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A continuous-time supply-driven inventory-constrained routing problem[☆]

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ABSTRACT

We address an inventory routing problem (IRP) in which routing and inventory decisions are dictated by supply rather than demand. Moreover, inventory is held in containers that act as both a storage container and a movable transport unit. This problem emanates from logistics related to biogas transportation in which biogas is transported in containers from many suppliers to a single facility. We present a novel and compact formulation for the supply-driven IRP which addresses the routing decisions in continuous-time in which inventory levels within the containers are continuous. Valid inequalities are included and realistic instances are solved to optimality. For all experiments, we found that the total transportation time is minimized when the storage capacity at each supplier is larger than or equal to the vehicle capacity. These routes are characterized by tours in which mostly single suppliers are visited. In 95% of the instances, the average content level of the exchanged containers exceeded 99.6%.

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1. Introduction

This paper addresses a Supply-Driven Cyclic Inventory-Constrained Routing Problem (SDCICRP) in which the routing, scheduling and inventory decisions are dictated by supply rather than demand. We are the first to address the routing and inventory collection decisions in continuous time in which the inventory contents are continuous in discrete containers. These containers are movable transport units, but also represent the storage capacity at a supplier. We show by solving it in an extensive design, how this problem leads to very specific routing and inventory patterns within each cycle. The problem is also known as a reverse inventory routing problem [39] and emerges when biogas is transported from biogas-producing farmers to a centralized green-gas upgrading facility. Biogas is produced from biomass such as manure at livestock farms. Producers are unable to adjust biogas production to demand due to the underlying processes. The produced biogas must be collected before the available storage capacity in containers is exceeded. Therefore, logistics decisions need to be orchestrated around biogas producers that

act as suppliers, creating the supply-driven characteristic. Another important aspect here is that biogas is compressed and stored in containers. These act as both local storage containers and movable transport units. A vehicle collects full or partly filled containers from biogas producers and exchanges these for empty ones. Given that production of biogas is stable, the problem is to find a repetitive schedule including a set of multiple tours that minimizes total transportation time and transports all produced gas and containers to the central facility. From a practical perspective, it is desirable that the schedule is cyclic in the sense that it is repeated after a fixed time period. From our experience in the Netherlands, realistic instances for this problem are typically small (up to 10 suppliers), since limited capacity of the centralized facility only enables a small set of suppliers to provide biogas. While instance size is generally expressed in the number of nodes, the number of vehicles or the number of time periods (for finite, multi-period planning horizons), our problem includes multiple tours for a set of nodes which increases difficulty and instance size.

Inventory routing problems (IRPs) have been studied in several papers and are a class of distribution problems that deal with both inventory replenishment and vehicle routing decisions simultaneously [22]. The main goal is to minimize the total inventory and transportation costs over the planning horizon [22,25]. While costs minimization is the most common objective function in the basic

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IRP literature, other functions can also be used. For example, the logistic ratio studied by Alvarez et al. [5], Archetti et al. [10] and Archetti et al. [9], service level and green house gas footprint by Rahimi et al. [45]. We also refer to Rahimi et al. [45] for a review of different objective functions in the literature. IRPs are also related to Vendor-Managed Inventory (VMI) problems in which the supplier decides on inventory policies for a set of customers and that enable cost savings by more effectively coordinating shipments and combining customer routes [22,55]. A separate class of IRPs are Cyclic IRPs (CIRP) that deal with finding repetitive routing schedules for long-term decision-making. In CIRPs, the time between replenishments is equal for each customer or the schedule for a fixed planning horizon repeats itself infinitely [20,42].

Whereas most literature relies on either discretization of time and continuous cycle times between visits, the main difference with other studies is that our formulation addresses both the optimal continuous-time and continuous-inventory solution. We identify the exact point in time at which a supplier is visited and the exact amount of inventory collected in a discrete number containers. In contrast to traditional IRP literature that focuses on the trade-off between inventory and routing costs, we do not consider any cost components. Since we focus on minimizing transportation time in which the routing and inventory decisions are constrained by the container and vehicle storage capacities, the problem can be referred to as an inventory-constrained routing problem. Therefore, we will refer to our problem as a Supply-Driven Cyclic Inventory-Constrained Routing Problem (SDCICRP).

This paper proposes a novel and compact formulation with valid inequalities for the SDCICRP that enables solving realistic instances to optimality. In a design with thousands of experiments we reveal the characteristic routing and inventory patterns provided by these solutions. This study differs from existing approaches by combining the following characteristics. Firstly, we address both continuous time and continuous inventory contents in each container enabling containers to be exchanged while partly filled. Secondly, we address the problem in which a container acts as both a storage container and a movable transport unit. Thirdly, we combine these aspects with multiple tours in cyclic schedules that can be repeated infinitely. Finally, we address an IRP that is supply-driven, rather than demand-driven in which the relationship between suppliers and a single customer is many-to-one, next to structures in existing literature that also include many-to-one [19,30] and many-to-many [e.g., 29].

This paper is organized as follows. A literature review is presented in Section 2. Section 3 delineates the proposed MIP formulation and provides valid inequalities to strengthen the continuous relaxation of the model. Section 4 provides detailed computational results of the model and experimental insights on storage capacities at each supplier and routing and inventory collection decisions. Section 5 provides concluding remarks.

2. Literature review

In this section, we position our problem in the literature on IRPs for each of the different characteristics related to our problem (see Table 1). IRPs are well-studied and we refer to Coelho et al. [22] for a detailed review. However, the IRP as developed in this paper has not yet been addressed in existing literature. This entails the combination of continuous time and continuous inventory contents, compartments which act as both storage capacity and movable transport units, cyclic schedules and supply-driven routing decisions.

Most IRPs are demand-driven, such that routes are designed in which one or many suppliers service customers that have a certain demand. Early research includes work by Bell et al. [13], Federgruen and Zipkin [26] and Golden et al. [28]. These basic versions

of IRPs consist of determining routes by minimizing inventory costs while meeting customer demands by shipping products from a single supplier to a number of customers [7,14,18]. Most work on the IRP includes extensions of the basic (demand-oriented) IRP to include multiple items [48,52,57] and heterogeneous fleet composition [21,40]. Other classifications include models with continuous time [1,3,4,7,21,35,54], discrete time [e.g., 14,23,50] and both finite [8,16,46] and infinite planning horizons [42,59]. Work that is supply-driven is still relatively limited.

In IRPs with discretized time periods, the precise time of a visit within a period does not affect the inventory level, as the demand is assumed to happen instantaneously at the begin or end of a period. These do not determine the exact point within a time period in which a node is visited and the exact amount of continuous inventory that is collected or replenished in discrete containers. Time is generally modeled in discrete time periods, which can either be fixed throughout the planning horizon [e.g., 6,8,33,34] or dynamic [1,2,41–43] in which the continuous cycle time between visits is calculated (see Table 1). This leads to decisions that are made for a specific time interval. Such schedules may be less accurate, because cyclic schedules in supply-driven problems depend strongly on the precise timing of arrivals in preventing the inventory levels to exceed storage limits. Discretization of time may have severe consequences [15]. We address this gap by optimally obtaining both the points in time in which a supplier is visited and the exact amounts of inventory collected in each container without affecting calculation times.

To the best of our knowledge, there is limited work available which addresses movable inventory containers that also act as local storage at a supplier and that can be picked up while partly filled. Larrain et al. [34] address an IRP in which ATMs are replenished using cassettes in which the old cassette is replaced by a new one with all of its contents. However, time is discretized in their model. Lahyani et al. [33] address a multi-compartment IRP in which vehicles collect olive oil that need to be loaded inside separate compartments. These compartments do not act as local storage containers at the producer and are fixed at the vehicle. In most other literature, the individual containers are not picked up for transport and are only fully emptied at the supplier site. This assumption is also made in papers related to fuel transportation, in which fuel is stored and transported in tanks [11,35,46,56]. Papers in which inventory levels are treated as continuous include [46,54] in maritime contexts and [58] in the liquefied gas industry.

IRPs with a more supply-driven nature relate to waste collection [11,32,39], olive oil collection [33] and shipping [3,4,21,54,58]. A number of reverse inventory routing problems can be considered both demand-driven and supply-driven. These exist in the context of returnable items in which the returned items that influence the routing decisions and inventory policies [36,37,51]. Many-to-one structures have also been explored in production and/or assembly contexts, for example, Chitsaz et al. [19], Hein and Almeder [30]. Guimarães et al. [29] address a many-to-many IRP with multiple echelons and depots. However, demand has remained the primary mechanism dictating the routing decisions in most of the existing IPR-related literature.

Cyclic scheduling aspects are also an important characteristic of our problem in which multi-tour schedules can be repeated infinitely. In existing literature, cycle time is usually addressed as the time between visits to a location [1,2,41–43,47]. In contrast, we replace cycle time by a planning horizon and obtain (multiple) exact times at which each supplier is visited within this planning horizon. The resulting schedule is repeated directly after this planning horizon. According to Vansteenkoven and Mateo [55], rudimentary contributions on CIRPs stem from Anily and Federgruen [7] and Gallego and Simchi-Levi [27]. Furthermore, Speranza and Ukovich [52] and Viswanathan and Mathur [57] considered

Table 1
Related themes in IRP literature .

Authors	Supply-driven	Continuous time	Cyclic IRP	Transport in discrete containers
Aghezzaf et al. [1]		✓	✓	
Aghezzaf et al. [2]		✓	✓	
Agra et al. [3]		✓		✓
Al-Khayyal and Hwang [4]	✓	✓		✓
Alvarez et al. [5]	✓			
Archetti et al. [8]				
Archetti et al. [10]				
Archetti et al. [9]				
Anily and Federgruen [7]		✓	✓	
Amponsah and Salhi [6]	✓			
Avella et al. [11]		✓		✓
Bard et al. [12]				
Bell et al. [13]				
Bertazzi et al. [14]				
Boudia and Prins [16]				
Campbell and Savelsbergh [17]				
Chien et al. [18]				
Chitsaz et al. [20]		✓	✓	
Chitsaz et al. [19]				
Christiansen [21]	✓	✓		
Diz et al. [23]				✓
Easwaran and Öster [24]	✓			
Ekici et al. [25]		✓	✓	
Federgruen and Zipkin [26]				
Gallego and Simchi-Levi [27]		✓	✓	
Golden et al. [28]				
Guimarães et al. [29]				✓
Hein and Almeder [30]				
Iassinovskaia et al. [31]	✓			
Johansson [32]	✓			
Lahyani et al. [33]	✓			✓
Larrazin et al. [34]				✓
Liu and Chung [38]	✓			
Li et al. [36]				
Li et al. [35]		✓		✓
Liu et al. [37]			✓	
Mes [39]	✓			
Persson and Göthe-Lundgren [40]				
Savelsbergh and Song [46]				✓
Shyshou et al. [47]			✓	✓
Sindhuchao et al. [48]		✓		
Solyaliand Süral [50]				
Solyali and Süral [49]				
Soysal [51]	✓			
Speranza and Ukovich [52]			✓	
Srivastava [53]				
Stanzani et al. [54]	✓	✓		✓
Raa and Aghezzaf [42]		✓	✓	
Raa and Aghezzaf [43]		✓	✓	
Raa and Dullaert [44]			✓	
Raa [41]			✓	
Vansteenwegen and Mateo [55]		✓	✓	
Vidović et al. [56]				✓
Viswanathan and Mathur [57]			✓	
Yamashita et al. [58]	✓	✓		✓
Zhao et al. [59]		✓		

¹Adapted from Coelho et al. [22]

repetitive transportation plans for multiple products. Aghezzaf et al. [1] consider multiple tours and vehicles for the CIRP. In their work, vehicles are assigned to specific regions and service a subset of customers in a repetitive manner where the cycle times (i.e., time between replenishments for each customer) are equal for a given solution. Vansteenwegen and Mateo [55], and [20] address a similar problem and treat cycle time as a key decision variable [25]. deal with cyclic schedules by setting beginning inventory levels equal to ending inventory levels for a predefined planning horizon, a condition that we adopt in this paper. Even though cycle times in the literature are modeled in continuous time, existing models have not addressed the routing and inventory collection decisions in continuous-time with continuous inventory contents in discrete containers.

3. Problem description

In this section, we describe in detail the SDCICRP and introduce a mathematical formulation. We consider a logistics system in which biogas from decentralized suppliers is transported to a single depot or customer. We summarize the definitions of sets, parameters and variables in Table 2.

3.1. The SDCICRP

The SDCICRP defined in this paper is represented on an undirected graph $G = \{V, E\}$ in which $V = \{0, \dots, N+1\}$ represents the set of suppliers $V' = \{1, \dots, N\}$ and includes the departure depot $\{0\}$ and arrival depot $\{N+1\}$. $V^0 = \{0, \dots, N\}$ is defined as the set

Table 2
Sets, parameters and decision variables

Sets	
$\mathcal{V}$	Set of all nodes which includes the origin and destination depot and suppliers $\mathcal{V} = \{0, \dots, N+1\}$
$\mathcal{V}'$	Set of all suppliers excluding the depots $\mathcal{V}' = \{1, \dots, N\}$
$\mathcal{V}^o$	Set of suppliers and the origin depot $\mathcal{V}^o = \{0, \dots, N\}$
$\mathcal{V}^d$	Set of suppliers and the destination depot $\mathcal{V}^d = \{1, \dots, N+1\}$
$\mathcal{K}$	Set of tours $\mathcal{K} = \{0, \dots, P\}$
$\mathcal{H}$	Set of containers at each supplier $\mathcal{H} = \{1, \dots, E\}$
i, j	Indices on the set of nodes $i, j \in \mathcal{V}, i \neq j$
k	Index on the set of tours $k \in \mathcal{K}$
h	Index on the set of containers $h \in \mathcal{H}$
Parameters	
Q	Vehicle capacity in number of containers
T_{ij}	Transportation time associated with edge $(i, j) \in \mathcal{E}$
U	Container capacity
W_i	Production rate of supplier $i \in \mathcal{V}'$
D	Length of planning horizon
Variables	
x_{ijk}	Binary variable indicating whether or not edge $(i, j) \in \mathcal{E}, i \neq j$ is included in tour $k \in \mathcal{K}$
l_{ihk}	Binary variable indicating whether or not container $h \in \mathcal{H}$ has been exchanged at supplier $i \in \mathcal{V}'$ for tour $k \in \mathcal{K}$
s_{ik}	Arrival time of the vehicle at node $i \in \mathcal{V}$ of tour $k \in \mathcal{K}$
g_{ik}	Inventory at supplier $i \in \mathcal{V}'$ for tour $k \in \mathcal{K}$ after pickup
z_{ik}	Remaining travel time when arriving at node $i \in \mathcal{V}$ for tour $k \in \mathcal{K}$
b_k	Waiting time of the vehicle at the depot before embarking on tour $k \in \mathcal{K}$
a_{ik}	Amount of cargo picked up at supplier $i \in \mathcal{V}'$ for tour $k \in \mathcal{K}$
y_i	Starting inventory level at supplier $i \in \mathcal{V}'$

Fig. 1. General graphical representation of the SDCICRP for a single example tour k with different container inventory levels ($Q = 3$ and $E = 4$).

of suppliers which includes the origin depot and $\mathcal{V}^d = \{1, \dots, N+1\}$ as the set of suppliers that includes the destination depot. Both the origin and destination depot are assumed to be at the same location. $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ denotes the set of edges. Each edge is associated with a transportation time T_{ij} . Each supplier $i \in \mathcal{V}'$ produces at a constant rate W_i per unit of time. Furthermore, there exists an unordered set of available inventory containers $\mathcal{H} = \{1, \dots, E\}$ at each supplier in which full containers are given the lowest numbers. The container capacity at each supplier in which gas inventory can be stored is represented by U . For each tour k in a set of tours $\mathcal{K} = \{0, \dots, P\}$, a single vehicle with capacity Q (in number of containers) exchanges empty container(s) for (partly) filled one(s) at a subset of suppliers. The model assumes that each tour k cannot visit the same node twice, but two different tours can visit the same node. These tours occur during a pre-defined planning horizon D . Fig. 1 illustrates one feasible solution for a single example tour k in one arbitrary instance of 3 suppliers in which containers at each supplier have different content levels.

The route is represented by a set of binary variables x_{ijk} that specify whether edge $(i, j) \in \mathcal{E}$ is included in tour $k \in \mathcal{K}$. s_{ik} represents the time at which the vehicle arrives at node $i \in \mathcal{V}$ in

tour $k \in \mathcal{K}$ and is calculated as the sum of transportation times for edges traversed earlier in tour $k \in \mathcal{K}$. z_{ik} denotes the remaining travel time after the visit at supplier i before reaching the depot in tour $k \in \mathcal{K}$. g_{ik} represents the inventory after gas has been picked up, whereas a_{ik} denotes the amount of gas picked up at supplier $i \in \mathcal{V}'$ for tour $k \in \mathcal{K}$. Moreover, b_k represents the vehicle waiting time at the depot before embarking on tour $k \in \mathcal{K}$. l_{ihk} indicates whether container $h \in \mathcal{H}$ is exchanged at supplier $i \in \mathcal{V}'$ for tour $k \in \mathcal{K}$. Finally, variable y_i denotes the starting inventory levels for each supplier $i \in \mathcal{V}'$ at the beginning of the planning horizon. The main objective is to minimize total transportation time, which consists of the sum of transportation times related to the traversed edges.

The mathematical formulation is as follows.

$$\min \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \sum_{k \in \mathcal{K}} x_{ijk} T_{ij} \quad (1)$$

$$\text{s.t.} \sum_{j \in \mathcal{V}} x_{ijk} \leq 1 \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (2)$$

$$\sum_{i \in \mathcal{V}} x_{ijk} \leq 1 \quad \forall j \in \mathcal{V}, k \in \mathcal{K} \quad (3)$$

$$\sum_{i \in \mathcal{V}} x_{i, N+1, k} = 1 \quad \forall k \in \mathcal{K} \quad (4)$$

$$\sum_{j \in \mathcal{V}} x_{0jk} = 1 \quad \forall k \in \mathcal{K} \quad (5)$$

$$x_{iik} = 0 \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (6)$$

$$\sum_{i \in \mathcal{V}^o} x_{ijk} - \sum_{i \in \mathcal{V}^d} x_{jik} = 0 \quad \forall j \in \mathcal{V}', k \in \mathcal{K} \quad (7)$$

$$s_{jk} - s_{ik} \leq T_{ij} + M(1 - x_{ijk}) \quad \forall i \in \mathcal{V}^o, j \in \mathcal{V}^d, k \in \mathcal{K} \quad (8)$$

$$s_{jk} - s_{ik} \geq T_{ij} - M(1 - x_{ijk}) \quad \forall i \in \mathcal{V}^o, j \in \mathcal{V}^d, k \in \mathcal{K} \quad (9)$$

$$s_{0k} = 0 \quad \forall k \in \mathcal{K} \quad (10)$$

$$z_{ik} = \sum_{m \in \mathcal{V}} \sum_{j \in \mathcal{V}} x_{mjk} T_{mj} - s_{ik} \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (11)$$

$$g_{ik} = g_{ik-1} + W_i(z_{ik-1} + s_{ik} + b_k) - a_{ik} \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \setminus \{0\} \quad (12)$$

$$g_{i0} = y_i + W_i(s_{i0} + b_0) - a_{i0} \quad \forall i \in \mathcal{V}' \quad (13)$$

$$g_{ik} + a_{ik} \leq UE \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (14)$$

$$g_{ik} \leq UE - \sum_{h \in \mathcal{H}} U l_{ihk} \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (15)$$

$$g_{lk} = 0 \quad \forall l \in \{0, N+1\}, k \in \mathcal{K} \quad (16)$$

$$a_{ik} \leq \sum_{h \in \mathcal{H}} U l_{ihk} \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (17)$$

$$a_{ik} \geq U(-1 + \sum_{h \in \mathcal{H}} l_{ihk}) + \epsilon \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (18)$$

$$\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \sum_{k \in \mathcal{K}} x_{ijk} T_{ij} + \sum_{k \in \mathcal{K}} b_k = D \quad (19)$$

$$\sum_{i \in \mathcal{V}'} \sum_{h \in \mathcal{H}} l_{ihk} \leq Q \quad \forall k \in \mathcal{K} \quad (20)$$

$$\sum_{h \in \mathcal{H}} l_{ihk} \leq E \sum_{j \in \mathcal{V}} x_{ijk} \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (21)$$

$$y_i = g_{ip} + W_i z_{ip} \quad \forall i \in \mathcal{V}' \quad (22)$$

$$\sum_{h \in \mathcal{H}} l_{ihk} \geq \sum_{j \in \mathcal{V}} x_{ijk} \quad \forall i \in \mathcal{V}', k \in \mathcal{K} \quad (23)$$

$$\sum_{j \in \mathcal{V}} x_{jik} \geq l_{ihk} \quad \forall i \in \mathcal{V}', h \in \mathcal{H}, k \in \mathcal{K} \quad (24)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i, j \in \mathcal{V}, k \in \mathcal{K} \quad (25)$$

$$l_{ihk} \in \{0, 1\} \quad \forall i \in \mathcal{V}, h \in \mathcal{H}, k \in \mathcal{K} \quad (26)$$

$$s_{ik} \geq 0 \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (27)$$

$$g_{ik} \geq 0 \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (28)$$

$$a_{ik} \geq 0 \quad \forall i \in \mathcal{V}, k \in \mathcal{K} \quad (29)$$

$$y_i \geq 0 \quad \forall i \in \mathcal{V} \quad (30)$$

$$z_i \geq 0 \quad \forall i \in \mathcal{V} \quad (31)$$

$$b_i \geq 0 \quad \forall i \in \mathcal{V} \quad (32)$$

Constraints (2) ensure that the vehicle can only visit one destination node from origin i , whereas constraints (3) ensure that the vehicle can only come from one origin i to destination j for each tour. Constraints (4) and (5) ensure the vehicle starts at the origin depot 0 and ends at the destination depot $N+1$. Constraints (6) ensure that destination node is not the same node as the origin node i . Constraints (7) ensure that two edges are incident to every node that is visited by connecting nodes in a tour.

Constraints (8) and (9) ensure that the difference between the time of arrival at node i and node j for each tour k is equal to the transportation time between i and j when $x_{ijk} = 1$. The big M represents a VRP solution cost in which the transportation time of a single tour is maximized, which is calculated heuristically using a furthest neighbor approach. Constraints (10) ensure that the arrival time at the origin depot is 0 for each tour k . Constraints (11) determine the remaining transportation time for each visited node i before reaching the depot in tour k .

Constraints (12) determine the inventory levels after inventory has been picked up when visiting supplier i for each tour k . Constraints (13) calculate the inventory levels for each supplier i once that supplier is visited for tour $k=0$. Constraints (14) ensure that the inventory level before being picked up does not exceed the total capacity of the available containers at each supplier i . Constraints (15) ensure that the gas inventory after a pickup does not exceed the available capacity of the non-exchanged containers at supplier i for each tour k . Constraints (16) ensure that the gas inventory is equal to zero at the origin and departure depot for each tour k .

Constraints (17) ensure the amount that is picked up at supplier i does not exceed the available storage capacity in containers at that supplier for each tour k . Constraints (18) ensure that no additional empty containers are collected. For cases in which

the vehicle capacity exceeds the total amount of gas collected during a tour (measured in containers), these constraints prevent exchanging additional empty containers which do not affect the objective value. The value of ϵ was set at a small number, 10^{-4} . Constraint (19) ensures that the sum of transportation times and waiting times are equal to the planning horizon. Constraints (20) ensure that the number of containers that can be exchanged in tour k does not exceed the vehicle capacity. Constraints (21) ensure that the number of containers exchanged at supplier i are less or equal to the available containers when visiting that supplier in tour k . Constraints (22) ensure that the starting inventory level at each supplier i is equal to the final inventory level at that supplier measured at the end of the last tour. Constraints (23) ensure that at least a container has to be exchanged at supplier i if that supplier is visited by tour k . Constraints (24) ensure that if a container exchange is performed at a supplier, that supplier must be included in a tour.

Constraints (25) and (26) define the route x_{ijk} and exchanged containers l_{ihk} to be binary. Finally, constraints (27) to (32) represent the non-negativity constraints of the decision variables.

3.2. Time windows

In practice, it may not be desirable for vehicles to arrive at a supplier at certain times outside working hours (e.g., at night). Therefore, our model can be extended to consider time windows in which a supplier can and cannot be visited. We define a set of time windows $f \in \mathcal{F}$, where $\mathcal{F} = \{0, \dots, F\}$. Let t_{ik} denote the actual (cumulative) time at which supplier i is visited in tour k .

$$t_{in} = \sum_{k=1}^n s_{ik} + b_k + z_{ik-1} \quad \forall n \in \mathcal{K}, i \in \mathcal{V}' \quad (33)$$

$$t_{i0} = s_{i0} \quad \forall i \in \mathcal{V}' \quad (34)$$

We define u_f^- and u_f^+ as the beginning and end of time window f respectively. Moreover, we introduce an auxiliary binary variable r_{fk} which specifies whether t_{ik} occurs before u_f^- or after u_f^+ . The following constraints ensure that these windows are avoided in the solution. The big M value was set as $M = D$.

$$t_{ik} \leq u_f^- + M(1 - r_{fk}) \quad \forall i \in \mathcal{V}', k \in \mathcal{K}, f \in \mathcal{F} \quad (35)$$

$$t_{ik} \geq u_f^+ - Mr_{fk} \quad \forall i \in \mathcal{V}', k \in \mathcal{K}, f \in \mathcal{F} \quad (36)$$

3.3. Valid inequalities

Since multiple optimal solutions exist for a single instance, we provide valid inequalities (VIs) in order to strengthen the continuous relaxation of the model and reduce the number of optimal solutions without loss of generality. The valid inequalities are based on the relationship between exchanged containers, supplier visits and flows which leave the depot. These extend the model as described in Section 3.1.

A tour cannot be made after arriving at the destination depot and before departing at the origin depot.

$$\sum_{j \in \mathcal{V}} x_{N+1,jk} = 0 \quad \forall k \in \mathcal{K} \quad (37)$$

$$\sum_{i \in \mathcal{V}} x_{i0k} = 0 \quad \forall k \in \mathcal{K} \quad (38)$$

The vehicle must either move from i to j or from j to i .

$$x_{ijk} + x_{jik} \leq 1 \quad \forall i, j \in \mathcal{V}', k \in \mathcal{K} \quad (39)$$

A container exchange also implies that a vehicle must have left the depot to a supplier. Hence, we can express this as follows.

$$\sum_{i \in \mathcal{V}'} x_{0ik} \geq l_{jhk} \quad \forall j \in \mathcal{V}', h \in \mathcal{H}, k \in \mathcal{K} \quad (40)$$

Table 3
Summary of the experiments.

Set	Number of instances	Number of suppliers	Storage capacity	Vehicle capacity	Time windows	Supplier and depot coordinates	Production rates
1	45	4	2–4	3–5	No	Random	Random
2	45	5–9	3–5	3–5	No	Fixed	Fixed
3	3200	5	2–5	2–5	No	50 random combinations ^a	4 combinations
4	3200	5	2–5	2–5	Yes	The same as set 3	The same as set 3
5	3	4	2–4	4	No	Fixed	Fixed

^a 50 instances of randomly generated locations for which the combinations of storage capacity, vehicle capacity and production rates are varied

Table 4
Valid inequalities improvements with 4 suppliers .

<i>E</i>	<i>Q</i>	Solution	Time (seconds)		Gap (%)		Time reduction (%)	Production rates ^a			
			Cuts	No cuts	Cuts	No cuts		1	2	3	4
2	3	885.3	407.4	12000.4	0.0	9.7	96.6	10	8	8	7
2	3	782.1	745.6	12000.4	0.0	12.1	93.8	6	7	9	8
2	3	924.1	1174.2	12000.4	0.0	3.2	90.2	7	9	9	8
2	3	880.4	924.1	12000.2	0.0	1.4	92.3	9	9	10	6
2	3	412.6	21.9	6741.8	0.0	0.0	99.7	6	10	8	6
2	4	915	1133.9	12000.1	0.0	10.0	90.6	10	8	8	7
2	4	877.2	147.6	12000.3	0.0	10.5	98.8	6	7	9	8
2	4	431.2	110.1	12000.2	0.0	5.5	99.1	7	9	9	8
2	4	754.3	2566.4	12000.1	0.0	18.6	78.6	9	9	10	6
2	4	933.1	1019.8	12000.1	0.0	22.1	91.5	6	10	8	6
2	5	741.3	45.1	12000.2	0.0	1.6	99.6	10	8	8	7
2	5	660.4	2387.3	4748.3	0.0	0.0	49.7	6	7	9	8
2	5	773.8	458.1	12000.1	0.0	7.8	96.2	7	9	9	8
2	5	452	651.0	12000.1	0.0	11.3	94.6	9	9	10	6
2	5	713.9	42.6	12000.2	0.0	3.1	99.6	6	10	8	6
3	3	949	1092.4	12000.6	0.0	7.9	90.9	10	8	8	7
3	3	906.3	507.6	12000.1	0.0	6.2	95.8	6	7	9	8
3	3	530.4	3.3	12000.2	0.0	0.3	100.0	7	9	9	8
3	3	926.2	2.2	12000.2	0.0	6.4	100.0	9	9	10	6
3	3	731.1	10.7	12000.2	0.0	7.8	99.9	6	10	8	6
3	4	528.7	100.5	12000.1	0.0	3.2	99.2	10	8	8	7
3	4	515.9	33.2	12000.3	0.0	0.3	99.7	6	7	9	8
3	4	520.5	206.0	12000.2	0.0	2.6	98.3	7	9	9	8
3	4	822.1	25.3	12000.4	0.0	18.2	99.8	9	9	10	6
3	4	605.2	34.5	12000.2	0.0	0.9	99.7	6	10	8	6
3	5	434	24.3	12000.2	0.0	4.0	99.8	10	8	8	7
3	5	496.7	197.3	12000.3	0.0	4.6	98.4	6	7	9	8
3	5	715.1	239.1	12000.1	0.0	9.3	98.0	7	9	9	8
3	5	284.6	6.5	546.1	0.0	0.0	98.8	9	9	10	6
3	5	427.8	3.9	1370.1	0.0	0.0	99.7	6	10	8	6
4	3	801.2	26.4	12000.3	0.0	5.8	99.8	10	8	8	7
4	3	651.8	12000.5	12000.4	6.1	11.0	n.a.	6	7	9	8
4	3	534.2	30.9	12000.2	0.0	2.6	99.7	7	9	9	8
4	3	463.9	6887.1	12000.2	0.0	13.7	42.6	9	9	10	6
4	3	918.4	74.8	12000.2	0.0	15.6	99.4	6	10	8	6
4	4	638	2.4	12000.1	0.0	5.4	100.0	10	8	8	7
4	4	684.7	18.6	12000.5	0.0	8.5	99.8	6	7	9	8
4	4	731.7	1.5	12000.5	0.0	9.6	100.0	7	9	9	8
4	4	333.4	86.0	12000.5	0.0	14.1	99.3	9	9	10	6
4	4	676.8	46.2	12000.3	0.0	17.0	99.6	6	10	8	6
4	5	518	9.9	12000.5	0.0	0.1	99.9	10	8	8	7
4	5	668.9	334.4	12000.2	0.0	15.7	97.2	6	7	9	8
4	5	676.7	12.3	12000.9	0.0	10.7	99.9	7	9	9	8
4	5	535.4	28.4	12001.0	0.0	0.2	99.8	9	9	10	6
4	5	365.1	14.0	4328.3	0.0	0.0	99.7	6	10	8	6
Average		660.6	753.2	11061.0	0.1	7.1	93.0				

^a Number of containers per week

The following inequalities are related to leaving the depot and visiting other nodes. If a tour is performed to serve a supplier, then that tour must have left the depot to another supplier.

$$\sum_{i \in \mathcal{V}'} x_{0ik} \geq x_{mjk} \quad \forall m, j \in \mathcal{V}', k \in \mathcal{K}. \tag{41}$$

In order to ensure that the final inventory levels do not exceed the initial inventory levels, a minimum number of container exchanges during the planning horizon are those for which full con-

tainers are exchanged. Therefore, the total number of container exchanges for each supplier is larger than or equal to the minimum amount of exchanges.

$$\sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{K}} l_{ihk} \geq \left\lceil \frac{DW_i}{U} \right\rceil \quad \forall i \in \mathcal{V}'. \tag{42}$$

For each supplier, the number of visits in which a tour from the any supplier to the depot is performed should be larger than or

Table 5

Percentage time increase due to omitting valid inequalities relative to all valid inequalities activated.

Instance	Time (s)	Percentage time increase due to omitting VI relative to all VIs activated																	
	All cuts	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
1	429.0	-7.9	10.9	59.2	22.9	-5.3	443.6	68.8	69.1	-14.7	-18.4	126.3	48.3	20.0	-6.4	57.9	184.2	67.9	235.6
2	761.3	-13.5	-33.7	9.7	-16.2	29.6	84.6	57.6	99.7	-4.5	1476.3	-43.6	49.5	50.1	-26.2	-36.7	19.1	-16.1	13.4
3	1208.9	9.0	225.4	136.2	6.6	103.4	73.0	144.2	1.5	54.7	892.7	127.6	-69.5	38.4	-31.2	8.1	22.8	73.2	71.1
4	979.9	17.8	-36.2	8.2	17.8	0.0	26.9	69.9	243.6	-14.5	51.9	20.6	-30.4	-27.8	-53.6	-25.3	-68.1	-33.2	2.9
5	20.7	-45.2	-0.8	-47.8	-48.8	-53.0	-60.0	-43.8	-39.9	-50.4	-50.9	-56.1	-49.3	-35.4	-46.5	-58.6	-23.7	8.4	-62.3
6	1117.0	26.5	20.8	65.6	65.4	147.8	115.7	-57.4	11.5	36.9	974.3	72.9	159.8	47.4	-30.0	29.5	36.0	22.7	70.3
7	157.5	133.2	-13.4	39.4	103.2	-35.6	73.5	80.3	184.9	123.4	5932.7	150.9	6.8	6.7	79.4	185.2	75.4	70.9	100.6
8	113.9	60.3	-13.7	-41.0	-29.2	-5.0	17.8	-25.3	296.1	13.6	770.7	-55.0	-53.3	-23.0	-11.0	-31.7	-32.4	-41.0	25.0
9	2646.3	58.1	74.6	155.8	39.5	108.6	17.4	-27.1	353.5	173.8	353.5	116.8	7.3	4.7	125.3	288.4	226.6	105.8	-14.5
10	1008.5	-10.1	-52.8	34.2	-52.1	-42.8	-44.3	-51.7	213.9	-3.5	1089.9	-52.5	-55.7	-45.5	-30.4	-32.7	-23.6	-60.8	-45.2
11	47.5	12.4	64.9	4.5	1.7	7.2	26.2	-14.8	282.0	-24.9	1821.8	51.6	30.8	119.9	-7.4	28.0	-22.0	-16.3	2.2
12	2234.0	58.9	3.1	41.3	-35.8	17.6	-56.4	-45.3	-49.5	-29.0	29.5	-36.5	-46.9	27.8	26.0	-54.0	14.3	-42.4	54.1
13	453.5	29.2	50.7	-7.9	35.3	249.5	7.0	50.3	89.5	-30.3	2546.2	-24.6	-3.3	47.7	76.7	-0.7	30.0	-2.2	5.6
14	648.8	209.9	-8.6	111.6	0.3	23.7	240.6	407.2	314.2	90.4	1749.8	393.3	443.0	219.5	30.1	8.6	212.6	60.8	434.3
15	45.5	11.3	-1.5	19.1	74.7	16.0	36.4	19.3	154.4	47.8	920.8	61.0	65.6	26.2	9.9	6.1	-1.6	57.2	-6.7
16	1160.8	-1.6	-59.7	-58.5	-19.4	1.5	26.5	-38.2	37.2	24.5	688.4	-56.9	-45.5	183.5	35.3	-21.1	27.1	-48.3	16.2
17	511.4	96.8	27.2	21.0	22.6	19.0	78.5	-15.1	108.9	57.7	2108.5	12.1	-11.5	188.2	38.8	144.2	-2.9	-27.7	15.7
18	3.7	150.0	-5.2	238.7	370.1	460.1	316.1	290.4	32.3	53.9	343.0	-18.0	459.5	331.2	254.6	17.2	18.7	34.2	192.9
19	2.1	51.1	-10.3	-8.2	-64.4	-49.4	17.3	784.1	117.1	-24.9	30.4	67.1	52.8	37.6	164.1	115.2	146.2	-8.1	27.6
20	10.8	25.6	16.0	184.5	32.8	4.6	-14.0	35.8	-0.2	-12.6	37.1	149.6	11.1	240.4	2.9	-8.9	3.4	5.0	13.0
21	100.9	-0.9	-8.5	-26.9	-38.4	-12.5	39.4	5.9	107.4	-25.8	329.7	-24.9	-23.2	23.0	-27.4	-31.7	36.3	-26.6	113.7
22	32.8	3.2	-23.8	-39.5	-8.3	-22.5	-4.1	-1.0	85.3	-19.8	1472.2	-34.5	-20.8	-35.9	-21.1	-23.3	-21.2	-6.4	-17.4
23	202.4	203.1	109.3	99.2	76.6	187.2	95.2	135.3	410.7	80.2	5829.3	130.7	225.6	597.1	37.4	189.7	59.3	48.8	139.3
24	26.6	19.6	35.7	2.4	-21.7	-14.6	-7.3	23.0	-12.1	13.3	-16.7	-17.3	11.2	-2.8	39.3	35.1	-14.9	-23.3	27.5
25	32.9	87.9	20.9	130.0	-3.5	35.6	26.9	94.1	110.0	10.0	65.0	21.9	4.6	28.3	20.7	9.1	15.1	26.7	2.5
26	25.3	-28.8	-2.5	-47.0	-3.9	-61.9	-57.4	-43.6	-49.1	-65.5	632.2	-55.7	-52.9	-48.1	-39.4	-51.8	-62.7	-42.8	-22.1
27	200.8	58.2	-5.5	22.0	-36.0	-31.0	-70.9	-40.9	612.1	-46.9	633.4	-52.4	-70.3	-43.8	-38.4	-65.0	-54.4	-44.0	38.9
28	238.7	46.8	23.5	-40.9	28.3	-54.9	-48.6	-54.5	213.6	-25.7	2466.2	29.0	11.4	-53.5	49.0	-38.8	-42.1	-8.4	7.3
29	6.1	-37.0	24.6	-64.0	-59.7	-44.5	-26.0	161.3	-42.6	-10.1	-43.2	-52.4	-9.2	-31.5	17.5	-56.4	41.6	-50.4	409.1
30	4.4	6.0	20.9	41.2	0.0	97.1	65.1	-8.2	101.7	-28.9	232.0	-3.3	23.6	38.8	40.2	33.8	35.3	33.0	25.9
31	27.2	94.4	-4.8	57.3	-0.7	239.2	16.4	344.4	76.2	24.2	62.7	-19.0	17.5	-22.9	11.7	71.3	322.2	26.3	27.3
32	12000.7	0.0	-0.0	0.0	0.0	-0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0
33	32.1	-26.6	-58.9	-29.7	-8.1	-43.4	-25.3	10.3	-26.4	-38.0	35.9	-33.7	-47.0	254.5	5.9	-1.3	-23.7	8.1	-3.6
34	7144.3	68.0	-70.4	68.0	-88.6	68.0	68.0	-88.5	68.0	68.0	-46.7	68.0	68.0	68.0	68.0	68.0	-63.2	68.0	68.0
35	86.4	6.4	176.3	167.1	201.9	43.9	35.4	164.6	409.4	9.8	434.7	136.7	263.0	769.4	151.2	-26.8	55.1	166.2	163.8
36	3.2	56.6	83.1	98.0	18.3	87.0	219.3	380.7	47.0	26.7	282.3	-26.7	-33.4	45.9	33.0	11.1	22.8	76.7	32.4
37	19.1	16.3	11.3	70.4	19.7	-5.7	-9.6	50.7	20.1	-0.5	851.1	43.8	8.5	28.4	-16.8	104.9	-6.1	8.7	-18.2
38	1.8	59.1	39.4	53.1	7.3	60.6	5.0	547.8	108.7	59.5	420.4	61.2	-20.0	408.8	101.1	201.4	13.2	-2.8	102.8
39	85.5	79.7	143.9	9.3	-54.0	2.0	-7.8	-7.5	267.0	2.9	673.4	-81.2	65.4	-41.9	-13.5	-10.5	-27.3	-12.3	-43.7
40	45.5	-10.4	-29.5	-24.0	-56.4	-21.3	-17.4	-42.3	118.9	-3.5	682.3	-41.1	-21.2	-22.6	-57.6	-19.3	27.3	0.3	-27.7
41	14.8	-39.5	-39.6	-23.6	-47.9	-45.4	-11.9	70.2	-24.1	-49.7	71.2	-43.4	-37.7	-11.2	-37.6	-40.9	-40.2	-31.2	-40.8
42	329.8	88.4	-48.3	-34.1	-18.5	-43.2	-42.4	18.3	0.4	-66.7	3539.2	-61.0	-46.4	-21.1	-31.9	-22.5	-39.1	-15.5	-26.2
43	12.8	153.9	16.1	105.4	18.3	63.5	45.6	98.4	94.4	32.1	789.5	74.4	69.2	49.8	64.2	71.5	34.7	27.8	51.8
44	29.3	452.1	39.9	31.9	113.6	74.6	1.8	-22.4	216.9	234.4	17103.4	13.7	-0.6	118.6	89.0	-16.1	25.3	75.6	12.0
45	14.2	-2.9	-22.7	-20.0	-18.3	-42.3	-30.8	-3.4	23.9	46.5	158.4	-20.2	-30.5	-16.2	-34.1	-24.0	-37.0	-8.2	-27.3
Average	761.7	49.5	15.3	34.9	12.2	33.6	37.4	77.4	121.3	15.4	1297.9	22.6	29.4	78.6	22.5	21.9	24.4	11.2	47.7

Table 6

Gap improvement relative to optimal solution in % points for including each set of valid inequalities, while solving the continuous relaxation.

			Gap improvement in % points of the lower bound																	
Instance	Solution	No cuts	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
1	885.3	330.0	0.0	0.7	0.0	13.7	0.0	0.7	0.0	4.3	10.2	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0
2	782.1	291.4	0.5	0.8	7.6	42.8	10.5	2.9	33.4	2.2	32.6	0.8	3.2	4.3	3.8	4.3	5.8	0.5	13.7	3.4
3	924.1	501.4	-1.9	0.2	10.3	26.1	-5.5	-0.8	13.4	1.3	24.0	0.3	1.1	-1.2	-0.7	-0.6	-2.0	-1.8	-0.5	-1.8
4	880.4	506.6	-3.9	-1.8	6.0	13.2	2.5	-1.5	12.0	1.6	12.7	-3.9	-1.6	0.2	-3.9	7.4	-3.9	-3.9	-3.6	0.4
5	412.6	352.2	-0.2	0.3	0.5	11.6	5.7	0.1	8.3	-0.3	11.5	0.3	0.3	-0.0	0.2	1.5	-0.4	-0.3	3.1	-0.5
6	915.0	575.8	0.0	0.0	1.4	16.1	4.7	0.2	16.1	1.4	17.3	0.0	1.0	1.0	0.0	0.0	0.0	0.0	4.3	0.0
7	877.2	557.0	-2.0	0.6	1.9	23.3	10.1	5.4	22.0	2.1	20.1	0.9	2.3	2.3	1.0	0.7	0.7	0.0	8.5	-1.2
8	431.2	288.9	2.6	-2.1	3.0	-0.2	-9.3	-2.4	5.3	1.8	-3.3	-5.0	-2.0	-3.8	-3.9	-2.4	2.6	0.0	1.4	2.6
9	754.3	400.1	0.4	1.0	3.6	19.8	2.6	2.7	19.6	1.3	17.4	2.0	0.6	0.6	-0.1	2.0	2.0	0.0	3.8	0.3
10	933.1	358.3	-0.4	0.0	1.9	28.6	6.5	-0.9	25.3	-0.0	28.9	-1.1	-0.0	1.9	-0.9	0.0	0.0	0.0	7.1	0.0
11	741.3	580.6	1.0	3.1	3.9	11.9	4.2	1.3	9.4	3.2	11.5	0.4	1.2	0.7	1.0	3.1	3.1	0.0	4.1	-0.4
12	660.4	446.6	-0.0	-0.0	-0.0	0.9	-0.1	1.6	-0.3	-0.0	0.9	-0.0	-0.0	0.1	-0.5	0.1	-0.0	0.0	0.3	-0.7
13	773.8	538.5	0.0	0.0	1.0	11.8	2.7	4.6	12.5	2.6	11.8	0.0	1.8	1.0	0.0	1.0	0.0	0.0	2.6	0.0
14	452.0	227.9	0.0	0.0	0.0	11.0	4.6	0.0	11.2	0.0	11.0	-1.7	-2.7	-1.7	-2.4	0.0	-1.6	0.0	4.7	0.0
15	713.9	572.8	0.0	0.0	0.3	8.7	-13.0	0.3	8.8	0.2	5.8	-0.3	0.3	-0.9	0.0	-7.2	-13.5	0.0	6.5	0.0
16	949.0	420.7	0.6	0.4	4.6	42.8	18.5	1.4	35.8	3.3	41.9	-0.5	4.0	3.5	2.3	-2.9	0.8	4.6	8.4	0.7
17	906.3	348.6	1.1	-0.2	5.4	46.3	14.2	-3.6	39.5	8.2	42.1	1.1	2.5	3.4	-0.3	-2.7	1.8	-1.2	-3.1	-6.4
18	530.4	330.3	-1.5	-0.2	5.5	31.5	14.5	-3.3	30.9	1.4	31.5	0.9	1.2	2.9	-2.9	-11.2	-3.7	-0.6	-13.6	-2.1
19	926.2	299.1	5.1	6.8	6.8	61.3	26.8	3.3	67.0	5.9	61.3	4.8	13.8	12.8	5.4	4.6	9.5	4.1	8.0	7.1
20	731.1	339.0	1.1	4.3	0.2	44.8	20.0	-0.2	43.2	2.3	45.7	-0.7	4.8	2.8	3.4	-2.7	0.2	-0.6	7.3	1.7
21	528.7	387.0	-1.8	1.4	-4.5	4.2	-3.4	-7.2	10.3	-0.9	10.2	-1.1	2.1	-0.3	-7.3	-7.3	-7.2	-7.3	-7.0	-0.1
22	515.9	267.4	0.0	0.0	0.0	17.4	0.0	0.2	23.8	3.6	13.2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
23	520.5	308.3	-3.5	0.4	3.2	1.6	-11.5	-4.3	15.1	0.1	1.0	-2.5	-1.8	0.3	2.4	-7.2	1.8	0.0	-2.7	-1.3
24	822.1	311.4	0.0	0.0	2.9	32.0	0.0	0.2	31.8	2.4	28.7	0.0	0.0	0.0	0.0	0.0	0.2	0.0	1.5	0.0
25	605.2	204.0	0.1	0.0	8.2	21.0	2.4	0.1	10.7	0.0	22.5	0.0	0.0	0.0	0.1	0.0	0.1	0.0	2.4	0.0
26	434.0	307.5	-0.0	0.0	0.2	22.6	-12.6	0.1	10.8	-0.7	22.1	-1.0	-4.1	-3.3	0.2	0.2	-10.5	0.0	-1.0	-0.3
27	496.7	305.5	0.0	0.0	8.4	12.4	1.8	0.2	9.1	4.3	11.2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	1.1	0.0
28	715.1	348.0	-3.9	-3.5	6.6	1.3	-8.9	0.0	16.1	1.1	1.3	-3.7	-1.2	-0.1	-2.1	-3.9	3.8	-1.8	-0.7	-3.9
29	284.6	227.7	0.0	0.0	0.3	-6.9	-15.6	0.5	9.3	-3.0	-6.4	-5.2	0.6	0.6	0.0	0.0	-7.9	-2.8	3.0	0.0
30	427.8	273.9	0.0	13.6	13.6	22.7	9.1	0.3	18.4	9.1	21.2	0.0	1.2	1.2	0.0	10.7	13.7	0.0	7.9	9.4
31	801.2	349.9	7.8	9.8	4.6	21.0	-0.2	-2.5	15.8	-2.7	22.8	3.1	4.5	-0.3	9.9	1.8	9.5	2.3	0.4	0.1
32	651.8	157.8	2.7	7.0	13.5	11.9	0.2	0.7	1.8	1.3	11.3	0.1	0.0	0.3	13.4	-0.1	1.3	-0.0	-0.0	-0.1
33	534.2	213.9	12.0	10.1	2.2	35.9	5.2	4.1	18.5	2.4	31.5	3.5	2.2	4.1	9.9	-0.0	2.5	2.7	6.8	2.0
34	463.9	173.1	4.2	4.9	-0.4	34.8	-1.0	2.7	23.3	-3.4	33.6	-1.3	-2.0	-3.9	4.0	-3.2	-6.1	-0.9	3.7	0.4
35	918.4	283.3	4.2	3.7	2.2	50.0	10.4	0.4	52.0	1.9	50.0	0.7	3.0	2.0	7.4	0.9	3.8	0.1	11.2	0.7
36	638.0	341.5	-19.8	1.5	4.9	31.7	8.6	-8.5	36.6	5.2	31.7	-4.2	2.5	2.9	-0.3	2.5	-7.0	0.0	-5.2	1.6
37	684.7	291.1	-15.0	-3.1	7.2	22.2	0.8	-14.2	32.9	3.4	22.2	-15.0	-4.6	4.2	-12.3	-15.0	-8.4	0.0	-7.4	1.5
38	731.7	175.0	15.1	20.8	22.8	66.5	34.1	10.2	73.3	20.4	66.5	19.6	24.6	24.6	22.1	16.5	7.4	0.0	16.2	15.0
39	333.4	119.8	0.0	0.0	0.0	46.9	8.6	0.8	48.3	6.2	48.5	0.0	7.2	6.1	0.5	4.1	8.7	0.0	0.9	0.0
40	676.8	166.4	0.2	0.1	3.1	57.4	16.7	1.5	60.6	7.0	54.7	2.2	7.8	7.4	1.8	3.5	4.5	0.0	19.3	0.4
41	518.0	235.4	-5.7	-9.9	0.1	34.8	3.8	-6.3	37.7	2.8	34.7	-9.9	-2.7	1.1	0.5	-3.1	-2.7	0.0	13.3	0.8
42	668.9	171.2	-0.0	-0.0	-0.0	26.9	-0.0	0.1	27.0	2.3	26.9	0.0	-0.0	-0.0	0.0	-0.0	2.0	0.0	-0.0	0.0
43	676.7	176.3	2.2	-8.8	5.2	52.9	11.0	7.2	61.0	4.1	52.9	0.1	6.4	6.4	3.0	-1.2	2.1	0.0	14.9	-2.0
44	535.4	224.1	-1.2	-1.0	0.4	16.1	-3.1	2.4	28.5	2.1	16.1	-0.7	-1.0	1.1	1.4	0.1	-8.6	0.0	1.5	-1.2
45	365.1	170.1	10.4	2.9	17.9	28.1	-3.6	2.1	28.4	2.8	28.1	-2.2	0.3	8.8	8.8	1.4	8.3	0.0	6.6	8.2
Average	660.6	321.2	0.2	1.4	4.1	25.1	3.8	0.1	24.8	2.5	24.3	-0.4	1.7	2.1	1.4	-0.1	0.3	-0.2	3.3	0.8

Table 7
Increasing the amount of suppliers .

Suppliers	E	Q	Solution	Time (s)	Gap (%)
5	3	3	621	593.0	0.0
5	3	4	539.7	6030.5	0.0
5	3	5	470.5	2544.5	0.0
5	4	3	621	314.4	0.0
5	4	4	461	81.9	0.0
5	4	5	418.7	527.2	0.0
5	5	3	621	336.0	0.0
5	5	4	461	59.6	0.0
5	5	5	389.8	782.8	0.0
6	3	3	849.5	8301.1	0.0
6	3	4	739.7	12000.7	3.3
6	3	5	637.1	12000.4	2.9
6	4	3	849.5	4402.2	0.0
6	4	4	671	12000.4	3.7
6	4	5	585.6	12000.3	0.2
6	5	3	849.5	5101.0	0.0
6	5	4	671	12001.1	3.9
6	5	5	538.5	12000.4	2.8
7	3	3	991.7	10809.9	0.0
7	3	4	822.7	12000.8	7.8
7	3	5	703.5	12000.7	10.2
7	4	3	991.7	12000.7	2.1
7	4	4	763.2	12000.5	4.7
7	4	5	660.9	12000.4	8.1
7	5	3	991.7	12000.6	2.3
7	5	4	763.2	12000.7	4.8
7	5	5	612.3	12000.4	8.1
8	3	3	1271	12000.9	2.9
8	3	4	1050.6	12000.7	12.6
8	3	5	895.9	12000.9	17.2
8	4	3	1270.9	12000.9	2.9
8	4	4	985.9	12001.7	9.4
8	4	5	835.2	12001.7	14.4
8	5	3	1270.9	12002.1	2.1
8	5	4	978.7	12001.7	8.9
8	5	5	798.4	12000.9	13.1
9	3	3	1548.7	12001.1	5.3
9	3	4	1295.4	12000.9	13.3
9	3	5	1108.4	12000.8	17.6
9	4	3	1521.1	12001.7	3.7
9	4	4	1159	12001.0	8.3
9	4	5	1012.8	12000.8	16.9
9	5	3	1521.1	12001.2	3.4
9	5	4	1159	12001.0	8.3
9	5	5	940.1	12000.8	12.3

equal to the minimum number of visits $\gamma = \lceil \frac{\sum_{i \in \mathcal{V}'} DW_i/U}{Q} \rceil$. The minimum number of visits is represented by single tours in which a single supplier is visited and Q containers are picked up while completely filled.

$$\sum_{i \in \mathcal{V}'} \sum_{k \in \mathcal{K}} x_{i,N+1,k} \geq \gamma. \quad (43)$$

Since each supplier must be visited at least once in the planning horizon, we can select a specific supplier v , being the one that has the lowest production rate, to be included in the last tour to reduce the number of optimal solutions.

$$\sum_{i \in \mathcal{V}} x_{ivP} = 1. \quad (44)$$

No containers can be exchanged at each supplier when tour k is empty and no suppliers are visited.

$$l_{ihk} + x_{0,N+1,k} \leq 1 \quad \forall i \in \mathcal{V}', h \in \mathcal{H}, k \in \mathcal{K}. \quad (45)$$

Empty tours in which no suppliers are visited can be scheduled successively at the start of the planning horizon to reduce the number of optimal solutions without loss of generality.

$$x_{0,N+1,k+1} \leq x_{0,N+1,k} \quad \forall k \in \{0, \dots, P-1\}. \quad (46)$$

Since empty tours occur at the beginning of the planning horizon, the last tour must include one visit from the depot to a supplier and back and this tour cannot be empty.

$$\sum_{i \in \mathcal{V}'} x_{i,N+1,P} = 1 \quad (47)$$

$$x_{0,N+1,P} = 0. \quad (48)$$

Exchanged containers can be arranged such that container h must be exchanged before container $h+1$.

$$l_{ihk} \geq l_{i,h+1,k} \quad \forall i \in \mathcal{V}', h \in \{1, \dots, E-1\}, k \in \mathcal{K}. \quad (49)$$

Since container h must be exchanged before container $h+1$ and a container can only be exchanged when supplier i is visited, we can enforce l_{i1k} to always be one when supplier i is visited.

$$l_{i1k} - \sum_{j \in \mathcal{V}'} x_{ijk} = 0 \quad \forall i \in \mathcal{V}', k \in \mathcal{K}. \quad (50)$$

The number of visits to supplier i during the planning horizon must be larger than or equal to the minimum number of visits required in order to pick up all inventory at that supplier.

$$\sum_{j \in \mathcal{V}^d} \sum_{k \in \mathcal{K}} x_{ijk} \geq \left\lceil \frac{DW_i}{UQ} \right\rceil \quad \forall i \in \mathcal{V}'. \quad (51)$$

In any given tour, the number of visited suppliers cannot exceed the vehicle capacity in containers.

$$\sum_{i \in \mathcal{V}'} \sum_{j \in \mathcal{V}^d} x_{ijk} \leq Q \quad \forall k \in \mathcal{K}. \quad (52)$$

Since waiting times before empty tours in which no suppliers are visited can have any value, we can set the waiting time b_k to zero for any empty tour to reduce the number of optimal solutions. We can define the big M as the maximum waiting time, which is denoted by the time to completely fill the minimum of the available storage capacity E or truck capacity Q minus the transport time from the depot to the fastest producer f and back, $M = (\min\{E, Q\}U)/W_f - 2T_{0f}$.

$$b_k \leq M(1 - x_{0,N+1,k}) \quad \forall k \in \mathcal{K}. \quad (53)$$

The difference between the total quantity produced and the total quantity of gas collected during the planning horizon must be zero.

$$W_i D - \sum_{k \in \mathcal{K}} a_{ik} = 0 \quad \forall i \in \mathcal{V}'. \quad (54)$$

3.3.1. Time windows

For the time windows extension of Section 3.2, we introduce the following valid inequalities in order to enhance the performance of the model.

An arrival in tour k that occurs before the beginning of time window u_f^- always occurs before the beginning of time window u_{f+1}^- .

$$r_{f+1,k} \geq r_{fk} \quad \forall f \in \{0, \dots, F-1\}, k \in \mathcal{K}. \quad (55)$$

Since empty tours are scheduled in the beginning of the planning horizon for which the cumulative arrival time at each supplier in that tour t_{ik} is zero, empty tours should always occur before an arrival to any time window.

$$x_{0,N+1,k} \leq r_{fk} \quad \forall f \in \mathcal{F}, k \in \mathcal{K}. \quad (56)$$

Fig. 2. Storage capacity at each supplier and average lower and upper bounds ($Q = 4$).

4. Computational results

In this section, we analyze the solution quality and computational efficiency of the model, and obtain experimental insights. In Section 4.1, we define the instance sets. In Section 4.2, we assess the solution quality and computational efficiency of the model and evaluate the impact of the valid inequalities (37) to (54). Section 4.3 evaluates the impact of the number of suppliers on computation times. Section 4.4 provides the general experimental insights. Finally, Section 4.5 provides a detailed illustrative example of the resulting routing and inventory decisions.

4.1. Sets of instances

The instance sets for the SDCICRP (see Table 3) are based on a real-life example. The instances are grouped in 5 sets. Sets 1 and 2 are used to assess the solution quality and computational efficiency. Set 1 is the benchmark set and set 2 is used to test the impact of larger numbers of suppliers. Set 3, 4 and 5 are included to gain experimental insights and determine parameter sensitivity. All instances in the sets have a cyclic planning horizon of one week. We consider production rates that represent an integer number of container equivalents produced during the planning horizon. The production rates reflect rates that are common in practice. The transportation times between nodes are symmetric. The total number of tours P in the set of tours $\mathcal{K}$ was set to the minimal number

of tours needed γ , increased by 4. Based on computational tests, 5 additional tours did not lead to a change in objective values for the instances in set 1. Therefore, we assume that 4 additional tours is a sufficient number.

The first instance set comprises small-scale instances with 4 suppliers with randomly generated locations. The coordinates of both the suppliers and depot are randomly chosen within a range of 0 and 60 min travel time to calculate the transportation time matrix. The container size is set at 1500 which reflects the size in m^3 of compressed gas in movable containers for tube trailers in practice. The storage capacity at each supplier ranges between 2 to 4 containers and the vehicle capacity ranges between 3 to 5 containers, which represent existing configurations for trucks. Each combination of storage and vehicle capacities is in turn combined with 5 levels of randomly obtained production rates sampled from the range between 6 and 11 containers per week at each supplier reflecting rates of existing biogas producers.

The instances in the second set comprise an increased number of suppliers ranging between 5 and 9. The supplier coordinates of 9 suppliers have been fixed and the instances with less suppliers form a subset of these. The production speeds have been fixed for all instances in this set, in order to examine the effect of the amount of suppliers on calculation times. Both the storage capacity at each supplier and vehicle capacity range between 3 to 5 containers.

Fig. 3. Time windows, storage capacity at each supplier and average lower and upper bounds ($Q = 4$).

Based on the computational tests, 5 suppliers were used in the third set of instances. In this set, we generate 3200 instances as follows. We apply 4 levels for the storage capacity, vehicle capacity, and production rates. For each combination of these 3 variables, we solve 50 pregenerated instances with random depot and supplier coordinates within a range of 0 and 60 min on the x and y axis. The storage capacities and vehicle capacities both range between 2 to 5 containers. The 4 levels for production rates for the 5 suppliers consist of a combination with low mean and low variance (9, 5, 7, 8, 6), low mean, high variance (5, 6, 11, 12, 4), high mean, low variance (9, 11, 10, 13, 12) and high mean, high variance (5, 9, 12, 14, 13). The high production rates are on average 31% higher than the low production rates.

In set 4, the 3200 instances of set 3 were solved with time windows activated in the model. In the planning horizon of one week, 7 time windows were implemented in which the vehicle can only visit a supplier between 06:00 and 18:00 during the day. It is assumed that the planning horizon starts on the first day at 06:00.

Set 5 consists of 3 instances that are solved in order to give a detailed illustration of routing and inventory decisions for existing locations of biogas producers. With fixed supplier coordinates and a vehicle capacity of 4 containers, we vary storage capacities between 2 and 4 containers.

4.2. Computational results and valid inequalities

We present the computational results of the model for the instances of set 1 by using the valid inequalities presented in

Section 3.3. These were solved using Gurobi 7.0.2 within a computation time limit of 12,000 s and a memory limit of 4Gb. The instances without valid inequalities required a memory limit of 8Gb.

4.2.1. Impact of all valid inequalities

Table 4 provides the results to generate multi-tour cyclic schedules for 4 suppliers. Columns 1 and 2 show the storage capacity at each supplier and the vehicle capacity in the number of containers. Column 3 (Solution) reports the objective value of the obtained solution, which is either the optimal solution (bold) or the upper bound in case the instance could not be solved to optimality. Columns 4 and 5 report the calculation times for the instances of set 1 with and without the proposed valid inequalities (cuts). Columns 6 and 7 report the optimality gap which is calculated as (Upper Bound - Lower Bound)/Lower Bound. The last column illustrates the percentage reduction in calculation time as a result of the valid inequalities. If an instance could not be solved, the last column indicates that a time reduction is not available. If only the instances with cuts could be solved in the time limit, the last column indicates the percentage time reduction relative to the time limit.

The results in Table 4 show that the valid inequalities highly affect the calculation times with an average time reduction of 93%. The valid inequalities also highly affect the ability to solve the instances to optimality within the specified time limit. Only 5 out of 45 instances could be solved without valid inequalities compared to 44 out of 45 instances with the valid inequalities within the

Fig. 4. Average number of suppliers visited per tour for different vehicle capacities.

time limit. For the instances with valid inequalities enabled, 34 out of the 45 instances could be solved in less than 10 min.

4.2.2. Impact of individual sets of valid inequalities

In order to assess the impact of each individual set of valid inequalities, we have performed additional experiments for each instance in set 1 (see Table 3). Firstly, Table 5 shows the percentage time increase for each set of VIs when it was omitted relative to the computation time in which all VIs were activated. In these experiments, a time limit of 12,000 s was set in which all instances except instance 32 and 34 could be solved to optimality. Secondly, Table 6 shows the gap improvement in % points of the lower bound for the instances in set 1 in which the continuous relaxation was solved. This enables identifying the extent to which a set of VIs contributes to an improved lower bound in the root node.

The experiments in Table 5 indicate that omitting any of the sets of VIs leads to a positive increase in computation times on average, even though the effects vary across instances. In particular, Table 5 shows that omitting each of the VIs (37), (43), (44), (46), (49) and (54) lead to the strongest performance effect in which the average computation time increases range between 48% to 1298% relative to having all VIs activated. VIs (46) break symmetry in which empty tours are scheduled sequentially and in the beginning of the planning horizon. These are most effective with a 1298% time increase when omitted. VIs (44) which schedule the slowest producer in the last tour are also very effective (121%). VIs (49) sequentially arrange the order of container exchanges (79%). VIs (43) provide lower bounds on the number of tours (77%). VIs

(37) ensure that tours departing from the destination depot cannot be made in each tour (50%). Omitting VIs (54) which enforce all gas to be collected leads to a 48% computation time increase.

The experiments in Table 6 show that the optimality gap improvement in % points of the lower bound in the root node as a result of each VI is strongest for VI (40), (43) and (45) and ranges between 23% and 25%. In contrast to the results in Table 5, the remaining VIs lead to either no improvement or a very small improvement up to 4.1% even though these VIs contribute much more strongly to a reduction in computation times when the model was solved to optimality (see Table 5). This suggests that the remaining VIs are more effective at lower branches in the branch-and-bound tree.

In sum, VIs (37), (43), (44), (46), (49) and (54) related to depot departures and arrivals, scheduling of empty tours, setting which of the containers are exchanged and setting the difference between production and gas collected are most effective in solving the model.

4.3. Computational results and number of suppliers

We present the computational results of the instances in set 2 that varies the number of suppliers. For all instances in set 2, the valid inequalities are applied. The experiments were again performed with a time limit of 12,000 s and 4Gb memory limit.

Table 7 shows that the number of suppliers highly affect calculation times, as expected. Moreover, the model with the valid inequalities can solve instances containing 7 suppliers within the

Fig. 5. Average number of containers exchanged per supplier per tour for different vehicle capacities.

time limit of 12,000 s. All instances with 5 suppliers could be solved to optimality within the time limit. Three out of 9 instances with 6 suppliers could be solved to optimality and one instance with 7 suppliers could be solved. This confirms the increasing difficulty of the problem for instances with more suppliers. Overall, 6 out of 45 instances could be solved optimally in less than 10 min, all with 5 suppliers. Overall, the remaining instances could not be solved within the time limit. The optimality gaps remain below 17.6% for these instances. It should be emphasized that instance size is not fully reflected by the number of suppliers as multiple tours are required for each supplier within the planning horizon as the experimental insights will show.

4.4. Experimental insights

The experiments of set 3 are executed to examine the effect of storage capacity, vehicle capacity and production rates at each supplier on the total transportation time and other indicators which reflect inventory collection patterns. These include the number of suppliers visited per tour, the number of exchanged containers per supplier, the average content levels of the collected containers and the number of performed tours.

Out of the 3200 instances with 5 suppliers, 2174 instances could be solved to optimality within a 12,000 s time limit.

Overall, the results show that both the transportation time and the number of suppliers per tour are reduced for increasing storage capacities until the storage capacity equals the vehicle capacity. In contrast, the number of collected containers increases with

increasing storage capacity levels. In 95% of the instances, the average content level of the exchanged containers exceeded 99.6%. Results indicate that the average transportation time is reduced for increasing vehicle capacities. Detailed results are described next.

4.4.1. Storage and vehicle capacity.

Fig. 2 shows lower and upper bounds for the transportation time, when the vehicle capacity Q is 4 containers. The average lower and upper bounds decrease for higher storage capacities, as expected. In particular, the total average transportation times stop decreasing when the storage capacity becomes larger or equal to the vehicle capacity. This was found for all vehicle capacities and across all production rate combinations.

The behavior of the transportation time as a result of storage capacity appears to be similar for low and high production rates and low and high variance of the production rates. Moreover, higher production rates require more tours to be performed which increases the average transportation time during the planning horizon.

Fig. 2 reveals a larger gap between the lower and upper bounds to the solution for storage capacities which are below the vehicle capacity for high production rates, indicating that such instances appear to be more difficult to solve within the time limit of 12,000 s. For example, the average gaps for storage capacities of 2 and 3 containers are 2.2% and 0.7% respectively for high production rates. This can be explained by the larger number of tours required for lower storage capacities at each supplier.

Fig. 6. Average number of tours performed for different vehicle capacities.

4.4.2. Time windows

Fig. 3 shows the lower and upper bounds for the transportation time when time windows are activated and for a vehicle capacity of 4 containers. Out of the 3200 instances with time windows activated, 1642 could be solved to optimality within the time limit. Moreover, the average optimality gap with time windows of 2.4% in experiment set 4 is 1.3% points higher than the experiments without time windows in set 3. This illustrates the increasing complexity associated with introducing time windows in the formulation.

Fig. 3 indicates that the upper bounds are slightly higher when time windows are activated compared to the experiments in set 3. For a storage capacity of 2, a vehicle capacity of 4 containers and low mean production rates, the average percentage increases in upper bounds are 3.6% and 6.7% for the low and high variance cases of production rates respectively. For high mean production rates, these average percentage increases were 5% and 2.5%. The average increase in transportation time is reduced for higher storage capacity levels up to 5 containers. This shows that introducing time windows leads to only slightly higher transportation times when storage capacity is limited relative to vehicle capacity. This effect may be overcome by installing sufficient storage capacity levels.

4.4.3. Number of suppliers visited per tour.

Fig. 4 indicates that the average number of suppliers visited per tour decreases for higher storage capacity levels at each supplier until the storage capacity is larger than or equal to the vehicle capacity.

This effect was observed for all vehicle capacity levels and production rate combinations. The average number of suppliers visited per tour remains constant when vehicle capacity is larger than or equal to the storage capacity at each supplier. In that case, tours in which a single supplier is visited are most efficient. The vehicle capacity can then be fully used to collect all inventory at a single supplier and this limits the required number of visits to a supplier and the total transportation time during the planning horizon.

4.4.4. Number of exchanged containers per supplier.

Figs. 5 illustrates again that it is most efficient to collect multiple containers at a single supplier if the vehicle capacity and storage capacity enable doing so. The average number of containers exchanged per supplier per tour increases for higher storage capacity levels until it approaches the vehicle capacity. The number of exchanged containers does not fully reach the vehicle capacity for any combination of vehicle capacity and storage capacity. This is due to the fact that some tours are necessary in which multiple suppliers are visited, since the number of containers to be collected at a supplier during the planning horizon is not always a multiple of vehicle capacity.

4.4.5. Content level of collected containers.

In none of the instances, the average content level of exchanged containers was below 86.7%. In 95% of the instances, the average content level exceeded 99.6%. In 50% of the instances, the average content level even exceeded 99.5%. In instances with the lowest

Fig. 7. Suppliers visited per tour and the order of visit for different storage capacities.

Fig. 8. Inventory levels and the related tours for a storage capacity of 2 containers.

Fig. 9. Inventory levels and the related tours for a storage capacity of 3 containers.

average content levels, partially filled containers were collected at suppliers that did not require a significant detour on a tour visiting other suppliers.

4.4.6. Number of tours required.

Fig. 6 shows that the average number of tours scheduled during the planning horizon decreases for larger storage capacities for all vehicle capacities. We observe that the number of tours required is mostly affected when storage capacity is smaller than the vehicle capacity. Notice that the scales on the vertical axis had to be adapted for each vehicle capacity. For a storage capacity of 5, a vehicle capacity of 5 does reduce the average number of tours by 56.5% compared to a capacity of 2 containers. But as more of the tours visit a single supplier (Fig. 4), the increased storage capacity still reduces transportation time. In this case, more storage capacity does not lead to less tours but better tours.

4.5. Illustrative example

In order to illustrate the relationships between storage capacities, vehicle capacities and the total transportation time, we consider a single example based on four locations of biogas producers in the Netherlands. In all experiments, we consider a planning horizon of one week and a vehicle capacity of 4 containers which is the most common in practice when using tube trailers. The container size is again set at 1500 m^3 of gas in uncompressed equivalents. The production rates are 12, 13, 10, and 11 container equivalents produced during the planning horizon for producers 1, 2, 3, and 4 respectively. Moreover, we test the storage capacities at each supplier between 2 and 4 containers.

4.5.1. Routes.

Fig. 7 illustrates the supplier visits. The numbers in the graph represent the order of visit in each tour for storage capacities of 2 to 4 containers, i.e. the last tour in the left-most graph, number 12, which first visits supplier 3 and next supplier 4. The following aspects can be seen in more detail for this example.

Consistent with the overall experimental results, the number of suppliers visited per tour decreases for increased storage capacities. For example, up to 4 suppliers are visited per tour for a storage capacity of 2 containers and at most 2 suppliers are visited per tour for a storage capacity of 4 containers.

For a storage capacity of 2, 2 producers are often combined in a single tour, collecting 2 containers from both. When the storage capacity increases to 3, it appears to be more efficient to regularly collect all 3 containers of a single producer and not make a detour to collect a fourth at another producer.

As a consequence, more tours are executed for a storage capacity of 3 containers than for a storage capacity of 2 containers, that is, 14 and 12 tours respectively. The experimental results in 4.4 have shown that, overall, multiple tours are not performed more often for higher storage capacities.

4.5.2. Inventory levels.

Fig. 8–10 show the inventory levels for each of the suppliers. The numbers in the graph specify the tour in which the inventory is depleted. The timing is such that all suppliers nearly reach their full storage capacity again at the next tour. For these tours, the supplier with the highest production rate dictates the departure time of the vehicle. Since the storage capacity limit of that supplier is reached first in consecutive tours, the containers of that

Fig. 10. Inventory levels and the related tours for a storage capacity of 4 containers.

supplier are collected full, whereas the containers of the next visited supplier are collected while partly filled. All suppliers were combined in the fourth tour to create a match between produced and collected gas within the planning horizon.

Notice that thus the optimal solution has 6 and 7 visits to supplier 1 and 3 respectively, while the storage capacity of 2 containers would enable 5 and 6 visits to collect their weekly productions of 10 and 12 containers respectively.

For a storage capacity of 3 containers, we recognize mostly single supplier visits in which all 3 available containers are collected (see Fig. 9). The vehicle capacity is not completely filled for these tours, and carries 3 out of 4 containers. For tours in which multiple suppliers are visited, the suppliers with the most similar production rates are visited in pairs (see tour 2, 10, 12 and 14). Because a storage capacity of 3 containers now enables the collection of 3 containers at the same supplier, we observe collected gas combinations of 1 and 3 containers (in tour 2 and 14) and 2 times 2 containers in tour 10 and 12.

For a storage capacity of 4 containers, Fig. 10 illustrates that all 4 containers are collected in almost all tours in which single suppliers are visited per tour and then in tour 11, only 3 containers are collected at supplier 3. In tour 12, supplier 3 and 4 are combined for 2 containers each to ultimately realize a match between weekly production and collection of gas. This is required to set inventory levels at the end of the planning horizon equal to starting inventory levels. Notice that supplier 4 is always visited just before reaching its storage capacity limit as 3 times 4 containers are collected, while supplier 4 produces 11 full containers each week. Supplier 1 is producing 12 containers each week and is visited 3 times when exactly reaching its storage capacity limit.

5. Conclusion

In this paper, we have addressed the formulation of a continuous-time supply-driven inventory routing problem in which the routing and inventory collection decisions are dictated by supply rather than demand. To the best of our knowledge, this study is the first to optimally solve the combination of routing schedules in continuous time with continuous inventory contents in discrete containers. The inventory is stored in containers that act as both movable transport units and the available storage capacity at each supplier. This problem occurs when biogas needs to be transported from a set of decentralized suppliers to a single processing facility during a cyclic schedule. We have proposed a novel and compact formulation that enables solving realistic instances to optimality. We have developed valid inequalities that have a large impact on running times and have provided experimental insights by examining the routing and inventory collection decisions for different storage and vehicle capacities at each supplier.

The computational results illustrate the complexity of the problem. The valid inequalities are a necessity in solving instances of a realistic size and reduced running times with 93% on average. The analyses of the optimal solutions indicate that these approach the lowest level of transportation time when the storage capacity equals at least the vehicle capacity. This enables the more efficient solution to visit fewer suppliers per tour. Both the average transportation time and the average number of suppliers visited per tour generally decrease for larger storage capacities at each supplier until the storage capacity is larger than or equal to the vehicle capacity. For fixed storage capacity levels, a larger vehicle capacity reduces the average number of tours and the transportation

time required. In 95% of the instances, the average content level exceeded 99.6%. An in-depth analysis of the routes and inventory patterns generated for a small set of instances reveals some remarkable characteristics of optimal schedules that would not have been expected by transport planners. These particularly relate to the requirements of weekly repeating schedules, while weekly production quantities do not allow for full truckloads.

Overall, the solution method is shown to be able to generate optimal solutions for real-life instances and points out where inefficiencies occur due to insufficient storage capacity at each supplier and the need to create cyclic schedules. Inefficiencies may relate to 1) routes having to combine multiple suppliers, 2) vehicles that drive while not fully loaded and 3) storage capacities which are not fully used.

Avenues for future research may include extending the approach with multiple vehicles and the problem case where the fleet is heterogeneous. For the case with time windows, future research may formulate a different model which is solved with a branch-and-price algorithm.

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Supplementary material

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