpc, as seen in Fig. 6.2. Two more clusters are found in this region of IOM space: NGC 5824 and Whiting 1, which had previously been tentatively associated with the dwarf by Bellazzini et al. (2003) and Law & Majewski (2010), respectively.

The progenitor of the Helmi streams

Recently, Koppelman et al. (2019) used Gaia data to characterise the progenitor of the Helmi streams (hereafter H99, Helmi et al., 1999). According to these authors and based on their dynamical properties (and a comparison with a numerical simulation that best reproduces the properties of the streams), the seven GCs shown in orange in Fig. 6.2 could be associated to this object. Interestingly, these seven clusters were shown to follow a tight and low-normalisation AMR.

To explore whether or not additional members could exist, we started from the dynamical criteria suggested in their work, and revisited the location of the selection boundaries while requiring consistency with the AMR. The following criteria: i) $350 < L_Z < 3000$ km/s kpc, ii) $1000 < L_{\text{perp}} < 3200$ km/s kpc, and iii) $E < -1.0 \times 10^5$ km$^2$/s$^2$ seem to be the most appropriate. Allowing for lower values of $L_Z$ leads to the inclusion of very old clusters (not consistent with the AMR of the core members) whereas increasing the upper limit leads to including two disc clusters lacking an age estimate (Pal 1 and BH 176). Increasing the $E$ limit would mean including a cluster with $apo > 100$ kpc, whereas the
typical value for the core members is $\sim 30$ kpc. Finally, the limits on $L_{\text{perp}}$ are given by Sagittarius clusters on one side and by the consistency with the AMR on the other.

Out of the ten GCs that would be associated to H99 according to the above criteria, there is no age information for three of them: Rup 106, E3, and Pal 5, and therefore we consider them to be tentative members (orange open symbols in Fig. 6.2). We note that Pal 5 and E3 have the lowest $e_c$ ($\sim 0.2$) in the set, with E3 having the lowest $Z_{\text{max}}$ ($\sim 7$ kpc), and Rup 106 the largest apo ($\sim 34$ kpc). Also in comparison to the H99 stars (see Koppelman et al., 2019, for details), E3 and Pal 5 have more extreme orbital properties, with Rup 106 being on a looser orbit. However, this does not necessarily preclude membership since GCs are expected to be less bound than the stars.

**Gaia-Enceladus**

To look for GCs associated to Gaia-Enceladus (G-E hereafter, Helmi et al., 2018), we directly compare the distribution in IOM space of GCs to that of field stars with 6D kinematics from Gaia, as shown in Fig. 6.3.

Based on this comparison, we associate clusters to G-E according to the following criteria: i) $-800 < L_Z < 620$ km/s kpc, ii) $-1.86 \times 10^5 < E < -0.9 \times 10^5$ km$^2$/s$^2$, and iii) $L_{\text{perp}} < 3500$ km/s kpc. This selection associates 28 GCs to G-E. With the exception of NGC 7492 (its apo being $\sim 28$ kpc), all of them have apocenters $< 25$ kpc, as reported for G-E stars (see Deason et al., 2018). The resulting AMR is remarkably tight (see below), and this tightness can be used to explore the energy boundaries. By decreasing the lower limit of $E$, a very old globular cluster enters the selection which is significantly off the AMR described by the other members. This energy limit is slightly higher than that used by Myeong et al. (2018a, who adopt the same Galactic potential as ours) to define the clusters belonging to the progenitor of the same accretion event, that they call Gaia-Sausage (see also Belokurov et al. 2018). Moving the upper limit to $E = -1.1 \times 10^5$ km$^2$/s$^2$ excludes Pal 2 and Pal 15, which we therefore consider to be tentative members (open symbols in Fig. 6.2).

Some of the associated clusters are located in regions of the IOM space that are shared by stellar debris of different progenitors. Therefore, the associations are more uncertain and deserve further discussion (see e.g. Borsato et al., 2019). For example, two clusters (NGC 5904 and NGC 5634) lie near the region occupied by H99 debris. They have $e_c \sim 0.8$, and their $Z_{\text{max}}$ and apo are somewhat larger but not inconsistent with those typical of G-E clusters. While NGC 5634 has no age estimate, the age of NGC 5904 (11.5 Gyr old) is consistent with both the AMRs. We therefore consider them as tentative members to both progenitors.

Also of particular interest is the overdensity of stars seen in Fig. 6.3 at the location $L_Z \sim -3000$ km/s kpc and $E \sim -10^5$ km$^2$/s$^2$, which has been associated to G-E by Helmi et al. (2018) because of its resemblance to a feature seen in numerical simulations of a merger event with similar characteristics to G-E (Villalobos & Helmi 2008). Two GCs (NGC 3201 and NGC 6101) are located in this region of IOM space, and therefore we mark them as tentative members but discuss them further in the following section.
With our selection criteria, \(\omega\)-Cen\(^2\) is the cluster with the highest binding energy among those associated to G-E. This is in good agreement with the idea that this cluster is in reality the remnant of the nuclear star cluster of an accreted dwarf (e.g. Bekki & Freeman 2003) as suggested by its peculiar chemistry.

After these considerations, we are left with 26 possible members of G-E (and 6 tentative ones). Although this number is large, it is consistent within the scatter with the relation between the number of GCs and host halo mass (van Dokkum et al., 2017), given the mass estimate of \(6 \times 10^{10}\) M\(_{\odot}\) from Helmi et al. (2018).

**Sequoia**

Recently, Myeong et al. (2019) proposed the existence of merger debris from a galaxy named *Sequoia* that would have been accreted about 9 Gyr ago. Based on clustering

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\(^2\)for which we used the metallicity of the most metal-poor and oldest population, see Bellini et al. 2017.
algorithms performed over a scaled action space, these authors found five GCs likely associated to this system. We find seven GCs to be possibly associated when considering a selection box in $E$ and $L_Z$ corresponding to the stars from Sequoia according to Myeong et al. (2019), namely $-3700 < L_Z < -850$ km/s kpc and $-1.5 \times 10^5 < E < -0.7 \times 10^5$ km$^2$/s$^2$. Three of these GCs are in common, namely FSR 1758, NGC 3201, and NGC 6101. The other four (IC 4499, NGC 5466, NGC 7006 and Pal 13) were excluded by Myeong et al. because of their slightly larger eccentricity ($\langle ecc \rangle \sim 0.75$, compared to their initial estimate of $\sim 0.6$). Three clusters have known ages and these follow a low-normalisation AMR similar to the H99 GCs, which is consistent with the low stellar mass estimated for Sequoia (Myeong et al. 2019).

As mentioned earlier, the Sequoia IOM selection has some overlap with the arch-like overdensity ascribed to G-E debris by Helmi et al. (2018). Discerning which is the actual progenitor of NGC 3201 and NGC 6101 is therefore not possible, and so in Table A.1 we link them to both systems. Nonetheless, there may be a slight preference for Sequoia given their ages and metallicities. On the other hand, Myeong et al. (2019) associated $\omega$-Cen and NGC 6535 to Sequoia because of their location in IOM space. However, these two GCs follow a much higher AMR, typical of clusters from more massive progenitors. For this reason, we prioritise the association of $\omega$-Cen to G-E and of NGC 6535 to one of the groups described below, though we acknowledge both the interpretations in Table A.1.

The remaining clusters

We were not able to associate 36 of the 151 GCs with full phase-space information to known merger events. From their distribution in the IOM space (Fig. 6.2), it is clear that at least a significant fraction of them (25) could tentatively be part of a structure at low energy, with $E < -1.86 \times 10^5$ km$^2$/s$^2$, low $L_{\text{perp}}$, and with $L_Z \sim 0$ km/s kpc (pink symbols in Fig. 6.2); we label these L-E.

The remaining 11 GCs all have high energy ($E > -1.5 \times 10^5$ km$^2$/s$^2$, in cyan in Fig. 6.2), but span a very large range in $L_Z$ and $L_{\text{perp}}$. Therefore, they cannot have a common origin. Most likely instead, they have been accreted from different low-mass progenitors which have not contributed debris (field stars) to the Solar vicinity (as otherwise we would have identified corresponding overdensities in Fig. 6.3). For convenience only, we use a single label for all these objects (H-E, for high energy) in Table 1. Upcoming datasets, especially of field stars with full phase-space information across the Galaxy, could be key to understanding their origin given their heterogeneous properties.

6.3.3 Age–metallicity relation

The various panels in Fig. 6.4 show the AMR of the clusters associated to the different structures discussed so far, colour-coded as in Fig. 6.2. Although consistency with the AMR of each group is checked after the IOM selection, it is quite remarkable that the dynamical identification of associations of GC results in AMRs that are all well-defined and depict different shapes or amplitudes.
Fig. 6.4: AMRs for the 69 GCs with age estimates, colour-coded as in Fig. 6.2. The corresponding progenitors are marked in the labels. Individual age uncertainties are also plotted. In the AMR for Main Progenitor clusters (upper-left panel), diamonds represent bulge clusters while circles describe disc clusters. The two red-circled black symbols are the two clusters that satisfy the disc membership criteria but that are excluded because they are near the boundary of the respective IOM region and their location in the AMR. The green curves show AMRs predicted by a leaky-box model, illustrating that a simple model can explain the AMRs shown by the progenitors.
The clusters of the Main progenitor constitute the largest group and have the highest normalisation, that is the most metal-poor oldest clusters were born in the Galaxy itself. The G-E AMR is remarkably tight and has a high normalisation, though as expected this is not as high as that of the Main progenitor. Similarly, the L-E group depicts a reasonably coherent AMR, with a high normalisation, which seems even higher than that of G-E clusters, thus possibly suggesting the presence of yet-to-be-discovered merger debris located preferentially in the Galactic bulge and originating in a more massive object. On the other hand, the AMR of the H99 members is remarkably low in terms of normalisation, and this is consistent with the fact that this progenitor is less massive ($M_\star \sim 10^8 \, M_\odot$, Koppelman et al. 2019).

We can describe the various AMRs with a leaky-box chemical evolution model (Prantzos, 2008; Leaman et al., 2013; Boecker et al., 2019), where the metallicity of the system evolves as

$$Z(t) = -p \ln \mu(t) = -p \ln \frac{t_f - t}{t_f - t_i} \quad \text{for } t \geq t_i,$$

where $\mu(t) = M_g(t)/M_g(0)$ is the gas fraction, and $p$ the (effective) yield. We obtained this expression by assuming a constant star formation rate starting at time $t_i$ after the Big Bang and ending at time $t_f$, which we take to be the time of accretion (which is constrained by estimates in the literature and which we took to range from 3.2 Gyr for Sequoia to 5.7 Gyr for Sagittarius). For the yield $p$ we assumed the dependence on $M_\star$ derived by Dekel & Woo (2003) for star-forming dwarf galaxies (see also Prantzos, 2008). The time $t_i$ is a free parameter which we varied for each progenitor to obtain a reasonable description of the observed points. The only constraint we apply is that more massive progenitors should start forming stars earlier, which led to $t_i$ values in the range of 0.5 Gyr (for the Main progenitor) to 1.1 Gyr (for Sequoia). The resulting curves for each progenitor are shown in the panels of Fig. 6.4 with green solid lines. We stress that these curves do not represent fits, but merely show that a simple leaky box chemical evolution model is reasonably adequate to describe the AMRs found for each set of clusters associated to the individual progenitors. The scatter around each curve (typically < 0.5 Gyr) is qualitatively consistent with the individual age uncertainties. This might indicate that for the clusters lacking an age uncertainty, our assumed value of 0.75 Gyr (see Appendix A.2) was too conservative, especially when considering relative ages rather than absolute ones.

### 6.4 Summary and Conclusions

In this Chapter we exploit the complete kinematic information for 151 Galactic GCs, in combination with metallicity and homogeneous age estimates for a subset of 69 GCs. Our goal is to elucidate which GCs formed in situ and which could have been accreted, associating the latter to a particular progenitor based on their dynamical properties and, where needed, on the shape of the AMR.
We found that 62 GCs likely formed in the Milky Way, and we separated them into disc and bulge clusters based on their orbital parameters. Among the accreted clusters, we assessed their possible associations with the progenitor of four known merger events: Gaia-Enceladus, the Sagittarius dwarf, the Helmi streams, and the Sequoia galaxy. We identified 26 (and an additional 6 tentative) GCs associated to Gaia-Enceladus. This large number as well as the high normalisation of their AMRs are consistent with Gaia-Enceladus being the most massive among these four objects. According to our findings, ω-Centauri would be its nuclear star cluster. We further identified eight clusters associated with the Sagittarius dwarf, possibly ten clusters to the progenitor of the Helmi streams, and a plausible seven to Sequoia. Despite not being very populated, the AMRs of these two groups are consistent with the lower literature estimates for their mass. There is an inherent uncertainty to these assignments, because debris from different progenitors partly overlaps in IOM space, as in the case of Sequoia and G-E, and to a lesser degree for G-E and the Helmi streams.

Interestingly, the specific frequency per unit of galaxy mass ($T_N$) as defined by Zepf, & Whitmore (1993) for each progenitor closely follows the relation reported in Zaritsky et al. (2016) for a sample of galaxies in the same stellar mass range. For example, we find $\log T_N = [1.8, 2.0, 1.2, 2.1]$ for G-E, the Helmi streams, Sgr, and Sequoia, while the expected values using Zaritsky et al. (2016) are respectively, $[1.8, 2.2, 1.8, 2.4]$, which is well within the observed scatter of the relation ($\sigma(\log T_N) \sim 0.7$). The values of $\log T_N$ were computed using stellar mass estimates of $[5, 1, 5] \times 10^8 M_\odot$ for G-E, the Helmi streams, and Sagittarius. These were derived with different methods, using chemistry in the first case (Helmi et al. 2018) and N-body models for the latter two (Koppelman et al. 2019; Dierickx, & Loeb 2017). In the case of Sequoia, we used the mass estimated by Myeong et al. (2019) on the basis of the specific frequency, and it is therefore less surprising to find good agreement (although our estimate uses a different number of associated clusters).

The 36 clusters that we have not associated to known debris can be split in two groups based on orbital energy. While the class of GCs with low binding energy is very heterogeneous and likely has several sites of origin, the low-energy (highly bound) group with 25 tentative members is relatively highly clustered in its dynamical properties and shows a reasonably tight and high-normalisation AMR, possibly suggesting the presence of debris towards the Galactic bulge from a large hitherto unknown galaxy. This finding is consistent with the conclusion that another significant accretion event is required to explain the overall AMR of Galactic GCs; for example, a merger with a “Kraken”-like galaxy (Kruijssen et al., 2019). We note however, that most of the clusters reported in Kruijssen et al. (2019) as possible members of Kraken are not dynamically coherent, since three are in common with the H99 GCs, seven with G-E, two with Sequoia, and one with the Main Progenitor. Therefore, although the L-E group is Kraken-like, the associated clusters are different. It transpires that taking into account the dynamical properties is fundamental to establishing the origin of the different GCs of our Galaxy.

6.4 Summary and Conclusions
The next data release of the *Gaia* mission will provide improved astrometry and photometry for all the GCs, as well as for a much larger sample halo stars. This will be crucial to achieving a complete and accurate sample of GCs with absolute ages. Moreover it will lead to a better understanding of the debris of the known progenitors, and possibly to the discovery of new ones. The combination of these factors will result in significant progress in the field and allow to better pin down tentative associations and possibly also to assess under which conditions the different clusters formed, such as for example whether formation was prior to or during the different merger events.

**Acknowledgements**

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**Bibliography**

Appendix 6.A Details of the sample of globular clusters

A.1 Kinematical properties
To put together the sample of GCs, we started from the 75 GCs analysed by Gaia Collaboration, Helmi et al. (2018), who combined the Gaia measured proper motions with distances and line-of-sight velocities available from the compilation by Harris (1996, 2010 edition). We then added the data for the remaining GCs from Vasiliev (2019a), who determined the 6D coordinates combining Gaia measurements with line-of-sight velocities also from Baumgardt et al. (2019).

We then transformed the observed measurements of the kinematics of the resulting catalogue of 151 clusters to the Galactocentric reference frame. To this end we assumed a Local Standard of Rest velocity $V_{\text{LSR}} = 232.8 \, \text{km/s}$ (McMillan, 2017), a solar motion $(U, V, W) = (11.1, 12.24, 7.25) \, \text{km/s}$ (Schönrich et al., 2010), and a distance from the Sun to the Galactic Centre of $R_0 = 8.2$ (McMillan, 2017).

A.2 Homogeneous cluster ages
Many methods have been applied to determine the absolute age of a GC. Photometric errors, poor calibration, and uncertainties on the cluster distance and reddening however, can affect such age estimates. To overcome these, it has often been preferred to determine relative ages (e.g. Buonanno et al. 1989; Bono et al. 2010; Massari et al. 2016), although these then need to be calibrated to some absolute scale (e.g. Marín-Franch et al. 2009). This explains why available age compilations of GCs in the literature are so heterogeneous, and therefore dangerous to blindly combine together as different methods can result in systematic differences that amount to several gigayears.

Figure A.1 highlights the need for care when combining different clusters age datatsets. The top panel shows the difference in metallicity between the VandenBerg et al. (2013) estimates based on the spectroscopic scale of Carretta et al. (2009), in comparison to those of Forbes & Bridges (2010). This difference is constant for $[\text{Fe/H}] \lesssim -1.1$, above which it rises steeply with metallicity to about +0.5 dex. The bottom panel of Fig. A.1 shows the effect of this trend on age, namely that the clusters with $[\text{Fe/H}] \gtrsim -1.1$ (red symbols) appear to be systematically older by $\sim 2$ Gyr. When excluding these clusters, the mean difference between the age estimates is $\Delta t = 0.08$ Gyr, with a spread of $\sigma_t = 0.75$ Gyr (r.m.s = 0.14 Gyr), and thus fairly consistent with zero. Therefore, we only consider clusters from the Forbes & Bridges (2010) sample with $[\text{Fe/H}] \lesssim -1.1$, and assign an uncertainty to their age estimates (which lack errors) that equals the observed spread around the mean difference ($\sigma_t = 0.75$ Gyr). As for metallicities, we adopted the scale from Carretta et al. (2009).
We also studied the estimates reported in Rosenberg et al. (1999), Dotter et al. (2011), Roediger et al. (2014), but found either poorly constrained values (with ages as old as 15 Gyr and very large uncertainties), or no new entries. Moreover, we decided not to include age estimates performed on single objects because of the impossibility of having systematic effects under control. The only exception is NGC 5634, estimated to be as old as NGC 4372 by Bellazzini et al. (2002), and for which we used NGC 4372 age estimate by VandenBerg et al. (2013).
Tab. A.1: List of Galactic GCs and associated progenitors. M-D stands for Main-Disc; M-B stands for Main-Bulge; G-E stands for Gaia-Enceladus; Sag stands for Sagittarius dwarf; H99 stands for Helmi Streams; Seq stands for Sequoia galaxy; L-E stands for unassociated Low-Energy; H-E stands for unassociated High-Energy. Finally XXX mark clusters with no available kinematics. Most of these are bulge GCs too extincted to be observed by Gaia.

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A massive mess:
When a large dwarf and a Milky Way-like galaxy merge

Based on: Koppelman, Bos, and Helmi (2020), submitted to A&A

Abstract

Circa 10 billion years ago the Milky Way merged with a massive satellite, Gaia-Enceladus. To gain insight into the properties of its debris we analyse in detail the suite of simulations from Villalobos & Helmi (2008), which includes an experiment that produces a good match to the kinematics of nearby halo stars inferred from Gaia data. We compare the kinematic distributions of stellar particles in the simulations and study the distribution of debris in orbital angular momentum, eccentricity and energy, and its relation to the mass-loss history of the simulated satellite. We confirm that Gaia-Enceladus probably fell in on a retrograde, 30° inclination orbit. We find that while 75% of the debris in our preferred simulation has large eccentricity (> 0.8), roughly 9% has eccentricity smaller than 0.6. Star particles lost early have large retrograde motions, and a subset of these have low eccentricity. Such stars would be expected to have lower metallicities as they stem from the outskirts of the satellite, and hence naively they could be confused with debris associated with a separate system. These considerations seem to apply to some of the stars from the postulated Sequoia galaxy. When a massive discy galaxy merges, it leaves behind debris with a complex phase-space structure, a large range of orbital properties, and a range of chemical abundances. Observationally, this results in substructures with very different properties, which can be misinterpreted as implying independent progeny. Detailed chemical abundances of large samples of stars and tailored hydrodynamical simulations are critical to resolving such conundrums.

7.1 Introduction

The ultimate goal of Galactic archaeology is to determine the series of events that have led to the formation of the Milky Way. The arrival of the full phase-space dataset of Gaia (Gaia Collaboration et al., 2016, 2018; Katz et al., 2019) has resulted in many insights that have given a boost to this field. One of these insights is that the local stellar halo formed predominantly through a merger with a massive dwarf galaxy named Gaia-Enceladus (Helmi et al., 2018, or Gaia-Sausage, Belokurov et al. 2018).
It remains unclear, however, which other objects on retrograde orbits have contributed debris to the local stellar halo. Although several smaller retrograde structures have been found that appear to be chemically and dynamically different from Gaia-Enceladus (e.g. Koppelman et al., 2018; Mackereth et al., 2019; Matsuno et al., 2019; Myeong et al., 2019), their origin and linkage are not always clear. For example, Myeong et al. (2019) advocate that several of these structures originate in a single (moderately) massive dwarf galaxy: Sequoia. However, Koppelman et al. (2019) based on their chemistry and orbital properties, suggest that there must be at least two progenitors of the structures.

The debris of a low-mass satellite orbiting in a static potential phase-mixes at roughly constant mean orbital energy (Helmi & White, 1999). This makes the integrals of motion powerful tools to identify halo structures and determine accretion history (Helmi & de Zeeuw, 2000; McMillan & Binney, 2008). For massive satellites, this picture becomes muddled because of dynamical friction and the tidal interaction between the satellite and host (see e.g. Jean-Baptiste et al., 2017). Dynamical friction is relevant for high mass-ratios between the satellite and its host. It affects the mean orbit of a satellite and makes it sink to the centre of the host (Quinn & Goodman, 1986). Since a satellite is stripped of its mass mostly in discrete events at its orbital pericentre (and this evolves because of dynamical friction), the result is a complex energy distribution of the debris. For example, the material from the satellite’s outskirts is lost early and hence has high orbital energy. On the other hand, its core sinks to the centre becoming more bound (Tormen et al., 1998; Van Den Bosch et al., 1999). Furthermore, massive satellites with disc-like morphology produce very complex tidal features (Quinn, 1984) which must give rise to an intricate structure of the remaining debris. These considerations are pertinent for understanding the left-overs from a system like Gaia-Enceladus.

Further complexity is expected from the chemical perspective, since dwarf galaxies also display chemical gradients (e.g. Kirby et al., 2011; Ho et al., 2015). These gradients are typically negative with radius because the star-formation-rate in the centre of galaxies is more intense due to higher gas density. Moreover, star-formation in the outskirts may be quenched first because this is where gas is stripped more easily. This means that merger debris should probably also reveal chemical gradients. Such a gradient has been found for Sagittarius (Ibata et al., 1994) as its streams have older stellar populations (Bellazzini et al., 2006), and are more metal-poor by \( \sim 0.7 \) dex than the core (see Dohm-Palmer et al., 2001; Martínez-Delgado et al., 2004; Chou et al., 2007, and Hayes et al. 2020, for more references).

Motivated by the above considerations, in this Letter we use numerical simulations to study the dynamical gradients that naturally occur in mergers of massive satellites. The results are particularly important to interpret the debris of Gaia-Enceladus and its relation to the other substructures discovered in the halo, such as the Sequoia galaxy.
7.2 Methods

We analyse the merger simulations of Villalobos & Helmi (2008, 2009) designed to study the establishment of a thick disc through heating of a thin disc by a merger. The simulation suite comprises a set of 1 : 10 and 1 : 5 mass-ratio mergers of varying orbital inclination and of both spheroidal and discy satellites. After the merger, part of the host’s disc is heated to a plausible thick disc. At the same time, a significant fraction of the original disc (15 – 25%) remains thin and cold. We focus on these simulations because Helmi et al. (2018) noted that the kinematic distribution of the satellite in one of the experiments (denoted here as $r_{30}$) matches very well that of nearby Gaia-Enceladus stars.

Since the host galaxy in the simulations is smaller than the Milky Way, and to facilitate comparison to observations, we scale the velocities such that the rotational velocity of the simulated disc at the solar position $v_{\phi,\odot,\text{sim}}$, corresponds to that measured for the Milky Way’s thick disc ($v_{\phi,\odot} = 170$ km/s, according to Morrison et al., 1990). Following Villalobos & Helmi (2009) we place the Sun at $2.4R_{D}$, and we scale the velocities by the factor $v_{\phi,\odot}/v_{\phi,\odot,\text{sim}}$. For example for the experiment $r_{30}$, $R_{D} \approx 2.16$ kpc (see Fig. 14 of Villalobos & Helmi, 2008), $v_{\phi,\odot,\text{sim}} \sim 132$ km/s, and the scaling factor is $\sim 1.3$. 

Fig. 7.1: Velocity distribution of the local (< 1 kpc) stellar halo using Gaia data (large panel) and in the Villalobos & Helmi (2008) simulations, where the insets indicate the orbital inclination ($0^\circ$, $30^\circ$ or $60^\circ$) and whether the merger was prograde or retrograde. Stellar particles from the discy (spherical) satellites are shown in blue (red). These stars’ distinct velocity distributions betray the progenitor’s properties and the merger geometry.
Fig. 7.2: Velocity distribution for the discy $r_{30}$ simulation 10 Gyr after infall. The stellar particles are coloured according to their final eccentricity (left) and energy ($E_f$, right). The extended ‘arch’-like structure (with very negative $v_\phi$) is persistent in time, for example, compare with Fig. 7.1.

The top-left panel of Fig. 7.1 shows the velocity distribution of a sample of halo stars in Gaia DR2 (see Appendix A.1 for details). Halo stars with $v_\phi < 75$ km/s (to the left of the dashed line) are highlighted with blue dots. For illustrative purposes, we indicate the location of the thin disc with a random subset of 50 000 stars (black dots). The remaining panels in Fig. 7.1 show the velocity distributions for all the 1 : 5 mass-ratio simulations, with the stellar particles from the host in black and those from the satellite in blue for discy and red for spheroidal progenitors. These particles are located inside a volume of 2.5 kpc radius centred on the ‘equivalent’ solar position at the end of the simulations, i.e. 4 Gyr after infall. Because of particle resolution limitations, this volume is relatively large compared to that used for the data, especially because it has not been scaled.

As reported in Helmi et al. (2018), the arch seen in the Gaia data and shown in the top-left panel of Fig. 7.1 is well reproduced in the discy simulation with the inset $r_{30}$, suggesting that the progenitor of Gaia-Enceladus was discy and merged on a retrograde orbit of $\sim 30^\circ$ inclination. However, because this simulation was only run for 4 Gyr, to establish the robustness of this conclusion and the arch’s origin, we have integrated the simulation for a total of 10 Gyr, which is approximately the time of the merger with Gaia-Enceladus.

7.3 Results

7.3.1 Origin of the velocity arch

Figure 7.2 shows the velocity distribution in a representative solar volume of the $r_{30}$ simulation 10 Gyr after infall. Although the shape of the distribution depends slightly on
Fig. 7.3: Velocity distribution in a solar volume (left, same as in Fig. 7.2) and distribution of $e$-$L_z$ of all satellite stellar particles (right) both at $t = 10$ Gyr. The colours indicate the location of various selections made in $e$-$L_z$, following the three clear ridges identifiable in this space.

The azimuthal location of the volume (because the remnant thick disc in the simulations is somewhat triaxial), the arch structure in velocity space is seen to persist in time. This implies that the conclusions that are drawn on the basis of Fig. 7.1 still stand.

To characterise the arch we have coloured the stars in Fig. 7.2 by their final eccentricity $e$ (left panel) and orbital energy $E_f$ (right panel), see Appendix 7.B for details. Note the large range of values of both quantities and how they appear to correlate with the velocities, although not perfectly. Since its discovery, Gaia-Enceladus has been conflated with stars on very eccentric ($e \gtrsim 0.8$) orbits (e.g. Belokurov et al., 2018; Mackereth et al., 2019). Even though this might be true for the bulk of the stars, we find that this is not necessarily true for all of its debris. Roughly 75% of the stars ends up on orbits of $e > 0.8$, while $\sim 9\%$ have eccentricities $e < 0.6$. 

7.3 Results
Figure 7.3 shows the link between the local structure in velocity space (left) and the orbital properties of all of the debris in $e-L_z$ space. The latter is a function of velocity ($L_z = Rv_\phi$) and is an integral of motion often used to identify Gaia-Enceladus’ debris (e.g. Helmi et al., 2018; Matsuno et al., 2019; Massari et al., 2019). The satellite’s debris displays a large amount of substructure as shown in the panels on the right, which is clearly linked to that present in velocity space in the solar volume. For example, the arch in velocity space appears to be the local manifestation of the large retrograde structures marked in the middle and bottom rows of Fig. 7.3. On the other hand, the top row of this figure highlights that some stars have relatively prograde angular momentum despite the initially retrograde orbit of the satellite.

We investigate the origin of the structures in $e-L_z$ space in Fig. 7.4, where we plot the spatial evolution of the satellite between 1 – 1.7 Gyr after infall. In the first panel, at $t \sim 1$ Gyr, most of the satellite’s dark matter has been lost but the stellar component remains largely bound. A significant fraction of the stars is lost during/shortly after the second pericentric passage which takes place at $t \sim 1.5$ Gyr (Villalobos & Helmi, 2008). The core (traced largely by the magenta points) continues to spiral inwards for two to three more passages and then fully dissolves. During this process, it couples more strongly to the disc and becomes more prograde (as we saw in the top right panel of Fig. 7.3). To appreciate the high complexity of the mass loss process, an animation of Figure 7.4 is available online\(^1\).

When cold rotationally supported disc galaxies merge they produce complex tidal tails (Toomre & Toomre, 1972; Eneev et al., 1973; Quinn, 1984; Barnes, 1988). These tidal tails can be quite different from those originating in spherical, dispersion supported systems. This is also what we find in our simulations as shown in Fig. 7.4. The bottom row of this figure zooms in on the discy satellite and reveals that some of the stars in green, which end up on high eccentricity, very retrograde orbits (i.e. the ridge in the bottom panel of Fig. 7.3) are lost first. These are followed by some stars in cyan with lower final eccentricity and slightly less retrograde motions since they are part of the ridge in the middle panel of Fig. 7.3. We also see that there is some alternation in green and cyan stars amongst the material that becomes unbound slightly later. Since a discy satellite has internal as well as orbital angular momentum, mass loss depends on the exact configuration of the merger (e.g. inclination, spin) and not only on the internal binding energy of the stars as in the case of a spheroidal system. A difference in eccentricity then arises from these additional degrees of freedom. For a given internal binding energy stars have a range of internal angular momenta, which translates in debris with high- and low orbital angular momentum, resulting in more circular or more radial final orbits, respectively.

On top of these effects, dynamical friction causes the satellite’s core to spiral inwards. This enhances the prominence of the only tidal arm apparent in Fig. 7.4, which trails behind the core.

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\(^1\)https://www.astro.rug.nl/~ahelmi/MassiveMess
Fig. 7.4: Evolution of the satellite around the time of the second pericentric passage, which takes place at $t \approx 1.5$ Gyr. The colours trace the three structures identified in the final orbital distribution of the debris, see Fig. 7.3, and evidence the complexity of the mass loss process. This is clearly seen in the bottom two panels which zoom-in onto the satellite at two specific times, and where its spin is indicated with a curled arrow. We encourage the reader to also inspect the animation online:
7.3.2 Some implications

We have just shown that the stars in the velocity arch constitute debris lost early on in the merger. This probably has also implications on the chemical abundances of its stars since dwarf galaxies are known to depict metallicity gradients. For $M_* \sim 10^{9.6} \, M_\odot$ the gradients have an amplitude $\sim 0.064$ dex/kpc according to Ho et al. (2015). Thus if Gaia-Enceladus had a physical extent of 5 kpc, this could imply a metallicity difference of 0.32 dex between its centre and outskirts.

Myeong et al. (2019) studied some of the stars in the velocity arch and argued that these, together with other retrograde stars, were part of a different accreted galaxy which they named Sequoia. These authors have shown that the stars are more metal-poor on average than the debris of Gaia-Enceladus by $\sim 0.30$ dex, which suggests they formed in a ten times smaller object (Matsuno et al., 2019). However, an alternative explanation supported by our simulations is that (some of) these stars stem from the outskirts of Gaia-Enceladus, and their lower metallicities could be explained as being due to internal gradients in Gaia-Enceladus. Also the lower values of [Mg/Fe] and [Ca/Fe] at similar [Fe/H] in comparison to Gaia-Enceladus reported by Matsuno et al. (2019) could be due to the typically lower star formation rates found in the outskirts of sizeable galaxies.

This interpretation is tentatively supported by Fig. 7.5 where we plot [Mg/Fe] vs [Fe/H] for members of Gaia-Enceladus and Sequoia, coloured by their orbital energy $E$. The chemical abundances are from a cross-match of Gaia DR2 with APOGEE DR16 (Ahumada et al., 2020), using the membership criteria described in Koppelman et al. (2019) (see the Appendix for more details). This figure shows that most Sequoia stars are accompanied by a star from Gaia-Enceladus that has very similar orbital energy and chemical abundance, making it difficult to argue on the basis of these data that the stars have a different origin.

7.4 Conclusions

We have analysed simulations of the merger between a Milky Way-like galaxy and a massive satellite. On the basis of comparisons to the kinematics of stars in the nearby halo, we confirm the conclusions by Helmi et al. (2018) that the most likely progenitor of Gaia-Enceladus was a discy dwarf galaxy that fell in on a retrograde orbit of $\sim 30^\circ$ inclination.

Because of the relative mass of the dwarf galaxy (which implies it was subject to dynamical friction), and its initial configuration (a cold, rotationally supported disc), its debris depicts a rather intricate morphology, quite different from that usually associated to small, spherical, non-rotating satellites. Star particles are found to have a broad range of orbital eccentricities, with 75% having $e > 0.8$, and 9% $e < 0.6$. We also find that even though the initial orbital motion of the satellite is very retrograde, some of its debris ends up on prograde orbits. Low eccentricity and prograde stars originate in the core of the satellite which has sunk in via dynamical friction. We may conclude that a large amount of information about the merger remains encoded in the phase-space structure of the debris.
Fig. 7.5: Chemical abundances for stars in Gaia-Enceladus and Sequoia, identified using selection criteria from Koppelman et al. (2019). The stars are coloured by their orbital energy calculated in the McMillan (2017) potential implemented in AGAMA. The background shows the location of the disc-stars in this diagram with a 2D-histogram. The histogram is coloured by the logarithm of the stars per bin.

Because large galaxies have chemical gradients, different portions of the debris may reveal different chemistry. This complicates the interpretation, and can possibly mimic what might be expected for debris from an independent system. For example, some of the stars in the arch (which according to our simulations must have been lost early) are dynamically similar to those that have been associated with Sequoia. On the other hand, the abundances of Sequoia stars do not appear to differ much from highly energetic/less bound (and hence potentially lost early) stars from Gaia-Enceladus.

Taken at face value, the simulation would predict that the core of the Gaia-Enceladus dwarf should be found on a mildly prograde, lower eccentricity orbit. However, this prediction depends strongly on whether the configuration of the merger is matched in detail by the simulations. In particular, the simulated host galaxy has a fixed size, which is unrealistic for the Milky Way, which in the last 10 Gyr has grown a very significant cold disc. Simulations with varying orientation of the discs' angular momentum vectors, and including gas and star formation physics are necessary to fully exploit the data that is currently available, and that which will be collected in the context of upcoming spectroscopic surveys like WEAVE (Dalton et al., 2012), 4MOST (de Jong et al., 2012), SDSS-V Kollmeier et al. (2017), and DESI (DESI Collaboration et al., 2016). Such a combined approach seems to be necessary as a massive satellite can give rise to substructures that appear to be both chemically and dynamically distinct. Massive mergers are messy.
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Bibliography

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Appendix 7.A Observational datasets used

A.1 The RVS sample

In Fig. 7.1, we use the subset with full phase-space information from Gaia DR2 as comparison and to interpret the simulations. The dataset, known as the RVS sample, contains 7,224,631 stars. We only consider stars with $\text{parallax}_{\text{over}} > 5$, $\text{parallax} > 1.0$ mas, and RUWE < 1.4 (see Lindegren, 2018), which leaves 2,784,095 sources.

For this set of stars, we calculate cylindrical space velocities using the transformations implemented in \texttt{vaex} (Breddels & Veljanoski, 2018). We place the Sun at $X = -8.2$ kpc, correct for the motion of the local standard of rest (LSR) using $v_{\text{LSR}} = 232.8$ km/s (both based on McMillan, 2017), and for the solar motion with respect to the LSR using $(U_{\odot}, V_{\odot}, W_{\odot}) = (11.1, 12.24, 7.25)$ km/s (Schönrich et al., 2010). Finally, we use a
kinematic selection to filter stars from the disc. All the stars with \(|V - V_{\text{LSR}}| < 210 \text{ km/s}\) are removed. The resulting set of stars comprises 5 371 sources that we label as halo stars.

A.2 Known halo structures and their abundances

The structures that are shown in Fig. 7.5 originate from Koppelman et al. (2019). In summary, the selection criteria for stars belonging to Gaia-Enceladus and Sequoia are defined as

1. Gaia-Enceladus: \(-1.5 < En < -1.1\) in units of \([10^5 \text{ km}^2/\text{s}^2]\) and \(-0.20 < \text{circ} < 0.13\);

2. Sequoia: \(-1.35 < En < -1.0\) in units of \([10^5 \text{ km}^2/\text{s}^2]\) and \(-0.65 < \text{circ} < -0.4\).

Here, \(En\) is the total energy, using the McMillan (2017) potential and \(\text{circ}\) (i.e. circularity) is defined as \(\text{circ} = \frac{L_z}{L_{z,\text{circ}}}\), where \(L_{z,\text{circ}}\) is the angular momentum of a circular orbit with the same energy as \(L_z\). The circularity parameter ranges from \(-1\) to 1 and indicates how circular an orbit is.

The abundances that are being shown in the figure come from a cross-match of Gaia DR2 with APOGEE DR16 (Ahumada et al., 2020). Following the work of Hayes et al. (2020), we filter stars with STARFLAG bitmask values of 0, 3, or 4 and ASPCAPFLAG bitmask values of 10 or 23. Furthermore, we only show stars with \(\text{VERR} < 0.2\), \(\text{SNR} > 70\), and \(\text{TEFF} > 3700\).

Appendix 7.B Computation of the orbital parameters for the simulations

To calculate orbital properties (i.e. energy and eccentricity) we fit a static potential to the simulation snapshot at \(t = 10 \text{ Gyr}\) after infall. In this snapshot, the debris of the satellite is sufficiently mixed for the whole system to be approximated by a smooth, rigid potential.

We fit the potential with AGAMA (Vasiliev, 2019) using a Multipole expansion to approximate each component (dark matter and stellar) of both the host and satellite separately. For the flattened stellar particle distributions we set \(\text{gridsizeR} = 50\), \(\text{gridsizez} = 50\), \(\text{mmax} = 5\), and \(\text{lmax} = 25\). We tested using a CylSpline for the flattened stellar systems but found the Multipole to reproduce the density profile better. The dark matter profile of the host is forced to be spherically symmetric, the other profiles are allowed to be triaxial.

The energy of the particles is computed for this potential at this same, \(t = 10 \text{ Gyr}\), and labelled \((E_f)\), as is the angular momentum \(L_z\). The eccentricity is derived by integrating the stellar particles’ orbits for 20 Gyr in this same potential.
Part II

Dynamics
The Reduced Proper Motion
selected halo: methods and
description of the catalogue

Based on: Koppelman and Helmi (2020), submitted to A&A

Abstract

The Gaia mission has provided the largest ever astrometric chart of the Milky Way. Using it to map the Galactic halo is helpful for disentangling its merger history. The identification of halo stars in Gaia DR2 with reliable distance estimates requires special methods because such stars are typically farther away and scarce. We apply the reduced proper motion (RPM) method to identify halo main sequence stars on the basis of Gaia photometry and proper motions. Using the colour-absolute-magnitude relation for this type of stars, we calculate photometric distances. Our selection results in a set of \( \sim 10^7 \) tentative main sequence halo stars with typical distance uncertainties of 7\% and with median velocity errors of 20 km/s. The median distance of our sample is \( \sim 4.4 \) kpc, with the faintest stars located at \( \sim 16 \) kpc. The spatial distribution of the stars in our sample is centrally concentrated. Visual inspection of the mean velocities of stars on the sky reveals large-scale patterns as well as clear imprints of the GD-1 stream and tentative hints of the Jhelum and Leiptr streams. Incompleteness and selection effects limit our ability to interpret the patterns reliably as well as to identify new substructures.

We define a pseudo-velocity space by setting to zero the line-of-sight velocities of our sample stars. In this space, we recover several known structures such as the footprint of Gaia-Enceladus (i.e. the Gaia-Sausage) as well as the Helmi streams and some other retrograde substructures (Sequoia, Thamnos). We show that the two-point velocity correlation function reveals significant clustering on scales smaller than 100 km/s, of similar amplitude as found for the 6D Gaia halo sample. This clustering indicates the presence of nearby streams that are predominantly phase-mixed. Spectroscopic follow up of our halo main sequence sample is bound to yield unprecedented views of Galactic history and dynamics. In future Gaia data releases the level of systematics will be reduced and the astrometry will be more precise, which will allow the identification of more substructures at larger distances.
8.1 Introduction

Galactic cartography has gained a huge boost with the advent of astrometric, photometric, and spectroscopic data of unprecedented volume and quality from the Gaia mission (Gaia Collaboration et al., 2016; Gaia Collaboration, Brown et al., 2018), in combination with spectroscopic surveys like APOGEE (Wilson et al., 2010; Abolfathi et al., 2018), RAVE (Kunder et al., 2017), LAMOST (Cui et al., 2012). These datasets have revealed numerous substructures in different Galactic components. For example, large wave-like patterns and sharp ridges have been found in the stellar disc (Antoja et al., 2018; Kawata et al., 2018; Katz et al., 2018). These arches give rise to intricate structures in action-space (Trick et al., 2019), and their imprint varies with location in the disc (Ramos et al., 2018). They are likely due to a combination of perturbations of satellite flying by (Minchev et al., 2009; Antoja et al., 2018; Laporte et al., 2018) and internal dynamical processes like resonances with the bar and spiral structures (Monari et al., 2019; Hunt et al., 2019; Khanna et al., 2019; Chiba et al., 2019).

Studies like those mentioned above show how complex and intertwined the phase-space structure of the Milky Way is. The expectation has been that the Galactic halo would be similarly complex as a result of the mergers experienced by the Milky Way (Helmi & White, 1999). In fact, the analysis of Gaia DR2 has revealed that besides several small features, the local stellar halo is largely dominated by two structures. These are a large, radially-anisotropic, slightly retrograde kinematic structure and a hot thick disc (Belokurov et al., 2018; Koppelman et al., 2018; Haywood et al., 2018). The origin of the former is related to the accretion of a massive ($M_\star \sim 10^9 M_\odot$) dwarf galaxy known as Gaia-Enceladus (Helmi et al., 2018, or Gaia-Sausage, c.f. Belokurov et al. 2018). The accretion of Gaia-Enceladus took place 9-11 Gyr ago (Helmi et al., 2018; Di Matteo et al., 2019; Mackereth et al., 2019; Chaplin et al., 2020) and lead to heating of the proto-Milky Way disc (Helmi et al., 2018; Gallart et al., 2019; Chaplin et al., 2020).

A large fraction of the results listed above stem from the Gaia subsample containing full velocity information (Katz et al., 2019), also known as the 6D sample. Although impressive in size, the 6D sample is small relative to the 5D sample (i.e. without line-of-sight velocities). It comprises $\sim 7$ million sources compared to a staggering $\sim 1.3$ billion sources with parallaxes and proper motions. Most stars in Gaia DR2 have relatively large errors in their parallax measurements, as ‘only’ $\sim 150$ million sources have distances with relative errors $< 20\%$. Also the Gaia DR2 parallaxes are known to suffer from a zero-point offset of $\sim -0.029$ mas with an RMS of 0.03 – 0.05 mas that varies with location on the sky (Arenou et al., 2018; Gaia Collaboration, Brown et al., 2018; Lindegren et al., 2018). For brighter stars the typical off-set may be closer to $\sim 0.05$ mas (Schönrich et al., 2019; Leung & Bovy, 2019; Zinn et al., 2019a; Chan & Bovy, 2019). The poorer parallaxes in combination with the missing line-of-sight velocities therefore complicate the utilisation of the entire Gaia DR2 sample for dynamical studies.

A general approach to cope with poorly constrained or missing distances is to use the luminosity of the sources that are known as ‘standard candles’, such as RR-Lyrae. Their
period-luminosity relation can be used to derive distances that are typically accurate up to $\sim 5\%$. This makes RR-Lyrae outstanding targets to study the morphology of the disc and stellar halo (e.g. Watkins et al., 2009; Sesar et al., 2010; Drake et al., 2012; Hernitschek et al., 2018; Iorio & Belokurov, 2019; Zinn et al., 2019b). The downside of RR-Lyrae stars is that they are not abundant. Another approach is to identify stars of a specific type such as BHBs (Xue et al., 2008, 2011; Deason et al., 2012; Fukushima et al., 2017; Lancaster et al., 2019), or RGB stars (Morrison et al., 2009) whose absolute magnitude can be derived using isochrones when knowledge of their $\log g$ or metallicity is available, such as from spectroscopic surveys (e.g. Leung & Bovy, 2019; Conroy et al., 2019; Cargile et al., 2019).

In this Chapter, we will use main sequence (MS) stars to study the Milky Way halo. The reason for focusing on MS stars is twofold. Firstly, they follow a relatively simple absolute magnitude relation as a function of colour, which can be used to calculate a photometric distance (e.g. Juric et al., 2008; Ivezić et al., 2008; Bonaca et al., 2012). Secondly, we can select them using only their photometry and proper motions (i.e. without knowing the distance to the stars) through a property known as the reduced proper motion (e.g. Jones, 1972; Smith et al., 2009).

We describe the methods that we use in Sec. 8.2, and the selection of MS halo stars and calibration of their distances in Sec. 8.3. In Sec. 8.4 we explore the spatial distribution of the halo stars in the sample. We combine the spatial coordinates with the proper motions of the stars to calculate the space velocities. In Sec. 8.5 we inspect the velocities of the full sample and in Sec. 8.6 we focus on the velocity distribution of a local sample. Finally, in Sec. 8.7 we summarise and discuss our results, and present our conclusions.

8.2 Methods

In this section, we will describe the tools that we require to select halo MS stars and to calibrate photometric distances. The description of the data, selection, and calibration will be carried out in Sec 8.3.

8.2.1 Photometric distance estimates for MS stars

Distances are notoriously difficult to measure in astronomy. Only about $\sim 10\%$ of the parallaxes released in Gaia DR2 are precise (i.e. those with $\text{parallax}\_\text{over}\_\text{error} > 5$). Another way of calculating distances is through the luminosity of a star. For specific types of stars, for which the intrinsic luminosity is known, we can calculate a photometric distance from the apparent luminosity. The relation between the intrinsic and apparent magnitude of a star in the Gaia $G$-band is given by

$$M_G = m_G - 5 \log_{10}\left(\frac{d}{\text{kpc}}\right) - 10 - A_G,$$

(8.1)
where $M_G$ is the absolute magnitude of the star, $m_G$ is its apparent magnitude, $d$ is its distance, and $A_G$ is the extinction in the $G$-band. For most sources, $M_G$ is unknown and Eq. (8.1) cannot be used to calculate a distance $d$.

In this Chapter, we will use the close to linear relation of the colour and absolute magnitude of MS stars to derive a distance. We note that the MS is only approximately linear in optical pass-bands, and in the near-infrared this approximation breaks down (e.g. Fig. 9 of Gaia Collaboration, Babusiaux et al., 2018). Because of the strong correlation between $M_G$ and colour of the MS in the Hertzsprung-Russel diagram (HRD) we can find the distance independent relation

$$M_G = f(G - G_{RP}).$$ (8.2)

In this Chapter, we will use the Gaia $G - G_{RP}$ colour because it is less prone to systematic effects than $G_{BP} - G_{RP}$, especially in crowded fields (Gaia Collaboration, Brown et al., 2018). Because $G - G_{RP}$ is distance independent, we can use Eq. (8.1) to calculate a distance that is a function of the apparent magnitude and colour only

$$d = 10^{(m_G - M_G - 10 - A_G)/5}.$$ (8.3)

When propagating the error in $d$, assuming that the error in $m_G$ can be neglected, we find that the relative distance error is

$$\epsilon_d/d = 0.2 \log(10) \epsilon_{M_G},$$ (8.4)

where $\epsilon_{M_G}$ is the error in the absolute magnitude.

### 8.2.2 Selecting MS stars

The method of determining distances described above is valid for MS stars. Giants and stars at the MS turn-off (MSTO) describe a sequence in the HRD that is too vertical or degenerate to find a reliable relation between $M_G$ and colour. Therefore, we need a (distance independent) way of selecting MS stars.

We will identify MS stars using a combination of the proper motion and photometry known as a reduced proper motion (RPM, Jones, 1972, see also Smith et al. 2009). The RPM of a star, in the Gaia $G$-band, is defined as

$$H_G = m_G + 5 \log_{10}(\frac{\mu}{\text{mas/yr}}) - 10 - A_G,$$ (8.5)

where $\mu = \sqrt{\mu_\ell^2 + \mu_b^2}$ is the amplitude of the proper motion. We note that this equation is similar to Eq. (8.1). In fact, the two equations can be combined to gain some insight

$$H_G = M_G + 5 \log_{10}\left(\frac{v_{\text{tan}}}{4.74057 \text{ km/s}}\right),$$ (8.6)
where $v_{\text{tan}}$ is the tangential velocity of a star given by

$$v_{\text{tan}} = 4.74057 \text{ km/s} \left( \frac{\mu}{\text{mas/yr}} \right) \left( \frac{d}{\text{kpc}} \right),$$

where $d$ is the heliocentric distance to the star.

When plotted as a function of colour, the $H_G$–colour diagram (which we will refer to as the RPM diagram) of a stellar population is equal to the HRD - but with an offset dependent on $v_{\text{tan}}$. If all the stars in the population have the same $v_{\text{tan}}$, their sequence in RPM diagram and HRD will look exactly the same. However, if the stellar population has a mean $v_{\text{tan}}$ plus a few km/s dispersion, its sequence in the RPM diagram will be broadened by the logarithm of the velocity dispersion. Furthermore, populations with characteristic, specific tangential velocities will split into parallel sequences.

We exploit this splitting of the MS to select halo stars by identifying the region where the MS stars with high $v_{\text{tan}}$ are located. Since $v_{\text{tan}}$ for the disc is small, even when considering the dispersion, the halo should appear as a separate sequence. Eqs. (8.5) and (8.6) imply that, for fixed $v_{\text{tan}}$ populations, $H_G$ will only be a function of $G - G_{\text{RP}}$ and can readily be computed. At the core of our selection method, we aim to locate high-$v_{\text{tan}}$, MS stars in the RPM diagram.

This type of selection is conceptually not too different from a kinematic selection performed in the Toomre diagram, which is often used to identify halo stars in the 6D sample of Gaia (e.g. Nissen & Schuster, 2010; Bonaca et al., 2017; Posti et al., 2018; Koppelman et al., 2018). Reduced proper motion diagrams are often used to select white dwarfs and classify them as belonging to the halo or the disc (e.g. Kalirai et al., 2004; Kilic et al., 2006; Fusillo et al., 2015; Torres et al., 2019; Geier, 2020). However, it is important to note that the RPM selection has a clear bias: halo stars with a small tangential velocity will not be selected (e.g. halo stars moving only along the line-of-sight).

### 8.3 Data selection and calibration

#### 8.3.1 Data and quality cuts

We start from the full subsample of Gaia DR2 with multi-band photometry. For the method outlined in Sec. 8.2, we have to rely on the photometry of the sources. Therefore, we impose the following cuts on the photometric quality of the stars

- $\text{phot}_g_{\text{mean}}_{\text{flux}}_{\text{over}}_{\text{error}} > 50$
- $\text{phot}_r_{\text{p}}_{\text{mean}}_{\text{flux}}_{\text{over}}_{\text{error}} > 20$
- $\text{phot} \_bp\_rp\_{excess\_factor} < 1.3 + 0.06 \cdot (\text{bp} \_rp)^2$
- $\text{phot} \_bp\_rp\_{excess\_factor} > 1.0 + 0.015 \cdot (\text{bp} \_rp)^2$

where $\text{bp} \_rp = \text{phot} \_bp\_mean\_mag - \text{phot} \_rp\_mean\_mag$. Besides cleaning the photometry, these cuts also remove sources with a bad astrometric solution (Arenou et al., 2018).
To further clean the sample we use the re-normalised unit weight error (RUWE). When DR2 came out, the Gaia Data Processing and Analysis Consortium (DPAC) recommended using the unit weight error (UWE) to filter sources with a bad astrometric solution (e.g. Lindegren et al., 2018; Arenou et al., 2018). However, the original UWE varies with colour and magnitude. Therefore, DPAC (Lindegren, 2018) recommends the use of the re-normalised UWE (RUWE), which does not depend on stellar properties. Following their suggestion, we remove all the stars that have RUWE > 1.4.

### Extinction correction

Independently of the quality of the photometry, our sources will be prone to extinction caused by absorption from dust in the interstellar medium (ISM) along the line-of-sight. We will correct for the extinction using the Schlegel et al. (1998) maps.

These maps provide the extinction factor integrated along the entire line-of-sight. For distant stars we can assume that all the absorbing ISM clouds lie in the foreground. However, for nearby stars we have to be careful, as some of the extinction in the Schlegel et al. maps might come from regions in the ISM behind the stars. Therefore, we use the approach outlined by Eqs. (10) & (11) from Binney et al. (2014) to calculate the amount of foreground dust for each star as a function of its parallax and location on the sky. The amount of extinction is given by

\[
A_V(\ell, b, s) = A_{V,\infty}(\ell, b) \frac{\int_0^s \rho(x(s'))ds'}{\int_\infty^0 \rho(x(s'))ds'}, \tag{8.8}
\]

where \(A_{V,\infty}(\ell, b)\) is the extinction given by the Schlegel et al. maps, \(s\) is the heliocentric distance to the star, and \(x\) is the position vector of the star that lies at distance \(s\) in the direction of \((\ell, b)\) on the sky. For the dust density we follow the model presented by Eq. (16) of Sharma et al. (2011)

\[
\rho_{\text{Dust}}(R, z) = \exp \left( \frac{R - R_{\odot}}{h_R \left( k_{\text{flare}} s_z \right)} \right) \tag{8.9}
\]

where \(z_{\text{warp}}\) and \(k_{\text{flare}}\) describe the warping and flaring of the disc

\[
k_{\text{flare}}(R) = 1 + \gamma_{\text{flare}} \text{Min}(R_{\text{flare}}, R - R_{\text{flare}}) \tag{8.10}
\]

and

\[
z_{\text{warp}}(R, \phi) = \gamma_{\text{warp}} \text{Min}(R_{\text{warp}}, R - R_{\text{warp}}) \sin(\phi) \tag{8.11}
\]

with values \(h_R = 4.2\) kpc, \(h_z = 0.088\) kpc, \(\gamma_{\text{warp}} = 0.18\) kpc\(^{-1}\), \(R_{\text{warp}} = 8.4\) kpc, \(R_{\text{flare}} = 1.12 R_{\odot}\), and \(\gamma_{\text{flare}} = 0.0054\) kpc\(^{-1}\). These values are based on the model of Robin et al. (2003). Effectively, we derive in this way an extinction fraction \(\frac{A_V(\ell, b, s)}{A_{V,\infty}(b, \ell)}\) which encodes what fraction of the full extinction should be applied.
Following Binney et al. (2014), we scale the Schlegel et al. maps because the reddening in the regions $E(B-V) > 0.15$ is overestimated (e.g. Arce & Goodman, 1999). The correction factor that we apply is

$$ f(E(B-V)) = 0.6 + 0.2 \left[ 1 - \tanh \left( \frac{E(B-V) - 0.15}{0.3} \right) \right]. \quad (8.12) $$

This factor scales the highly reddened regions by a factor of 0.6. We note that the results presented in this Chapter are not affected by this scaling.

For stars with parallaxes $< 0.1$ mas we set the extinction fraction to 1.0 because they are likely to be distant stars. For all the other stars we invert the parallaxes to obtain an estimate for the distance. We ignore the error on the parallax because we only are looking for an estimate of the distance. On average, the parallax-errors increase with heliocentric distance. Nearby sources, for which the correction fraction is essential, will have relatively good parallaxes. The correction fraction is $> 0.90$ for $> 90\%$ of the sources, only $1.6\%$ of the stars receive a correction of $< 0.50$. The resulting weighted $A_V$ values are transformed to $A_G, A_{BP}, A_{RP}$ using the relations given by Malhan et al. (2018) (they originate from the Padova model\(^1\) and are originally based on Cardelli et al., 1989).

As a final quality selection criterion we remove sources located in areas on the sky where the extinction is larger than $A_V > 2.0$. These highly extinct sources are mostly found close to the plane of the disc, so the cut acts as a filter for the Galactic disc.

### 8.3.2 Fitting the MS

Our method is contingent upon having a reliable fit for the absolute magnitude of halo MS stars as a function of colour. Gaia Collaboration, Babusiaux et al. (2018) have shown that tangential velocities can be used to identify nearby halo stars (i.e. $v_{\text{tan}} > 200$ km/s). This set of high-$v_{\text{tan}}$ stars is characterised by two sequences in the HRD. They are known as the blue and red sequence (e.g. Gaia Collaboration, Babusiaux et al., 2018). The red sequence is kinematically reminiscent of the slower-rotating, hotter tail of the thick disc and the blue sequence of a classic halo that has a close to zero rotation (e.g. Koppelman et al., 2018; Haywood et al., 2018; Di Matteo et al., 2019; Gallart et al., 2019).

For the purpose of mapping the Galactic halo, we are mainly interested in the stars in the blue sequence. We expect the red sequence to be more important closer to the plane of the disc and in the inner Galaxy. It is currently not possible to avoid contamination from the red sequence (metallicity information could help, e.g. Gallart et al. 2019). Its stars are, both kinematically and photometrically, too similar to the stars in the blue sequence. However, for the fitting, we will strive to keep the contamination from the red sequence to a minimum. One way of doing this is to increase the cut in $v_{\text{tan}}$ to a larger value. However, even at $v_{\text{tan}} > 300$ km/s the contribution of the red sequence is still $\sim 22\%$ (Sahlholdt et al., 2019).

\(^1\)http://stev.oapd.inaf.it/cmd
Figure 8.1: Hertzsprung-Russel diagram (HRD) of a local sample of stars with large tangential motions ($v_{\text{tan}} > 200$ km/s) and very high-quality parallaxes ($\text{parallax}_{\text{over}}_{\text{error}} > 50$). Overlayed is an 11 Gyr age and $[\text{M/H}] = -0.5$ metallicity isochrone (red). This isochrone is shifted to the left by 0.01 mag in $G - G_{\text{RP}}$ to split the two sequences that are shown.

Figure 8.1 shows an HRD of all the stars in Gaia DR2 that survive after imposing the quality cuts described in Sec. 8.3.1 and two additional criteria: ($\text{parallax}_{\text{over}}_{\text{error}} > 50$) & ($v_{\text{tan}} > 200$ km/s). For illustrative purposes we use here $v_{\text{tan}} > 200$ km/s rather than 300 km/s because this brings out the two sequences better. However, for the fitting procedure we will use $v_{\text{tan}} > 300$ km/s.

After the $v_{\text{tan}}$ cut we impose a $M_G$-colour cut to remove the last bit of red sequence contamination. For this cut we use a synthetic isochrone that was first used by Gaia Collaboration, Babusiaux et al. (2018) to describe the red sequence. The isochrone, overlayed in Fig. 8.1, describes a stellar population of a metallicity $[\text{M/H}] = -0.5$ and age of 11 Gyr. This isochrone is obtained from Marigo et al. (2017), after enhancing the $\alpha$ elements by 0.23 (Salaris et al., 1993). We note that this isochrone is not a fit, but simply describes well the valley between the two sequences when shifted by 0.01 mag in $G - G_{\text{RP}}$. All the stars to the right of the isochrone (i.e. those belonging to the red sequence) are removed. As a final quality cut we remove a handful of sources that are offset from (i.e. are below) the MS. For this cut we remove the stars with (($G - G_{\text{RP}} < 0.65$) & ($M_G > 8$)) OR (($G - G_{\text{RP}} < 0.8$) & ($M_G > 10$)).
For the final part of this section, we will fit the cleaned MS in two ways: a simple 3-component, piece-wise linear fit and a more accurate fit using a running mean and standard deviation of the MS. The 3-component fit has a straightforward parametrisation, which is ideal for the construction of the MS selection. We will be selecting the MS stars from the full dataset of Gaia, so a computationally efficient parametrisation is beneficial. On the other hand, the running mean is the most accurate, which is crucial for the calculation of the photometric distances. We have tested using a synthetic isochrone like that overlaid in Fig. 8.1 instead of fitting the MS. However, the isochrone does not perfectly trace the MS over the full colour range. The fit on the data is more precise, given that there are enough stars per bin.

The three components of the linear fit describe the MS in the absolute-magnitude ranges: \((4 < M_G < 6), (5 < M_G < 8), \) and \((M_G > 8)\). For the running mean we split the MS into 128 bins in the range of \(0.35 < (G - G_{RP}) < 1.1\) and remove stars with \((M_G < 4)\), which is roughly where the MSTO occurs. In each colour-bin, we calculate the mean absolute magnitude and the standard deviation. The resulting fit closely describes the width and amplitude of the absolute magnitude as a function of \(G - G_{RP}\). Both fits are run on a sample of high-\(v_{\text{tan}}\) stars with good parallaxes: \((\text{parallax}_\text{over_error} > 50) \& (v_{\text{tan}} > 300 \text{ km/s})\).

Figure 8.2 shows both the 3-component linear fit (red, dashed) and the running mean (blue). In the background, we show the sample that is being fit. The two fitting procedures agree well with the data. The top panels show the MS sample that we fit on before (left) and after (right) the photometric cleaning described in this section.

### 8.3.3 Inferring distances for MS stars

We use the running-mean-fit from Sec. 8.3.2 to calibrate photometric distances. For each star we find the colour-bin in which it falls. The bins sizes are sufficiently small so we do not perform any interpolation. We assume that the absolute magnitude of the star is the same as the mean value found for the specific bin. Using Eqs. (8.3) and (8.4), we calculate the distance and its relative error.

The bottom panel of Fig. 8.2 shows the typical error, based on the width of the MS in absolute magnitude. Overall, the expected error in the photometric distance is quite small, averaging 7% for a large fraction of the MS. The relative distance error of 10% given for stars at the MS turn-off (MSTO) \((0.35 < G - G_{RP} < 0.45)\) is somewhat misleading. Since the sequence here is close to vertical, the range in possible magnitudes is larger than what is indicated by the error bars. The fit of the MS aims to trace the faint part of the MSTO, since we do not fit for stars brighter than \(M_G < 4\). The fitted absolute magnitude is comparable to or smaller than the true absolute magnitude. Therefore, distances for MSTO stars that are far away from the fitted sequence are typically underestimated - and on average not overestimated. For comparison, if the intrinsic brightness of a source is underestimated by one magnitude, which is typical for the vertical extent of the MSTO, the distance will be underestimated by 37%. Another systematic bias that is not included in the relative error is the difference of half a magnitude that is typical for the offset
between the red and blue sequence. This offset results in distances for red-sequence stars that are systematically underestimated by 20%.

For faint MS stars, at $G - G_{RP} \approx 0.7$, there is a break in the MS. This break is a known feature in the faint MS for low-mass stars (Saumon et al., 1994; Cassisi et al., 2000). It is caused by a low effective temperature in combination with collisionally induced absorption. The feature is observed in globular clusters (Bono et al., 2010, who use it to determine the age of NGC 3201) and in the Galactic bulge (Zoccali et al., 2000).
Fig. 8.3: Reduced proper motion diagram for all sources in Gaia that pass the quality cuts of Sec. 8.3.1. The sample of sources within the red lines are selected to be tentatively halo stars. The red lines are drawn based on the 3-component fit of the MS. The location of the box is chosen to select halo stars with \(200 \text{ km/s} < v_{\text{tan}} < 800 \text{ km/s}\). The dark region above the red box comprises mainly disc stars (they make up \(\gg 99\%\) of all stars in the Milky Way).

The width of the MS increases in the region redder than this break. As a result, the photometric distances are less reliable for stars with \(G - G_{\text{RP}} > 0.715\). The brightest of these (\(M_G \sim 8\)) are only visible out to \(\sim 4 \text{ kpc}\) (assuming a Gaia limiting magnitude of 21 in the G-band). Our interest in the halo lies mainly with stars more distant than 4 kpc and with stars with reliable distances. Therefore, when using the newly calibrated photometric distances, we will often constrain ourselves to stars in the range of \(0.45 < G - G_{\text{RP}} < 0.715\) only. This range is indicated in the bottom panel of Fig. 8.2 with vertical dashed lines. Of course, the distance calibration described in this section only works for MS stars. Therefore, the last step in the construction of our sample is to remove contamination from other types of stars (e.g. giants and white dwarfs).

### 8.3.4 Selecting MS stars

The method of selecting halo MS stars in the RPM diagram is best understood when visualised. Figure 8.3 shows the RPM as a function of colour for the full Gaia data set after imposing the quality cuts from Sec. 8.3.1. To find the location of MS stars with a large tangential velocity we place the 3-component linear fit from Sec. 8.3.2 on top of the density map in Fig. 8.3. We add the \(v_{\text{tan}}\)-based offset from Eq. (8.6) to the line. The
upper line of the box in Fig. 8.3 is given by the fit plus an offset of 200 km/s, the lower has an offset of 800 km/s. Therefore, the stars that fall between the horizontal lines are those that, based on their location in the RPM diagram, are on the MS and have a velocity in the range of 200 km/s < $v_{\text{tan}}$ < 800 km/s. The vertical lines of the box are set to 0.35 < $G - G_{\text{RP}}$ < 1.1. The blue limit (0.35 < $G - G_{\text{RP}}$) is chosen because there are no MS stars bluer than this in the halo, see for example Fig. 8.1. At the other end, we truncate the selection at $G - G_{\text{RP}} = 1.1$ because this is approximately where the MS ends. We note that the truncation is beyond the red limit where the distances become less reliable. However, these stars are still likely halo stars and therefore we add them to the sample.

### 8.3.5 Final quality checks

#### Removing white dwarfs

Figure 8.4 shows the HRD of a subset of the RPM sample with parallax_over_error > 5, where the $M_G$ is calculated using the parallaxes. The red line is the 3-component fit that is also shown in Fig. 8.2, and the blue is this same line, but offset by 2 mag in $M_G$. Below the blue line, there is contamination from faint sources. Amongst these sources is a population of white dwarfs, visible at 12.5 < $M_G$ < 15. The purpose of the blue line is to filter this contamination, that is, we remove all the sources below the line (and have parallax_over_error > 5).
Based on the number of sources with reliable parallax over error > 5, we estimate that the contamination is at most 0.2%. White dwarfs are intrinsically faint objects, the brightest few in our sample have an absolute magnitude of $M_G \sim 13$. As a result, we expect no contamination of white dwarfs farther out than $\sim 0.40$ kpc, beyond which they fall below the magnitude limit of Gaia. Because white dwarfs can only be observed in the close vicinity of the Sun, we expect that all of them have relatively good parallaxes. Therefore our filter shown in Fig. 8.4 should remove all contamination from white dwarfs.

Giants are an unlikely source of contamination because of their intrinsic brightness and even uncertainties in proper motions or photometry are not large enough. Therefore, we estimate that the fraction of contamination of non-MS stars in our final RPM sample is negligible.

**Quality of photometric distances**

As a last check before exploring the properties of the RPM halo sample, we test the quality of the photometric distances. Figure 8.5 shows the quality of the photometric distances for the same sample of stars used for fitting the MS. The quality is displayed as a function of the colour $G - G_{RP}$, the amplitude of the proper motion, and $G$-magnitude. The vertical dashed lines in the top right panel of Fig. 8.5 indicate the limit for reliable distances that is described in Sec. 8.3.3. The stars that are outside of this range (in grey) are not included in the other panels. The typical parallax-distance error in the sample that is shown can be neglected, it is $\lesssim 2\%$. These figures confirm that there is no dependence and that the distance derivation works properly. The photometric distances agree well with the parallaxes. In each panel of Fig. 8.5, we overlay two blue horizontal dashed-lines that correspond to photometric distances that are 10% off from the parallaxes.

### 8.4 Spatial distribution of the RPM sample

The final sample obtained using the above described procedures comprises 11 711 399 tentative MS halo stars. The subset with reliable photometric distances (i.e. stars in the range $0.45 < G - G_{RP} < 0.715$, which excludes MSTO and faint-end stars) comprises 7 117 555 stars.

We now inspect the spatial distribution of the stars in the RPM sample and check for signs of substructure. Because MS stars are intrinsically faint objects, Gaia can only observe them up to $\sim 16$ kpc, assuming the brightest star has $M_G \sim 5$ and Gaia’s limiting magnitude in $G$ is $\sim 21$ mag. However, at the faint-end, Gaia is far from complete. Only 7384 sources ($\sim 0.1\%$) in the sample have a photometric distance larger than 10 kpc - the median photometric distance of the sample is 4.39 kpc.

Figure 8.6 shows the distribution of the galactocentric distance, G-magnitude, and photometric distance. The top panel shows a centrally concentrated distribution. Close to 83% of the stars are located inside of the solar radius ($r_{gc} < 8.2$ kpc). This high concentration towards the centre is to be expected, the stellar halo is known to have a steep density profile (e.g. Juric et al., 2008; Deason et al., 2011).
Fig. 8.5: Inspection of the quality of the photometric distances compared to the parallax (top, left), and as a function colour (top, right), proper motion (bottom, left), and extinction corrected $G$-magnitude (bottom, right). As a reference, we overlay dashed lines indicating a difference of $\pm 10\%$ in the distance. The quality of the photometric distances does not depend on any of the values showed here, as is to be expected.
Figure 8.6: Top: Distribution of galactocentric distances. The dashed line indicates the location of the Sun. Bottom: Distribution of G-magnitudes (left) and photometric distances (right) of the stars in the RPM sample.

Figure 8.7 shows the distribution of the stars in the RPM sample in a Mollweide map, colour-coded by the logarithm of the number of stars per pixel. The maps are created at a healpix level of 7, resulting in 196 608 pixels of equal area. The left panel shows the distribution for the complete sample and the right panel shows all sources with a photometric distance larger than 5 kpc (∼ 35% of the sample). Several globular clusters are visible in both panels as small yellow dots, they are highlighted with white markers. These globular clusters are all nearby, they are NGC 7099, NGC 362, NGC 5904, NGC 6341, NGC 5466, and NGC 288.

Gaia’s scanning-pattern (c.f. Lindegren et al., 2018; Arenou et al., 2018, where these patterns are shown and discussed) is prominent in both maps but is most clearly seen in the right panel. The sinusoidal band with an amplitude of ∼ 60° in $b$ is a known artefact in the Gaia DR2 data related to insufficient subtraction of zodiacal light (c.f. Fig. 18 of Evans et al., 2018). Besides these systematic effects, the right panel shows significant signs of incompleteness, most clearly apparent in the number of bins with...
Fig. 8.7: Sky-maps of RPM selected sample of halo stars. The left panel shows the full RPM sample, and the right panel shows a selection of the 35% most distant stars. Strong signatures of the Gaia scanning pattern are visible in both panels. Sources in the plane of the disc are filtered by our quality cuts. The markers in the left panel encompass six globular clusters that are picked up by the RPM selection.
Fig. 8.8: The spatial distribution of the RPM sample in heliocentric Cartesian coordinates. The sample is distributed uniformly, and the density of sources decreases with heliocentric distance, as is expected for a magnitude-limited sample. A combined effect of dust extinction and the scanning pattern of Gaia creates stripes. Sources with low-\(b\) are removed (i.e. stars in the plane of the disc).

zero sources (white pixels). The pixels with no stars, in the left panel, are mostly related to high-extinction regions. See, for example, the disc region, but also the area near the Magellanic Clouds, which are at \((\ell, b) \sim (-60^\circ, -40^\circ)\).

The spatial distribution of the sample, shown in Fig. 8.8, displays a similar level of structure. The coordinate system is oriented such that the Galactic Centre is in the positive \(X\)-direction and the rotation of the disc is in the positive \(Y\)-direction. To mitigate the contamination from thick disc we have removed all the stars with \(|b| < 20^\circ\). The combined effects of dust extinction, the scanning pattern of Gaia and the errors in the photometric distance create the radial features seen in the figure.

Maps like those shown in Fig. 8.8 project 3D-structure onto a 2D-plane. As a result, most small-scale structure is smoothed out. A simple method to inspect the internal 3D-structure is to minimise the smoothing effect by dividing the sample in thin slices. In Fig. 8.9 we project wedges in \(\ell\) in heliocentric cylindrical \((R, Z)\) coordinates, each wedge is 36° in width. The maps are binned \((128 \times 256\) bins\) and coloured by the logarithm of the number of stars per bin. Low-latitude areas \((|b| < 20^\circ)\) are highly affected by extinction, which introduces systematic errors in the distance estimate. Therefore, these areas are coloured in greyscale to focus the attention to the less affected parts. We overlay overdensities found in SDSS (i.e. Table 4 of De Jong et al., 2010), see the caption of the figure for more information. Most of the large overdensities like the Virgo Over Density (VOD) (Newberg et al., 2002; Juric et al., 2008; Bonaca et al., 2012), the Hercules-Aquila Clouds (HAC) (Belokurov et al., 2007; Simion et al., 2014), TriAnd (Majewski et al., 2004; Rocha-Pinto et al., 2004; Deason et al., 2014) and similar structures (e.g. De Jong et al., 2010; Grillmair & Carlin, 2016) are too distant to be detected in our sample.

The distribution of stars is not isotropic as, for example, shown in the lower-left panel of Fig. 8.9 which depicts a strong North-South asymmetry. This asymmetry is likely caused by incompleteness of Gaia because of its scanning pattern. There is an asymmetry in the
number of faint stars ($G > 19$) when comparing the source counts in that specific wedge for $b > 0$ versus the $b < 0$. Some of the asymmetries are related to the photometric quality cuts that we impose, or are linked to the Gaia scanning pattern. They, for example, correlate with the Gaia DR2 catalogue parameter visibility_periods_used. The complexity of the structure in the data makes it non-trivial to compare the structure against a smooth model, which makes the significance of the structure present in the maps here unclear.

We check for spatial trends with extinction in $G$-mag in Fig. 8.10 where the mean extinction is shown in bins projected on the $XY$- and $XZ$-plane. As expected, the extinction strongly correlates with latitude $b$. Stars with low-latitude (the disc region) are strongly affected by extinction. The pattern in the $XY$-plane is less intuitive to understand. The low-latitude features in $b$ correlate non-uniformly with features in $\ell$. Towards the anti-centre ($X < 0$) the extinction is less significant. From the right panel, it is clear that a
significant fraction of the high-extinction sources is easily removed by applying a simple cut in latitude $b$. Although these low-latitude stars are removed in Fig. 8.8, some of the stripes still seem to correspond to the high-extinction features.

### 8.5 Velocity content of the RPM sample

The spatial distribution of the RPM sample is smooth and homogeneous, although affected by signs of incompleteness and selection effects. However, we can combine spatial information with that encoded in the proper motions to filter halo structures from the smooth background. This will be easier when such structures have velocity vectors that are sufficiently different from the “background” (in the particular region of the sky). The degree of distinction will depend also on the magnitude of the velocity errors.

#### 8.5.1 Binned velocity moments

A powerful, yet simple, tool is to bin moments of the velocity distribution on the sky. Using the photometric distances, we convert the proper motions to space velocities using

$$v_j = 4.74057 \text{ km/s} \left( \frac{\mu_j}{\text{mas/yr}} \right) \left( \frac{d}{\text{kpc}} \right), \tag{8.13}$$

where $j = (\ell, b)$. These space velocities can be corrected for the solar reflex motion

$$v_j^* = v_j + v_{j,\odot}, \tag{8.14}$$
Fig. 8.11: Distribution of the mean, solar-motion-corrected $v_\ell$ (top) and $v_b$ (bottom) velocities binned on the sky for distant halo stars ($d > 6$ kpc). Large scale-streaming motions as well as several small streams stand out. This map includes MSTO stars, which might have large uncertainties in the distances. See Appendix 8.A and Fig. A.1 for a version without MSTO stars.

with

$$v_{\ell,\odot} = -U_\odot \sin \ell + (V_\odot + v_{\text{LSR}}) \cos \ell,$$

(8.15a)

$$v_{b,\odot} = W_\odot \cos b - \sin b \cdot (U_\odot \cos \ell + (V_\odot + v_{\text{LSR}}) \sin \ell),$$

(8.15b)

where we use the Schönrich et al. (2010) solar motion $(U_\odot, V_\odot, W_\odot) = (11.1, 12.24, 7.25)$ km/s and the McMillan (2017) motion of the local standard of rest (LSR) $v_{\text{LSR}} = 232.8$ km/s.

For our sample of MS stars, the median velocity errors taking into account the error in the distance as well as that in the proper motions, are $\epsilon(v_\ell) \sim 22$ km/s and $\epsilon(v_b) \sim 15$ km/s. For MSTO stars and those farther away than 6 kpc, the median errors are larger, namely $\epsilon(v_\ell) \sim 42$ km/s and $\epsilon(v_b) \sim 28$ km/s.

Figure 8.11 shows the mean $v^*_\ell$ and $v^*_b$ velocities binned on the sky. We only consider stars with a photometric distance $d > 6$ kpc because we are interested in picking up distant streams (and because nearby streams will not appear as coherent, thin structures on the sky). Besides large scale velocity patterns, which we discuss below, a few stream-like features stand out because members of a stream move in the same direction and with similar mean velocity, or at least sufficiently distinct from that of the background given the velocity errors. The most conspicuous is the GD-1 stream (Grillmair & Dionatos, 2006), but also Jhelum is apparent as well as tentatively Leiptr (c.f. Fig. 6 of Ibata et al., 2019).

As mentioned above, we have included all of the stars in our sample also those near the MSTO, even though their distances will on average be underestimated by up to $\sim 40\%$ (see Sec. 8.3.5). Yet, because they are intrinsically brighter they allow us to probe the farthest into the halo. This means that the mean velocities shown in the figure might
not be very accurate, yet including these MSTO stars allows us to probe distant streams whose motions are sufficiently different from those of the background stars in a similar portion of the sky, as appears to be the case for GD-1 and the Jhelum streams.

The large all-sky patterns that are seen in Fig. 8.11 are reminiscent of a rotation signal (particularly in $v^*_\ell$), and this is plausibly related to contamination from the hot thick disc. Although the correction for the solar motion, and in particular the contribution from the LSR motion, could potentially affect these velocities (since the correction is dependent on the photometric distance), the effect is unlikely to be significant. This view is supported by the large-scale pattern seen in $v^*_b$ in the figure, which cannot be due to the solar motion correction (because of its dependence on galactic longitude). The pattern is most likely due to very radial motions of a large fraction of the halo, as we discuss next.

Let us now inspect the cross-correlation between the velocity components projected on the sky. In previous work, Iorio & Belokurov (2019) using a sample of RR-Lyrae stars, have shown that the Pearson correlation of $\mu_\ell$ and $\mu_b$ creates a pattern on the sky indicative of the halo being radially anisotropic, see for example their Fig. 5. Figure 8.12 shows the mean correlation coefficient in bins on the sky for our RPM sample. The coefficient is calculated for proper motions after correcting for the solar motion (i.e. using Eqs. (8.15) and the inverse of Eq. (8.13)), it is defined as

$$\text{Corr}(\mu^*_\ell, \mu^*_b) = \frac{\text{cov}(\mu^*_\ell, \mu^*_b)}{\text{std}(\mu^*_\ell)\text{std}(\mu^*_b)}. \quad (8.16)$$

The Pearson correlation ranges from $[-1, 1]$, given the Cauchy-Schwarz inequality. Figure 8.12 clearly shows that there exists a strong correlation between the two proper
motion components. If the velocity ellipsoid of the stars in our sample were isotropic there would be no signal and if it were biased towards circular orbits it would have a very different signal. The correlation pattern that is observed is similar to that detected by Iorio & Belokurov (2019) and this is direct evidence that stars on radial orbits dominate the halo sample.

8.5.2 Global kinematic maps

Using the \((v_\ell^*, v_b^*)\) space velocities defined above, we can calculate pseudo-3D velocities. These are not the true 3D velocities because we assume that the line-of-sight velocities of all the stars are zero (i.e. \(v_{\text{los}}^* = 0\)). This assumption is valid if the velocity distribution is centred on zero in the galactocentric frame of rest. However, they will not be zero on average for local regions on the sky because of the imprint of the motion of the Local Standard of Rest around the Galactic Centre, following a \(\sin \ell \cos b\) pattern.

The equations for the pseudo-Cartesian velocities are

\[
\tilde{v}_x = -v_\ell^* \sin \ell - v_b^* \cos \ell \sin b,
\]

\[
\tilde{v}_y = v_\ell^* \cos \ell - v_b^* \sin \ell \sin b,
\]

\[
\tilde{v}_z = v_b^* \cos b.
\]

We adopt the notation \((\tilde{v}_x, \tilde{v}_y, \tilde{v}_z)\) for this set of velocities to make clear they are not the true Cartesian velocities. Subsequently, we calculate galactocentric cylindrical velocities and adopt a similar notation \((\tilde{v}_\phi, \tilde{v}_R, \tilde{v}_z)\). To obtain these coordinates, we place the Sun at \(X = -8.2\) kpc (McMillan, 2017, which agrees well with the recently determined distance to the massive black hole in the centre of our Galaxy by GRAVITY Collaboration et al. 2018).

Proceeding with the search for structure in the binned velocity moments, we show the mean \(\tilde{v}_\phi\) on the sky in Fig 8.13. Again, only stars with a photometric distance larger than 6 kpc (including MSTO stars) are taken into account to calculate the mean \(\tilde{v}_\phi\). As in Fig. 8.11, we note clear large-scale patterns some of which may be due to the missing line-of-sight velocities. Again clearly visible in this plot is the GD-1 stream because of its very retrograde nature. Another retrograde stream, Leiptr (c.f. Ibata et al., 2019), is visible on the left.

Besides looking for narrow streams, we can also use the velocity information to investigate the footprint of some interesting selections in velocity space. The two regions that we inspect are the retrograde halo, selected as all the stars with \(\tilde{v}_\phi < -150\) km/s, and the radial halo, selected as \(|\tilde{v}_\phi| < 50\) km/s and \(|\tilde{v}_R| > 200\) km/s. We do not select stars with a small space velocity (i.e. \(|\tilde{v}_R| < 200\) km/s) because the missing line-of-sight velocity moves stars towards zero velocities, so this is where we expect to find significant contamination.

Figure 8.14 shows the distribution on the sky of the full sample (top), all retrograde stars (middle), and stars on radial orbits (bottom). These maps reveal a centrally concentrated halo, with most of the stars near the Galactic Centre (yellow colours). Both
Fig. 8.13: Mean $\tilde{v}_\phi$ velocity of distant stars ($d > 6$ kpc) binned on the sky. The annotations highlight several (tentative) structures that are visible in these maps. This map includes MSTO stars, which might have large uncertainties in the distances. See Appendix 8.A and Fig. A.2 for a version without MSTO stars.
Fig. 8.14: Distribution of halo stars projected on the sky. Top: All of the stars in the sample with reliable distances (see Sec. 8.3.3). Middle: A subset of retrograde halo stars. Bottom: A subset of halo stars on radial orbits. The footprints seen in these panels are mostly due to selection effects.
the retrograde maps (middle row) and radial maps (bottom row) show a clear footprint. At first sight, the footprint of the radial halo is very similar to the full-sky maps made of nearby Gaia-Enceladus stars, shown in Helmi et al. (2018) (see also the skymaps presented in Iorio & Belokurov, 2019). However, this footprint is affected by the selection effects of the RPM sample. The kinematic selections imposed in the middle and bottom panels can also impact the distribution of the stars on the sky, depending on what the intrinsic velocity distributions are.

In Appendix 8.B we explore how the selection biases that are introduced by the incomplete velocity information affect the maps shown in Figure 8.14. We use the Gaia 6D sample, but apply a high-tangential velocity selection criterion and set the line-of-sight velocities to zero. This analysis shows that such selections can produce the footprints that are very similar to those shown in Figure 8.14. Therefore, the maps shown in this figure cannot be simply interpreted at face value.

### 8.6 The velocity distribution of the local halo

After highlighting the streams and structures in the distant halo, we will now focus on exploring the velocity distribution of the local halo. Velocity space is very suitable to look for local streams in the form of overdensities. In small volumes, in which the (orbital) velocity gradients are small, stars with similar velocities will have similar orbits. In this section we will inspect only stars with a heliocentric distance smaller than 2 kpc, or even smaller volumes when indicated.

#### 8.6.1 Toomre selection

The RPM sample is selected to contain a high fraction of halo stars, but we have seen in Sec. 8.4 that there is still some fraction of contamination by thick disc stars. Therefore we will now impose a second selection to filter thick disc stars based on their kinematics, namely, we remove stars that have $|\tilde{V} - V_{\text{LSR}}| < 250$ km/s. This selection is similar to a ‘Toomre’ cut, which is often used to differentiate disc from halo stars (e.g. Bonaca et al., 2017; Koppelman et al., 2018; Posti et al., 2018). We adopt a rather strict limit of 250 km/s here because the $\tilde{v}_i$ are not the true 3D velocities. In total, we are left with 3 223 725 high-quality halo stars ($\sim 30\%$ of the total sample). This number is in concordance with the fact that the red sequence (hot thick disc) contributes about 50$\%$ to the sample of stars with $v_{\text{tan}} > 200$ km/s (e.g. Sahlholdt et al., 2019, see also Amarante et al. 2020).

#### 8.6.2 Consistency check with RVS sample

In Sec. 8.5.2 and Appendix 8.B we have seen that the RPM selection method introduces selection effects in subsamples that are selected kinematically. These features vary with location on the sky (i.e. the ‘blindspots’). We will check if the missing line-of-sight velocities also create features in velocity space locally. For this check, we use a control sample of nearby halo stars selected from the Gaia 6D sample, which includes both giants
Fig. 8.15: Velocity distribution of halo stars in the solar neighbourhood (distance < 2 kpc) selected in the Gaia 6D subsample. The velocities calculated using the full phase-space information of the stars are shown in the left panels, while on the right they have been computed by setting the line-of-sight velocities to zero. Most of the structures present in the full 6D sample are strongly diluted in the 5D case.
and main sequence stars. See Appendix 8.C for a summary of this halo sample, with stars within < 2 kpc. We aim to compare the RPM selection method to one using the full phase-space information. After applying the (pseudo) Toomre selection, we find about 6,700 halo stars based on 6D information and roughly 7,700 when artificially setting the line-of-sight velocity to zero.

Figure 8.15 shows the velocity distributions of halo stars in the 6D sample. Both the true velocities (left) and the pseudo-velocities (right) are shown. Only those stars with line-of-sight velocities close to zero are found in the same location left and right. The disc in these spaces is centred on \( v_\phi \approx 230 \text{ km/s} \), the horseshoe shape (top panel) is an artefact of its removal (the Toomre cut). Subtle structures present in the true velocity distributions are diluted in the pseudo-velocities. Even the footprint of Gaia-Enceladus (Helmi et al., 2018; Belokurov et al., 2018), which creates an elongated structure in \( v_R \) at constant \( v_\phi \), is blurred. For a much larger sample of stars we should be able to find faint imprints of the structures present in the true velocities, simply because by chance there will be stars with close to zero radial velocities. Comforted by the fact that no artificial structures are created by the introduction of the pseudo-velocities we turn back to the RPM sample.

### 8.6.3 Local streams in velocity space

We start with the analysis of the velocity distribution, see Fig. 8.16. The sample that is shown streams from the RPM sample after applying the Toomre criterion. The top row shows the 2D-histogram of the velocities in \( 256 \times 256 \) bins. The elongation in \( \tilde{v}_R \) at constant \( \tilde{v}_\phi \) of the velocity distribution is reminiscent of the footprint of Gaia-Enceladus (c.f. Fig. 8.15). All three panels show relatively smooth distributions, with only subtle hints of substructures. To enhance these structures, we inspect the asymmetry of the distributions in the horizontal axis of each panel, see the middle row. We define the asymmetry as

\[
\text{Asymmetry} = \frac{H'_+ - H'_-}{H'_+ + H'_-},
\]

where \( H'_{+, -} \) are smoothed histograms given by

\[
H'_{+, -} = \mathcal{N}(H_{+, -} + 1, 2).
\]

Here \( H_+ \) is the histogram as it appears in the top row, \( H_- \) is its mirrored counterpart with the sign of \( \tilde{v}_R \) or \( \tilde{v}_z \) flipped, and \( \mathcal{N}(H, 2) \) is a Gaussian filter of the histogram \( H \) with kernel-size 2. The histograms range from \(-500 \) km/s to \( 500 \) km/s in both directions with \( 250 \times 250 \) bins. A Gaussian kernel of 2 corresponds to a standard deviation of \( \sim 8 \) km/s, which is roughly the mean velocity error of this sample. Positive values (red) indicate an overdensity and negative (blue) an under density. By construction, each overdensity is matched by a conjugate underdensity mirrored with respect to the panel’s \( x = 0 \) axis.
Fig. 8.16: Velocity distributions of local (d < 2 kpc) halo stars calculated without the line-of-sight component. For each combination of the cylindrical velocity components a 2D-histogram is shown in the top row, while the asymmetry (defined as in Eq. 8.18) with respect to the velocity plotted on the x-axis is shown in the middle row. We compare the maps to the asymmetry originating from a few structures that have been identified in the 6D sample in the bottom row. The asymmetry fleshes out structures that are asymmetric in either \( \tilde{v}_R \) or \( \tilde{v}_\phi \). Structures that are clearly present in the RPM sample, see middle row, are the Helmi streams (green dots) and Sequoia (red/purple dots).
The asymmetry maps reveal both small overdensities and large patterns. These overdensities imply the presence of non-phase-mixed debris. Any population in the halo that is sufficiently phase-mixed will not display an asymmetry in the (local) velocity distribution. To shed light on the origin of the asymmetries, we map several structures detected in the 6D sample in the bottom row. These maps are based on the structures found by Koppelman et al. (2018). We choose to focus on these structures because they are asymmetric in the true velocities and therefore might also be in the 5D sample.

To establish if there is a link between the structures in the 6D and 5D samples, we first need to bear in mind the following issues: i) The substructures from the 6D samples only comprise few stars each and hence they do not sample the sky densely; ii) The values of the pseudo-velocities $\tilde{v}$ depend on location on the sky, so we cannot use these stars directly to predict where stars in the streams in the 5D sample would be located in velocity space; iii) We expect the streams to be broad enough for the member stars to be isotropically distributed on the sky. With these caveats in mind we aim to enhance the number of sources per group in the 6D sample by resampling each star using the following method:

- Each of the 103 stars is re-sampled 1000 times,
- For each realisation, we generate a random location on a sphere of 2 kpc in radius that is centred on the solar position (uniformly distributed on the surface),
- We assign the random location with the unchanged velocity vector (in galactocentric coordinates),
- We calculate the $\tilde{v}_i$ velocities for each star based on its new location.

We assume here that the Galactic potential does not vary over the volume in which we resample the structures. In this approximation, the orbits of the stars are mostly determined by the amplitude and direction of the velocities since to first order the location-dependent potential term is constant. The resampling is thus a simple way of modelling more members of the stream. That is, to add stars on the same orbits as the detected streams.

This resampling shows what the footprint of the structures identified in the 6D sample could look like in the pseudo velocity-space. The stars are coloured by the original labels of Koppelman et al. (2018). The structure indicated with green dots has been associated with the Helmi Streams (e.g Helmi et al., 1999; Koppelman et al., 2019a). Recent studies suggest that the structures indicated with red/purple dots are part of a structure in velocity space known as Sequoia (Myeong et al., 2019), and that the blue/orange structures are part of a structures labelled Thamnos (Koppelman et al., 2019b).

We overlay the expected asymmetry over the newly sampled members of the structures identified in the 6D set (blue and red lines in the bottom row). These asymmetry contours do a surprisingly good job in explaining the asymmetries seen in the 5D sample. Therefore, we conclude that the features that are shown in the middle row for this sample are mainly due to the structures previously detected in the Gaia sample with full phase-space information.
Apart from these structures, there are several other overdensities visible of ~ few km/s in size (small red and blue dots). To check the statistical significance of the asymmetries found we shuffle the data. We randomly shuffle the velocities of Fig. 8.16 (top panel) and for each random set we create an asymmetry map. The Helmi streams and a handful of other groups (those with the darkest colours) have a strong asymmetry. However, similar levels of asymmetry are found in the random realisations of the data. We note that this is a very crude estimate of the statistical significance as only the amplitude and not the extent of the asymmetries are taken into account. For example, Sequoia (purple and red dots in the bottom panel) overlaps with several of the asymmetries seen in the middle row. It spans most of the retrograde part of the diagram ($v_\phi < -100$ km/s). Therefore, we surmise that this asymmetry is due to the debris of Sequoia (Myeong et al., 2019) and possibly also Thamnos (Koppelman et al., 2019b). Of course, the asymmetries seen in Fig. 8.16 can also be partly due to unidentified halo structures in velocity space.

We perform an additional test to measure the overall level of asymmetry in the dataset (rather than of a given feature). We compute the “total level of asymmetry” per map as the sum over all the bins, of the absolute values of the asymmetries per bin both for the data as well as for 1000 randomly reshuffled samples. For these samples, we compute the average and its dispersion $\sigma$. Finally we measure a significance value as $(\text{TotalAsymmetry}_{\text{data}} - \langle \text{TotalAsymmetry}_{\text{rand}} \rangle) / \sigma$. We find that the significance levels vary per map (i.e. combination of cylindrical velocities), taking values 10.2$\sigma$, 12.5$\sigma$, and 5.8$\sigma$ from left to right in Fig. 8.16. These results imply that, while individual asymmetries might be caused by random clustering of the stars, the total level of asymmetry found in the data is far from random.

### 8.6.4 Excess of pairs

As final part of this work, we will quantify the clustering of stars in velocity space with a two-point correlation function. That is, we check if the stars are randomly distributed in velocity space, or whether they tend to cluster on some velocity scale. We use a correlation function of the form

$$\xi(\Delta v) = \frac{DD(\Delta v)}{RR(\Delta v)},$$

where $DD(\Delta v)$ is the number pairs with a (pseudo) 3D velocity difference of $\Delta v$ and, similarly, $RR(\Delta v)$ is the number of pairs in a randomised sample. We obtain the randomised sample by shuffling the velocities and we count the pairs in 64 bins ranging from 0 to 800 km/s. For computational reasons, we only calculate the correlation function for stars inside a volume of 1 kpc.

Figure 8.17 shows the velocity correlation function $\xi$ for the RPM sample (black) and for a control sample of stars with full phase-space information (blue). Similar to analysis presented in Sec. 8.6.2, we show the effect of the missing line-of-sight velocities with the control sample (gray curve). The error bars indicate the error in $\xi$, calculated as the Poisson error in the number of pairs. A correlation of $\xi > 1$ indicates grouping of stars
in excess of random clustering. All curves show significant clustering on velocity scales $\Delta v \lesssim 100$ km/s. Clustering of velocities on small scales hints at the presence of streams.

The correlation excess at large velocity scales is less intuitive to interpret. However, a similar effect is seen in the 5D version of the 6D control sample. The grey and black curve are strikingly similar. Thus, the missing velocity component artificially enhances the correlation of the velocities on large scales and to a lesser extent also on small scales.

It is interesting that the black and grey curves match so well. This must mean that in spite of the roughly ten times smaller size of the 6D sample (2,945 stars), compared to the RPM sample, in both cases, most of the local streams are well resolved and contain at least a few stars per stream. This would be consistent with the early predictions by (e.g. Helmi & White, 1999) who estimated that order of 300 - 500 streams should be present near the Sun for a halo with a pure accretion origin. The advantage of a larger sample is that the amplitude of the correlation will be measured more accurately, as the (Poisson) uncertainty decreases with sample size.

Recently, Simpson et al. (2019) have investigated the velocity correlation function for Milky Way-like galaxies in the Aurigaia mock catalogues (Grand et al., 2018). Several of the halos analysed by these authors show similar correlation functions, both on small and large scales, to the one observed in the RPM sample. An excess of pairs can be very
difficult to interpret if there are too few particles, also if the volume is not localised. Pairs always appear as a consequence of substructure and this can be accreted but also \textit{in situ} (mergers do produce features in the \textit{in situ} population as well, e.g. Gómez et al. (2012); Jean-Baptiste et al. (2017)).

8.7 Discussion and Conclusions

We have explored the use of a reduced proper motion (RPM) diagram to identify tentative main sequence (MS) halo stars in \textit{Gaia} DR2. This method makes only use of the \textit{Gaia} photometry and proper motions. Most conventional methods rely on distance information or spectroscopic observations of the stars, which limits the sample size substantially. With the RPM selection method we find 11 692 245 tentative halo MS stars.

For these stars, we calculate photometric distances using the relatively simple colour-magnitude relation of the main sequence in the \textit{Gaia} photometric bands. These distances have typical errors of \(\sim 7\%\), which makes them much more reliable than inverted parallaxes. The distances are especially accurate for stars with colours \(0.45 < G - G_{RP} < 0.715\). Stars with bluer colours are near the MS turn-off and their magnitudes are a very steep function of colour. On the other hand, the MS broadens significantly for redder colours, which causes the error in the photometric distance calibration to increase. The above-mentioned colour range reduces our sample to 7 177 041 MS halo stars with good distances. The median velocity errors for these stars are \(\sim 20\) km/s.

A restriction of dealing with a sample of halo MS stars is that they are intrinsically faint. The most distant MS star in our particular sample is found at \(\sim 16\) kpc. A star with \(M_G \approx 5\) (the brightest given the colour cut) at this distance would have the limiting magnitude of the \textit{Gaia} DR2 catalogue. Beyond a heliocentric distance of \(\sim 5\) kpc, the sample suffers from incompleteness, and thus halo overdensities like Virgo and Hercules-Aquila cannot be studied with our dataset.

The spatial distribution of the RPM sample displays large-patterns and structures but it is not trivial to disentangle real structure from artefacts of the selection method. Nonetheless, full-sky maps of the mean velocities \(v_\ell\) and \(v_b\), reveal the presence of some known streams, such as GD-1 (Grillmair & Dionatos, 2006). Since pixels on the sky that overlap with a stream may have a mean velocity that is different from that of the background, and a smaller velocity dispersion, this promises to be an interesting approach to identifying substructures. Future \textit{Gaia} data releases will be less affected by systematics due to, for example, the scanning-pattern, and will provide more precise proper motions leading to easier distinction between stream and background stars.

Another promising approach is to calculate (pseudo) space velocities of the stars by assuming their line-of-sight velocity to be zero. Most of the structure in velocity space is smoothed out because of the missing velocity component. However, it can occur that by chance, some stars have a true line-of-sight velocity of zero, particularly if the sample is large enough. In that case, we may still expect to find some structure in velocity space.
In fact, in pseudo-velocity space for nearby MS halo stars we find clear imprints of the Helmi streams. Also the footprint of Gaia-Enceladus (i.e. the Gaia-Sausage) is found in several of the velocity maps that we explore. Moreover, the retrograde halo shows a strong asymmetry in the velocity distribution, reminiscent of the accreted structures that have been previously reported in the 6D sample (Koppelman et al., 2018, 2019b; Myeong et al., 2019). The total level of non-phased-mixed substructure in pseudo-velocity space, as measured by an asymmetry parameter, is very significant ($\gtrsim 6\sigma$), in comparison to randomised samples.

Through a two-point velocity correlation function we measure a very significant excess of stars with small velocity differences ($\Delta v < 100$ km/s). The amplitude of this clustering signal for the RPM sample is similar to that obtained when using 6D Gaia sample, implying that this sample is already large enough for a true quantification of the amount of clustering. This appears to be consistent with the expectation from theoretical models that hundreds of debris streams are crossing the solar vicinity (e.g. Helmi & White, 1999), as only if the sample is large enough will the streams be populated by enough stars to produce a signal. It will be particularly interesting to study this excess of close velocity pairs in more detail and, in particular, to do spectroscopic follow-up of the stars in pairs as these likely originate from very localised regions in the phase-space of accreted galaxies.

The RPM catalogue we have built provides interesting targets for spectroscopic follow-up, for example for chemical tagging. Even low/intermediate resolution spectroscopy would be highly valuable because it would provide the missing line-of-sight velocity but also because even a metallicity and $[\alpha/\text{Fe}]$ abundance are extremely useful to disentangle merger events from one another and to construct basic chemical enrichment histories. Halo main sequence stars are easy to identify as shown here. Moreover, they have the advantage of being very numerous and that the elements found in their atmospheres are directly representative of the elements in their birth material (e.g. Tolstoy et al., 2009), which is not necessarily true for giants, which might have undergone mixing.

Finally, a sample like the one presented here could be used to map the (local) mass-distribution of the Milky Way, the density profile of the stellar halo as well as its dynamical properties. In all applications it is important to bear in mind that our sample is kinematically biased by construction, and that it misses halo stars with small proper motions and large line of sight velocities.
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Appendix 8.A  Velocity maps without MSTO stars

In Sec. 8.5.1 we show the velocity moments of the RPM sample binned on the sky. To flesh out distant streams, like GD-1, we have included sources that are near the MSTO. However, the MSTO stars might have distances that are underestimated by up to 40%. For this reason, we show here the same figures, but without these MSTO stars, see Figs A.1 and A.2 (and Figs. 8.11 and 8.13 for comparison). Clearly, the overall contours are the same with and without the MSTO stars. However, the figures shown in this appendix do not show any (clear) narrow streams.

Appendix 8.B  Selection Effects

If, for simplicity, we assume that all of the halo stars are on radial orbits, then there would be blind spots in the RPM sample towards the Galactic Centre and anti-centre. In these directions, stars on purely radial orbits move along the line-of-sight. Therefore, their proper motions are close to zero, and they will not be present in the RPM selection. The blind spot towards the Galactic Centre is smaller than its conjugate spot towards the anti-centre.

In Fig. B.1 we investigate this selection effect by showing the sky-distribution of halo stars in the 6D sample. We show a sample of halo stars selected using the full phase-space information that is available (left column) and a sample of halo stars selected using only tangential velocities (right column). The sample that is used is identical to the one described in Appendix 8.C. The layout (per column) is the same as the layout of Fig. 8.14. The similarity between the footprints of the red dots and the distribution displayed in Fig. 8.14 are striking. We have to conclude that the large structures that are seen in Fig. 8.14 are dominated by selection effects.
We do note that the distance distribution of the stars in the full phase-space sample is more local. Typically these stars are within 3 kpc, while the RPM sample has a median distance of 4.39 kpc. Therefore, the maps of Fig. B.1 might look different when a larger (volume-wise) sample is used. Although, up to first order they should be the same. These maps mostly show where the motion aligns the most with the line-of-sight, which is a sky-dependent phenomenon and not distance dependent.
Fig. B.1: Selection effects of the RPM halo-selection, illustrated with a sample of halo stars for which the full phase-space information is available (see Appendix 8.C). The layout of the panels is based on Fig. 8.14. The left columns shows the distribution on the sky of halo stars selected using full phase-space information. On the right, we show the distribution of a halo sample selected without the line-of-sight velocity.
Appendix 8.C  RVS sample

In Sec. 8.6.2 we have used the RVS (Katz et al., 2019) subset of Gaia DR2 as a control sample. This sample of stars comprises full phase-space information. For the construction of this sample we start with the full RVS sample and impose the following quality criteria:

- $\text{phot}_g\_\text{mean}\_\text{flux}\_\text{over}\_\text{error} > 50$,
- $\text{phot}_rp\_\text{mean}\_\text{flux}\_\text{over}\_\text{error} > 20$,
- $\text{visibility}\_\text{periods}\_\text{used} \geq 5$,
- $\text{parallax}\_\text{over}\_\text{error} > 5$.

We calculate Cartesian coordinates and velocities in the similarly as described in Sec. 8.5 and Sec. 8.6, as well as tangential space velocities according to Eq. (8.7). We then select a halo sample as stars with $v_{\text{tan}} > 200$ km/s. Because for this set there are line-of-sight velocities available, we also calculate the true and pseudo-velocities, and then impose the following cut in velocity space:

$$|V - V_{\text{LSR}}| > 250 \text{ km/s}. \quad (C.1)$$

We calculate this cut separately for the true and pseudo-velocities. As a result we end up with two halo samples, one based on full phase-space information and one without line-of-sight velocities. For a local sample (< 2 kpc), we find 6 718 (using true velocities) and 7 749 (using pseudo-velocities) tentative halo stars.
Determination of the escape velocity using a proper motion selected halo sample

Based on: Koppelman and Helmi (2020), submitted to A&A

Abstract

The Gaia mission has provided the largest catalogue ever of sources with tangential velocity information. Using this catalogue for dynamical studies is, however, difficult because most of the stars lack line-of-sight velocity measurements. Recently, we presented a selection of \( \sim 10^7 \) halo stars with accurate distances that have been selected based on their photometry and proper motions. Using this sample, we model the tail of the velocity distribution in the stellar halo, locally and as a function of distance. Our goal is to measure the escape velocity, and herewith to constrain the mass of our Galaxy. We fit the tail of the velocity distribution with a power-law distribution, a commonly used approach first established by Leonard & Tremaine (1990). For the first time ever we use tangential velocities measured accurately for an unprecedented number of halo stars to estimate the escape velocity. For the solar neighbourhood we determine a very precise lower limit to the escape velocity: \( 497^{+40}_{-24} \) km/s. Our best guess is that this value is underestimated by \( \sim 10\% \). For a spherical NFW halo in a Milky Way potential, this translates into a lower limit on the mass of \( M_{200} = 0.67^{+0.30}_{-0.15} \times 10^{12} \, M_\odot \) and concentration parameter \( c = 15.0^{+2.6}_{-2.3} \). When correcting for the underestimation of 10\%, we find \( M_{200}^{10\%} = 1.11^{+0.50}_{-0.23} \times 10^{12} \, M_\odot \) and \( c^{10\%} = 11.8^{+1.2}_{-2.1} \). The behaviour of the escape velocity towards the inner Galaxy follows expectations from Milky Way dynamical models, but curiously we find that its value appears to increase with distance beyond the solar radius. This behaviour is due to a change in the shape of the velocity distribution, and could be related to the presence of velocity clumps. A tentative analysis of the escape velocity as a function of \((R,z)\) shows that it decreases too slowly compared to what is expected for a spherical halo using standard values for the characteristic parameters describing the galactic disc.

9.1 Introduction

Numerous studies have attempted to measure the mass of the Milky Way, yet it has been notoriously difficult to obtain precise, model independent constraints. Most works now agree that the mass of the Milky Way’s dark matter halo is \( 10^{12} \, M_\odot \) within a factor of
two (see Fig. 7 of Callingham et al., 2019, for a recent compilation). The kinematics of globular clusters, dwarf galaxies, and halo stars have often been used in such studies (Kochanek, 1996; Xue et al., 2008; Watkins et al., 2010; Deason et al., 2012; Fragione & Loeb, 2017; Posti & Helmi, 2019; Callingham et al., 2019; Fritz et al., 2018). The timing argument and the properties of debris streams such as those from the Sagittarius dwarf (e.g. Dierickx & Loeb, 2017; Zaritsky et al., 2019) have provided additional, yet similar constraints. In this Chapter, we aim to derive a very precise lower limit to the escape velocity near the Sun, and hence under further assumptions to the mass of the Milky Way.

The escape velocity is the maximum velocity that stars can have while still being bound to the Galaxy. In principle, the single fastest moving bound star places a lower limit on the escape velocity. However, in practice, individual stars might be affected by large measurement errors or they might be outliers (such as escapees). A more robust approach is to fit the velocity distribution as a whole as put forward by Leonard & Tremaine (1990, hereafter LT90), who describe the tail of the velocity distribution with a power-law.

Several works have used the LT90 method in the past. For example, Smith et al. (2007) and Piffl et al. (2014b), hereafter S07 and P14, estimated the escape velocity locally to lie in the range of $[500 – 600]$ km/s, using only radial velocity information from RAVE (Steinmetz et al., 2006). The analysis of Williams et al. (2017, hereafter W17) supports these values and these authors also show that the escape velocity drops to $\sim 300$ km/s at a distance of 50 kpc. The advent of full phase-space information with Gaia DR2 has not led to a reduction in the estimated range for the escape velocity in the solar neighbourhood: it is still $[500 – 640]$ km/s (Monari et al., 2018; Deason et al., 2019, hereafter M18 and D19), a result that can largely be attributed to the different underlying assumptions used by the authors.

In this Chapter, we will use a sample of halo stars with only tangential velocities from Gaia DR2 to infer the escape velocity applying also the LT90 method. This sample comprises orders of magnitude more halo stars than any other sample used before. Samples making use of only tangential velocities have not been popular for this kind of studies in the past because of the large uncertainties in the velocities, particularly induced by the distance errors. Even more dramatic was the lack of (accurate) proper motion measurements for large numbers of stars. However, Gaia DR2, containing about $\sim 200 \times$ more stars with proper motions than radial velocities, makes this kind of study feasible now. We proceed in this Chapter as follows. We describe the data used and its properties in Sec. 9.2 and the methods used in Sec. 9.3. In Sec. 9.4 we test the method for determining the escape velocity using mock data and cosmological simulations. In Sec. 9.5 and Sec. 9.6 we present our results in the solar neighbourhood and as a function of galactocentric distance, respectively. In Sec. 9.7 we use the estimated escape velocity to derive a lower limit for the mass of the dark halo of the Milky Way, and to identify likely unbound stars. In Sec. 9.8 we present our conclusions.
9.2 Data

The determination of the escape velocity is contingent upon having a sample of halo stars with high-quality measurements and large velocity amplitudes. Most of the data used in this Chapter is provided by the Gaia mission (Gaia Collaboration et al., 2016; Gaia Collaboration, Brown et al., 2018). We will mainly use the sample of halo stars selected and analysed in Koppelman & Helmi (2020, KH20 hereafter). This sample comprises $\sim 10^7$ Main Sequence (MS) halo stars, and we refer to it as the reduced proper motion or the 5D sample hereafter. To verify our results, we will also make use of a set of nearby halo stars with full phase-space information.

9.2.1 Velocity information

To transform the observed motions (proper motions and radial velocities when available) into space velocities we proceed as follows. We compute the space velocity of a star by combining the proper motion and its distance as

$$v_j = 4.74057 \text{ km/s} \left( \frac{\mu_j}{\text{mas/yr}} \right) \left( \frac{d}{\text{kpc}} \right),$$

where $j = (\ell, b)$. These velocities are then corrected for the solar motion using the values for the motion of the Sun with respect to the local standard of rest (LSR) given by Schönrich et al. (2010) and the motion of the LSR given by McMillan (2017); they are $(U_\odot, V_\odot, W_\odot) = (11.1, 12.24, 7.25) \text{ km/s}$ and $v_{LSR} = 232.8 \text{ km/s}$ respectively. The transformations to correct the tangential velocities are

$$v_j^* = v_j + v_{j, \odot},$$

where $v_{\ell, \odot}$ and $v_{b, \odot}$ are defined as

$$v_{\ell, \odot} = -U_\odot \sin \ell + (V_\odot + v_{LSR}) \cos \ell,$$

$$v_{b, \odot} = W_\odot \cos b - \sin b \cdot (U_\odot \cos \ell + (V_\odot + v_{LSR}) \sin \ell).$$

Finally, the tangential velocity in the Galactic frame of rest as observed from the Sun is calculated as

$$v_t = \sqrt{(v_\ell + v_{\ell, \odot})^2 + (v_b + v_{b, \odot})^2}.$$
To derive space velocities we use the following expressions:

\[ v_x = v^*_\text{los} \cos \ell \cos b - v^*_\ell \sin \ell + v^*_b \cos \ell \sin b, \]  
(9.6a)

\[ v_y = v^*_\text{los} \sin \ell \cos b + v^*_\ell \cos \ell \sin b, \]  
(9.6b)

\[ v_z = v^*_\text{los} \sin b + v^*_b \cos b. \]  
(9.6c)

To transform the coordinates to a galactocentric frame we place the Sun at \( X = -8.2 \) kpc (McMillan, 2017). We use this value for the distance to the Galactic centre because it is consistent with the McMillan (2017) potential that we will employ later, and the same is true for the LSR velocity. We note however that the McMillan values agree well with the more recent determination of the distance to the Galactic Centre by the GRAVITY Collaboration et al. (2018) and circular velocity at the position of the Sun by Eilers et al. (2019).

To isolate a halo sample using the \textit{Gaia} DR2 data, we consider stars with velocity vectors that deviate more than 250 km/s from the velocity vector of the LSR (i.e. the velocity vector of a typical disc star), namely \(|\vec{v} - \vec{v}_{\text{LSR}}| \geq 250 \text{ km/s}\). This type of selection is known as a ‘Toomre’ selection.

When no line-of-sight velocity information is available, we use Eqs. (9.6) setting \( v_{\text{los}} \) to zero. In that case, we refer to the velocity vector as \((\tilde{v}_x, \tilde{v}_y, \tilde{v}_z)\) to stress that these are not the true Cartesian velocities. For this set of stars, which constitute the majority of our sample, we use an adapted Toomre selection to isolate a halo sample, namely \(|\vec{v} - \vec{v}_{\text{LSR}}| \geq 250 \text{ km/s}\).

### 9.2.2 Sample with full phase-space information

In the solar neighbourhood, we will use a sample of stars with full phase-space information from \textit{Gaia} known as the 6D or the RVS sample (Katz et al., 2019). We extend this dataset by adding sources with radial velocities observed by APOGEE (Wilson et al., 2010; Abolfathi et al., 2018), LAMOST (Cui et al., 2012), and RAVE (Kunder et al., 2017), see Sec. 2 of Koppelman et al. (2019) for a full description of this catalogue. The cross-matches with APOGEE and RAVE have been obtained from the \textit{Gaia} archive (Marrese et al., 2018).

For this sample and in line with M18 and D19, we use the quality cuts described in Marchetti et al. (2018), namely

- astrometric\_gof\_al < 3,
- astrometric\_excess\_noise\_sig ≤ 2,
- \(-0.23 \leq \text{mean\_varpi\_factor\_al} \leq 0.32,\)
- visibility\_periods\_used > 8,
- rv\_nb\_transits > 5,

and also impose the following quality criteria:

- ruwe < 1.4,
- parallax\_over\_error > 5.
For the additional spectroscopic data we use the same quality cuts, with exception of the criterion on $rv_{nb\_transits}$. Additionally, we use survey-specific quality constraints. For APOGEE we use

- $SNR > 20$,
- $STARFLAG == 0$,
- $abs(SYNTHVELIO\_AVG - OBSVHELIO\_AVG) < 50$,
- $NVISITS > 2$,

for RAVE

- $eHRV < 10$,
- $Algo\_Conv\_K! = 1$,
- $SNR\_K > 20$,

and for LAMOST

- $snri > 20$,
- $snrg > 20$.

Several studies have reported that the sources in the RVS sample, and bright sources in general, contain a parallax offset of $\sim 0.05$ mas (see Schönrich et al., 2019; Leung & Bovy, 2019; Zinn et al., 2019; Chan & Bovy, 2019). Therefore, we correct the parallaxes in the 6D sample for an offset of $-0.054$ mas as estimated by Schönrich et al. (2019). Following these authors, we increase the parallax errors by $0.006$ mas to account for the error in the offset and by $0.043$ mas to account for the RMS in the offset reported by Lindegren et al. (2018), both of which are added in quadrature.

Nonetheless, to mitigate the effects of the parallax offset we only consider sources within 2 kpc. As explained earlier we select halo stars as those with $|\vec{v} - \vec{v}_{\mathrm{LSR}}| > 250$ km/s.

Finally, we remove the star with Gaia DR2 source_id 5932173855446728064 since its radial velocity reported in Gaia DR2 is known to be incorrect (Boubert et al., 2019). The final sample comprises 2067 high-quality stars, of which 495 are from the Gaia RVS sample, 10 from APOGEE, one from RAVE, and 1561 from LAMOST.

Since the spectroscopic surveys add a considerable number of stars, mostly from LAMOST, we have checked that they do not bias our results. In fact, these are fully consistent with using only Gaia RVS sources. The stars from the spectroscopic surveys do not dominate the determination of $v_{\mathrm{esc}}$ because they, in general, have larger uncertainties. However, they do help in closing the likelihood contours, as we will see in Sec. 9.5.

### 9.2.3 The reduced proper motion sample

For the complete description of the reduced proper motion (RPM) sample we refer the reader to the KH20 paper. Here we will summarise the details that are relevant for this Chapter. By virtue of the selection method, the RPM sample comprises only MS stars. These types of stars have a relatively simple relation between their colour and absolute magnitude. This relation can be used to calculate photometric distances with typical uncertainties of 7%.
The quality cuts described in KH20 already filter many stars. In this Chapter we prune the sample even further. In summary, we:

1. Target the most pure set of halo stars: $|\vec{v} - \vec{v}_{\text{LSR}}| > 250$ km/s (see Sec. 9.2.1).
2. Select stars with large tangential velocities: $v_t > 250$ km/s.
3. Isolate stars that are the least affected by extinction, that is we consider only those with $A_G < 0.2$.
4. Select stars in the colour range where the photometric distances have the smallest error: $0.50 < G - G_{\text{RP}} < 0.71$. The blue limit here is stricter than in KH20, to be absolutely certain that there is no contamination from the MS turn-off.
5. Select stars at high latitudes to remove contamination from the disc: $|b| > 20^\circ$.

Some stars in the RPM sample have less precise photometric distance than trigonometric distance (i.e. parallax from Gaia). Furthermore, some stars may have been excluded because they did not satisfy the last three quality cuts described above, even though their trigonometric parallaxes are accurate. Therefore we add such stars back to the sample. We also replace the photometric distances with trigonometric distances for stars with $\text{parallax \_over \_error} > 10$, if the latter has a smaller uncertainty than the first, and we only consider stars with parallaxes > 0.5 mas.

As mentioned above, the trigonometric parallaxes from Gaia DR2 are known to contain a zero-point offset that has a complex dependence on other observational parameters (e.g. the colour and magnitude of the stars). Because most of the stars in the 5D sample (without radial velocities) are fainter than those in the 6D sample, we correct their parallaxes with a different offset. Following Lindegren et al. (2018), we use a value of $-0.029$ mas for the parallax offset and increase the parallax uncertainties by 0.043 mas (the errors are added in quadrature) to account for variations in the offset.

Within 1 kpc about 90% of the distances stem from the Gaia trigonometric parallaxes, and at 2 kpc this percentage drops to $\sim 50\%$. The final, pruned sample comprises 197 449 sources of which 18 236 have trigonometric distances.

### 9.2.4 Inspection of the data sample

Figure 9.1 shows the spatial distribution of the stars in the sample. The maps are coloured by the logarithm of the counts per bin. The quality cuts described in the previous section affect the spatial distribution of the stars, most notably by removing low-latitude stars. The overdensity at the solar neighbourhood (centre of the figure) is caused by the addition of sources with accurate parallaxes. The median heliocentric distance of the sample is 3.6 kpc.

In Fig. 9.2 we show the tail of the tangential velocity distribution as a function of galactocentric distance by slicing the sample in uniformly spaced overlapping bins, ranging from 4 – 12 kpc, of 1 kpc width, which is larger than the typical error in the distances. A visual inspection reveals only small variations in the distributions. These clearly resemble a power-law (as anticipated) but with a slight tendency to become more exponential with distance from the Galactic Centre.
Fig. 9.1: Spatial distribution of the RPM sample used in this work, in heliocentric coordinates and for stars with \( v_t > 250 \) km/s. The Galactic Centre is located at \( X = 8.2 \) kpc as indicated. The concentration of stars near the origin is caused by a small subset of stars with very accurate trigonometric parallaxes.

Fig. 9.2: Tail of the tangential velocity distributions for different galactocentric distances. The annotations in the panels indicate the central distance and number of stars per bin. The black marks give the error in the counts and the mean error in \( v_t \) for each bin.
Fig. 9.3: Distribution of velocity errors shown separately for sources with photometric (in blue) and trigonometric (in green) distances.

The distribution of the relative errors in the tangential velocity $v_t$ is shown in Fig. 9.3, separately for the photometric (in blue) and trigonometric distances (in green). On average, the tangential velocities derived from the trigonometric parallaxes are slightly more accurate than those based on the photometric distances. This is a selection effect since only sources with very accurate parallaxes are included in our sample. When propagating the errors in the velocities, we find that the uncertainty distribution for sources with photometric distances peaks at 7% and has a median of 8%. For the trigonometric distances, the distribution in the velocity uncertainties peaks at $\sim 5\%$. The distribution of $v_t$-errors has a tail towards higher uncertainties due to errors in the proper motions, and there is only is a small dependence with magnitude at the faint end (i.e. for $G \gtrsim 20$).

9.3 Methods

9.3.1 Determining $v_{\text{esc}}$

As described earlier, we will use here the LT90 method to determine the escape velocity, denoted hereafter as $v_{\text{esc}}$. The idea behind this method is that the tail of the velocity distribution can be described by a power-law, and $v_{\text{esc}}$ is the velocity at which the probability of finding a star goes to zero. Although we follow closely Sections IIa and IIc from LT90 and adopt their notation, the formalism we use reveals some differences.

As just stated, the probability of finding a star in a local volume with a velocity in the range $(v, v + dv)$ is described close to the escape velocity as a power-law

$$f(v|v_{\text{esc}}, k) = \begin{cases} A(v_{\text{esc}} - v)^k, & v_{\text{cut}} < v < v_{\text{esc}} , \\ 0, & v \geq v_{\text{esc}}, \end{cases}$$

(9.7)
where \( k \) is the index of the power-law, \( v_{\text{esc}} \) is the escape velocity, and \( v_{\text{cut}} \) is a threshold velocity below which the distribution is not well-represented by a power-law. It is important to set \( v_{\text{cut}} \) accordingly such that only the tail of the distribution is fit. The normalisation constant is defined as \( A = \frac{k+1}{(v_{\text{esc}}-v_{\text{cut}})^{k+1}} \), which is obtained from \( A \int_{v_{\text{cut}}}^{v_{\text{esc}}} f(v | v_{\text{esc}}, k) \, dv = 1 \).

The expression in Eq. (9.7) describes the distribution of velocities corrected for the solar motion (including peculiar and LSR), for example at the location of the Sun. Note that if \( f dv \) is the probability of finding a star with velocity \( v \) in the range \( (v, v + dv) \), this implies that there exists some distribution \( g(\vec{v}) \) such that \( \int_{v}^{v_{\text{esc}}} g(\vec{v}) \, dv = 4\pi v^2 g(v) \, dv = f(v) \, dv \) under the assumption that the velocity distribution is isotropic.

We now wish to obtain the probability distribution for the tangential velocity, i.e. \( f_{t}(v_{t} | v_{e}, k) \). This can be derived from the joint distribution \( f_{r,t}(v_{r}, v_{t} | v_{e}, k) \) which gives the probability of finding a star with a given line-of-sight velocity and tangential velocity as \( f_{r,t}(v_{r}, v_{t} | v_{e}, k) \, dv_{r} \, dv_{t} \). By performing a transformation of variables

\[
f_{r,t}(v_{r}, v_{t} | v_{e}, k) = \int g(\vec{v} | v_{e}, k) \delta(v_{r} - \vec{v} \cdot \hat{n}) \delta(v_{t} - |\vec{v} \times \hat{n}|) \, dv,
\]

where \( \hat{n} \) is a unit vector along the line-of-sight. To express the distribution function in terms of only \( v_{t} \) we integrate over the line-of-sight component (and over angle)

\[
f_{t}(v_{t} | v_{e}, k) = \frac{1}{2\pi} \int g(\vec{v} | v_{\text{esc}}, k) \delta(v_{t} - |\vec{v} \times \hat{n}|) \, dv.
\]

The distribution \( f_{t} \, dv_{t} \) gives the probability of finding a tangential velocity \( v_{t} \) in the range \( (v_{t}, v_{t} + dv_{t}) \).

Evaluating this integral in spherical coordinates, with \( \hat{n} \) aligned with the z-axis (implicitly assuming the stars are distributed isotropically), we obtain

\[
f_{t}(v_{t} | v_{e}, k) = \int \int f(v) \delta(v_{t} - v \sin \theta) \, v^2 \sin \theta \, dv \, d\theta,
\]

which, by changing the order of integration and substituting \( u = v_{t} - v \sin \theta \), reduces to

\[
f_{t}(v_{t} | v_{e}, k) = -\int_{v_{t}}^{v_{\text{esc}}} f(v) \left[ v_{t}^{-2} - v^{-2} \right]^{-\frac{1}{2}} \, dv.
\]

The resulting integral for \( f(v) \) given by Eq. (9.7) can be solved with Mathematica (and depends on the regularised hypergeometric \( _{2}F_{1} \) function). When evaluating the Taylor series expansion of \( v_{t} \to v_{e} \) for the integral, we obtain

\[
f_{t}(v_{t} | v_{\text{esc}}, k_{t}) \propto \begin{cases} (v_{\text{esc}} - v_{t})^{k+t+\frac{1}{2}}, & v_{\text{cut}} \leq v_{t} < v_{\text{esc}}, \\ 0, & v_{t} \geq v_{\text{esc}}, \end{cases}
\]

\[
(9.12)
\]
which can be normalised by multiplying with the constant $A_t = \frac{k_t + 1.5}{(v_{\text{esc}} - v_{\text{cut}})^{k_t + 1.5}}$, which is derived from the requirement that $A_t \int_{v_{\text{cut}}}^{v_{\text{esc}}} f_t(v_t | v_{\text{esc}}, k_t) \, dv_t = 1$, and where we have replaced $k$ with $k_t$ for clarity. The reason for this is that only in the case of $v_t \to v_e$ are the two power-law indices of Eq. (9.7) and Eq. (9.12) related, and $k_t = k$. It is thus best to think of $f_t(v_t | v_{\text{esc}}, k_t)$ in Eq. (9.12) simply as a power-law description of the tangential velocity tail, an approximation which is supported by Fig. 9.2. We will see in Sec. 9.4.1 that it is in general not quite true that $k_t = k$ for the $v_{\text{cut}}$ values that are typically considered in the literature. In what follows, we thus reserve the notation $k_t$ for the power-law index of $f_t$, use $k$ for the index using the distribution from Eq. (9.7) and use $k_r$ to indicate the index for a sample using only line-of-sight velocities (e.g. when comparing to values in the literature).

So far we have assumed that the velocities ($v$ and $v_t$) are the true velocities. However, in reality we are dealing with ‘measured’ velocities, which are a combination of the true velocity and some unknown uncertainty. To account for the uncertainty we smooth the velocities by convolving them with an error distribution $\epsilon(v_t - v'_t, \sigma_t)$, where $\sigma_t$ is the error in $v'_t$. If we assume that $v'_t$ and $v_b$ have Gaussian errors, then the distribution $\epsilon(v_t - v'_t, \sigma_t)$ follows the Beckmann distribution (i.e. it is non-Gaussian). However, if evaluated far away from the origin ($v'_t/\sigma_t >> 0$), this distribution is well-approximated by a Gaussian. This gives us another reason to choose a sufficiently large $v_{\text{cut}}$. Therefore, in what follows we approximate $\epsilon(v_t - v'_t, \sigma_t)$ by a Gaussian $f_G(v_t - v'_t, \sigma_t)$. The convolution of the power-law from Eq. (9.12) and the Gaussian is given by

$$C(v'_t, v_{\text{esc}}, k_t, \sigma_t) = \int_{v_{\text{cut}}}^{v_{\text{esc}}} f_t(v_t | v_{\text{esc}}, k_t)f_G(v_t - v'_t, \sigma_t) \, dv_t. \tag{9.13}$$

We note that we have taken as the integration lower-limit $v_{\text{cut}}$ and not zero as in Eq. (17) of LT90. Since the velocity distribution below $v_{\text{cut}}$ is not well-described by a power-law, but by a different distribution function $f_G^\dagger(v_t)$, the convolution over the range $0 < v_t < v_{\text{cut}}$, would take the form

$$C^\dagger(v'_t, \sigma) = \int_{0}^{v_{\text{cut}}} f_G^\dagger(v_t)\epsilon(v_t - v'_t, \sigma) \, dv_t. \tag{9.14}$$

which does not depend on $v_{\text{esc}}$ nor on $k$. As we will see below, we may thus ignore this part of the velocity distribution. This also means that we also ignore stars that have an apparent $v'_t$ below the cut, but with a finite probability of having a true $v_t$ above it. We will see in Sec. 9.4 that these assumptions do not affect the method’s ability to infer $v_{\text{esc}}$.

By normalising Eq. (9.13) we find $P(v'_t | v_{\text{esc}}, k_t, \sigma_t, v_{\text{cut}})$, the probability of finding a star with $v'_t$ in the range $(v'_t, v'_t + dv'_t)$

$$P(v'_t | v_{\text{esc}}, k_t, \sigma_t) = \frac{C(v'_t, v_{\text{esc}}, k_t, \sigma_t)}{\int_{0}^{\infty} C(v'_t, v_{\text{esc}}, k_t, \sigma_t) \, dv'_t}. \tag{9.15}$$
By definition, because both $f_t$ and $f_G$ are pre-normalised, the integral in the denominator is unity, as the area under a convolution is $\int (f \otimes g) dt = [\int f(u) du] [\int g(t) dt] = 1$.

Through Bayes’ theorem, we find that the likelihood of finding $v_{esc}$ and $k_t$ is given by

$$L_m(v_{esc}, k_t | \sum_i^n \mu_i, \sigma_i) \propto P(v_{esc})P(k_t) \prod_{i=1}^n P(v_i' | v_i', v_{esc}, k_t, \sigma_t),$$  (9.16)

where $P(v_{esc})$ and $P(k_t)$ are priors for $v_{esc}$ and $k_t$. For numerical reasons the logarithm of the modified likelihood is evaluated rather than $L_m$ itself and the likelihood is not normalised. The latter would mean evaluating a double integral for $10^5$ sources. Ignoring the normalisation means that we can only relatively compare models and cannot make statements concerning the overall fit of the power-law to the data.

The procedure that is outlined above implicitly makes the following assumptions:

1. The tail of the velocity distribution is populated up to the escape velocity.
2. The tail of the velocity distribution is smooth.
3. There are no unbound stars in our sample.
4. And there is no contamination from a rotating (disc-like) population, which would break the isotropy on the sky.

Perhaps the most problematic assumption is the first one. There is no guarantee that the velocity distribution locally, or at any other location in the Milky Way, extends up to the escape velocity. Most likely it is truncated at some lower value. As a result, the LT90 method is prone to underestimate the true $v_{esc}$. For example, cosmological simulations show velocity distributions that are truncated at 90% of $v_{esc}$ (e.g. S07). The exact location of the truncation depends on the assembly history of the galaxy and quite possibly also on the resolution of the simulation. We will quantify the truncation of the velocity distribution using mock data in Sec. 9.4. We stress that, because of this truncation, whatever value we derive for $v_{esc}$ it most likely is a lower limit.

The second assumption has recently been tested by Grand et al. (2019), who find that clustering in the velocity distribution biases the estimation of $v_{esc}$, and can result both in under and overestimates. Nonetheless, these authors show that the estimated $v_{esc}$ is typically underestimated by 7%. To emphasise the importance of this bias: a difference of 7% in the escape velocity results in a 21% bias in the estimated mass.

It seems unlikely that our sample contains many unbound stars, since hyper-velocity stars are typically young stars ejected from the Galactic Centre and not old stars in the halo (e.g. Brown, 2015; Boubert et al., 2018). Furthermore, the velocity distributions shown in Fig. 9.2 are relatively smooth, suggesting the presence of a single population dominated by main sequence halo stars. Nonetheless, it would be interesting to follow-up spectroscopically stars near the escape velocity. In Sec. 9.7.3 we will revisit possible outliers in the solar neighbourhood.
9.3.2 Adopting a prior on \( v_{\text{esc}} \) and \( k_t \)

In line with the literature, we assume a simple prior on \( v_{\text{esc}} \) of the form \( P(v_{\text{esc}}) \propto 1/v_{\text{esc}} \). For \( k \) (we will use the notation \( k \) here, understanding that it only compares to \( k_t \) and \( k_r \) in the limiting case) there is some debate in the literature on what to assume, and since \( v_{\text{esc}} \) and \( k \) are highly degenerate (see next section), the prior assumed might bias the resulting \( v_{\text{esc}} \). For example, the M18 and D19 estimates of \( v_{\text{esc}} \) differ by \( \sim 50 \) km/s mainly because of the different ranges considered for \( k \). Attempting to estimate \( v_{\text{esc}} \) and \( k \) simultaneously is only possible with a large sample with very small uncertainties. For example, LT90 estimated that a sample of > 200 stars with high-quality radial-velocities above \( v_{\text{cut}} \) is necessary to estimate both values simultaneously.

LT90 argue that \( k \)-values should be in the range \([0.5 – 2.5]\), because this brackets \( k = 1.5 \), which is the value expected for a system that has undergone violent relaxation (Aguilar & White, 1986; Jaffe, 1987; Tremaine, 1987). S07 have compared stellar halos in cosmological simulations of Milky Way-like galaxies and found a range of \([2.7 – 4.7]\) to be more appropriate. P14 building on more recent such simulations reduced this range to \([2.3 – 3.7]\), which is also the range used by M18. D19 updated the criteria for finding Milky Way analogues based on recent discoveries regarding the merger history of the Milky Way (Belokurov et al., 2018; Helmi et al., 2018). When using cosmological simulations, the range \([1.0 – 2.5]\) was found to be more favoured. Using a sample of BHB stars, K-giants, and MSTO stars from SDDS with only line-of-sight velocities, W17 determine both \( k \) and \( v_{\text{esc}} \) simultaneously. They report a value for \( k_r \) of \( 4 \pm 1 \) for the local stars.

The above paragraph shows that no consensus has been reached on the value of \( k \) for the Milky Way. To complicate matters, the ranges mentioned above were determined for the stellar halo at the position of the Sun (in the simulations). Although it is not clear whether \( k \) remains constant as a function of distance to the Galactic Centre, in this Chapter, we will also rely mainly on the estimate of the power law index at the location of the Sun. This is where our sample contains many stars with reliable parallax information and which are approximately isotropically distributed on the sky. For this local sample of stars, we calculate the marginal posterior distribution for \( k_t \). We will apply this posterior as a prior to other distance bins in which we estimate \( v_{\text{esc}} \). In doing so, we assume that \( k_t \) does not vary (much) over the distance range that we probe, which is also justified by the analysis we carry out in Sec. 9.4.2.

9.4 Validating the method

Before applying the method to the data we will attempt to establish the accuracy of the method, the sample size required to estimate both \( v_{\text{esc}} \) and \( k_t \) at the same time, and the effect of the velocity cut-off \( v_{\text{cut}} \). We first look at mock data and then apply the method on cosmological simulations.
Fig. 9.4: Maximum likelihood estimation of $v_{\text{esc}}$ and $k_t$ derived using tangential velocities for a mock data sample drawn from a power-law distribution in the velocity modulus, and convolved with realistic errors. The input parameters $v_{\text{esc}}$ and $k$ are indicated with a red marker.

### 9.4.1 Tests with mock data

The mock data are drawn from an idealised power-law distribution. We sample velocities according to the distribution given by Eq. (9.7) assuming $v_{\text{esc}} = 550$ km/s and $k = 2.3$. We then set one of the three velocity components (i.e. the ‘line-of-sight velocity’) artificially to zero by assuming that the velocities are isotropically distributed. In practice, we randomly distribute the velocities on a sphere with radius equal to the velocity modulus. Next, we artificially set one of the 3D components to zero and compute the new velocity modulus, which thus corresponds to the tangential velocity. Finally, we convolve the resulting distribution with realistic errors drawn from the distribution of errors (i.e. that shown in Fig. 9.3), for the photometric distances sample.

Figure 9.4 shows the results of applying the formalism described in Sec. 9.3 to this dataset for three different sample sizes (see annotation) above $v_{\text{cut}} = 250$ km/s. The contours indicate the level where the likelihood has dropped to 1% of the maximum value and the red marker shows the input value for $v_{\text{esc}}$ and $k$. Decreasing the number of stars (from 10 000 to 500) results in higher uncertainties in the estimates of the $v_{\text{esc}}$ and $k_t$ parameters. A sample with $\sim 10^4$ stars is sufficiently large to determine both $v_{\text{esc}}$ and $k_t$ at the same time, given the amplitude of the velocity uncertainties.
Figure 9.5: Mock tangential velocity distributions drawn using Eq. (9.7), for two cut-off velocities (250,500) km/s (left and right panels, respectively). The red line indicates the distribution that is expected when $v_t \rightarrow v_{esc}$ (i.e. Eq. (9.12)).

Figure 9.4 shows that there is a slight tension between the estimated value of $k_t$ and the input value $k$, which we already anticipated in Sec. 9.3.1. Figure 9.5 reveals that the reason for this is that the power-law index of the $f_t(v_t)$ distribution is not exactly $k$, since this relation holds only for values of $v_t$ approaching $v_{esc}$. Although this does not invalidate our approach at all since $v_{esc}$ is robustly determined without any biases, we nonetheless have to be cautious when comparing the value of $k_t$ obtained using tangential velocities. Similar considerations are in order when applying the LT90 method to a sample of line-of-sight velocities only.

9.4.2 Tests on Aurigaia Milky Way-like halos

We now test the method on two halos from the Aurigaia suite of mock Gaia catalogues (Grand et al., 2019). We explore here whether the tail of the velocity distribution is well described by a power-law, the effect of velocity clumps, the behaviour of $k_t$ as a function of distance, and the power of the method given the typical errors in the tangential velocity in our sample.

The Aurigaia catalogues have been generated from the Auriga suite of Milky Way-like galaxies (Grand et al., 2017) – which is a suite of high-resolution, zoom-in re-simulations based on galaxies extracted from a dark-matter-only counterpart of the EAGLE simulations (Schaye et al., 2014). The re-simulations are carried out with the AREPO moving-mesh code (Springel, 2010). The mock catalogues that we analyse correspond to halos 6 and 27, have the bar at 30 degrees orientation, and were generated with the SNAPDRAGONS code (Hunt et al., 2015). We will refer to these simulations as Au-06 and Au-27. These specific halos are chosen somewhat at random, although Au-06 is the closest example to the MW
Fig. 9.6: Truncation of the velocities in the Aurigaia halos as a function of galactocentric distance. The velocity distribution in the halos is truncated at $\sim 95\%$, except for a few bins in the outer regions of Au-27. The black markers show the stars that are the closest to the escape velocity. To indicate how densely populated the high-velocity tail also the 10th star fastest star is shown (grey markers).

The high-velocity tail in Aurigaia

Figure 9.6 shows the velocity of the fastest moving stars as a fraction of the escape velocity as a function of distance. We see that typically, the fastest star moves at 90 – 100% of the true $v_{\text{esc}}$ throughout the range of galactocentric distances probed. The escape velocity according to halo spin’ according to Grand et al. (2018). The halo of Au-06 has a similar mass as the Milky Way (i.e. $M_{200} \approx 10^{12} \, M_\odot$), whereas that of Au-27 is slightly more massive: $M_{200} \approx 1.7 \cdot 10^{12} \, M_\odot$ (Grand et al., 2017). Both halos are mildly prograde ($\sim 30 – 70 \, \text{km/s}$), as measured by the mean rotational velocity of accreted stars with ‘heliocentric’ distances smaller than 1 kpc.

Because the original Auriga simulations do not have the resolution of Gaia DR2 ($\sim 10^9$ stars), the SNAPDRAGONS code has been used to artificially increase the number of objects, whereby simulated stellar particles are split into multiple ‘stars’. This leads to artificial enhancement of the clustering of stars in phase-space, which can lead to biases in the determination of the escape velocity. Therefore, here we only use unique stellar particles by filtering all duplicates using the true HCoordinates and HVelocities parameters in the Aurigaia catalogue.
Fig. 9.7: Determination of \( v_{\text{esc}} \) and \( k_t \) as a function of galactocentric distance. The results for both a fixed (blue) and adaptive (green) cut-off velocity are shown. The yellow contour in the background shows the true escape velocity, calculated as \( \sqrt{2|\text{GravPotential}|} \). The grey contour has been shifted downwards by 10%.

has been calculated here as the velocity needed to reach \( r \to \infty \) for the potential given in the Aurigaia catalogue (parameter: GravPotential), and using

\[
v_{\text{esc}}(r \to \infty) \equiv \sqrt{2|\Phi(r) - \Phi(\infty)|} = \sqrt{2|\Phi(r)|}.
\] (9.17)

In Fig. 9.6 the black markers correspond to the fastest star while the grey markers indicate the location of the 10th fastest star and provides an idea of the steepness of the velocity tail.

Interestingly, for the halo of Au-27 the velocity distribution is truncated close to the escape velocity around a galactocentric radius of \( \sim 15 \) kpc. In the inner regions, particularly for Au-6 but also to some extent for Au-27, the difference between the fastest and the 10th fastest moving star shows typically less scatter, indicating that there are many stars near the truncation of the velocity distribution.

**Determination of the escape velocity in Aurigaia**

We follow a similar procedure as for the data to select stars with large tangential velocities from the Aurigaia halos. Firstly, the tangential velocities are convolved with errors drawn from the ‘observed’ error distribution shown in Fig. 9.3 (and as in Sec. 9.4.1). We then select stars that have \( v_t > 200 \) km/s. Next, we artificially set the line-of-sight velocities to zero and select stars with \( |\vec{v} - \vec{v}_{\text{LSR}}| > 250 \) km/s (as in Sec. 9.2.3). Although the Aurigaia catalogues do not exactly represent the Milky Way, these velocity cuts serve to remove the thin disc and (most of) the thick disc present in the simulations.
Fig. 9.8: Velocity distribution in Au-27 in two distance ranges. Left: a smooth distribution of stars in the range 9 – 11 kpc that truncates shortly before the escape velocity indicated by the dashed vertical line. Right: a clearly non-smooth velocity distribution for stars in the range 15 – 17 kpc that reaches up to the escape velocity. This figure shows the full velocities (and not $v_t$) to emphasise the clumpiness.

We then determine $v_{\text{esc}}$ and $k_t$ in concentric shells of 1 kpc in width centred on the galaxy’s centre, with radii ranging from 2 – 21 kpc. For both Auriga halos the cut-off velocity is set at $v_{\text{cut}} \approx 250$ km/s. This is well below the escape velocity in all the distance bins we probe. We also test a heuristic procedure to determine $v_{\text{cut}}$ by taking the maximum of 250 km/s and the velocity of the 10 000th fastest moving star (20 000th for Au-27). For bins with a large number of stars, this pushes the cut-off to higher values.

We noticed that for the Au-27 halo, the top 20 000 stars works better to determine $v_{\text{cut}}$ than the top 10 000. Since this halo is more massive than that of Au-06, its escape velocity is higher and there are more stars with extreme velocities. However, because the Au-06 halo is more similar to the Milky Way, we expect that 10 000 is a realistic number of stars for the Milky Way.

Figure 9.7 shows the results of fitting the tangential velocity tail in the halos of Au-06 (left) and Au-27 (right). In yellow we show the mean $v_{\text{esc}}$ that is calculated from Eq. (9.17) by using the pre-computed potential energies of every particle (i.e. the GravPotential parameter). The width of the yellow region indicates that there is a range of escape velocities at a fixed radius. This range exists because the potential is not spherically symmetric. Stars closest to the disc experience a stronger potential than those slightly farther away.

The results obtained from a fixed $v_{\text{cut}}$ are indicated with blue markers, while green markers are for the adaptive $v_{\text{cut}}$. The top panels of Fig. 9.7 show that the estimates are systematically too low compared to the expected $v_{\text{esc}}$ for both halos. However they match well with the grey curve which has been obtained by lowering by 10% the yellow
This is a reflection of a truncation in the tangential velocity distribution, in that it does not extend all the way to $v_{\text{esc}}$. Note that in both halos, the features in the $v_{\text{esc}}$ curve are matched closely by the velocity features apparent for the 10th fastest stars shown in Fig. 9.6. An interesting result is that $k_t$ varies only weakly over distance as can be seen from the bottom panels of Fig. 9.7.

The above results mean that the determination of $v_{\text{esc}}$ with the method described in Sec. 9.3 is sensitive to the behaviour of the tail of the velocity distribution. This is particularly clear for Au-27 which shows a bump in $v_{\text{esc}}$ at $d \gtrsim 15$ kpc. In fact there is an excess of stars (a clump) in the halo of Au-27 that is moving at a velocity close to $v_{\text{esc}}$ as can be seen by comparing the panels of Fig. 9.8, which plot the velocity distributions for the distance ranges 9 – 11 kpc and 15 – 17 kpc.

In summary, the analysis of the Aurigaia experiments analysis shows that

• $v_{\text{cut}}$ can be determined from the top 10 000 stars.
• We may assume that $k_t$ varies only weakly over the distance range probed by the RPM sample.
• On average the method underestimates $v_{\text{esc}}$ by $\sim 10\%$. This is slightly more than the 7% estimated by Grand et al. (2019), which might be related to differences in the method (e.g. the convolution with an error distribution and the typically large uncertainties on $v_t$).
• By determining $v_{\text{esc}}$ over a range of galactocentric distances we can check for local ‘biases’.

### 9.5 Results: solar neighbourhood

We determine the escape velocity at the solar position using the two samples of stars described in Sec. 9.2, one with full 6D information and the other with only tangential velocities (5D). We consider only stars with a heliocentric distance of 2 kpc or less. We evaluate the modified likelihood function (Eq. (9.16)) on a grid of $100 \times 100$ points ranging from $400 \text{ km/s} < v_{\text{esc}} < 800 \text{ km/s}$ and $1 < k_t < 6$ (both for the 5D and 6D cases). These ranges bracket the values that are presented in the literature. For the 5D sample the cut-off velocity is based on the 10 000th fastest star and set to 317 km/s, and for the 6D sample it is 250 km/s. Although the results for the 6D sample are consistent when $v_{\text{cut}}$ is set to 317 km/s, in this case the inference on $k$ is less strong.

The likelihood contours for the two samples are presented in Fig. 9.9. For the sample with full phase-space information we plot the results for the Gaia-only data (6D) and also including the additional data from ground-based spectroscopic surveys (6D+). The arrows in the figure indicate the maximum likelihood values for each sample. The contours correspond to the levels where the likelihood has dropped by [50%, 90%, 99%] of the maximum value. For reference, these levels roughly correspond to 1-, 2-, 3$\sigma$ contours if the likelihood function would have been a Gaussian. The side-panels show the marginalised distributions $P(v_{\text{esc}})$ and $P(k)$ ($P(k_t)$ for the 5D sample). These distributions are the best constrained for the 5D sample (in blue) because of its large number of stars.
Fig. 9.9: Likelihood contours obtained by applying the LT90 method to the 5D and 6D samples in the solar neighbourhood. For each curve the 99%, 90%, and 50% confidence levels are shown and the arrows indicate the maximum likelihood values. The side panels show the marginalised posterior distributions for $P(v_{\text{esc}})$ and $P(k)$. For the 6D sample we show the results for both the augmented dataset and when using Gaia data only. For the 5D sample, recall that the method determines $k_t$, and this is what is shown on the y-axis of the main panel, while the blue curve in the right panel represents $P(k_t)$. 

9.5 Results: solar neighbourhood
Tab. 9.1: Escape velocity ($v_{\text{esc}}$), power-law index ($k_t$, for the 5D and $k$ for the 6D samples) and the number of stars ($N_{\text{stars}}$) for different distance estimates in the solar neighbourhood. The errors in $v_{\text{esc}}$ and $k_t$ (or $k$) are given by the marginalised 99% confidence intervals. The upper limits on the Gaia-only estimation are calculated for a larger range of $k$ values than what is shown in Fig. 9.9 because otherwise the contours are not closed.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$v_{\text{esc}}$ [km/s]</th>
<th>$k_t/k$</th>
<th>$N_{\text{stars}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5D</td>
<td>497$^{+40}_{-24}$</td>
<td>3.4$^{+1.4}_{-0.9}$</td>
<td>10 000</td>
</tr>
<tr>
<td>6D+</td>
<td>497$^{+53}_{-20}$</td>
<td>3.0$^{+1.2}_{-0.6}$</td>
<td>2067</td>
</tr>
<tr>
<td>6D (Gaia only)</td>
<td>505$^{+214}_{-32}$</td>
<td>3.0$^{+4.4}_{-1.0}$</td>
<td>495</td>
</tr>
</tbody>
</table>

The marginal distributions of $v_{\text{esc}}$ agree very well with each other for all samples. The slight tension in the 5D and 6D curves (the contours are however consistent within the $\sim 90$%-level), is driven by the anticipated differences that are the result of using the full velocity modulus or tangential velocity information only, i.e. $k \neq k_t$ for $v_{\text{cut}}$ far from $v_{\text{esc}}$ (c.f. the contours and red marker in Fig. 9.4).

Marginalising over $k_t$, we find a maximum likelihood value $v_{\text{esc}} = 497^{+40}_{-24}$ km/s for the 5D sample, which we stress is a very precise lower-limit to the actual $v_{\text{esc}}$. The quoted errors correspond to the marginalised 99% contour levels (i.e. the $\sim 3\sigma$ level). Table 9.1 presents $v_{\text{esc}}$ and $k_t$ (or $k$) derived for all the samples considered and curves shown in Fig. 9.9.

9.6 Results: Beyond the solar neighbourhood

9.6.1 Determination of $v_{\text{esc}}$

We now proceed to determine $v_{\text{esc}}$ as a function of galactocentric distance. As we have seen in the Aurigaia halos, the behaviour of $v_{\text{esc}}$ as a function of distance can help in identifying local ‘biases’ or issues. We will here assume as prior for $k_t$ the marginalised distribution obtained for the solar neighbourhood $P(k_t)_{\text{SN}}$, and shown in the right panel of Fig. 9.9 with the blue curve. Therefore, we implicitly assume that $k_t$ remains constant over the distance range probed. This assumption is justified by the Aurigaia simulations, as shown in Fig. 9.7.

We sliced the data in 16 concentric shells of 1 kpc width, with $4 < r < 12$ kpc and centred on the Galactic Centre (as in Fig. 9.2). The number of stars per bin, with velocities larger than $v_{\text{cut}}$, varies from 58 744 to 1128. In each shell, $v_{\text{cut}}$ is determined adaptively by selecting the top 10 000 fastest stars. We do note that the results do not change significantly when the cut-off is fixed to $v_{\text{cut}} = 250$ km/s.

Figure 9.10 shows the trend of our estimate of $v_{\text{esc}}$ with galactocentric distance. In each bin, the likelihood map has been marginalised over the range $2.6 < k_t < 4.8$, after applying $P(k_t)_{\text{SN}}$. This range in $k_t$ corresponds to the 99% interval of the posterior
distribution of $P(k_t)_{SN}$. Note that this is a very similar range to that assumed in S07. The use of the $P(k_t)_{SN}$ prior beyond the solar neighbourhood has helped in the determination of $v_{esc}$ for all the distance bins considered, despite the sometimes relatively small number of stars used. With the size of the samples presently available we could not have constrained both $k_t$ and $v_{esc}$ simultaneously for all radial bins.

The behaviour of the escape velocity in the inner halo ($r < 8$ kpc) matches well the expectation from several Milky Way models. This can be seen by comparison to the predicted escape velocity plotted in the background of Fig. 9.10 for the Piffl14, McMillan17, BT08 (model I), and MW14 potentials (Piffl et al., 2014a; McMillan, 2017; Binney & Tremaine, 2008; Bovy, 2015, all computed using the implementation from AGAMA, Vasiliev 2019). The behaviour for the estimated $v_{esc}$ shows small variations: a slight elevation at $\sim 6$ kpc and a dip at $\sim 4.5$ kpc, although it is fully consistent with a smooth increase towards the inner Galaxy. Furthermore, the amplitude of these variations is of a similar level as what we observed in the Aurigaia simulations. Curiously, our estimate of $v_{esc}$ increases beyond the solar radius (i.e. distance $> 8$ kpc). This cannot be driven by the mass profile of the Milky Way and can only mean that something is biasing the determination of $v_{esc}$, as we discuss in detail next.

**Fig. 9.10:** Escape velocity as a function of galactocentric distance (top). We also shown in grey the expected behaviour of the escape velocity for four often-used Milky Way models. The bottom panel shows the logarithm of the number of stars for each distance bin. The blue marker indicates the $v_{esc}$ that we determined using a local sample of stars, see Sec. 9.5. The slight difference is due to a small difference in the size of the volume considered.
Fig. 9.11: Two-point correlation function $\xi$ of the pseudo Cartesian velocities of the stars, binned by galactocentric distance. A correlation of $\xi > 1$ indicates an excess of pairs compared to a random sample. The random sample $\langle RR \rangle$ is obtained by randomly shuffling the velocities in the galactic rest frame.

9.6.2 What drives the increase of $v_{esc}$

Several effects could give rise to an increasing $v_{esc}$ outside the solar radius, namely (i): biases in the data (e.g. in the distance estimate); (ii) biases in the method (e.g. sample size), and (iii) variations in the dynamical properties of the stars with distance. We already explored the biases introduced by the first two categories in Sec. 9.2 (see also KH20) and Sec. 9.4.1. Nevertheless, we also tested that when the sample is downsized to a random subset of 5 000 stars and bins with fewer stars are excluded, the results do not change. We therefore now focus on the third possibility.

Careful inspection of Fig. 9.2 shows that the velocity distribution is not contaminated by single outliers, even though in a relative sense (to the absolute number of objects) there seem to be more extreme velocity values in the outer radial bins. However, as mentioned earlier, the figure does show that the distributions seem to become more exponential with distance.
Curiously, we have seen a similar increase for Au-27 in the \( v_{\text{esc}} \)-profile as observed for the 5D sample, see Fig. 9.7. In that case, the increase in \( v_{\text{esc}} \) was tentatively attributed to the presence of tidal debris (or at least lumpiness) moving with speeds close to the true escape velocity.

With a two-point velocity correlation function, we test the statistical clustering of the stars in the tail of the velocity distribution for our 5D sample. An excess of pairs implies that the velocity distribution is not smooth. The two-point velocity correlation function is given by

\[
\xi(\Delta v) = \frac{DD(\Delta v)}{\langle RR(\Delta v) \rangle},
\]

where \( DD(\Delta v) \) is the number of data-data pairs with a velocity separation of \( \Delta v \) and similarly \( \langle RR(\Delta v) \rangle \) is the mean number of random-random pairs obtained by randomly shuffling the velocities 100 times. To this end, \( v_f \) and \( v_b \) are shuffled and the pseudo Cartesian velocities are re-calculated from Eqs. (9.6) (for \( v_{\text{los}} = 0 \)). Both the data and re-shuffled samples are cut-off at the velocity of the 10 000th star, or 250 km/s if there are not enough stars per bin. Because of this re-sampling, most of the bins have an equal number of stars, except for those at large radii.

Figure 9.11 shows the results of the correlation function \( \xi \) for the 5D sample for the same distance bins as used throughout this Chapter. The curves in Fig. 9.11 are coloured by the mean distance of the bin and the error bars show the uncertainty in \( \xi \) due to the Poisson error in the number of counts per bin. A value \( \xi = 1 \) indicates no excess correlation. We see that inside 8 kpc, \( \xi \) decreases with distance. Meanwhile, the bins just outside of this radius (light red) show the largest level of correlation over the full velocity range probed. The inner and outermost bins (dark colours) show the least correlation, although the uncertainties are large because of the low number of stars in these bins. There might also be an effect associated to the area of the shells increasing with distance squared, which results in the stars in the outer shells being physically more separated than those in the inner shells, and which could give rise to gradients in the trajectories of the stars and hence to lower correlation amplitudes.

The analysis of the velocity correlation function confirms that the properties of the velocity distribution change with distance. A hint of velocity clustering at \( r \sim 10 \) kpc in our 5D sample, similar (although of lower amplitude) to that seen for Au-27, could thus be responsible for this change.

### 9.7 Discussion

#### 9.7.1 Relating \( v_{\text{esc}} \) to the Milky Way’s potential

In Eq. (9.17) we defined the escape velocity as the velocity to reach \( r = \infty \). A more realistic definition is obtained by taking a different zero-point. No matter how one defines ‘escaping from the Milky Way’, stars do not have to travel to infinity to be considered as escapees. For example, stars escaping to M31 make a much shorter journey (\( r \approx 800 \) kpc).
Therefore, we use here the definition of P14, who take the escape velocity to be the velocity required to reach $3r_{340}$

$$v_{\text{esc}}(r \to 3r_{340}) \equiv \sqrt{2|\Phi(r) - \Phi(3r_{340})|},$$

(9.19)

where $r_{340}$ is the radius within which the average halo density is $340 \times \rho_{\text{crit}}$ (which is equal to $3H^2/8\pi G$, and where we assume $H = 73 \text{ km/s/Mpc}$). We note that this zero-point is set somewhat arbitrarily, the ‘true’ value is directional dependent and might be a few km/s higher or lower. D19 use a different definition, which is for the star to escape to $2r_{200} \approx 2.5r_{340}$. At the solar position, these two definitions result in a difference of 5 km/s.

Because the potential is axisymmetric, $v_{\text{esc}}$ varies as a function of cylindrical $R$ and $z$ for a fixed spherical $r$. In the plane of the disc, where the potential is the steepest, the escape velocity is the highest. Using the McMillan (2017) potential, we estimate that $v_{\text{esc}}$ decreases by $\sim 20 \text{ km/s}$ when moving 5 kpc away from the plane of the disc, whereas at 10 kpc the difference is about 50 km/s.

To develop some intuition on how properties like the mass of the Milky Way are related to $v_{\text{esc}}$ we use the following equations. For a spherical potential, the gradient $d v_{\text{esc}}/dr$ is related to the mass, circular velocity, and potential as

$$\frac{d\Phi(r)}{dr} = -v_{\text{esc}}(r) \frac{dv_{\text{esc}}(r)}{dr} = \frac{v_{\text{circ}}^2(r)}{r} = \frac{GM(r)}{r^2}. \quad (9.20)$$

Another insightful equation, given by Eq. (2-22) of Binney & Tremaine (1987) is

$$v_{\text{esc}}(r_{\odot})^2 = 2v_{\text{circ}}(r_{\odot})^2 + 8\pi G \int_{r_{\odot}}^{\infty} r \rho(r) \, dr, \quad (9.21)$$

(see S07). The circular velocity $v_{\text{circ}}$ at the solar position is a direct measure of the mass inside of the solar radius. On the other hand, the escape velocity $v_{\text{esc}}$ is a measure of the total gravitational potential. The two are related through a factor of $\sqrt{2}$ only if there is no mass outside of the radius where both are measured. In other words, the difference $v_{\text{esc}}^2 - 2v_{\text{circ}}^2$ at the solar neighbourhood probes the potential, and with it the mass distribution beyond the solar radius.

9.7.2 A lower limit to the mass of the Milky Way’s halo

We will now use our very precise lower limit to the escape velocity at the position of the Sun to provide a precise lower limit to the mass of the halo of the Milky Way. The escape velocity and the gravitational potential of the Milky Way are related through Eq. (9.19). A straightforward procedure to derive the mass of the Milky Way is to take an existing model and adjust the parameters of the halo such that it matches the $v_{\text{esc}}$ measured for the solar neighbourhood. We follow closely the procedure outlined in Sec. 5 of D19,
however here we will use the McMillan (2017) potential and vary only the parameters of its dark halo, which is represented by an NFW profile (Navarro et al., 1997).

The only issue with this procedure is that $v_{\text{esc}}(r_\odot)$ is mostly sensitive to the mass outside of the solar radius. As a result, fitting $v_{\text{esc}}$ constrains only weakly the concentration of mass inside the solar radius. A solution is to use the circular velocity ($v_{\text{circ}}$), which is sensitive to the mass inside the solar radius, as an additional constraint. That is when fitting $v_{\text{esc}}(r_\odot)$ we force the model to have a certain $v_{\text{circ}}(r_\odot)$.

The best-fitting potential is defined as the one that minimises

$$
(v_{\text{esc}}(r_\odot) - v_{\text{esc}}^{\text{est}})^2 + (v_{\text{circ}}(r_\odot) - 232.8 \text{ km/s})^2,
$$

where we take $v_{\text{esc}}^{\text{est}}$ to be the maximum likelihood value found in the solar neighbourhood for the 5D sample (see Table 9.1). The value for the circular velocity that we assume, $v_{\text{circ}}(r_\odot) = 232.8 \text{ km/s}$, is the value that was used in the original McMillan (2017) potential. We note that there is no freedom in choosing $v_{\text{circ}}(r_\odot)$ because the data is only consistent with the value above, as it is used in the correction for the solar motion.

Figure 9.12 shows the values for Eq. (9.22) for the ranges of $M_{200}$ and $c$ that we explore, namely $\log_{10}(M_{200}) \ [M_\odot] \in [11.5, 12.5]$ and $c \in [1, 30]$. The solid line marks all models that have a correct $v_{\text{circ}}(r_\odot)$ and the dashed lines mark all models that have
the correct $v_{\text{esc}}^\text{est}$. The best fitting potential lies at the intersection of the two lines. The curves illustrate the benefit of including the $v_{\text{circ}}$ in the fit, since as expected the dashed curve is only weakly sensitive to $c$. The orange marker highlights the combination of $M_{200}$ and $c$ that best fits $v_{\text{esc}}$ and $v_{\text{circ}}$. Therefore the best fitting lower limit for the mass is $M_{200} = 0.67^{+0.30}_{-0.15} \cdot 10^{12} \, M_\odot$ and the corresponding concentration parameter is $c = 15.0^{+2.6}_{-2.3}$. The errors are derived by calculating the best fitting $M_{200}$ and $c$ for the extreme cases of $v_{\text{esc}}^\text{est} + 40 \, \text{km/s}$ and $v_{\text{esc}}^\text{est} - 20 \, \text{km/s}$, which are the limits given by the 99\% level (e.g. Table 9.1).

Our lower limit on the mass of the Galactic halo is consistent with that in the original potential of McMillan (2017) in the sense that it is smaller and more concentrated. Moreover, this lower limit is also lower than most recent mass estimates (c.f. Fig. 7 Callingham et al., 2019, for a recent compilation). If we now use the results from the analysis of the Aurigaia simulations and adjust for the 10\% underestimation of $v_{\text{esc}}$ (grey dashed curve in Fig. 9.12), we find that best-fit mass and concentration parameter are $M_{200} = 1.11^{+0.50}_{-0.23} \cdot 10^{12} \, M_\odot$ and $c = 11.8^{+1.2}_{-2.1}$.

### 9.7.3 Stars that might be unbound

A possibly interesting follow-up project is to measure the radial velocities of the stars in the 5D sample that lie near the truncation of the best-fit power-law. Using the maximum likelihood fit of the velocity distribution we can calculate which stars have a high probability of being unbound. Given the apparent $v_t'$ and its error, we can calculate the probability of these stars having a true $v_t$ larger than $v_{\text{esc}}$. We note that strictly speaking the errors are non-Gaussian, see also Sec. 9.3. However, we assume that the errors are small enough such that they may be approximated to be Gaussian.

For the set of stars that have apparent tangential velocities larger than the estimated $v_{\text{esc}}$ we calculate the probability of the star being bound as

$$P_{\text{bound}} = \sum_{v_{\text{esc}}^i} P_{\text{SN}}(v_{\text{esc}}^i) \int_{0}^{v_{\text{esc}}^i} f_G(v_t', v_t, \sigma_t) dv_t,$$

where $P_{\text{SN}}(v_{\text{esc}})$ is the posterior of $v_{\text{esc}}$ marginalised over $k_t$ (i.e. the blue curve in the right panel of Fig. 9.9). The error distribution $f_G(v_t', v_t, \sigma_t)$ is defined such that it gives the probability of finding the star with a true velocity $v_t$ and error $\sigma_t$ with an apparent velocity in the range $(v_t', v_t' + dv_t')$. The probability of the star being unbound is simply $P_{\text{unbound}} = 1 - P_{\text{bound}}$.

The list of sources that fall outside of the maximum likelihood value of $v_{\text{esc}}$ is given in Table 9.2. We stress that, very likely, the actual $v_{\text{esc}}$ is higher than our best estimate. The values for $P_{\text{unbound}}$ given here should therefore be considered as upper-limits. To emphasise this we also calculate the probability of these stars being unbound after correcting $v_{\text{esc}}$ for a 10\% offset, based on our analysis in Sec. 9.4.2. Only two sources remain unbound in this case, of which one barely. The source with the largest probability of being unbound, with identifier Gaia DR2 source_id 2655054950237153664, has been
flagged in the faststars\footnote{https://faststars.space/ (Guillochon et al., 2017)} database as a potential hyper-velocity star. The source was first identified by Du et al. (2019) based on its large tangential velocity.

About half the sources in Table 9.2 have an inward-pointing velocity vector, based on the pseudo velocities in the galactocentric frame. This makes it likely that the majority of these stars are bound to the Milky Way. Of course, there remains a possibility that the stars’ velocity vectors will all point radially outwards when line-of-sight velocities are measured. However, for some stars the vectors will always point inwards even in the extreme case of $v_{los} = \pm 500 \text{ km/s}$. We do note that the velocity vector of the most unbound star (source_id 2655054950237153664) barely changes, and always points outwards, even for adopted line-of-sight velocities of $\pm 500 \text{ km/s}$ - and therefore might truly be unbound.

### 9.7.4 $v_{esc}$ as tracer of the mass distribution

The luminous components of the Milky Way are most definitely not spherically symmetric. Because the escape velocity traces the potential we should ultimately measure it in axisymmetric coordinates rather than as a function of spherical radius. By estimating $v_{esc}$ as a function of $z$ we can perhaps constrain the flattening of the halo, although with the
current sample we are more sensitive to the contribution of the disc to the total potential of the Milky Way. Therefore, such an analysis would benefit from a large sample of stars probing deeper into the Milky Way’s halo, such as what may become available with Gaia (e)DR3.

Because of the large number of sources in our 5D sample, it is for the first time possible to explore the escape velocity as a function of cylindrical $R$ and $z$. We slice our 5D sample in overlapping bins of $8 \times 11$ volumes of $|R_c| < 1$ kpc and $|z_c| < 1$ kpc, where $R_c$ and $z_c$ are the centres of the volumes. Assuming that the Milky Way is perfectly axisymmetric, we include sources independent of their azimuthal angles. Bins with less than 500 stars are discarded. We use the same method to determine the escape velocity as we used in
Sec. 9.6 and presented in Fig. 9.10. That is, we again assume the posterior distribution of \( P_{SN}(k_t) \) from the solar neighbourhood as prior on \( k_t \). For computational reasons, we have decreased the size of the grid on which the modified likelihood function is evaluated to \( 50 \times 22 \) points ranging from \( 400 \text{ km/s} < \nu_{\text{esc}} < 800 \text{ km/s} \) and \( 2.6 < k_t < 4.8 \) (which corresponds to the 99% levels in the solar neighbourhood).

Figure 9.13 shows the escape velocity in these volumes (coloured, large markers) with a colour map corresponding to the escape velocity predicted by the McMillan17 model, with the updated lower limit of the halo mass computed in Sec. 9.7.2. Therefore this model is based on the lower limit for \( \nu_{\text{esc}} \) and we use it to predict what this lower limit would be at other locations for a spherical NFW halo. The large ‘+’ markers in Fig. 9.13 indicate volumes in which the 99% levels of the \( \nu_{\text{esc}} \) include the expected value. The large ‘×’ markers indicate volumes where the \( \nu_{\text{esc}} \) expected from the updated McMillan17 potential lies outside of the 99% confidence level of the maximum likelihood determination of \( \nu_{\text{esc}} \). Interestingly, the distribution is not fully symmetric in \( z \). The fact that \( \nu_{\text{esc}} \) does not match the expected value in many locations could potentially indicate a bias in the estimated \( \nu_{\text{esc}} \) at the solar neighbourhood. Another possibility is that the decrease in the strength of the potential with \( z \) is less steep than expected for a spherical halo (e.g. pointing to a prolate halo or less strong influence from the disc).

### 9.8 Conclusions

We have used a sample of halo stars with large tangential velocities to constrain the escape velocity in the vicinity of the Sun and as a function of galactocentric distance. We have applied the well-known LT90 method, which fits the high-velocity tail (i.e. above some velocity \( \nu_{\text{cut}} \)) of the velocity distribution with a power law of the form \((\nu_{\text{esc}} - \nu)^k\). In the process of applying the method, we identified a number of shortcomings.

The study presented here constitutes the first application of the method to a sample of stars using tangential velocities only. We have found that in practice, the estimated value for the parameter \( k \) is not exactly what is predicted by LT90 (namely \( k_t = k \)), except really in the tail of the distribution, i.e. where \( \nu_{\text{cut}} \) differs by 10% from \( \nu_{\text{esc}} \). Unfortunately, the value of \( \nu_{\text{cut}} \) typically chosen is farther away from \( \nu_{\text{esc}} \) because enough stars (\( \sim 10^4 \)) with high velocity need to be present in the sample for a precise estimate of \( \nu_{\text{esc}} \). A similar conclusion may be reached when applying the method to radial velocity samples. Therefore, care is necessary when comparing the values of \( k \) for different studies in the literature. On the other hand, no bias is present in the estimation of \( \nu_{\text{esc}} \).

In addition, and as previously discussed in the literature, the \( \nu_{\text{esc}} \) determined via the LT90 method is most likely a lower limit. To get a handle on this bias we have tested the method on two mock Gaia catalogues from the Aurigaia project (Grand et al., 2018). In these simulated galaxies the estimated \( \nu_{\text{esc}} \) are \( \sim 10\% \) lower than the true values, close to the 7% bias found in a similar study by Grand et al. (2019). Based on this result, when reporting our estimates of the escape velocity, we also quote the value obtained by applying a 10% correction. However, we note that there is no guarantee that the Milky
Way’s halo is truncated at a similar level as the Aurigaia halos. The truncation of the velocity distribution will be dependent on the (recent) assembly history of the Galaxy and for the simulations, it might depend on the numerical resolution.

In the solar neighbourhood, using a 5D sample, we determine a very precise lower limit to the escape velocity, $v_{\text{esc}} = 497^{+40}_{-24}$ km/s, and a power-law index $k_t = 3.4^{+1.4}_{-0.9}$. The quoted errors are given by the level where the likelihood has dropped to 99% of the maximum value (i.e. the $\sim 3\sigma$ level). These values agree well with previous works, but this is the first time, we can determine (a lower limit to) the escape velocity with such high confidence. This value for $v_{\text{esc}}$ agrees remarkably well that obtained when we use a local sample of halo stars with full phase-space information. Applying the 10% fix would mean that the true $v_{\text{esc}}^{+10\%} = 552$ km/s.

We also determine $v_{\text{esc}}$ as a function of galactocentric distance. We find that the escape velocity increases towards the inner halo matching well the behaviour expected from smooth Milky Way models. However, for radii beyond 8 kpc, $v_{\text{esc}}$ increases with distance (see Fig. 9.2).

Interestingly, we find that the increase in the estimated $v_{\text{esc}}$ is paired with an evolution of the velocity distribution. For example, the tail of the velocity distribution becomes more exponential (and less power law-like) with galactocentric distance (see Fig. 9.2). Also, the velocities in the bins outside of 8 kpc show a higher degree of correlation as measured by the velocity correlation function. Therefore, we conclude that the increase in $v_{\text{esc}}$ towards the outskirts is likely driven by a change in the kinematic properties of the sample as a function of galactocentric distance. Coincidentally, we found a similar effect in one of the Aurigaia halos analysed, where a velocity bump (presumably related to a clump or a non-phase-mixed structure in the halo) dominates the tail near the escape velocity.

The estimated $v_{\text{esc}}$ can be used to provide a very precise lower limit to the mass of the halo of the Milky Way. To this end, we have adjusted the halo component of the McMillan (2017) Milky Way potential, while keeping the other components fixed. The halo parameters that best fit the estimated $v_{\text{esc}}(r_{\odot})$ are $M_{200} = 0.67^{+0.30}_{-0.15} \cdot 10^{12} M_{\odot}$ and $c = 15^{2.6}_{-2.3}$, where we used $v_{\text{circ}}(r_{\odot})$ as an additional constraint. When we apply the tentative 10%-fix we find that the best fitting halo has $M_{200}^{+10\%} = 1.11^{+0.50}_{-0.23} \cdot 10^{12} M_{\odot}$ and $c^{+10\%} = 11.8^{1.2}_{-2.1}$.

The method to determine $v_{\text{esc}}$ consists in fitting the tail of the velocity distribution with a parametrised model. Using the best fitting model obtained, we can also establish if there are any unbound stars in the solar neighbourhood. That is, we may calculate which stars have a high probability of having a true velocity that is larger than the determined escape velocity. We list these stars in Table 9.2. Their pseudo velocities (without the line-of-sight velocity), however, suggest they are not all unbound: their velocity vectors point both inwards and outwards. If these high-velocity stars were truly escaping we would expect them to all be on radially outbound trajectories. Nonetheless, it might be interesting to follow-up these stars. When taking into account the tentative 10%-fix only
one candidate with a large probability of being unbound remains: Gaia DR2 source_id 2655054950237153664. This star was first flagged as being unbound by Du et al. (2019).

Finally, we discuss a tentative method to probe the mass distribution of the Milky Way by determining $v_{\text{esc}}$ as a function of ($R, z$). We find that escape velocity values that are weakly asymmetric with respect to the galactic plane, and also tentative indication that the halo may be prolate. However, for more robust conclusions a larger sample with more accurate distances and that probes deeper into the Milky Way is necessary. We hope that such a sample will become available with Gaia (e)DR3.

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Time evolution of gaps in stellar streams in axisymmetric Stäckel potentials

Based on: Koppelman and Helmi (2020), submitted to A&A.

Abstract

When a subhalo interacts with a cold stellar stream it perturbs its otherwise nearly smooth distribution of stars, and this leads to the creation a gap. The properties of such gaps depend on the parameters of the interaction. Their characterisation could thus lead to the determination of the mass spectrum of the perturbers and possibly reveal the existence of dark subhalos orbiting the Milky Way. Our goal is to construct a fully analytical model of the formation and evolution of gaps embedded in streams orbiting in a realistic Milky Way potential. To this end, we extend the model of Helmi & Koppelman (2016) for spherical potentials, and predict the properties of gaps in streams evolving in axisymmetric Stäckel potentials. We make use of action-angles and their simple behaviour to calculate the divergence of initially nearby orbits slightly perturbed by the interaction with a subhalo. Our model, corroborated by N-body experiments, predicts that the size of a gap grows linearly with time. We obtain analytical expressions for the dependencies of the growth rate on the orbit of the stream, the properties of the subhalo (mass, scale-radius), and the geometry of the encounter (relative velocity, impact parameter). We find that the density at the centre of the gap decreases with time as a power-law in the same way as the density of a stream. This results in the density-contrast between a pristine and a perturbed stream on the same orbit asymptotically reaching a constant value dependent only on the encounter parameters. We find that at a fixed age, smallish gaps are sensitive mostly to the mass of the subhalo, while gaps formed by subhalo flyby's with a low relative velocity, or when the stream and subhalo move parallel, are degenerate to the encounter parameters.

10.1 Introduction

The widely accepted $\Lambda$CDM model is very successful in reproducing the large-scale structure of the Universe (e.g. Davis et al., 1985), but it faces some key problems on small scales (e.g. Bullock & Boylan-Kolchin, 2017). For example, on the scales of individual galaxies, we observe much less substructure than what is predicted by dark matter only cosmological simulations (Klypin et al., 1999; Moore et al., 1999). Such simulations show that substructure exists down to very small scales and can be found at all radii,
although preferentially in larger numbers in the outskirts of galaxies’ halos (e.g. Diemand et al., 2008; Springel et al., 2008).

There exist several possibilities to solve this missing substructure conundrum. For example, adding baryonic physics to the simulations alleviates some of the problems, although mostly in the inner part of galaxies (e.g. D’Onghia et al., 2010; Zhu et al., 2016; Sawala et al., 2017). Adjusting the properties of the dark matter particle (e.g. self-interacting dark matter, warm dark matter, or fuzzy dark matter) can help in suppressing the formation of the smallest substructures (e.g. Spergel & Steinhardt, 2000; Hu et al., 2000; Bode et al., 2001; Vogelsberger et al., 2016; Bozek et al., 2016; Hui et al., 2017). Another solution is to assume that the structures are present but in a dark form. Dark structures only reveal their presence through gravitational interaction, rendering them very difficult to detect. Results from gravitational lensing support the existence of dark structures at a level that is compatible with ΛCDM (Dalal & Kochanek, 2002; Vegetti et al., 2010, 2012; Ritondale et al., 2019; Hsueh et al., 2020).

Establishing whether there exists a population of subhalos with masses < 10^8 M⊙ in and around the Galaxy is therefore of the utmost importance as it can lead to a better understanding of the nature of the dark matter particle. Clearly, the discrepancy between the predicted and observed small-scale structure could be hinting at a fundamental problem with our current cosmological paradigm.

In this Chapter, we will focus on a method to indirectly detect dark subhalos in our own Galaxy, through their possible interactions with cold stellar streams. Such streams are thin, almost one-dimensional elongated structures consisting of stars that originate from the tidal disruption of globular clusters or small dwarf galaxies. Because of their fragile nature, these streams are easily perturbed by gravitational interactions, making them promising probes of dark substructures (Ibata et al., 2002; Johnston et al., 2002). Occasional flyby’s of dark subhalos can lead to the creation of a gap in an otherwise relatively smooth distribution of stars (Yoon et al., 2011; Carlberg, 2013). Unfortunately, finding streams is challenging because of their low surface brightness, let alone finding gaps in streams. However, recent deep photometric surveys have identified a few dozen of narrow streams (e.g. Belokurov et al., 2006; Bernard et al., 2016; Shipp et al., 2018). The analysis of Gaia DR2 Gaia Collaboration et al. (2016, 2018) has also yielded another dozen streams (Malhan et al., 2018; Ibata et al., 2019). So far, only two of these streams have been claimed to contain gaps: GD-1 (Grillmair & Dionatos, 2006) and Palomar 5 (or Pal 5) (Odenkirchen et al., 2001), although several other streams show peculiarities (Bonaca et al., 2019a; Shipp et al., 2019; Li et al., 2020).

GD-1, is a promising stream to probe for gaps because of its length and coldness. It is known to contain several non-smooth features (Carlberg & Grillmair, 2013; De Boer et al., 2018; Price-Whelan & Bonaca, 2018). The origin of these features, or gaps, is currently highly debated in the literature. For example, they could have been formed by an interaction with a massive - dark - object of 10^6 – 10^8 M⊙ that might have once been part of the Sagittarius system (Bonaca et al., 2019b, 2020, see also Banik et al. 2019). On the other hand, it has been argued that the presence and nature of a nearly periodic spatial
distribution of gaps is an indication that these could be explained by internal dynamics without the need to recur to interactions with dark structures (Ibata et al., 2020).

Pal 5’s stream has been tentatively shown to host two gaps and several other features that would be consistent with being induced by subhalos in the range of $10^6 - 10^8 \, M_\odot$ (Erkal et al., 2017; Bovy et al., 2017). The inferred number of interactions appears to agree with the expected number predicted by CDM-only simulations (e.g. Sanderson et al., 2016). Unfortunately, Pal 5’s stream is not ideally suited to look for gaps due to dark structures because of its proximity to the Galactic Centre. The high baryon density in this region can lead to the formation of irregularities in the stream’s profile, for example, due to interactions with the bar (Pearson et al., 2017), globular clusters and with other baryonic structures (Banik & Bovy, 2018). Moreover, some of the gaps and features found in Pal 5’s stream may be explained by survey incompleteness (Thomas et al., 2016).

Since the expectation is that in the near future many gaps in many different streams will be detected, it is imperative to develop an in-depth understanding of the characteristics and evolution of these gaps. With such an understanding we may be able to link the population of gaps to an underlying population of dark substructures. For example, we need to establish the relation between subhalos and gap sizes, the growth rate of gaps and the dependence of their properties on the encounter parameters as well as on the characteristics of the host potential. Clearly, the ultimate goal would be to infer the properties of the perturbers from the analysis of the gaps observed.

Erkal & Belokurov (2015a) developed a framework that predicts the evolution of gaps formed in streams that are orbiting on circular orbits. Using this model, Erkal & Belokurov (2015b) showed how to infer the properties of a subhalo from the properties of a gap, down to a degeneracy in subhalo mass and relative velocity. A more recent model by Sanders et al. (2016) focuses on modelling gaps in angle-frequency space, allowing for eccentric orbits (see also Bovy et al., 2017). The authors validate several, but not all aspects of Erkal & Belokurov (2015a), and argue for example that the velocity dispersions in the underlying stream affect the evolution of the gap, and thus should be taken into account. A caveat of all these models is that they are not fully analytical - and thus always rely on numerical exploration of the parameter space - or they are limited to circular orbits only. For this reason, we presented a fully analytical model for the evolution of gaps in streams (Helmi & Koppelman, 2016, HK16 hereafter) orbiting in spherical potentials.

In this Chapter, we extend the HK16 model to streams orbiting in axisymmetric potentials. The model presented in this Chapter not only predicts the behaviour of the size of the gap as a function of time but also its central density and their dependence on the characteristic parameters of the encounter. This Chapter is structured as follows. In Sec. 10.2, we describe the model in detail and its predictions for the properties of gaps. In Sec. 10.3 we validate our model with N-body experiments. Subsequently, in Sec. 10.4 we analyse the dependencies of the gap’s properties on the collision parameters and investigate possible degeneracies in the parameters. Finally, we present a discussion and conclusions in Sec. 10.5.
10.2 Methods

The main reason to extend our HK16 model, which only works for spherical potentials, is that the Milky Way is more realistically described as an axisymmetric system. From a dynamical point of view, breaking the spherical symmetry will add a degree of freedom to the system.

The notation we use here is very similar to that employed in HK16. It builds on the action-angle stream description of Helmi & White (1999, HW99 hereafter), see also Helmi & Gomez (2007).

10.2.1 Choice of the potential

We are somewhat restricted in our choice for a potential for the Milky Way because our approach is based on the use of action-angle variables. These can only be calculated in potentials that are separable in the coordinates. For this reason, we will use Stäckel potentials, which are separable in ellipsoidal coordinates and are fully integrable (in fact, they are the only type of potentials with this property).

Because the (inner part of the) Milky Way is best described as an oblate system, we will use a set of prolate spheroidal coordinates \((\lambda, \phi, \nu)\) which we adopt from de Zeeuw (1985). The coordinate \(\phi\) is the azimuthal angle and the other two coordinates, \(\lambda\) and \(\nu\), are the roots for \(\tau\) in

\[
\frac{R^2}{\tau + \alpha} + \frac{z^2}{\tau + \gamma} = 1,
\]

where \(R = x^2 + y^2\), and \(\alpha\) and \(\gamma\) are constants related to the shape of the spheroid. The most general form of a Stäckel potential in these coordinates is

\[
\Phi(\lambda, \nu) = \frac{(\nu + \gamma)G(\nu) - (\lambda + \gamma)G(\lambda)}{\lambda - \nu},
\]

where \(G(\tau)\) determines the exact shape of the potential. For \(G(\tau)\) we choose a two-component Kuzmin-Kutuzov potential, which takes the following form

\[
G(\tau) = \frac{GM_h}{\sqrt{\tau + c_h}} + \frac{GM_d}{\sqrt{\tau - q + c_d}},
\]

where \(q\) is a parameter set by the choice of the different axis ratios for the components taking into account the constraint that the sum remains a Stäckel potential: \(\lambda_h - \nu_h = \lambda_d - \nu_d\), or \(\lambda_d = \lambda_h - q\) and \(\nu_d = \nu_h - q\), where

\[
q = c_h^2 \frac{\epsilon_h^2 - \epsilon_d^2}{1 - \epsilon_d^2}, \quad \text{with } q \geq 0.
\]

Here the ratio of the semi-major \(a\) and semi-minor \(c\) axis \(\epsilon^2 = \alpha/\gamma\) (i.e. the flattening of the system) is a free parameter for each component, where \(\alpha = -a^2\) and \(\gamma = -c^2\).
Finally, we define the fraction of the mass of the disc with respect to the total mass as $k = M_d / M_{\text{tot}}$, with $M_{\text{tot}} = M_d + M_h$. We recommend the interested reader to consult Dejonghe & de Zeeuw (1988) for more details on axisymmetric Stäckel potentials.

The resulting potential is therefore described by five parameters, namely the total mass $M_{\text{tot}}$, the fraction of mass in the disc $k$, the scale length of the halo component $a_h$, and the flattening parameters of the halo $\epsilon_h$ and disc $\epsilon_d$. Here we set these parameter values to: $M_{\text{tot}} = 4.0 \cdot 10^{11}$ $M_{\odot}$, $k = 0.11$, $a_h = 7.0$ kpc, $\epsilon_h = 1.02$, $\epsilon_d = 75.0$ (which are based on Batsleer & Dejonghe, 1994; Famaey & Dejonghe, 2003, interested readers might want to consult also Reino et al. 2020, where two-component Stäckel potentials are fit to several streams around the Milky Way using Gaia DR2). The resulting potential matches reasonably well the circular velocity curve of the Milky Way, as can be seen from Fig. 10.1 (solid black line). This can be inferred by comparison to the recently estimated circular velocity curve from Eilers et al. (2019) (in blue).

10.2.2 Impulse approximation

Before diving into the model, we will first describe the impact that a subhalo has on a cold stream. The gravitational interaction of a subhalo is well described by the impulse approximation\(^1\) (Yoon et al., 2011; Carlberg, 2013). We define a reference system where the stream is aligned along the y-axis, and moves in the positive y-direction (similar

\(^1\)see Sec. 8.2 from Binney & Tremaine (2008).
to the system of Erkal & Belokurov, 2015a, c.f. their Fig. 2). In this co-moving frame the relative velocity vector of the subhalo is \( \mathbf{w} = w(-\cos \theta \sin \alpha, \sin \theta, \cos \theta \cos \alpha) \), or \( \mathbf{w} = (-w_{\perp} \sin \alpha, w_\parallel, w_{\perp} \cos \alpha) \), where \( w_{\perp} = w \cos \theta \) and \( w_\parallel = w \sin \theta \). Figure 10.2 illustrates the geometry of the stream-subhalo encounter.

The change of velocity (i.e. the impulse) of a particle along the stream due to the encounter is

\[
\Delta v_i = \int_{-\infty}^{\infty} a_i(x, w, M, r_s) \, dt, \tag{10.5}
\]

where \( i = (x, y, z) \). The acceleration \( a_i \) is a function of the relative velocity \( \mathbf{w} \), the distance to the point of impact \( y \), and of the subhalo mass \( M \) and scale radius \( r_s \). We model the subhalos as Plummer spheres but the expressions can be generalised for other profiles (Sanders et al., 2016). The change in velocities in all three coordinates at the time of the impulse according to Eq. (10.5) is

\[
\frac{\Delta v_x}{2GM} = \frac{yw_{\perp}w_\parallel \sin \alpha}{w\left(r_s^2 w^2 + y^2 w_{\perp}^2\right)} = \frac{y \cos \theta \sin \theta \sin \alpha}{w\left(r_s^2 + y^2 \cos^2 \theta\right)}, \tag{10.6a}
\]

\[
\frac{\Delta v_y}{2GM} = -\frac{w_{\perp}^2 y}{w\left(r_s^2 w^2 + y^2 w_{\perp}^2\right)} = -\frac{y \cos^2 \theta}{w\left(r_s^2 + y^2 \cos^2 \theta\right)}, \tag{10.6b}
\]

\[
\frac{\Delta v_z}{2GM} = -\frac{yw_{\perp}w_\parallel \cos \alpha}{w\left(r_s^2 w^2 + y^2 w_{\perp}^2\right)} = -\frac{y \cos \theta \sin \theta \cos \alpha}{w\left(r_s^2 + y^2 \cos^2 \theta\right)}. \tag{10.6c}
\]
The above expressions are valid for direct encounters, that is when the impact parameter \( b = 0 \). Eqs. (1-3) in Erkal & Belokurov (2015a) provide a more general form for the velocity changes which take into account the parameter \( b \). This parameter enters into the equations above through \( r^2_s \rightarrow r^2_s + b^2 \).

We assume that the stream is linear over the scale where the impulse is significant. Moreover, the equations above assume that the stream is a 1D-structure. This approximation is sufficient when the width of the stream is smaller than the scale radius of the subhalo. However, the expressions can be generalised to the full 3D case, for which we find

\[
\Delta v(x) = -\frac{2GM}{w} \frac{w^2 x_i - w_i(x \cdot w)}{(r^2_s + x \cdot x)w^2 - (x \cdot w)^2},
\]

(10.7)

with \( i = (x, y, z) \). To gain insight into the model we will use the equations of the 1D approximation in this section. However, when evaluating the model we will use the full 3D equations.

From Eq. (10.6) we can find the maximum kick in velocities \( \Delta v_{i}^{\text{max}} \) and at what distance \( y_{\text{max}} \) to the centre of impact it occurs,

\[
\Delta v_{i}^{\text{max}} = -\frac{2GM}{w} \frac{w^2 x_i - w_i(x \cdot w)}{(r^2_s + y_{\text{max}}^2)w^2 - (y_{\text{max}}w_y)^2},
\]

(10.8a)

where \( x_i = [0, y_{\text{max}}, 0] \) and

\[
y_{\text{max}} = \frac{wr_s}{\sqrt{w^2 - w_y^2}} = \frac{r_s}{\cos \theta}.
\]

(10.8b)

Typical profiles of \( \Delta v_y(y) \) are shown and discussed in Sec. 10.3, see Fig. 10.11.

### 10.2.3 Action-Angle variables

This section aims to serve as a brief introduction to these variables, and it is by no means exhaustive or comprehensive. For more details on action-angle variables, the reader could consult Goldstein et al. (2002); Binney & Tremaine (2008).

Orbits in smooth and simple potentials (e.g. spherical, axisymmetric, triaxial) have a number of integrals of motion: properties that do not change in time and serve to characterise them. For a spherical potential, the integrals of motion are the total energy (or the Hamiltonian) and the angular momentum vector. Orbits in axisymmetric systems (e.g. disc galaxies) typically have up to three integrals of motion: the total energy, the momentum in the azimuthal direction, and a non-classical integral which in most cases does not take an analytic form.

For separable potentials (e.g. the Stäckel potentials discussed in Sec. 10.2.1) there exist three isolating integrals \( J \), known as the actions. Each action is paired with a conjugate coordinate \( \Theta \), the angles. Together, these coordinates make up the action-angle variables \( (\Theta, J) \). The actions uniquely define the orbit, that is, a point in action-space corresponds
to a complete orbit in phase-space. The conjugate angles define the phase, that is they specify where along the orbit a body is located at any given time.

To obtain the action-angle variables we make use of the Hamiltonian \( H \), which being an integral of motion must depend on the actions (i.e. \( H = H(J) \)). The rate of change of the angles \( \dot{\Theta} = \partial H / \partial J \) is known as the frequency \( \Omega(J) \). Therefore

\[
\Theta(t) = \Theta_0 + \Omega t,
\]

(10.9)

and hence the angles are linearly dependent on time. Finally, the actions \( J \) of a bound orbit in a separable potential are defined as

\[
J = \frac{1}{2\pi} \int p \cdot dq,
\]

(10.10)

where \((q, p)\) are any set of generalised phase-space coordinates and momenta.

### 10.2.4 Size of the gap using an actions-angles framework

The analytical framework of the method that we will use to describe the evolution of a gap in a stream with time, was first established by HW99. Originally, this framework was used to describe the divergence in the orbits of a distribution of nearby particles. It makes use of a linearised Taylor expansion around a central orbit. In our case, we will model the size of the gap as the spatial separation of two orbits: one on each side of the gap. These orbits are taken to be those of the particles that receive the largest impulse from the subhalo flyby. In practice, this is equivalent to modelling the (size of the) gap as twice the separation of the central orbit and one of the edges of the gap, as gaps are symmetric with respect to their centre.

#### Generalities

Let us consider a central orbit and some other orbit separated by \( \Delta X_0 \) and \( \Delta V_0 \), where the subscript is used to denote the time of the impact between the subhalo and the stream, \( t = t_0 \). To calculate the evolution of this separation vector we first transform it to action-angle variables

\[
\begin{bmatrix}
\Delta \Theta_0 \\
\Delta J_0
\end{bmatrix} = T_0 \begin{bmatrix}
\Delta X_0 \\
\Delta V_0
\end{bmatrix},
\]

(10.11)

where \( T_0 \) is a matrix calculated at \( t = t_0 \) that locally transforms from Cartesian coordinates to action-angles. In practice, the transformation is a product of matrices

\[
T_0 = T^{\text{AA} \leftarrow \text{st}}_0 T^{\text{st} \leftarrow \text{cyl}}_0 T^{\text{cyl} \leftarrow \text{xyz}}_0,
\]

(10.12)

where \( T^{\beta \leftarrow \alpha}_0 \) transforms the set of coordinates \( \alpha \) to the set \( \beta \), \( \text{xyz} \) indicating Cartesian coordinates, \( \text{cyl} \) cylindrical coordinates, \( \text{st} \) spheroidal coordinates used for the Stäckel potential, and \( \text{AA} \) action-angle variables.
Next, the separation vector in action-angles can be evolved in time by expanding linearly Eq. (10.9) and making use of the matrix $\Omega'$

$$
\Omega' = \begin{bmatrix} I_3 & \partial \Omega / \partial J_t \\ 0 & I_3 \end{bmatrix},
$$

(10.13)

At any point in time, the separation in action-angle coordinates can be transformed back to Cartesian coordinates locally, and therefore

$$
\begin{bmatrix} \Delta X_t \\ \Delta V_t \end{bmatrix} = T_t^{-1} \Omega' T_0 \begin{bmatrix} \Delta X_0 \\ \Delta V_0 \end{bmatrix},
$$

(10.14)

where $T_t^{-1}$ is the (local) transformation back to Cartesian coordinates at time $t$ and at the location of the central orbit of the gap.

Finally, the size of the gap can be taken as twice the separation calculated in Eq. (10.14). The initial separation of the two orbits describing the gap can be obtained assuming Eq. (10.8a) and Eq. (10.8b). Since the two orbits are typically separated a few kpc initially, we need to add the velocity gradient of the orbit to the separation in velocities, so $\Delta V_0 = \Delta v_{\text{max}} + \delta v_{\text{orbit}}$. We note that this is an ad hoc fix to the non-local nature (finite extent) of the stream. It takes into account that the velocity of the stream particles changes as a function of location.

**Long-term behaviour**

The growth rate of the size of the gap can be derived from Eq. (10.14) in a similar fashion as shown in HK16. In the limit where $t \gg t_0$ (or better $t/t_{\text{orb}} \gg 1$), this equation simplifies to

$$
\begin{bmatrix} \Delta X_t \\ \Delta V_t \end{bmatrix} \sim t \begin{bmatrix} T_{t,1}^{-1} \partial \Omega / \partial J \Delta J_0 \\ T_{t,3}^{-1} \partial \Omega / \partial J \Delta J_0 \end{bmatrix},
$$

(10.15)

where $T_{t,1}^{-1}$ is the upper left submatrix of $T_t^{-1}$, and $T_{t,3}^{-1}$ is the bottom left submatrix. The spatial separation of the two orbits is equal to the length of vector $\Delta X_t$

$$
|\Delta X_t| = \sqrt{\Delta X_t^\dagger \Delta X_t} \sim t \sqrt{\Delta J_0^\dagger f_{x,\Omega} \Delta J_0},
$$

(10.16)

In this equation $f_{x,\Omega} = \partial \Omega / \partial J T_{t,1}^{-1} \partial J T_{t,1}^{-1} \partial \Omega / \partial J$. Similarly, we can calculate the velocity difference of the two orbits $\Delta V$

$$
|\Delta V_t| = \sqrt{\Delta V_t^\dagger \Delta V_t} \sim t \sqrt{\Delta J_0^\dagger f_{v,\Omega} \Delta J_0},
$$

(10.17)

where $f_{v,\Omega} = \partial \Omega / \partial J T_{t,3}^{-1} \partial J T_{t,3}^{-1} \partial \Omega / \partial J$.

We note that the terms $f_{x,\Omega}$ and $f_{v,\Omega}$ are dependent on the orbit of the gap and its location, but they do not depend on the impact parameters. Both $|\Delta X_t|$ and $|\Delta V_t|$ are
linearly dependent on time $t$, similar to gaps orbiting in spherical potentials. Interestingly, the ratio of the two separations is constant with time - which potentially can be used to infer the properties of the gap at any time (as we will demonstrate also in Sec. 10.4).

**10.2.5 Density of stream gaps**

We now build further on the framework developed by HW99 and focus on modelling the evolution of the density of the gap. The impulse imparted on the stream by the subhalo increases the local velocity dispersion of the stars in the gap. This causes it to grow faster and thus appear as under dense region in comparison to the neighbouring parts of the stream. If we know the central orbit of the gap and the initial phase-space distribution around it, we can calculate the evolution of the density in the gap.

**Generalities**

We will describe this initial phase-space distribution as a multi-variate Gaussian distribution, in other words

$$ f(x, v) = f_0 \exp \left( \Delta_{\omega,0} \sigma_{\omega,0} \Delta_{\omega,0} \right), \quad (10.18) $$

where $f_0$ is the phase-space density at $t = t_0$, $\Delta_{\omega,0}$ is a separation vector: $\Delta_{\omega,0} = \xi_i - \xi_{c,i}$ and where $\xi_i = [x, y, z, v_x, v_y, v_z]$ and $\xi_{c,i}$ is the central point of the distribution (which we will take to be the location where the subhalo impacts the stream, or in the terminology previously used, the central orbit) at $t = t_0$. The matrix $\sigma_{\omega,0}$ is the inverse of the covariance matrix of the phase-space coordinates.

To compute the initial dispersion matrix of the gap, we start from the original, unperturbed distribution and add the impulse in the velocities according to Eq. (10.7). That is, we transform $\sigma_{\omega,0}^{\text{stream}} + \text{impulse} \rightarrow \sigma_{\omega,0}^{\text{gap}}$. Below we show how to calculate the new covariance matrix in the regime where the stream is approximated by a 1D-structure, but in Appendix 10.A we provide the full 3D expressions.

The most general form of the initial unperturbed covariance matrix $\Sigma_{\omega,0}$ is

$$ \Sigma_{\omega,0} = \sigma_{\omega,0}^{-1} = \begin{pmatrix} \sigma_x^2 & C(x, y) & \cdots \\ C(y, x) & \sigma_y^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad (10.19) $$

where $C(x, y)$ is the covariance of $x$ and $y$ and $\sigma_x$ is the standard deviation of $x$. The covariance matrix can be represented with $3 \times 3$ block matrices

$$ \Sigma_{\omega,0} = \begin{pmatrix} C_{xx} & C_{x,y} \\ C_{y,x} & C_{yy} \end{pmatrix}, \quad \text{where} \quad C_{x,y} = C_{y,x}^\dagger. \quad (10.20) $$

We now proceed to compute the perturbed covariance matrix by computing the changes of each individual element due to the encounter with the subhalo. The impulse only
affects the velocities. Therefore, the position block matrix \((C_{x,x})\) does not change during the encounter. The first element with a velocity term is in the \(C_{x,v}\) block matrix

\[
C(v_x, x) = \frac{1}{n} \sum_{i=1}^{n} (v_{xi} - \mu_{vx})(x_{i} - \mu_{x}),
\]

(10.21)

where \(n\) is the total number of particles, \(\mu_{vx}\) and \(\mu_{x}\) are the mean \(v_x\) and \(x\) of the distribution in the region around the gap. After applying the impulse, the new covariance element becomes

\[
C(v_x + \Delta v_x, x) = \frac{1}{n} \sum_{i=1}^{n} (v_{xi} + \Delta v_{xi} - \mu_{vx} - \Delta \mu_{vx})(x_{i} - \mu_{x}),
\]

(10.22)

where \(\Delta v_{xi}\) is the velocity change of particle \(i\), and \(\Delta \mu_{vx}\) is the shift of the mean velocity of all particles. Since the kicks are symmetric around the central point, the mean shift of velocities is zero \(\Delta \mu_{vx} = 0\), and we can rewrite the covariance term as

\[
C(v_x + \Delta v_x, x) = C(v_x, x) + \frac{1}{n} \sum_{i=1}^{n} \Delta v_{xi}(x_{i} - \mu_{x}).
\]

(10.23)

Considering that the covariance matrix describes the central density (i.e. positions close to the centre) we can express the kick \(\Delta v_x\) as a function that is only linearly dependent on \(y\), since the quadratic term in the denominator of Eq. (10.6a) is negligible (i.e. \(r_s^2 w^2 >> y^2 w^2_\perp\)). Moreover, since the velocity kicks are calculated in a frame where there is symmetry with respect to \(y\) (i.e. \(\mu_y = 0\)) we can rewrite the last term in the equation above as

\[
\frac{1}{n} \sum_{i=1}^{n} \Delta v_{xi}(x_{i} - \mu_{x}) = 2GM \frac{w_\perp w_\parallel \sin \alpha}{r_s^2 w^3} \frac{1}{n} \sum_{i=1}^{n} (y_{i} - \mu_{y})(x_{i} - \mu_{x}),
\]

(10.24)

which is equal to

\[
C(v_x + \Delta v_x, x) = C(v_x, x) + 2GM \frac{w_\perp w_\parallel \sin \alpha}{r_s^2 w^3} C(y, x).
\]

(10.25)

The new covariance term \(C(v_x + \Delta v_x, x)\) can be expressed as the old covariance term plus a new term that depends on the impact parameters. The procedure shown above can be extended to all covariance terms of the form \(C(\alpha, v_\beta + \Delta v_\beta)\) and \(C(v_\beta + \Delta v_\beta, \alpha)\), where \((\alpha, \beta) = (x, y, z)\).
Using similar arguments, it is easy to show that covariance terms in the velocity submatrix \((C_{v,v})\) take the following general form

\[
C(v_\alpha + \Delta v_\alpha, v_\beta + \Delta v_\beta) = C(v_\alpha, v_\beta) + C(\Delta v_\alpha, v_\beta) \\
+ C(v_\alpha, \Delta v_\beta) + C(\Delta v_\alpha, \Delta v_\beta).
\] (10.26)

For example, for \(\alpha = x\) and \(\beta = y\), and following similar procedures as above

\[
C(\Delta v_x, v_y) = 2GM \frac{w_\perp w_\parallel \sin \alpha}{r_0^2 w_3^3} C(y, v_y),
\] (10.27)

\[
C(v_x, \Delta v_y) = -2GM \frac{w_\perp^2}{r_0^2 w_3^3} C(v_x, y),
\] (10.28)

\[
C(\Delta v_x, \Delta v_y) = -\left(2 \frac{GM}{r_0^2 w_3^3}\right)^2 w_\perp w_\parallel \sin \alpha C(y, y).
\] (10.29)

We now have full expressions for the matrix \(\sigma_{\omega,0}\) in Eq. (10.18) representing the phase-space configuration around the gap at the time of the encounter \(t = t_0\). By transforming \(\sigma_{\omega,0}\) to action-angle coordinates as \(\sigma_{\omega,0} = T^{-1}_0 \sigma_{\omega,0} T^{-1}_0\), where \(T\) is the transformation matrix defined in Eq. (10.12), we can calculate the evolution in time of the covariance matrix in phase-space using Eq. (10.13). This allows us to describe the local density of the portion of the stream around the location of the impact by the subhalo (i.e. of the gap) as

\[
\rho(x_c, t) = \int f(x_c, v, t) \, d^3v,
\] (10.30)

where \(\rho(x_c, t)\) is the density of orbits in a location around the central orbit. In the principal axes, where the velocity covariance matrix is diagonal, this density takes a simple form

\[
\rho(x_c, t) = \rho_0 / \sqrt{\det |\sigma_v|} \propto \rho_0 \sigma_{v_1}(t) \sigma_{v_2}(t) \sigma_{v_3}(t),
\] (10.31)

where \(\rho_0\) is the central density at \(t = t_0\) and \(\sigma_{v_1}, \sigma_{v_2}, \sigma_{v_3}\) are the velocity dispersions along the three principal axes.

**Long-term behaviour of the density**

Using the above formalism, it is possible to show that the density of a stream (and thus also that of a gap) decreases as a power law of time which depends on the number of degrees of freedom of the orbit of the stream (Vogelsberger et al., 2008)

\[
\rho \propto t^{-n}, \text{ with } n = \text{d.o.f.}
\] (10.32)

Ultimately these degrees of freedom are determined by the number of independent frequencies, and this number generally is dependent on the functional form of the potential. For axisymmetric galaxies the number of d.o.f. is 3 for most (non-resonant)
orbits. On the other hand, for example, circular orbits only have one degree of freedom, implying that the density decreases much slower (i.e. \(1/t\)).

HW99 derived a general expression for the central density at late times for streams (and gaps) in a general Stäckel potential (see their Appendix C) and found

\[
\rho(x_c, t) = \frac{\rho_0 f_{\text{orb}}}{\sqrt{\det |\sigma_{\Theta_0}|}} t^{-3},
\]

(10.33)

where \(\rho_0\) is the initial density of the distribution, \(f_{\text{orb}}\) is a constant determined by the central orbit, and \(\sigma_{\Theta_0}\) is the angle submatrix at \(t = t_0\). This implies that the ratio of the density of a perturbed to unperturbed stream is a constant

\[
\delta\rho_{\text{gap}}^{\text{str}} = \sqrt{\frac{\det |\sigma_{\Theta_0}|_{\text{str}}}{\det |\sigma_{\Theta_0}|_{\text{gap}}}},
\]

(10.34)

as all other variables are independent on the impact parameters. We will refer later in the Chapter to this ratio of densities as the density contrast.

**10.2.6 Setting up the stream-subhalo encounter**

To verify our model predictions, we perform N-body simulations of the encounter of a subhalo with a stream orbiting in the Milky Way potential described in Sec. 10.2.1. To this end, we use a modified version of GADGET-2 (Springel, 2005), where we model the host as the rigid potential, and the subhalo as a rigid Plummer sphere which is centred on a particle with a negligible mass that is put on a trajectory in the host potential.

The progenitor of the stream is modelled with \(10^6\) test particles\(^2\) following a Gaussian distribution in 6D phase space, with \(\sigma_x = 0.2\) kpc and \(\sigma_v = 0.5\) km/s. These very low dispersions are chosen such that the stream has a high density even a few Gyr after forming. In comparison, globular clusters orbiting the Milky Way typically have a \(\sigma_v\) of a few km/s (e.g. Harris, 1996, 2010 edition).

The progenitor is put on an elongated orbit with maximum distance from the centre \(r_{\text{max}} \approx 20\) kpc and minimum distance \(r_{\text{min}} \approx 10\) kpc, reaching \(r_z = \pm 20\) kpc above the plane of the disc, as shown in Fig. 10.3. After the progenitor of the stream is evolved for 1 Gyr in the host potential, a subhalo is inserted on a trajectory that directly crosses the stream. We remove the subhalo after the collision to isolate a single interaction, and when its gravitational effect is sufficiently small that it no longer affects the stream.

Fig. 10.4 shows an example of a stream-subhalo interaction. The left panel shows both the stream and a subhalo at the time of the collision. The right panels show a stream with and without an encounter, 2 Gyr after the interaction with the subhalo. The perturbed stream clearly shows a gap of several kpc in size at the centre of the panel.

\(^2\)Because we use test particles, it is not strictly necessary to model their evolution using an N-body code such as Gadget.
Fig. 10.3: The orbit of the stream shown in galactocentric Cartesian coordinates, evolved for 10 Gyr forward in time in the Stäckel axisymmetric Milky Way-like potential. Top: orbit in coordinates aligned with the host galaxy (e.g. plane of the disc is $z = 0$). Bottom: coordinates aligned with the initial angular momentum vector of the orbit. The right bottom panel highlights the precession of the orbital plane, which is indicative of the non-spherical nature of the potential considered.

Tab. 10.1: The masses and scale radii of the subhalos. They are modelled as rigid Plummer spheres.

<table>
<thead>
<tr>
<th></th>
<th>subhalo 1</th>
<th>subhalo 2</th>
<th>subhalo 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ [$M_\odot$]</td>
<td>$10^6$</td>
<td>$10^7$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>$r_s$ [kpc]</td>
<td>0.35</td>
<td>0.59</td>
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Fig. 10.4: Left: Snapshot of a stream at the time of interaction with a subhalo. The stream is plotted with black dots and the centre of mass of the subhalo is shown as a red solid circle. The velocity vector of the stream is marked with a black arrow and that of the subhalo with a red arrow. Right: Both panels show the stream after 2 Gyr of evolution, in isolation in the top panel, and after the encounter with the subhalo in the bottom panel.
10.3 Results

We compare the predictions of the model presented in Sec. 10.2.4 and Sec. 10.2.5 with the gaps produced in the N-body experiments. We will first investigate gaps produced by subhalos of varying mass and size for a fixed encounter configuration (i.e. the same velocity and impact angle). Next, we focus on the effects of a varying configuration while keeping the subhalo properties fixed.

10.3.1 Size evolution

Figure 10.5 shows the evolution of gaps caused by interactions sharing the same configuration, but with different subhalo masses (see Table 10.1 for their properties). The model (solid lines) reproduces very well the size of the gap as measured in the N-body experiments.
Fig. 10.6: Left: A stream 1 Gyr after an interaction with a subhalo. The two red dots indicate the orbits that are being used to measure the size of the gap, the red arrow shows the distance on a straight line between the two dots. Right: The same stream, 3.6 Gyr after the encounter. At this point in time the two orbits (red dots) are close to their maximum separation.

simulation (coloured dashed lines). The latter is measured as the average separation of two groups of 50 particles on each side of the gap. These 50 particles are identified as those that experience the largest velocity change at the time of the impact. We use 50 particles to lower the effects of discreteness of the N-body simulation, but there is only very little difference when using the single particle with the maximum velocity change on each side of the gap. The bottom panel of Figure 10.5 shows the total distance of the gap to the centre of the host galaxy and gives an indication of its orbit. The frequency of $r(t)$ and the oscillations in the gap size are in antiphase. This is naturally expected since the gap will be stretched at pericentre and be smallest at apocentre.

Although Fig. 10.5 shows that the model reproduces the gap size in the N-body experiment very well, there appears to be an upper limit to its measured size. The largest difference is apparent for the encounter with the most massive subhalo at late times. This limit occurs because the size of the gap becomes comparable to the typical scale of the orbit and hence our method of measuring the size of the gap fails to work. The typical scale of the specific orbit that is used here is $\lesssim 40$ kpc, see Fig. 10.6. This value can also be determined analytically using the inverse of Eq. (10.11), and considering that the two orbits on each side of the gap are at a maximum separation at $\Delta \Theta = \pi$. The maximum distance between two particles on the same orbit but apart by $180^\circ$ in the angles, at any location in the orbit, is $\sim 35$ kpc. This value agrees very well with the ceiling reached by the red, dashed line measured from the N-body experiment in Fig. 10.5.
Fig. 10.7: Density evolution of an unperturbed stream (in blue) and of a gap in a stream following the same orbit (in green). The dashed curves are the densities obtained from the corresponding N-body experiments. The agreement between the solid and dashed curves is excellent. The subhalo that is used to create the gap in the stream has a mass of $10^7 \, M_\odot$.

The size of the gap in this regime is pushing the limits of our analytical model. The transformation from action-angles to Cartesian coordinates (i.e. Eq. 10.14) is only valid ‘locally’ near the central orbit, and therefore the approximation breaks down for such large gaps. Although it should be possible to extend the formalism to include such cases, this is not really necessary as there are no known streams with gaps of this size - nor is it likely to observe one such gap in the (near) future.

### 10.3.2 Evolution of the density

Now we compare the density as predicted by our model with the density measured in N-body experiments. For the latter we count the number of particles in a small volume in 6D space with $r < 0.1 \, \text{kpc}$ and $v < 7.5 \, \text{km/s}$. This velocity limit does not remove any particles from the stream at $t_0$ when the impact occurs, but it removes particles that may have drifted away (i.e. have a different orbital phase) at later times. The volume is centred on the central orbit, which is determined in a simulation of a stream with the same set-up, but without a subhalo interaction.

Figure 10.7 compares the predicted (solid lines) and measured density from the N-body simulation (dashed lines) for a stream with and without a gap. For the latter, we have simulated the exact same stream with and without an encounter with a subhalo. Although the peaks and troughs of the stream and the gap are always larger in our model, the figure shows that the model provides an excellent description of the N-body experiments. The small differences can be attributed to a resolution effect: in the N-body experiments we measure the density in a finite volume, whereas the model computes a density at a single location in space. If the density were measured in a smaller volume in our experiments...
the peaks would be sharper. However, the number of particles would drastically decrease and drop to less than a handful in less than 4 Gyr of evolution.

**Varying subhalo masses**

![Graph showing density contrast over time for different subhalo masses](image)

*Fig. 10.8:* Density contrast measured for a stream experiencing an encounter with subhalos of different mass and scale radius, see Table 10.1. The relative velocity of the impact is the same for all three interactions. The coloured solid lines show the density contrast according to our model, and the dashed lines are measured in the N-body experiments. The shaded areas indicate the Poisson error in the observed density. The subhalo of $10^8 M_\odot$ is shown in a different panel because of its slightly different set-up.

Next, we explore the evolution of the density contrast (i.e. the ratio of the density around the gap to that of the unperturbed stream) in Fig. 10.8. The figure shows the same experiments as those plotted in Fig. 10.5, with the density contrast of the most massive subhalo shown in a separate panel. For the most massive halo, we have modified slightly our set-up, instead of starting from the same initial conditions as the other experiments using the orbit shown in Fig. 10.3, we have used the location of the gap to determine its...
orbit. We used this as the central orbit both in our analytical model and for the N-body experiment representing the unperturbed stream. The reason for this is that when the subhalo and the stream interact, the stream receives an impulse that displaces it slightly from its original orbit. The effect is negligible for subhalos of $M \lesssim 10^7 M_\odot$, and is small but apparent for more massive objects, particularly after $\sim 3 – 4$ Gyr of evolution. This new set-up is actually more realistic since when attempting to model an observed stream or gap, its actual measured position and velocity in a suitable gravitational potential would be integrated (as it is not possible to have a priori access to the original initial conditions of the orbit of the stream, before it received the impact).

In Fig. 10.8, we show with solid lines the predicted density contrast from our model, and with dashed lines those measured in the N-body experiments. The Poisson errors on the ratio of the densities as measured in the N-body experiments are marked with shaded areas. In general, the amplitude of the density contrast is well reproduced by the model, with the difference in the amplitude of the narrow peaks at early times explained by the same resolution effects as described in Sec. 10.3.2.

Variation of encounter configuration

Finally, we check how our model performs for different configurations of the stream-subhalo encounter, keeping the subhalo at a fixed mass of $10^7 M_\odot$. We compare three different configurations which correspond to rotations of the same velocity vector, as listed in Table 10.2, with Configuration 1 being that used in the previous section. We note that the velocity vector is rotated in the rest frame, not in the co-moving frame of the stream. The resulting configurations thus have different velocity amplitudes in the co-moving frame.

Figures 10.9 and 10.10 show the time evolution of the size and density contrast. The layout of the figures is the same as in the previous section. Again, the model (solid lines) predicts the behaviour of the gaps as measured in the N-body experiments (dashed lines) extremely well.

Interestingly, Configuration 2 with a subhalo of $10^7 M_\odot$ gives rise to a density contrast of similar amplitude as the collision with the subhalo of $10^6 M_\odot$ (see Fig. 10.8) on Configuration 1, both producing a gap with a density contrast of $\sim 0.9$. However, if we examine the size of the gaps we notice that the gap caused by the $10^6 M_\odot$ subhalo (blue curve Fig. 10.5) is smaller than that for the $10^7 M_\odot$. This implies that by measuring both size and density, one may be sensitive to different parameters characterising the encounter, as we shall discuss in more detail in Sec. 10.4.

<table>
<thead>
<tr>
<th></th>
<th>Config. 1</th>
<th>Config. 2</th>
<th>Config. 3</th>
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<tr>
<td>$\theta$ [deg.]</td>
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<tr>
<td>$\alpha$ [deg.]</td>
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<td>$w$ [km/s]</td>
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<td>33.7</td>
<td>378.3</td>
</tr>
</tbody>
</table>

Tab. 10.2: Parameters of the different configurations, see Sec. 10.2.2 for their definition.
Fig. 10.9: Similar to Fig. 10.5 but for three different configurations of the stream-subhalo interactions. The gaps of all configurations are created by the same subhalo of size $10^7M_\odot$, but their relative velocity $w$ and impact angles $\theta$ and $\alpha$ are different.

Figure 10.9 shows that the gaps resulting from the encounter in Configurations 1 and 2 have initially approximately the same size. Interestingly the gap resulting in Configuration 3 is initially the largest and also remains the largest throughout its evolution in time (although the size is somewhat poorly modelled because of the change in velocities for this particular configuration is very shallow, see Fig. 10.11, which gives rise to some complications in identifying the correct particles to trace in the N-body).

Fig. 10.10: Similar to Fig. 10.8, but for three different stream-subhalo interactions with a subhalo of fixed mass, and whose size evolution is described in Fig. 10.9.
10.4 Exploration of the gap observables: dependencies and degeneracies

Now that we have validated our analytic model, we will use it to explore how the size and density of a gap depend on the collision parameters using Eqs. (10.16) and (10.33). We will consider hypothetical gaps formed in the stream presented in Sec. 10.2.6 and analysed in Sec. 10.3. To this end, we vary the characteristic parameters of the collision with a subhalo, namely \( w, \theta, \) and \( M, \) while keeping the angle \( \alpha \) fixed at some arbitrary value \( \alpha = 163^\circ. \) We consider \( w \) in the range \([0, 800]\) km/s and \( \theta \) in the range \([-90^\circ, 90^\circ]\). Instead of varying separately \( M \) and \( r_s, \) we use a relation for \( V_{\text{max}} \propto r_{\text{max}} \) for subhalos...
Fig. 10.12: Top row: size of the gap after 2.5 Gyr of evolution, as a function of the impact parameters (subhalo mass, angle $\theta$ and amplitude of the relative velocity $w$). Bottom row: density contrast as a function of the same parameters. The two panels on the left show the discrete variation of the curves - varying the velocity amplitude $w$ or collision angle $\theta$. The two panels on the right show two sets of lines, solid for a subhalo of mass $10^7 M_\odot$ and dashed for $10^8 M_\odot$.

found in the ‘Aquarius’ simulations by Springel et al. (2008), see Appendix 10.B for details.

Figure 10.12 shows the gap’s properties, namely size and density contrast, as a function of these characteristic parameters. Each panel shows the dependence of these two observable quantities with one of the three parameters: $M$, $w$, or $\sin\theta$. At the same time, we vary discretely a second parameter, which gives rise to the different curves in each subpanel, but keep fixed the third parameter. For example, in the leftmost panels we show the variation of gap size (top) and density contrast (bottom) 2.5 Gyr after impact as a function of mass of the subhalo $M$, for different values of $w$ as indicated by the colour bar, and for $\theta = \pi/4$.

Since the size of a gap varies depending on its orbital phase, we have checked that the dependencies shown in Figure 10.12 are robust. We have found them to be identical, except for an overall scaling of the amplitude that depends on the phase. Since it will be possible to establish the phase of a gap observationally (after assuming a suitable Galactic potential and integrating the orbit of the stream in which it is embedded), this implies that the trends observed here can be used to infer several of the characteristic properties of the encounter. The density contrast, meanwhile, does not vary along the orbit (because the density variations along the orbit for the gap are identical to those for the unperturbed stream). However, in the bottom panels of Figure 10.12, we have taken the late times limit of the density contrast given by Eq. (10.34).
We have shown in Eq. (10.8b) that the size of the gap at any point in time strongly depends on its initial magnitude (i.e. \( \propto r_s/\cos \theta \)) implying a dependence on the subhalo’s mass through the \( r_s \) parameter (with \( r_s \propto M^{2/5} \)), as shown in Appendix 10.B. This simple relation explains the curves in the top panels of Fig. 10.12 well, which show that the gap size depends strongly on the mass of the subhalo (two leftmost panels), with relatively little dependence on \( w \) and \( \sin \theta \), except for extreme values of these parameters (two rightmost panels). For example, when the subhalo moves along the stream (i.e. when \( \cos \theta \rightarrow 0 \)) the size of the gap is clearly not well defined. In this case the impulse approximation breaks down as the subhalo and stream interact for a long time, and, perhaps more importantly, the interaction affects a large part of the stream. Also for very low values of \( w \), the impulse approximation is no longer valid. Low relative velocities and extreme alignment must be rare because they only happen when the stream and subhalo move at a similar velocity and in the same direction. In summary, the top panels of Figure 10.12 suggest that given a gap size, it is possible to infer with some confidence the mass of the subhalo that perturbed it for most values of \( w \) and \( \theta \).

With knowledge of the mass, the density contrast could be used to infer some plausible encounter geometries. To understand the factors driving the density contrast, we use Eq. (10.34), which depends on the ratio of \( \det |\sigma_{\Theta 0}| \) for the stream and the gap. Although general analytic expressions can be obtained, these are rather cumbersome, so to gain further insight we simplify the initial configuration of the stream subhalo interaction and assume that this has occurred close to a turning point (see Appendix 10.C for full details). In this case we find

\[
\delta \rho_{\text{gap}}^{\text{str}} \propto 1 - \frac{M \cos \theta}{r_s^2} \frac{w}{w} f(\theta, \alpha, C_{v,v_0}^{\text{str}}),
\]

(10.35)

where \( f(\theta, \alpha, C_{v,v_0}^{\text{str}}) \) is a function that depends the angles characterising the encounter, and on the velocity covariance matrix of the unperturbed stream at the time of the impact. This relation implies that the density contrast becomes shallower with increasing \( w \), as can indeed be seen in Fig. 10.12. On the other hand, more massive subhalos create gaps with lower densities. Because of the different dependence of the gap size (top panel) and of the density contrast with the characteristic parameters of the encounter, \( (M, w, \sin \theta) \), this means that it is possible to break some of the degeneracies present using these two observable quantities, provided the time since the collision could be established (which is necessary for making use of the constraints provided by gap size).

Another time-invariant combination of observables is plotted in Fig. 10.13. This figure shows the spatial size of the gap relative to the separation in velocity space, normalised by its initial value at \( t_0 \). Note that although the ratio of \( \Delta X \) and \( \Delta V \) does vary with the orbital phase of the gap/stream, this phase can be established through orbit integrations as discussed earlier. The lines shown in Fig. 10.13 (measured at 2.5 Gyr) are for a gap that is near its apocentre. Evaluating the ratios \( \Delta X/\Delta V \) near the pericentre results in a similar figure, but where the \( y \)-axis is mirrored with respect to the line \( y = 0 \).
There are clear similarities between this ratio and the behaviour of the density contrast shown in the bottom row of Fig. 10.12, except for the third panel, which reveals a sensitivity for low $w$ velocity on the angle of the encounter $\theta$. Overall, this ratio will therefore be a good discriminator for low $w$ of $\sin \theta$.

In summary (smallish) gaps $\lesssim 25$ kpc are mostly dependent on the mass of the subhalo, while large gaps can either be due to a specific configuration (low relative velocity or angle of the encounter) or due to a large subhalo. Assuming the average encounter has a relative speed of $w > 200$ km/s it appears that per orbit/stream we can break the degeneracy of the interaction parameters using also the density contrast. We have checked that these conclusions (and dependencies) are robust and independent of the orbital characteristics of the stream (e.g. different inclinations with respect to the Galactic plane), but they are only strictly valid if the age of the gap can be well constrained.

10.5 Discussion

Action-angle variables have been previously used to describe streams and their gaps (e.g. Helmi et al., 1999; Helmi & Gomez, 2007; Bovy, 2014; Sanders et al., 2016; Helmi & Koppelman, 2016; Bovy et al., 2017). There is a trade-off to be made when using these variables: one may either make use of a numerical approach and obtain a (local) approximation for a generic potential (Binney, 2012; Sanders & Binney, 2015), or use a fully analytic approach and be restricted in the choice of the potential. In this Chapter, we take the latter approach such that we can express the properties of the gaps in physical space directly as a function of the encounter parameters.

In contrast to the work of Erkal & Belokurov (2015a), who argue that gaps grow at late times as $\sqrt{t}$ (for circular orbits), we find that both in our numerical experiments as in the analytic model, gaps grow linearly with time independently of the type of orbit or shape of the gravitational potential. We have thus extended the findings of HK16 who considered a spherical potential, and confirm also the results of Sanders et al. (2016).
Sanders et al. (2016) have found that the density contrast of a gap approaches a constant value at late times. Our fully analytical model allows us to verify their conclusion and we are also able to show why this happens and what the constant value depends upon (e.g. Eqs. 10.34 and 10.35). Sanders et al. (2016) also find that gaps grow differently in the leading and trailing arm. Judging from the expressions derived in this Chapter, there may be two reasons for the different growth rate: i) a difference in the local (velocity) dispersions of the particles in either the leading or trailing arm, as was already noted by Sanders et al. (2016); ii) the orbits of the leading and trailing stream have slightly different characteristic parameters (they are slightly offset in energy), and these affect the growth rate of gaps as well as the decline in their density.

Erkal & Belokurov (2015b) argue that there exists a degeneracy in the gap parameters with mass and velocity. The reported degeneracy of \( (M, w) \rightarrow (\lambda M, \lambda w) \) only exists if the scale radius \( r_s \) of the subhalo is kept fixed, but its mass is not. For example, the size of a gap depends on \( r_s / \cos \theta \), while the density contrast depends on \( M/r_s^2 \times \cos \theta/w \). Since a non-linear relation between \( r_s \) and \( M \) is known to exist for subhalos in cosmological simulations \( (r_s \sim M^{2/5}, \) Neto et al. 2007; Springel et al. 2008), we must conclude that the above degeneracy does not exist.

The model presented in this Chapter successfully describes gaps in streams in axisymmetric potentials. However, it builds on several key assumptions, namely:

- **The time of the collision/age of the gap.** Although this information is in principle encoded in the size of the gap, we have generally explored its properties at a fixed time. Keeping it open will add one more parameter to optimise. A rough estimate of the formation time could be obtained by integrating the two sides of the gap backwards in time and to see when they meet. Recall on the other hand, that if we may assume that the encounter happened sufficiently long ago, the density contrast is time-independent, and some of the encounter parameters can be constrained.

- **The potential of the host galaxy.** Changes in the host potential will change the central orbit of the gap and thus could change its size and density evolution. However, we note that the explicit time dependence of both the size and density of the gap will not change with (small) variations in the potential.

- **Knowledge of the pristine stream conditions, before the interaction.** In principle from observing the full stream morphology and knowing the age of the gap, it should be possible to derive the full 6D properties of the stream at the time of the collision.

- **The properties of the subhalo,** here assumed to be well-described by a Plummer sphere. This choice was made because of its simple mathematical expression, for which there is an analytic solution to the integral of the impulse approximation used to compute the velocity kicks. However, it is possible to compute this integral numerically for other profiles (e.g. Sanders et al., 2016).

We have shown here that some of the previously reported degeneracies in the space of parameters describing the encounter can be broken by making plausible assumptions on
the stream-subhalo configuration. A natural next step would be to consider probability distributions for the encounter parameters, much like, for example Erkal et al. (2016). Our model can then be used to quickly explore the parameter space as it can constrain the most likely encounter parameters given observation of a gap.

10.6 Conclusions

We have successfully extended the model of the evolution of gaps in spherical potentials, presented in HK16, to describe gaps in streams orbiting in axisymmetric Stäckel potentials. The model accurately predicts the evolution of both the size of the gap and its central density. The model is unique in that it is fully analytic, meaning that we can directly relate the stream-subhalo interaction parameters to the properties of the resulting gaps. In doing so, it provides some interesting insights into the evolution of gaps in streams.

We find that the sizes of the gaps in axisymmetric potentials grow linearly in time - and this dependence is independent of the shape of the Galactic potential. On the other hand, the density declines in time as $t^{-n}$ where $n$ denotes the number of independent frequencies characterising its orbit. The growth of the size and density of a gap depend on the subhalo properties (mass and scale radius), the properties of the stream at the time of the impact (velocity and positional differences of the particles), and on the central orbit of the gap.

We have shown that the size of the gap is correlated with the portion of the stream most affected by the subhalo flyby (the value $y_{\text{max}}$ in the impulse approximation). The density contrast of the gap, on the other hand, is more correlated with the amplitude of the interaction ($\Delta v_{\text{max}}$). These different correlations are in the end, what drives the ability to break the degeneracy of the encounter parameters. For example, for a given gap age, small gaps (< 25 kpc) are very dependent on the size of the subhalo, while a large gap can be caused by a large subhalo, or by an alignment of the orbit of the stream and subhalo. These results are encouraging and appear to be useful to constrain the properties of a population of dark subhalos if present in the halo of the Milky Way.

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Appendix 10.A  
\textbf{Covariance matrix - full 3D impulse}

In this section we derive the change of the covariance matrix when the full 3D morphology of the stream is taken into account. Similar to the 1D case, see Sec. 10.2.5, we assume that the change in velocity is a linear function of the spatial coordinates, meaning the denominator of the kicks ($w((r_s^2 + r^2)w^2 - (\vec{x} \cdot \vec{w})^2)) \approx r_s^2 w^3$. This approximation is in general true for the small volumes in which we measure the density, typically $<< 1 \text{ kpc}$. This assumption allows us to rewrite Eqs. (10.7) to

$$\Delta v_i(\vec{x}) = -2GM \frac{w^2 x_i - w_i (\vec{x} \cdot \vec{w})}{r_s^2 w^3}, \quad (A.1)$$

where the subscript $i = x, y, z$.

In a similar procedure as for the 1D approximation we can now compute the covariance terms. The velocity-position terms in the most general form are

$$C(v_i + \Delta v_i, x_j) = C(v_i, x_j) + \frac{1}{n} \sum \Delta v_i (x_j - \bar{x}_j) \quad (A.2)$$

with

$$\frac{1}{n} \sum \Delta v_i (x_j - \bar{x}_j) = -\frac{2GM}{r_s^2 w^3} \left[ w^2 C(x_i, x_j) - \sum_{k=x,y,z} w_i w_k C(x_k, x_j) \right]. \quad (A.3)$$

The velocity-velocity terms are a bit more cumbersome

$$C(v_i + \Delta v_i, v_j + \Delta v_j) = C(v_i, v_j) + \frac{1}{n} \sum \Delta v_i (v_j - \bar{v}_j) + \frac{1}{n} \sum (v_i - \bar{v}_i) \Delta v_j + \frac{1}{n} \sum \Delta v_i \Delta v_j \quad (A.4)$$

where

$$\frac{1}{n} \sum \Delta v_i (v_j - \bar{v}_j) = -\frac{2GM}{r_s^2 w^3} \left[ w^2 C(x_i, v_j) - \sum_{k=x,y,z} w_i w_k C(x_k, v_j) \right]. \quad (A.5)$$

The last term in Eq. (A.4) is

$$\frac{1}{n} \sum \Delta v_i \Delta v_j = \left( \frac{2GM}{r_s^2 w^3} \right)^2 \left[ w^4 C(x_i, x_j) - \sum_{k=x,y,z} \left( w^2 w_k [w_j C(x_i, x_k) + w_i C(x_j, x_k)] - w_j w_k w_i C(x_j, x_k) \right) \right.$$

$$+ w_i w_j \left( 2w_x w_y C(x, y) + 2w_x w_z C(x, z) + 2w_y w_z C(y, z) \right) \left. \right]. \quad (A.6)$$
These expressions take a much simpler form if the initial covariance matrix $\Sigma_{\infty,0}$ (e.g. Eq. 10.19) is diagonal. In this case, Eq. (A.3) simplifies to

$$\frac{1}{n} \sum \Delta v_i (x_j - \bar{x}_j) = \frac{2GM}{r_s^2 w^3} C(x_j, x_j) \left[ w^2 \delta_{ij} - w_i w_j \right], \quad (A.7)$$

where $\delta_{ij}$ is the Kronecker delta. Equation (A.5) simply vanishes, since it only features off-diagonal terms, and Eq. (A.6) reduces to

$$\frac{1}{n} \sum \Delta v_i \Delta v_j = \left( \frac{2GM}{r_s^2 w^3} \right)^2 \left[ w^4 C(x_i, x_j) - w^2 w_i w_j [C(x_i, x_i) + C(x_j, x_j)] \right]$$

$$+ \sum_{k=x,y,z} \left( w_i w_j w_k^2 C(x_k, x_k) \right). \quad (A.8)$$

### Appendix 10.B Subhalo scaling relations

For the scaling of the subhalos scale radius $r_s$ with mass $M$ we have used several scaling relations, they are listed below. To obtain the scaling we first relate the subhalo mass $M$ with the maximum circular velocity $V_{\text{max}} = V_c(r_{\text{max}})$. In Springel et al. (2008) (see their Fig. 27) we find

$$V_{\text{max}} = \left( \frac{M}{3.37 \cdot 10^7 M_\odot} \right)^{1/3.49} \cdot 10 \text{ km/s}, \quad (B.1)$$

which is an empirical scaling relation based on the subhalos down to the mass-range of $\lesssim 10^5 M_\odot$, identified in the ‘Aquarius’ simulations.

Next, $r_{\text{max}}$ is related to $V_{\text{max}}$ using Eqs. (6,8,9) from Springel et al.

$$r_{\text{max}} = V_{\text{max}} \left[ \frac{\delta_c H_0^2}{14.426} \right]^{-\frac{1}{2}} \cdot 0.62, \quad (B.2)$$

where the factor 0.62 is added based on the comment in the caption of Fig. 26 of Springel et al. (2008, see also below) and we assume $H_0 = 73 \text{ km/s/Mpc}$. The final missing piece is $\delta_c$, which is related to the concentration parameter $c$ by

$$\delta_c = \frac{200}{3} c^3 \left( \log(1 + c) - \frac{c}{1 + c} \right)^{-1}. \quad (B.3)$$

Typically $c$ is related to the subhalo mass $M$, motivated by Springel et al. (2008), we relate the two using an empirical scaling relation found by Neto et al. (2007) for relaxed halos

$$c = 5.26 \left( \frac{M}{h \cdot 10^{14}} \right)^{-0.10}, \quad (B.4)$$
where \( h = H_0/(100 \text{ km/s}) \). We note that Springel et al. (2008) find that the resulting scaling relation of \( V_{\text{max}} \sim r_{\text{max}} \) is lower than the relation found from extrapolating the results of Neto et al. (which is not calibrated for subhalos in the low-mass range that we consider here). The offset is 0.62, which is why we add this factor in Eq. (B.2).

With the equations above we can relate the subhalo mass \( M \) to \( V_{\text{max}} \) and a corresponding \( r_{\text{max}} \). The scale radius \( r_{s,NFW} \) is related to \( r_{\text{max}} \) simply as

\[
r_{\text{max}} = 2.163 \cdot r_{s,NFW},
\]

which is found numerically from calculating where \( V_c(r_{\text{max}}) = V_{\text{max}} \) (but see Eq. (11) of Diemand et al. (2007), where we originally found the relation).

Finally, in this Chapter we use a Plummer profile to describe the subhalos, rather than an NFW. Therefore, we relate the scale radii of the two profiles by equation the acceleration at \( r_{\text{max}} \)

\[
a_{\text{NFW}}(r_{\text{max}}) = a_{\text{Plummer}}(r_{\text{max}}) = -\frac{GM}{(r_{\text{max}}^2 + r_s^2)^{3/2}} \cdot r_{\text{max}}.
\]

The scale radius of the Plummer, \( r_s \), can be found by solving the equation above, which then is a function of \( M \) only. Finally, by fitting the scaling relation numerically we find that the scale radius depends on mass by \( r_s \propto M^{0.397} \sim M^{2/5} \).

### Appendix 10.C Computation of the density contrast at late times

As described in the main paper, the density contrast at late times takes the form

\[
\delta \rho_{\text{str}}^\text{gap} = \sqrt{\frac{\det |\sigma_{\Theta_0}|_{\text{str}}}{\det |\sigma_{\Theta_0}|_{\text{gap}}}},
\]

(see Eq. 10.34). To be able to establish its dependence on the characteristic parameters of the encounter, we need to determine the form of determinant of the matrix \( \sigma_{\Theta_0} \). The latter is a submatrix of \( \sigma_{\omega,0} \) that is described in Sec. 10.2.5. This matrix, following the notation of Sec. 10.2.4, takes the form

\[
\sigma_{\omega,0} = T_0^{-1} \sigma_{\omega,0} T_0^{-1},
\]

where \( T_0 \) is given by Eq. (10.12) and \( \sigma_{\omega}^{-1} = \Sigma_{\omega} \) is the inverse of the 6×6 covariance matrix.

Therefore, \( \sigma_{\Theta_0} \) for the stream depends on location as well as on the initial properties of the stream, and similarly for the gap. Note that in both cases, the location is set to be the same (because we compare the density of the stream and the gap on the stream
at the same location). This turns out to significantly simplify the computation of the determinant in Eq. (C.1). In fact, using Eq. (C.2) it can be shown that

$$\det |\sigma_{\Theta 0}| = h_{\text{orb}} \det \sigma_{v_0}^{2 \times 2}. \quad (C.3)$$

This expression has been worked out in detail by HW99 for example for the cases of spherical symmetry, axisymmetry for an Eddington potential (the last equation in their Appendix B) and for the Staeckel case (Eq. C13 in their Appendix C) for an initially isotropic Gaussian distribution in velocity space, and for a preferred location along the orbit, namely the apocentre.

In our case, we will assume no initial correlations between positions and velocities in the stream (i.e. a diagonal covariance matrix $\Sigma_{\omega 0}$), which means that the submatrix $\sigma_{v_0} = C^{-1}_{v,v_0}$ according to Eq. (10.19). We may now express

$$C_{v,v_0}^{\text{gap}} = C_{v,v_0}^{\text{str}} + \epsilon D(\theta, \alpha) \quad (C.4)$$

where the elements of $D(\theta, \alpha)$ are $\Delta(\theta, \alpha)_{ij} = \Delta v_i \Delta v_j$ which are given by Eqs. (10.6) (and depend on $\theta$ and $\alpha$), without the common multiplicative factor $\epsilon$ which (for $r_s^2 w^2 >> y^2 w_{\perp}^2$) is defined as

$$\epsilon = \frac{2GM}{wr_s^2} \cos \theta. \quad (C.5)$$

If we now return to Eq. (C.3), we see that we need the determinant of the $2 \times 2$ lower submatrix of $C_{v,v_0}^{\text{gap}}$. Since the inverse of a matrix $C_{v,v_0}^{-1} = 1/ \det C_{v,v_0} S$, where $S$ contains the cofactor elements of $C_{v,v_0}$, this means that the determinant of the submatrix

$$\det \sigma_{v_0}^{2 \times 2} = \frac{\det S_{2 \times 2}^{2 \times 2}}{(\det C_{v,v_0})^2}. \quad (C.6)$$

We may now compute the density contrast using Eqs. (C.1, C.3, and C.4)

$$\delta \rho_{\text{str}}^{\text{gap}} = \sqrt{\frac{\det \sigma_{v_0}^{2 \times 2}}{\det \sigma_{v_0}^{2 \times 2}}} \left|_{\text{gap}} \right. \frac{\det C_{v,v_0}^{\text{gap}}}{\det C_{v,v_0}^{\text{str}}} \frac{\det S_{2 \times 2}^{2 \times 2}}{\det S_{2 \times 2}^{2 \times 2}} \left|_{\text{gap}} \right. \quad (C.7)$$

The elements of $S|_{\text{gap}}$ are to first order in $\epsilon$, of the form

$$s_{ij}^{\text{gap}} = s_{ij}^{\text{str}} + \epsilon g_{ij}(\theta, \alpha), \quad (C.8)$$

which therefore means that

$$\det S_{2 \times 2}^{2 \times 2} \left|_{\text{gap}} \right. = \det S_{2 \times 2}^{2 \times 2} \left|_{\text{str}} \right. \det (1 + \epsilon S_{2 \times 2}^{-1} \left|_{\text{str}} \right. g(\theta, \alpha))) \quad (C.9)$$

$$\sim \det S_{2 \times 2}^{2 \times 2} \left|_{\text{str}} \right. \left[1 + \epsilon \text{tr}(S_{2 \times 2}^{-1} \left|_{\text{str}} \right. g(\theta, \alpha))) \right]. \quad (C.10)$$
Using Eq. (C.4), we similarly find that

\[ \det C_{\text{gap}}^{v,v_0} \sim \det C_{\text{str}}^{v,v_0} [1 + \epsilon \, \text{tr} (C_{v,v_0}^{-1,\text{str}} D_{(\theta,\alpha)})]. \]  

(C.11)

We may now use these last two equations in Eq. (C.7). After performing a first order Taylor expansion in \( \epsilon \), we derive for the density contrast

\[ \delta \rho_{\text{str}}^{\text{gap}} \sim 1 - \frac{M}{r_s^2} \cos \theta f(\theta, \alpha, C_{v,v_0}) \]  

(C.12)

where \( f(\theta, \alpha, C_{v,v_0}) \) is a function of the trace of the matrices \( C_{v,v_0}^{-1,\text{str}} D_{(\theta,\alpha)} \) and \( S_{2 \times 2}^{-1} |\text{str} g(\theta, \alpha)\).
Samenvatting

Sterrenstelsels ontstaan uit de materie die is gevormd in de Oerknal. De exacte eigenschappen van deze materie, met name die van de donkere variant, zijn nog niet geheel bekend. Ook de theorie die beschrijft hoe sterrenstelsels ontstaan is nog onvolledig. De Melkweg, het sterrenstelsel waarin wij leven, geeft ons een unieke blik in deze fundamentele onderwerpen. Omdat we ons er midden in bevinden, bestuderen we de Melkweg ster-voor-ster. Dit is (momenteel) onmogelijk voor andere sterrenstelsels.

Echter, we kunnen de Melkweg niet alleen per ster bekijken, we moeten ook! Het is niet mogelijk om een beeld van buitenaf te krijgen, zoals dat wel het geval is met andere sterrenstelsels. Door nauwkeurig de eigenschappen van zoveel mogelijk sterren in kaart te brengen, kunnen we toch een redelijk beeld krijgen. Onlangs heeft de Gaia-satelliet een revolutie teweeg gebracht met de uitgifte van een zeer nauwkeurige ‘kaart’ van de sterren van de Melkweg. Data van de Gaia-satelliet zijn veelvuldig gebruikt in dit proefschrift.

Sterrenstelsels groeien door fusies met andere stelsels en door het aantrekken van materie uit het intergalactisch medium. Beide processen worden gedreven door de zwaartekracht. Bij deze fusies kan het er wild aan toegaan. Kleine sterrenstelsels worden geheel uit elkaar gerukt door de zwaartekracht van het (grotere) sterrenstelsel waarmee het fuseert. Men spreekt ook wel van ‘galactisch kannibalisme’. Fusies van vergelijkbare massa’s zorgen voor veel spektakel waarbij flarden van materie naar buiten worden geslingerd. Het soort fusies van de Melkweg en een sterrenstelsel van enkele malen kleiner zijn interessant om te ontdekken en te onderzoeken. Deze fusies zijn klein genoeg om te kunnen spreken van een duidelijk herkenbare ‘Melkweg voor’ en ‘Melkweg na’ de fusie, maar soms zijn ze groot genoeg om een blijvende impact te hebben op de structuur van de Melkweg. De stelsels die zijn gefuseerd, noemen we ook wel de bouwblokken van de Melkweg.

De Melkweg als laboratorium

De geschiedenis van de Melkweg staat beschreven in haar sterren. Dat is dan wel alleen de geschiedenis in termen van miljarden jaren en op een schaal vele malen groter dan ons zonnestelsel. De dynamica van sterrenstelsels en de ontstaansgeschiedenis van de Melkweg behoren (voornamelijk) tot de klassieke natuurkunde: ze zijn deterministisch. We kunnen zeer nauwkeurig de banen van sterren berekenen binnen een sterrenstelsel. Dit soort berekeningen zijn bijna altijd tijdsomkeerbaar: we kunnen berekenen wat de banen van sterren zijn geweest en waar ze vandaan komen. Door het meten van de eigenschappen van alle sterren zouden we kunnen berekenen hoe de Melkweg er miljarden jaren geleden uitzag. Echter, dit werkt grotendeels alleen in theoretisch opzicht.
Ten eerste kunnen we nooit nauwkeurig genoeg alles meten en, ten tweede, is de meeste massa onzichtbaar als donkere materie. We kunnen deze dus niet direct meten. In de praktijk berekenen we voornamelijk alleen de banen van individuele sterren terug in de tijd, ervan uitgaande dat de algemene structuur van de Melkweg over die tijd niet veranderd is.

Sterren bevatten nog meer eigenschappen waaraan hun oorsprong te herleiden is: bijvoorbeeld hun chemische samenstelling en leeftijd. Sterren die tegelijk ontstaan hebben dezelfde leeftijd. Om de chemische samenstelling en evolutie te begrijpen moeten we terug gaan naar de Oerknal. De chemische elementen die ontstaan uit de Oerknal zijn: waterstof (ongeveer 75%) en helium (∼ 25%) en een minuscule hoeveelheid lithium. Alle andere elementen in het periodieke systeem worden pas later gevormd, bijvoorbeeld in sterren, of in laboratoria. We noemen de elementen zwaarder dan helium om deze reden ‘metalen’ in de sterrenkunde. De eerste sterren die ontstaan zijn metaal-arm. Sterren produceren metalen in hun kernen en verspreiden deze aan het einde van hun leven, bijvoorbeeld door te exploderen. Dit proces herhaalt zich vele malen en resulteert in de chemische evolutie van sterrenstelsels. De chemische samenstelling van een ster verandert weinig voor het grootste deel van zijn leven, behalve dan in de kern - maar die kunnen we niet observeren.

In dit proefschrift combineren we inzichten vanuit de dynamica en chemische eigenschappen om te werken aan onderzoeksvragen zoals: wat is de geschiedenis van de Melkweg? Hoeveel bouwblokken zijn er geweest? En wat waren hun eigenschappen? Wat is de structuur van de Melkweg? Maar ook: hoe zit het met die donkere materie? Vormt dat ook structuren? Hoe ontdekken we die?
Galactische archeologie in de halo

We zoeken naar de antwoorden op deze vragen in de halo van de Melkweg, zie het figuur ‘Anatomy of the Milky Way’. De halo bevat sommige van de oudste en meest metaal-arme sterren van de Melkweg. Deze sterren lijken wellicht het meeste op de eerste sterren die zijn ontstaan. En, belangrijk voor dit proefschrift, de restanten van de sterrenstelsels die al zijn gefuseerd met de Melkweg zijn voornamelijk te vinden in de halo. Het is niet zo dat ze de schijf van de Melkweg mijden. Maar het zou wel erg toevallig zijn als alle sterrenstelsels die zijn gefuseerd precies zo zijn opgeslokt door de Melkweg dat ze zich in de schijf bevinden. Niet alleen vinden we dus de restanten van deze fusies in de halo, maar ook zijn hier de dynamische tijdsschalen miljarden jaren lang, met name in de buitenste delen van de halo. Dit betekent dat het miljomen tot miljarden jaren duurt voor de Melkweg om volledig te fuseren met een ander stelsel en om de sterren daarvan te vermengen met haar eigen. We zouden dus nog steeds flarden van die stelsels moeten kunnen waarnemen, zoals die weergegeven in het figuur van NGC 474. Om deze redenen staan de sterren in de halo ook wel bekend als de galactische fossielen en de studie daarvan als galactische archeologie. In de halo komen ook ietwat specifieker onderzoeksvragen aanbod zoals: wat is de structuur van de halo? En wat voor substructuur is hier te vinden, in de buurt van de Zon? Hoe identificeren en bestuderen we halo-sterren, in het geval van incomplete data?

Bewijs voor fusies met de Melkweg

Met behulp van de gegevens van de Gaia-satelliet tonen we in dit proefschrift aan dat er een verscheidenheid aan substructuur is in de lokale halo van de Melkweg. Lokaal betekent hier in de buurt van de Zon, binnen drieduizend tot tienduizend lichtjaren. Vele van deze structuren doen erg denken aan de restanten van kleine sterrenstelsels die miljarden jaren geleden zijn gefuseerd met de Melkweg. Verrassend genoeg blijken de meeste van de sterren in de lokale halo onderdeel te zijn van één enkele, hele grote groep (of ‘blob’).

Deze blob van sterren lijkt erg veel op het eindresultaat van een computer simulatie die is ontwikkeld in 2008. Aan de hand van deze simulatie stellen we dan ook de volgende
hypothese op: de sterren in de blob zijn afkomstig van een relatief groot sterrenstelsel dat circa tien miljard jaar geleden is gefuseerd met de Melkweg. Als dit klopt dan moeten de gevolgen van deze fusie zeker nog te zien zijn in de Melkweg! Door de chemische composities van de blob-sterren te bestuderen hebben we kunnen vaststellen dat ze inderdaad afkomstig zijn uit een relatief groot sterrenstelsel. Let wel: relatief groot voor tien miljard jaar geleden, vergeleken met de huidige Melkweg is het vrij klein. We hebben we het stelsel Gaia-Enceladus genoemd, naar de satelliet Gaia en de Gigant Enceladus, de zoon van de ‘oermoeder’ Gaia volgens de Griekse mythologie.

Een groot deel van dit proefschrift is gewijd aan het bestuderen van de eigenschappen van Gaia-Enceladus en andere fusie-restanten van kleinere sterrenstelsels. Zo bestuderen we de mogelijke eigenschappen van het sterrenstelsel waarvan de restanten de huidige ‘Helmi-stroom’ vormen. Deze stroom is een van de eerste bewijzen van fusies in de (lokale) halo van de Melkweg en is ontdekt in 1999. Ook vinden we bewijs voor de aanwezigheid van in ieder geval twee andere restanten in het retrograde deel van de halo. Tevens bestuderen we in detail de simulaties die hebben geleid tot onze hypothese voor Gaia-Enceladus. Zo blijkt dat Gaia-Enceladus dusdanig groot is geweest dat er er mogelijk meerdere restanten te vinden zijn. Wat het geheel erg ingewikkeld maakt is dat deze restanten zowel dynamisch als chemisch andere eigenschappen kunnen hebben. Ze kunnen er dus uitzien als ongerelateerde restanten. Dit kan onderzoekers er toe misleiden ze te classificeren als restanten van aparte fusies.

Naast sterren, brengen fusies ook veel ander materiaal met zich mee, waaronder bolclusters. Deze bolclusters zijn zeer compacte systemen van honderdduizend tot een miljoen sterren die, in eerste instantie, behoorden tot het satellietsysteem van de gefuseerde sterrenstelsels. Als deze stelsels worden opgeslokt door de Melkweg blijven de bolclusters over. Ze zijn namelijk zo compact dat de zwaartekracht van de Melkweg ze niet (makkelijk) uit elkaar krijgt. We vinden inderdaad bewijs voor enkele groepen van bolclusters die lijken overeen te komen met de restanten van de opgeslokte sterrenstelsels die zijn ontdekt in de halo.

Massa van de Melkweg en donkere materie structuren

Na deze bevindingen, vervolgen we met een studie van een selectiemethode voor de identificatie van halo-sterren gebaseerd op incomplete data. Halo-sterren zijn relatief makkelijk te onderscheiden van schijf-sterren, hun banen zijn erg verschillend. Echter, voor slechts één per tweehonderd sterren is het momenteel mogelijk om de banen te berekenen. Voor de overige sterren zijn de waarnemingen niet compleet, of bevatten de afstanden grote meetfouten. De methode die wij gebruiken maakt alleen gebruik van data die beschikbaar zijn voor (bijna) alle sterren. Het is een manier om de incomplete informatie te omzeilen en alsnog miljoenen halo-sterren te identificeren.

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3 retrograad betekent dat ze in de tegenovergestelde richting bewegen ten opzichte van de schijf
We gebruiken deze set van halo-sterren om de lokale ontsnappingssnelheid te bepalen. Dit is een eigenschap van de Melkweg die interessant is om te bepalen omdat hij gerelateerd is aan haar massa. De ontsnappingssnelheid is de snelheid die nodig is voor een object om te ontsnappen aan de Melkweg en is afhankelijk van de locatie vanaf waar een object ontsnapt. Ontsnappen vanuit het Galactisch Centrum is moeilijker dan ontsnappen vanuit ergens ver daarbuiten. De ontsnappingssnelheid wordt voornamelijk bepaald door het de massa van de Melkweg, hoe zwaarder die is des te meer snelheid er nodig is. Volgens onze analyse is de ontsnappingssnelheid in de buurt van de Zon tenminste 493 kilometer per seconde. Omdat de methode die we gebruiken slechts een ondergrens meet betekent dat de echte ontsnappingssnelheid waarschijnlijk ergens boven de 500 kilometer per seconde ligt: erg snel. Op basis van deze limiet komen we uit op een massa van de Melkweg in de buurt van duizend-miljard (een biljoen) zonmassa's.

Het proefschrift eindigt met een studie naar stelsels van donkere materie (ook wel donkere satellieten genoemd) en hun invloed op de waarneembare sterrenstromen in de halo. Het soort interacties die we onderzoeken zijn die met dunne, ‘koude’ sterstromen. Koud betekent in het algemeen dat onderlinge snelheidsverschillen minimaal zijn, hier specifiek houdt dat in dat de sterren allemaal langs bijna dezelfde baan bewegen. Als een satelliet van donkere materie botst met, of in de buurt komt van, zo ’n koude stroom dan verstoort het de banen van de sterren. Deze verstoring groeit langzaam uit tot een soort gat. In dit proefschrift hebben we een model ontwikkeld dat heel precies de evolutie van dit gat beschrijft in een realistisch model van de Melkweg. De hoop is dat in de nabije toekomst er een groot aantal van dit soort gaten wordt waargenomen. Aan de hand van ons model is dan, wellicht, te berekenen wat voor soort donkere satelliet het gat heeft veroorzaakt. Uiteindelijk moet het model dus ‘licht werpen op’ de donkere satellieten in de halo van de Melkweg.
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Colophon

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