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Saturated control without velocity measurements for planar robots with flexible joints

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Abstract—In this work, we propose a saturated passivity-based controller that addresses the problem of set-point regulation for planar robots with two links and flexible joints. Moreover, the controller does not require velocity measurements. We implement the control law and present experimental results.

Index Terms—Port-Hamiltonian systems, passivity-based control, set-point regulation, nonlinear systems.

I. INTRODUCTION

The problem of set-point regulation for robots with flexible joints has been of interest to the robotics and control systems communities during the last decades, some works on this topic are [1], [2]. In this note, we are particularly interested in the design of controllers for robotic arms that can overcome two common issues that arise during practical implementation, namely, the lack of sensors to measure the velocities and the necessity of saturated signals to ensure the safety of the equipment. To that end, we represent the robotic arm using the port-Hamiltonian approach, which underscores the roles of the energy and the dissipation—see [3]. This, besides the easy identification of the passive output, suggests the passivity-based control (PBC) approach as a natural option to design the controller.

The main contribution of this work is the design of a controller that uses output feedback to produce a stabilizing, saturated control signal without using velocity measurements. Designing such a controller is challenging because of the nonlinear dynamics of these robots and the fact that flexibility in the joints introduces unactuated states. The controller proposed in this paper has the following appealing properties:

- Its design does not require the solution of partial differential equations (PDEs).
- Its components are constrained to a predefined interval. Thus, in implementation, no additional saturation is necessary to prevent damage to the motors.
- It only requires position measurements. Therefore, it can be implemented without filters nor observers to estimate the velocities.

Set-point regulation of flexible-joint manipulators has been achieved without velocity measurements in [4], and with saturated control signals in [5]. However, to the best authors knowledge, the technique described in this work is the first to address these issues simultaneously.

The outline of this paper is as follows: we provide the system's model and the problem formulation in Section II. Section III is devoted to the control design and the experimental results. We conclude this note with some remarks and future work in Section IV.

Caveat: We refer the reader to [arXiv:1812.08257 \[cs.SY\]](#) for an extended version of this document including elements that were removed due to space constraints.

II. MODEL AND PROBLEM SETTING

The system to be controlled consists of two links, each one attached to a motor shaft through a spring. As is stated above, we adopt a pH model to characterize the behavior of the system. Therefore, we consider as state vector the positions $q \in \mathbb{R}^4$ and the momenta $p \in \mathbb{R}^4$ related to the elements of the robot arm, namely,

$$q = \begin{bmatrix} q_l \\ q_m \end{bmatrix}, p = \begin{bmatrix} p_l \\ p_m \end{bmatrix}$$

where the vectors $q_l \in \mathbb{R}^2$, $q_m \in \mathbb{R}^2$ denote the angular position of the links and the motors, respectively; while, $p_l \in \mathbb{R}^2$ represent the momenta of the links, and the momenta of the motors are given by $p_m \in \mathbb{R}^2$. Hence, the system dynamics are expressed as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{4 \times 4} & I_4 \\ -I_4 & -R_2 \end{bmatrix} \begin{bmatrix} \nabla_q H(q, p) \\ \nabla_p H(q, p) \end{bmatrix} + \begin{bmatrix} \mathbf{0}_4 \\ B \end{bmatrix} u \quad (1)$$

$$H(q, p) = \frac{1}{2} p^\top M^{-1}(q_l) p + \frac{1}{2} \|q_l - q_m\|_{K_s}^2$$

with

$$M(q_l) = \begin{bmatrix} M_l(q_l) & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & M_m \end{bmatrix}, R_2 = \begin{bmatrix} D_l & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & D_m \end{bmatrix}, B = \begin{bmatrix} \mathbf{0}_2 \\ I_2 \end{bmatrix}$$

$$M_l(q_l) = \begin{bmatrix} a_1 + a_2 + 2b \cos(q_{l_2}) & a_2 + b \cos(q_{l_2}) \\ a_2 + b \cos(q_{l_2}) & a_2 \end{bmatrix}$$

$$M_m = \text{diag}\{\mathcal{I}_{m_1}, \mathcal{I}_{m_2}\}, D_l = \text{diag}\{D_{l_1}, D_{l_2}\}$$

$$D_m = \text{diag}\{D_{m_1}, D_{m_2}\}, K_s = \text{diag}\{k_{s_1}, k_{s_2}\},$$

where a_1 , a_2 and b are constants related to the moment of inertia (MoI) of the links; and $u \in \mathbb{R}^2$ is the input vector which corresponds to the torques of the motors. See Table III in Appendix for more info about the parameters.

To formulate the set-point regulation problem we, first, define the set of assignable equilibria as

$$\mathcal{E} := \{q \in \mathbb{R}^4 \mid q_l = q_m, p = \mathbf{0}_4\}.$$

Problem setting: the objective of this work is to design a control law that stabilizes system (1) at a constant point $x_* \in \mathcal{E}$ subject to the following constraints:

- Only q is measurable.
- Define $\mathcal{U} := [-u_{\max_1}, u_{\max_1}] \times [-u_{\max_2}, u_{\max_2}]$, then $u(t) \in \mathcal{U}$ for all $t \geq 0$.

In the following section, we design a control law that fulfills the mentioned requirements.

III. CONTROL DESIGN

In this section, we present three controllers that stabilize the planar robot. The first one is based on the PI-PBCs reported in [6], which are the starting point to develop saturated controllers. The second controller satisfies the requirements established in Section II, nonetheless, the experiments exhibit steady-state error. The third controller includes an integral-like term of the error in position to remove the steady-state error.

A. Preliminary PI controller

In [6], a constructive procedure to stabilize pH systems is proposed, an advantage of this approach over other PBC techniques is that the control law is obtained without the necessity of solving PDEs. Furthermore, the controllers derived from this approach can be interpreted as PI regulators, where the feedback signal is the passive output of the system. The following proposition provides a modified PI controller that stabilizes system (1) at the desired position.

Proposition 1: Consider system (1) in closed-loop with the controller

$$u = -K_{P_m} \dot{q}_m - K_I(q_m - q_*) - K_{P_l} \dot{q}_l \quad (2)$$

where $q_* \in \mathbb{R}^2$ is the desired position of the links, and the matrices $K_{P_m}, K_{P_l}, K_I \in \mathbb{R}^{2 \times 2}$ verify

$$K_{P_m} > 0, K_I > 0, D_m + K_{P_m} - \frac{1}{4} K_{P_l}^\top D_l^{-1} K_{P_l} > 0. \quad (3)$$

Then, the following statements hold true.

- (i) The closed-loop system admits a pH representation, that is

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{4 \times 4} & I_4 \\ -I_4 & J_{\text{PI}_2} - R_{\text{PI}_2} \end{bmatrix} \begin{bmatrix} \nabla_q H_{\text{PI}}(q, p) \\ \nabla_p H_{\text{PI}}(q, p) \end{bmatrix} \quad (4)$$

where

$$\begin{aligned} R_{\text{PI}_2} &:= \begin{bmatrix} D_l & \frac{1}{2} K_{P_l}^\top \\ \frac{1}{2} K_{P_l} & D_m + K_{P_m} \end{bmatrix}, \\ J_{\text{PI}_2} &:= \frac{1}{2} \begin{bmatrix} \mathbf{0}_{2 \times 2} & K_{P_l}^\top \\ -K_{P_l} & \mathbf{0}_{2 \times 2} \end{bmatrix} \\ H_{\text{PI}}(q, p) &:= H(q, p) + \frac{1}{2} \|q_m - q_*\|_{K_I}^2. \end{aligned}$$

- (ii) The point

$$x_* := (q_l, q_m, p_l, p_m) = (q_*, q_*, \mathbf{0}_2, \mathbf{0}_2)$$

is an asymptotically stable equilibrium of the closed-loop system with Lyapunov function $H_{\text{PI}}(q, p)$.

Remark 1: The natural damping of the system ensures that the term $-K_{P_l} \dot{q}_l$ can be included at the same time that the pH structure is preserved, which is desirable for analysis purposes and physical interpretation of the closed-loop system.

B. Saturated control without velocity measurements

The control law (2) renders asymptotically stable the desired point. However, it requires velocity information and the control signals are not bounded to the operation range of the motors. To overcome the aforementioned issues, we propose two modifications to the control law:

- Replace the integral term $K_I(q_m - q_*)$ with a saturated function.
- Perform damping injection without measuring the velocities.

In [7], the authors propose a method to inject damping without velocity measurements for mechanical systems, where the main idea is to propose a virtual state, which is linearly related to the positions, to inject damping into the closed-loop system. Proposition 2 establishes one of the main contributions of this paper, where we combine the damping injection approach of [7] with the PI of Proposition 1. An advantage of this combination—compared to [7]—is the absence of PDEs in the control design. Additionally, we improved the stability proof of [7].

Proposition 2: Let the controller state vectors $x_{c_l}, x_{c_m} \in \mathbb{R}^2$. Define the functions

$$\begin{aligned} z_l(q_l, x_{c_l}) &:= q_l - q_* + x_{c_l}, \\ z_m(q_m, x_{c_m}) &:= q_m - q_* + x_{c_m}, \\ \Phi_l(z_l) &:= \sum_{i=1}^2 \frac{\alpha_{l_i}}{\beta_{l_i}} \ln(\cosh(\beta_{l_i} z_{l_i})) \\ \Phi_m(z_m) &:= \sum_{i=1}^2 \frac{\alpha_{m_i}}{\beta_{m_i}} \ln(\cosh(\beta_{m_i} z_{m_i})) \end{aligned} \quad (5)$$

where $\alpha_{l_i}, \alpha_{m_i}, \beta_{l_i}, \beta_{m_i} \in \mathbb{R}_{>0}$. Consider the dynamics

$$\begin{aligned} \dot{x}_{c_l} &= -R_{c_l} \nabla_{x_{c_l}} \Phi_l(z_l(q_l, x_{c_l})) \\ \dot{x}_{c_m} &= -R_{c_m} [\nabla_{x_{c_m}} \Phi_m(z_m(q_m, x_{c_m})) + K_c x_{c_m}], \end{aligned} \quad (6)$$

where $R_{c_l}, R_{c_m}, K_c \in \mathbb{R}^{2 \times 2}$ are positive definite constant matrices verifying

$$R_{c_l} - \frac{1}{4} (D_l^{-1} + D_m^{-1}) > 0. \quad (7)$$

Consider the control law

$$u = -\nabla_{z_l} \Phi_l(z_l(q_l, x_{c_l})) - \nabla_{z_m} \Phi_m(z_m(q_m, x_{c_m})). \quad (8)$$

Then:

- (i) The elements of the input vector u are saturated.
- (ii) Consider system (1) in closed-loop with (8). Hence, the dynamics of the augmented state space $\zeta = [q^\top, p^\top, x_{c_l}^\top, x_{c_m}^\top]^\top$ admit a pH representation.
- (iii) The point $\zeta_* = (x_*, \mathbf{0}_2, \mathbf{0}_2)$ is an asymptotically stable equilibrium of the closed loop system with Lyapunov function

$$H_\zeta(\zeta) = H(q, p) + \Phi_l(z_l(q_l, x_{c_l})) + \Phi_m(z_m(q_m, x_{c_m})) + \frac{1}{2} \|x_{c_m}\|_{K_c}^2. \quad (9)$$

Proof: To prove (i), note that

$$\begin{aligned} \nabla_{z_l} \Phi_l &= \begin{bmatrix} \alpha_{l_1} \tanh(\beta_{l_1} z_{l_1}) \\ \alpha_{l_2} \tanh(\beta_{l_2} z_{l_2}) \end{bmatrix} \\ \nabla_{z_m} \Phi_m &= \begin{bmatrix} \alpha_{m_1} \tanh(\beta_{m_1} z_{m_1}) \\ \alpha_{m_2} \tanh(\beta_{m_2} z_{m_2}) \end{bmatrix}. \end{aligned} \quad (10)$$

Therefore, the control law (8) reduces to

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \alpha_{l_1} \tanh(\beta_{l_1} z_{l_1}) + \alpha_{m_1} \tanh(\beta_{m_1} z_{m_1}) \\ \alpha_{l_2} \tanh(\beta_{l_2} z_{l_2}) + \alpha_{m_2} \tanh(\beta_{m_2} z_{m_2}) \end{bmatrix}. \quad (11)$$

Thus,

$$-(\alpha_{l_i} + \alpha_{m_i}) \leq u_i \leq \alpha_{l_i} + \alpha_{m_i}. \quad (12)$$

To prove (ii) note that

$$\begin{aligned} \nabla_{q_l} \Phi_l &= \nabla_{x_{c_l}} \Phi_l = \nabla_{x_{c_l}} H_\zeta = \nabla_{z_l} \Phi_l, \\ \nabla_{q_m} \Phi_m &= \nabla_{x_{c_m}} \Phi_m = \nabla_{z_m} \Phi_m. \end{aligned} \quad (13)$$

Therefore, from (9) and (13), in closed-loop the dynamics of the momenta vector take the form

$$\begin{aligned} \dot{p}_l &= -\nabla_{q_l} H_\zeta - D_l \nabla_p H_\zeta + \nabla_{x_{c_l}} H_\zeta \\ \dot{p}_m &= -\nabla_{q_m} H_\zeta - D_m \nabla_p H_\zeta - \nabla_{x_{c_l}} H_\zeta. \end{aligned} \quad (14)$$

Moreover, (6) can be rewritten as

$$\dot{x}_{c_l} = -R_{c_l} \nabla_{x_{c_l}} H_\zeta, \quad \dot{x}_{c_m} = -R_{c_m} \nabla_{x_{c_m}} H_\zeta. \quad (15)$$

Hence, from (14) and (15), the closed-loop system takes the form

$$\dot{\zeta} = F_\zeta \nabla H_\zeta, \quad (16)$$

with,

$$F_\zeta := \begin{bmatrix} \mathbf{0}_{4 \times 4} & I_4 & \mathbf{0}_{4 \times 2} & \mathbf{0}_{4 \times 2} \\ -I_4 & -R_2 & \Gamma_\zeta & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 4} & -R_{c_l} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 2} & -R_{c_m} \end{bmatrix}; \quad \Gamma_\zeta := \begin{bmatrix} I_2 \\ -I_2 \end{bmatrix}. \quad (17)$$

thus preserving the pH structure. Furthermore, from (7), it follows that

$$\text{sym}\{F_\zeta\} = \begin{bmatrix} \mathbf{0}_{4 \times 4} & \mathbf{0}_{4 \times 4} & \mathbf{0}_{4 \times 2} & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{4 \times 4} & -R_2 & \frac{1}{2} \Gamma_\zeta & \mathbf{0}_{4 \times 2} \\ \mathbf{0}_{2 \times 4} & \frac{1}{2} \Gamma_\zeta^\top & -R_{c_l} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 4} & \mathbf{0}_{2 \times 2} & -R_{c_m} \end{bmatrix} \leq 0. \quad (18)$$

To prove (iii) note that, from (16) and (18), we have

$$\dot{H}_\zeta = (\nabla H_\zeta)^\top \dot{\zeta} = (\nabla H_\zeta)^\top F_\zeta \nabla H_\zeta \leq 0, \quad (19)$$

which implies that $H_\zeta(\zeta)$ is non-increasing. Moreover,

$$\begin{aligned} z_{l*} = \mathbf{0}_2 &\implies (\nabla_{q_l} \Phi_l)_* = (\nabla_{x_{c_l}} \Phi_l)_* = \mathbf{0}_2, \\ z_{m*} = \mathbf{0}_2 &\implies (\nabla_{q_m} \Phi_m)_* = (\nabla_{x_{c_m}} \Phi_m)_* = \mathbf{0}_2. \end{aligned} \quad (20)$$

Therefore,

$$(\nabla H_\zeta)_* = \begin{bmatrix} (\nabla H)_* \\ \mathbf{0}_2 \\ K_c x_{c_m*} \end{bmatrix} = \mathbf{0}_{12}. \quad (21)$$

Furthermore,

$$(\nabla^2 H_\zeta)_* = \begin{bmatrix} K_s + A & \mathbf{0}_{4 \times 4} & A \\ \mathbf{0}_{4 \times 4} & M_*^{-1} & \mathbf{0}_{4 \times 4} \\ A & \mathbf{0}_{4 \times 4} & A + K_c \end{bmatrix} > 0, \quad (22)$$

where

$$\begin{aligned} A &:= \text{diag}\{\beta_{l_1} \alpha_{l_1}, \beta_{l_2} \alpha_{l_2}, \beta_{m_1} \alpha_{m_1}, \beta_{m_2} \alpha_{m_2}\}, \\ K_s &:= \begin{bmatrix} K_s & -K_s \\ -K_s & K_s \end{bmatrix}, \quad K_c := \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & K_c \end{bmatrix}. \end{aligned}$$

Accordingly, from (21) and (22), $\arg \min\{H_\zeta(\zeta)\} = \zeta_*$. Thus, the stability property of ζ_* is proved by invoking Lyapunov theory. Moreover, to prove the asymptotic stability property, note that

$$\begin{aligned} \dot{H}_\zeta = 0 &\iff \begin{cases} \nabla_p H_\zeta = \mathbf{0}_4 \implies p = \mathbf{0}_4 \\ \nabla_{x_{c_l}} H_\zeta = \mathbf{0}_2 \implies z_l = \mathbf{0}_2 \\ \nabla_{x_{c_m}} H_\zeta = \mathbf{0}_2 \implies K_c x_{c_m} = -\nabla_{q_m} \Phi_m \end{cases} \\ &\implies \begin{cases} \nabla_{q_l} \Phi_l = \mathbf{0}_2 \\ \dot{p}_l = \mathbf{0}_2 \\ \dot{p}_m = \mathbf{0}_2 \end{cases} \implies \begin{cases} -K_s(q_l - q_m) = \mathbf{0}_2 \\ x_{c_m} = \mathbf{0}_2 \end{cases} \\ &\implies \begin{cases} q_m = q_l \\ z_m = \mathbf{0}_2 \end{cases} \implies \begin{cases} q_m = q_l = q_* \\ x_{c_l} = \mathbf{0}_2. \end{cases} \end{aligned} \quad (23)$$

Hence, the asymptotic stability property is proved by Barbashin Theorem. ■

Experimental results: the controller (8) is implemented in the robot arm *2 DOF serial flexible joint* by Quanser. We fix the following control parameters.

$$\begin{aligned} R_{c_m} &= \text{diag}\{25, 25\}, \quad \beta_{l_1} = 2, \\ R_{c_l} &= \text{diag}\{10, 40\}, \quad \beta_{l_2} = 1, \\ K_c &= \text{diag}\{5, 5\}, \quad \beta_{m_i} = 1; \end{aligned} \quad (24)$$

and we study three cases for different values of the parameters $\alpha_{l_i}, \alpha_{m_i}$ which are shown in Table I.

Fig. 1: Behavior of the positions for **C1**.

Fig. 2: Control signals for **C1**.

The three cases under study illustrate the effect of the term $\nabla_{z_l} \Phi_l(z_l(q_l, x_{cl}))$ in the behavior of the closed-loop system. This term is interpreted as damping in the links of the planar robot, therefore, is expected that as the values α_{l_i} increase, the oscillations in the response decrease. Accordingly, the experiments for the three cases are carried out under the same initial conditions, $\zeta(0) = \mathbf{0}_{12}$, and the same reference, $q_* = (-1, 1)$. Figures 1, 2 depict the results of case **C1**, Figures 3, 4 show the results of case **C2**; and Figures 5, 6 correspond to the results of case **C3**. From the aforementioned plots, we conclude the following:

- The existence of steady-state error can be observed for all the cases, this situation is probably a consequence of phenomena that are not taken into account in the model, e.g., nonlinear friction terms. Moreover, from the experiments, we notice the actuators are not able to provoke any displacement when $u_i \in [-0.12, 0.12]$. Therefore, the positions remain constant, even when the control signals are different from zero. This is particularly evident in Figure 2.
- There exists a trade-off between the damping injected to the links and the magnitude of the steady-state error. Furthermore, a similar relationship takes place between the magnitude of α_{l_i} and the oscillations, where a greater magnitude of these values yield an important attenuation in the oscillations. Both relations can be noticed in Figures 1, 3 and 5.

From a theoretical point of view, the control law (8) solves the problem stated in Section II, that is, the controller developed in this section is saturated and does not depend on the measurement of velocities. However,

Fig. 3: Behavior of the positions for **C2**.

Fig. 4: Control signals for **C2**.

a steady-state error exists. In the following section we propose an alternative to overcome this issue.

TABLE I: Cases for different values of the parameters α

Case	values for α_{l_i}	values for α_{m_i}
C1	0.8	0.4
C2	0.2	1
C3	0	1.2

Fig. 5: Behavior of the positions for **C3**.

Fig. 6: Control signals for case **C3**.

C. Eliminating the steady-state error

Customarily, the steady-state error is eliminated by the negative feedback of an integral term of the error between some measurements and the reference. In our case, such an error is the difference between the positions of the motors and the reference, that is, $(q_m - q_*)$. Note that the derivative of this error is given by $\dot{q}_m$, which is the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady-state error is given by a double integrator of the passive output. Proposition 3 provides an alternative control law that includes a double integral-like term, which eradicates the steady-state error and ensures that the control signals remain saturated, alas, the pH structure is not preserved.

Proposition 3: Consider the vector state $\sigma \in \mathbb{R}_2$ whose dynamics are given by

$$\dot{\sigma} = \nabla^2 \Phi_\sigma(\sigma)(q_m - q_*) - K_\sigma \sigma, \quad (25)$$

where the matrix $K_\sigma \in \mathbb{R}^{2 \times 2}$ and the function $\Phi_\sigma : \mathbb{R}^2 \rightarrow \mathbb{R}$ are defined as

$$\Phi_\sigma(\sigma) := \sum_{i=1}^2 \frac{\alpha_{\sigma_i}}{\beta_{\sigma_i}} \ln(\cosh(\beta_{\sigma_i} \sigma_i)) \quad (26)$$

$$K_\sigma := \text{diag}\{k_{\sigma_1}, k_{\sigma_2}\} \quad (27)$$

with $\alpha_{\sigma_i}, \beta_{\sigma_i}, k_{\sigma_i} \in \mathbb{R}_{>0}$. Consider F_ζ , given in (17) and assume that R_{c_l} verifies (7). Consider the control law

$$u = -\nabla_{z_{c_l}} \Phi_l(z_l(q_l, x_{c_l})) - \nabla_{z_{c_m}} \Phi_m(z_m(q_m, x_{c_m})) - \nabla \Phi_\sigma(\sigma), \quad (28)$$

with z_l, z_m, Φ_l, Φ_m defined as in (5). Fix the reference q_* and define the matrices

$$A_\sigma := \text{diag}\{\beta_{\sigma_1} \alpha_{\sigma_1}, \beta_{\sigma_2} \alpha_{\sigma_2}\}, \quad A_{\xi_1} := \begin{bmatrix} \mathbf{0}_6 \\ -A_\sigma \\ \mathbf{0}_4 \end{bmatrix}, \quad (29)$$

$$\mathcal{A} := \begin{bmatrix} F_\zeta(\nabla^2 H_\zeta)_* & A_{\xi_1} \\ A_{\xi_2}^\top & -K_\sigma \end{bmatrix}, \quad A_{\xi_2} := \begin{bmatrix} \mathbf{0}_2 \\ A_\sigma \\ \mathbf{0}_8 \end{bmatrix}.$$

where $(\nabla^2 H_\zeta)_*$ is given in (22). Then:

- (i) The elements of the input vector u are saturated.
- (ii) If the matrix $\mathcal{A}$ is Hurwitz, then $\xi_* = [q_*^\top \ q_*^\top \ \mathbf{0}_{10}^\top]^\top$ is a (locally) asymptotically stable equilibrium point of the closed-loop system (1), (28).

Proof: To prove (i), note that

$$\nabla \Phi_\sigma = \begin{bmatrix} \alpha_{\sigma_1} \tanh(\beta_{\sigma_1} \sigma_1) \\ \alpha_{\sigma_2} \tanh(\beta_{\sigma_2} \sigma_2) \end{bmatrix}. \quad (30)$$

Furthermore, from the proof of item (i) in Proposition 2 and (30), we get

$$u_i = -\alpha_{l_i} \tanh(\beta_{l_i} z_{l_i}) - \alpha_{m_i} \tanh(\beta_{m_i} z_{m_i}) - \alpha_{\sigma_i} \tanh(\beta_{\sigma_i} \sigma_i)$$

Moreover,

$$-(\alpha_{l_i} + \alpha_{m_i} + \alpha_{\sigma_i}) \leq u_i \leq \alpha_{l_i} + \alpha_{m_i} + \alpha_{\sigma_i}.$$

To prove (ii), define the new state space $\xi := [\zeta^\top \ \sigma^\top]^\top$, and the error $\tilde{\xi} := \xi - \xi_*$. Then, some lengthy but straightforward computations show that the linearization of closed-loop system, around ξ_* , is given by

$$\dot{\tilde{\xi}} = \mathcal{A} \tilde{\xi}.$$

The proof is completed by applying Lyapunov's Indirect Method, see Chapter 4 of [8]. ■

While the main theoretical contributions of this document are the results of Proposition 2, with the implementation of the control law (28) the closed-loop system exhibits a better performance in terms of steady-state error and oscillations. Below, we report the experimental results of this implementation.

Experimental results: the control law (28) is implemented in the robotic arm using the control matrices

$$R_{c_l} = \text{diag}\{25, 40\}, \quad K_c = 0.1I_2, \quad (31)$$

$$R_{c_m} = 0.25I_2, \quad K_\sigma = I_2.$$

We fix the reference $q_* = (-1, 1)$, and we perform experiments under the initial condition $\xi(0) = \mathbf{0}_{14}$. The rest of the control parameters is given in Table II. Note that with this selection of the controller parameters, we ensure that $\mathcal{A}$, defined in (29), is Hurwitz and consequently the point ζ_* is an asymptotically stable equilibrium of the closed-loop system.

TABLE II: Control parameters

$\alpha_{l_1} = 0.6$	$\alpha_{l_2} = 0.3$	$\alpha_{m_1} = 0.25$
$\alpha_{m_2} = 0.6$	$\alpha_{\sigma_1} = 0.35$	$\alpha_{\sigma_2} = 0.3$
$\beta_{l_1} = 3$	$\beta_{l_2} = 1$	$\beta_{m_1} = 1$
$\beta_{m_2} = 1$	$\beta_{\sigma_1} = 2.5$	$\beta_{\sigma_2} = 3$

Figure 7 and Figure 8 show that the errors in position and the control signals are close to zero. From the plots we conclude that the implementation of controller (28) has in overall a better performance than controller (8). Moreover, in this case the trade-off between the attenuation of the oscillations and the steady error seems to have been removed. The errors that persist can be result of several factors, e.g., slips in the motors or numerical errors.

IV. CONCLUSIONS AND FUTURE WORK

We present a controller that solves the set-point problem for planar robots with flexible joints. The control law signals are saturated and are designed without the necessity of solving PDEs nor velocity measurements. We remark the fact that by considering the natural damping, we significantly improve the performance of the controller.

To remove the steady-state error, we add an integral-like term of the error $(q_m - q_*)$. Additionally, this extra

Fig. 7: Behavior of the positions with the addition of an integral-like term.

Fig. 8: Control signals with the addition of an integral-like term.

term attenuates the oscillations in the response of the closed-loop system, while preserving the control signals saturated.

As future work, we seek to develop a proof for Proposition 3, that does not require the linearization of the closed-loop system. Furthermore, we aim to extend our results to physical systems in different domains that can be stabilized via PI-PBC, e.g., electrical circuits or fluid systems. Other areas to explore are the proposition of tuning rules for the parameters of the controller, and a solution to the current over-conservative saturation problem.

APPENDIX

Table III contains the details of the system parameters. These values were taken from the Quanser 2 DOF serial flexible joint robot arm datasheet and from [9].

TABLE III: System parameters

Parameter	Physical meaning	Value	Units
a_1	$\times$	0.148	$kg \cdot m^2$
a_2	$\times$	0.073	$kg \cdot m^2$
b	$\times$	0.086	$kg \cdot m^2$
$\mathcal{I}_{m_1}$	MoI of motor 1	0.217	$kg \cdot m^2$
$\mathcal{I}_{m_2}$	MoI of motor 2	0.007	$kg \cdot m^2$
D_{l_1}	Damping on link 1	0.038	$[N \cdot m \cdot s/rad]$
D_{l_2}	Damping on link 2	0.03	$[N \cdot m \cdot s/rad]$
D_{m_1}	Damping on motor 1	8.435	$[N \cdot m \cdot s/rad]$
D_{m_2}	Damping on motor 2	0.136	$[N \cdot m \cdot s/rad]$
k_{s_1}	Spring constant 1	9	$[N \cdot m/rad]$
k_{s_2}	Spring constant 2	4	$[N \cdot m/rad]$

The experiments reported in Section III were carried out using the Matlab/Simulink interface, where

$$u_{\max_1} = u_{\max_2} = 1.2[A].$$

In these experiments, the control signals are the currents supplied to the motors, nonetheless, if we, reasonably, assume the armature reaction to be insignificant, the currents and torques satisfy a linear relationship.

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