University of Groningen

Leader Tracking of Euler-Lagrange Agents on Directed Switching Networks Using a Model-Independent Algorithm

Ye, Mengbin; Anderson, Brian D. O.; Yu, Changbin

Published in:
IEEE Transactions on Control of Network Systems

DOI:
[10.1109/TCNS.2018.2856298](https://doi.org/10.1109/TCNS.2018.2856298)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2019

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Ye, M., Anderson, B. D. O., & Yu, C. (2019). Leader Tracking of Euler-Lagrange Agents on Directed Switching Networks Using a Model-Independent Algorithm. *IEEE Transactions on Control of Network Systems*, 6(2), 561-571. <https://doi.org/10.1109/TCNS.2018.2856298>

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Leader Tracking of Euler–Lagrange Agents on Directed Switching Networks Using a Model-Independent Algorithm

Mengbin Ye¹, Member, IEEE, Brian D. O. Anderson², Life Fellow, IEEE, and Changbin Yu¹, Senior Member, IEEE

Abstract—In this paper, we propose a discontinuous distributed model-independent algorithm for a directed network of Euler–Lagrange agents to track the trajectory of a leader with nonconstant velocity. We initially study a fixed network and show that the leader tracking objective is achieved semiglobally exponentially fast if the graph contains a directed spanning tree. By model independent, we mean that each agent executes its algorithm with no knowledge of the parameter values of any agent's dynamics. Certain bounds on the agent dynamics (including any disturbances) and network topology information are used to design the control gain. This fact, combined with the algorithm's model independence, results in robustness to disturbances and modeling uncertainties. Next, a continuous approximation of the algorithm is proposed, which achieves practical tracking with an adjustable tracking error. Last, we show that the algorithm is stable for networks that switch with an explicitly computable dwell time. Numerical simulations are given to show the algorithm's effectiveness.

Index Terms—Directed graph, distributed algorithm, Euler–Lagrange agent, model independent, switching network, tracking.

I. INTRODUCTION

COORDINATION of multi-agent systems using distributed algorithms has been widely studied over the past decade

Manuscript received February 8, 2018; revised May 12, 2018; accepted June 30, 2018. Date of publication July 16, 2018; date of current version May 28, 2019. This work was supported in part by the National Natural Science Foundation of China under Grant 61761136005, the Australian Research Council under grant DP-160104500, and Data61-CSIRO. The work of M. Ye was supported by an Australian Government Research Training Program (RTP) Scholarship. Recommended by Associate Editor F. Zhang. (Corresponding author: Changbin Yu.)

M. Ye is with the Institute of Advanced Technology, Westlake Institute for Advanced Study, Westlake University, Hangzhou 310024, China, also with the Research School of Engineering, Australian National University, Canberra ACT 2601, Australia, and also with the Faculty of Science and Engineering, ENTEG, University of Groningen, Groningen 9747 AG, The Netherlands (e-mail: m.ye@rug.nl).

B. D. O. Anderson is with the School of Automation, Hangzhou Dianzi University, Hangzhou 310000, China, and also with Australian National University, Canberra ACT 2601, Australia and, Data61-CSIRO, Canberra ACT 2601, Australia (e-mail: Brian.Anderson@anu.edu.au).

C. Yu is with the Institute of Advanced Technology, Westlake Institute for Advanced Study, Westlake University, Hangzhou 310024, China, and also with the Research School of Engineering, Australian National University, Canberra ACT 2601, Australia (e-mail: brad.yu@anu.edu.au).

Digital Object Identifier 10.1109/TCNS.2018.2856298

[1]. Of recent interest is the study of agents whose dynamics are described using Euler–Lagrange equations of motion, which from here onward will be referred to as Euler–Lagrange agents (in some literature known as Lagrangian agents). The nonlinear Euler–Lagrange equation can be used to model the dynamics of a large class of mechanical, electrical, and electromechanical systems [2]. Thus, there is significant motivation to study coordination problems with multiple Euler–Lagrange agents. The interacting agents collectively form a network, and may be modeled using a graph [1]. Directed networks capture unilateral interactions (e.g., sensing or communication) and are generally more plausible in real-world applications when compared to undirected networks.

To better place our results in context, two existing approaches for designing coordination algorithms for Euler–Lagrange networks are reviewed: model-dependent and adaptive algorithms. The aim is to give readers an idea of available works; the list is not exhaustive. The authors in [3]–[5] study different coordination objectives, such as consensus or leader tracking, using algorithms that require *exact knowledge of the agent models*. Specifically, each agent's algorithm requires knowledge of its own Euler–Lagrange equation in order to execute. The algorithms are, therefore, less robust to uncertainties in the model, e.g., some parameters in the Euler–Lagrange equation may be unknown or uncertain. Recently, the more popular approach is for each agent to use an *adaptive algorithm*. Specifically, an Euler–Lagrange equation can be linearly parameterized [2] with respect to a set of constant parameters of the equation, e.g., the mass of an arm on a robotic manipulator agent. This parameterization is then used in an adaptive algorithm to allow the agent to estimate its own set of constant parameters (which is assumed to be unknown) while simultaneously achieving the multi-agent coordination objective. Adaptive algorithms have been exploited to solve problems of containment control [6], [7], leaderless consensus [7], [8], and leader tracking [9]–[13].

In contrast to the aforementioned works, which rely on the direct knowledge (or adaptive identification) of an agent model, there have been relatively few works studying *model-independent* algorithms, that is, algorithms for obtaining robust controllers. Furthermore, most results study model-independent algorithms on *undirected* networks. The pioneering work in [14] considered leaderless position consensus, with time-delay

considered in [15]. Consensus to the intersection of target sets is studied in [16]. Leader-tracking algorithms are studied in [17]–[19]. Rendezvous to a stationary leader with collision avoidance is studied in [20]. For directed networks, several results are available. Passivity analysis in [21] showed that synchronization of the velocities (but not of the positions) is achieved on strongly connected directed networks. Rendezvous to a stationary leader and position consensus was studied in [22] and [23], respectively, but the papers assumed that the agents did not have a gravitational term in the dynamics. Leader tracking is studied in [24] but restrictive assumptions are placed on the leader. Preliminary work by the authors also appeared in [25], and is further analyzed below.

A. Motivation for Model-Independent Algorithms

Further study of model-independent algorithms is desirable for several reasons. Given a unique Euler–Lagrange equation, determining the minimum number of parameters in an adaptive algorithm is difficult in general [26]. Moreover, the adaptive algorithms require knowledge of the exact equation structure; the algorithms can deal with uncertain constant parameters associated with the agent dynamics but are not robust to unmodeled nonlinear agent dynamics. Model-independent algorithms are reminiscent of robust controllers, which stand in conceptual contrast to adaptive controllers. Stability and indeed performance is guaranteed given limited knowledge of upper bounds on parameters of the multi-agent system, and without use of any attempt to identify these parameters.

As will be shown in this paper, and similarly to [22] and [23], model-independent controllers are exponentially stable, with a computable minimum rate of convergence. Exponentially stable systems are desired over systems that are asymptotically stable, but not exponentially so, because exponentially stable systems offer improved rejection to small amounts of noise and disturbance. Some algorithms *requiring exact knowledge of the Euler–Lagrange equation* have been shown to be exponentially stable [3], [5]. Further, adaptive controllers will yield exponential stability if certain conditions are satisfied, e.g., persistency of excitation. However, the aforementioned detailed works using adaptive algorithms have not verified such conditions.

B. Contributions of This Paper

In this paper, we propose a discontinuous model-independent algorithm that allows a directed network of Euler–Lagrange agents to track a leader with arbitrary trajectory. First, we assume that the network is fixed and contains a directed spanning tree. Then, we relax this assumption to allow for a network with switching interactions. In order to achieve stability, a set of scalar control gains must be sufficiently large, i.e., satisfy a set of lower bounding inequalities. These inequalities involve limited knowledge of the bounds on the agent dynamic parameters, limited knowledge of the network topology, and a bound on the initial conditions (which may be arbitrarily large). This last requirement means the algorithm is semiglobally stable; a larger set of allowed initial conditions simply requires recomputing of the control gains. It is also shown that the algorithm is robust to heterogeneous, bounded disturbances for each individual agent.

We now record the points of contrast between this paper and the previously mentioned existing works. Several results have been listed studying leader tracking, but most involving model-independent algorithms have been studied on undirected, *not directed*, graphs. Those studying directed graphs primarily use *adaptive algorithms*, and motivation was provided previously as to why model-independent algorithms, such as the one proposed in this paper, are desirable. Most model-independent algorithms on directed networks consider position consensus or rendezvous to a stationary leader, e.g., [22] and [23]. Furthermore, [22] and [23] did not consider the gravitational term in the agent dynamics. This paper makes the nontrivial extension to incorporate both a moving leader and the gravitational term using an algorithm and associated theoretical analysis, which, due to the two extensions, is greatly expanded and novel from that appearing in [22] and [23]; the introduction of a moving leader greatly increases the difficulty of the problem due to the complex, nonlinear Euler–Lagrange dynamics, while the nonlinear gravitational term behaves similarly to a bounded disturbance. Feng *et al.* [24] studied a model-independent leader tracking algorithm on directed graphs, with the restrictive assumption that the leader trajectory is governed by a marginally stable linear time-invariant system, and the system matrix is known to all agents. In addition to considering directed switching networks (the previously discussed works all assume a static network), a model-independent algorithm, and the gravitational term, a *major contribution of this paper is to allow for any arbitrary leader trajectory that satisfies some mild and reasonable smoothness and boundedness properties*. In addition, [24] does not establish an exponential stability property, whereas the algorithm proposed in this paper does.

This paper significantly extends the preliminary version [25] in several aspects. First, an additional control gain is introduced allowing for an additional degree of freedom in algorithm design. Increasing the new gain ensures stability, but at the same time, it does not negatively affect convergence rate, unlike in [25]. Second, we address the issues arising from the discontinuous nature of the control algorithm by using an approximation of the signum function. An explicit expression relating the tracking error to the degree of approximation and control gain is derived. Additionally, switching topology is considered. Details of omitted proofs are also now provided.

This paper is structured as follows. Section II introduces mathematical preliminaries, and the problem. The problem with fixed network topology, and dynamic topology, is solved in Sections III and IV, respectively. Simulations are provided in Section V and this paper is concluded in Section VI.

II. BACKGROUND AND PROBLEM STATEMENT

A. Mathematical Notation and Preliminary Theory

First, definitions of notation and several results are provided. The Kronecker product is denoted as $\otimes$. Denote the $p \times p$ identity matrix as $\mathbf{I}_p$ and the n -column vector of all ones as $\mathbf{1}_n$. The l_1 -norm and Euclidean norm of a vector $\mathbf{x}$, and matrix $\mathbf{A}$, are denoted by $\|\cdot\|_1$ and $\|\cdot\|_2$, respectively. The signum function is denoted as $\text{sgn}(\cdot)$, and for a vector $\mathbf{x}$, the function $\text{sgn}(\mathbf{x})$ is defined element wise. A matrix $\mathbf{A} = \mathbf{A}^\top$ that is pos-

itive definite (respectively, nonnegative definite) is denoted by $\mathbf{A} > 0$ (respectively, $\mathbf{A} \geq 0$). For symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} > \mathbf{B}$ is equivalent to $\mathbf{A} - \mathbf{B} > 0$. For a matrix $\mathbf{A} = \mathbf{A}^\top$, the minimum and maximum eigenvalues are $\lambda_{\min}(\mathbf{A})$ and $\lambda_{\max}(\mathbf{A})$, respectively. The following inequalities hold:

- 1) $\lambda_{\min}(\mathbf{A}) > \lambda_{\max}(\mathbf{B}) \Rightarrow \mathbf{A} > \mathbf{B}$;
- 2) $\lambda_{\max}(\mathbf{A} + \mathbf{B}) \leq \lambda_{\max}(\mathbf{A}) + \lambda_{\max}(\mathbf{B})$;
- 3) $\lambda_{\min}(\mathbf{A} + \mathbf{B}) \geq \lambda_{\min}(\mathbf{A}) + \lambda_{\min}(\mathbf{B})$;
- 4) $\lambda_{\min}(\mathbf{A})\mathbf{x}^\top\mathbf{x} \leq \mathbf{x}^\top\mathbf{A}\mathbf{x} \leq \lambda_{\max}(\mathbf{A})\mathbf{x}^\top\mathbf{x}$.

Definition 1: A function $f(\mathbf{x}) : \mathcal{D} \rightarrow \mathbb{R}$, where $\mathcal{D} \subseteq \mathbb{R}^n$, is said to be positive definite in $\mathcal{D}$ if $f(\mathbf{x}) > 0$ for all $\mathbf{x} \in \mathcal{D}$, except $f(\mathbf{0}) = 0$.

Lemma 1 (The Schur Complement [27]): Consider a symmetric block matrix, partitioned as

$$\mathbf{A} = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^\top & \mathbf{D} \end{bmatrix}. \quad (1)$$

Then, $\mathbf{A} > 0$ if and only if $\mathbf{B} > 0$ and $\mathbf{D} - \mathbf{C}^\top\mathbf{B}^{-1}\mathbf{C} > 0$, or equivalently, if and only if $\mathbf{D} > 0$ and $\mathbf{B} - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^\top > 0$.

Lemma 2: Suppose $\mathbf{A} > 0$ is defined as in (1). Let a quadratic function with arguments $\mathbf{x}$ and $\mathbf{y}$ be expressed as $W = [\mathbf{x}^\top, \mathbf{y}^\top]\mathbf{A}[\mathbf{x}^\top, \mathbf{y}^\top]^\top$. Define $\mathbf{F} := \mathbf{B} - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^\top$ and $\mathbf{G} := \mathbf{D} - \mathbf{C}^\top\mathbf{B}^{-1}\mathbf{C}$. Then, there holds

$$\lambda_{\min}(\mathbf{F})\mathbf{x}^\top\mathbf{x} \leq \mathbf{x}^\top\mathbf{F}\mathbf{x} \leq W \quad (2a)$$

$$\lambda_{\min}(\mathbf{G})\mathbf{y}^\top\mathbf{y} \leq \mathbf{y}^\top\mathbf{G}\mathbf{y} \leq W. \quad (2b)$$

Proof: We obtain (2b) by recalling Lemma 1 and observing that $W = \mathbf{y}^\top\mathbf{G}\mathbf{y} + [\mathbf{y}^\top\mathbf{C}^\top\mathbf{B}^{-1} + \mathbf{x}^\top]\mathbf{B}[\mathbf{B}^{-1}\mathbf{C}\mathbf{y} + \mathbf{x}]$. An equally straightforward proof yields (2a). ■

Lemma 3: Let $g(x, y)$ be a function given as

$$g(x, y) = ax^2 + by^2 - cxy^2 - dxy \quad (3)$$

for real positive scalars $c, d > 0$. Then, for a given $\mathcal{X} > 0$, there exist $a, b > 0$ such that $g(x, y)$ is positive definite for all $y \in [0, \infty)$ and $x \in [0, \mathcal{X}]$.

Corollary 1: Let $h(x, y)$ be a function given as

$$h(x, y) = ax^2 + by^2 - cxy^2 - dxy - ex - fy \quad (4)$$

where the real positive scalars c, d, e , and f and two further positive scalars ε and ϑ are fixed. Suppose that for given $\mathcal{Y}$ and $\mathcal{X}$, there holds $\mathcal{Y} - \varepsilon > 0$, and $\mathcal{X} - \vartheta > 0$. Define the sets $\mathcal{U} = \{x, y : x \in [\mathcal{X} - \vartheta, \mathcal{X}], y > 0\}$ and $\mathcal{V} = \{x, y : x > 0, y \in [\mathcal{Y} - \varepsilon, \mathcal{Y}]\}$. Define the region $\mathcal{R} = \mathcal{U} \cup \mathcal{V}$. Then, there exist $a, b > 0$ such that $h(x, y)$ is positive definite in $\mathcal{R}$.

The results of Lemma 3 and Corollary 1 are almost intuitively obvious. However, the detailed statements in the proof (see [28, Sec. II-A]) lay out explicit inequalities for a and b . These inequalities will be used to show that for any given Euler-Lagrange network, *control gains can always be found* to ensure leader tracking is achieved.

B. Graph Theory

The agent interactions can be modeled by a weighted directed graph, which is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with the set of nodes $\mathcal{V} = \{v_0, v_1, \dots, v_n\}$, and with a corresponding set of ordered edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A directed edge $e_{ij} = (v_i, v_j)$ is outgoing

with respect to v_i and incoming with respect to v_j , and implies that v_j is able to obtain some information from v_i . The precise nature of this information will be made clear in the sequel. The weighted adjacency matrix $\mathcal{A} \in \mathbb{R}^{(n+1) \times (n+1)}$ of $\mathcal{G}$ has nonnegative elements a_{ij} . The elements of $\mathcal{A}$ have properties such that $a_{ij} > 0 \Leftrightarrow e_{ji} \in \mathcal{E}$, while $a_{ij} = 0$ if $e_{ji} \notin \mathcal{E}$ and it is assumed $a_{ii} = 0 \forall i$. The neighbor set of v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The $(n+1) \times (n+1)$ Laplacian matrix, $\mathcal{L} = \{l_{ij}\}$, of the associated digraph $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}$ for $j \neq i$ and $l_{ij} = \sum_{k=1, k \neq i}^n a_{ik}$ if $j = i$. A directed spanning tree is a directed graph formed by directed edges of the graph that connects all the nodes, and where every vertex apart from the root has exactly one parent. A graph is said to contain a spanning tree if a subset of the edges forms a spanning tree. We make use of the following standard lemma.

Lemma 4 ([29]): Let $\mathcal{G}$ contain a directed spanning tree, and suppose there are no edges that are incoming to the root vertex of the tree, which without loss of generality, is set as v_0 . Then, the Laplacian of $\mathcal{G}$ can be partitioned as

$$\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathcal{L}_{11} & \mathcal{L}_{22} \end{bmatrix} \quad (5)$$

and $\exists \mathbf{\Gamma} > 0$, which is diagonal and $\mathbf{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\mathbf{\Gamma} > 0$.

For future use, denote the i th diagonal element of $\mathbf{\Gamma}$ as γ_i and define $\bar{\gamma} \triangleq \max_i \gamma_i$ and $\underline{\gamma} \triangleq \min_i \gamma_i$.

C. Euler-Lagrange Systems

The i th Euler-Lagrange agent's equation of motion is

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) + \boldsymbol{\zeta}_i = \boldsymbol{\tau}_i \quad (6)$$

where $\mathbf{q}_i(t) \in \mathbb{R}^p$ is a vector of the generalized coordinates. Note that from here onward, we drop the time argument t whenever there is no ambiguity. The inertia matrix is $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{p \times p}$, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{p \times p}$ is the Coriolis and centrifugal force matrix, $\mathbf{g}_i \in \mathbb{R}^p$ is the vector of (gravitational) potential forces, and $\boldsymbol{\zeta}_i(t)$ is an unknown, time-varying disturbance. It is assumed that all agents are fully actuated, with $\boldsymbol{\tau}_i \in \mathbb{R}^p$ being the control input vector. For each agent, the k th generalized coordinate is denoted using superscript (k) ; thus, $\mathbf{q}_i = [q_i^{(1)}, \dots, q_i^{(p)}]^\top$. It is assumed that the systems described using (6) have the following properties.

- 1) The matrix $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric positive definite.
- 2) There exist scalar constants $k_m, k_M > 0$ such that $k_m\mathbf{I}_p \leq \mathbf{M}_i(\mathbf{q}_i) \leq k_M\mathbf{I}_p \forall i, \mathbf{q}_i$. It follows that $\sup_{\mathbf{q}_i} \|\dot{\mathbf{M}}_i\| \leq k_M$ and $k_m \leq \inf_{\mathbf{q}_i} \|\mathbf{M}_i^{-1}\|_2$ holds $\forall i$.
- 3) The matrix $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is defined such that $\dot{\mathbf{M}}_i - 2\mathbf{C}_i$ is skew symmetric, i.e., $\dot{\mathbf{M}}_i = \mathbf{C}_i + \mathbf{C}_i^\top$.
- 4) There exist scalar constants $k_C, k_g > 0$ such that $\|\mathbf{C}_i\|_2 \leq k_C \|\dot{\mathbf{q}}_i\|_2 \forall i, \dot{\mathbf{q}}_i$ and $\|\mathbf{g}_i\|_2 < k_g \forall i$.
- 5) There exists a constant k_ζ such that $\|\boldsymbol{\zeta}_i\|_2 \leq k_\zeta \forall i$.

Properties 1–4 are standard and widely assumed properties of Euler-Lagrange dynamical systems; see [2] for details. Property 5 is a reasonable assumption on disturbances.

D. Problem Statement

The leader is denoted as agent 0, i.e., vertex v_0 , with $\mathbf{q}_0(t)$ and $\dot{\mathbf{q}}_0(t)$ being its time-varying generalized coordinates and

generalized velocity, respectively. The objective is to develop a model-independent, distributed algorithm that allows a directed network of Euler–Lagrange agents to synchronize and track the trajectory of the leader. The leader tracking objective is said to be achieved if $\lim_{t \rightarrow \infty} \|\mathbf{q}_i(t) - \mathbf{q}_0(t)\|_2 = 0$ and $\lim_{t \rightarrow \infty} \|\dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_0(t)\|_2 = 0$ for all $i = 1, \dots, n$. By model independent, we mean that the algorithm does not contain $\mathbf{M}_i, \mathbf{C}_i, \mathbf{g}_i \forall i$ nor make use of an associated linear parameterization. Two mild assumptions are now given.

Assumption 1: The leader trajectory $\mathbf{q}_0(t)$ is a $\mathcal{C}^2$ function with time derivatives $\dot{\mathbf{q}}_0$ and $\ddot{\mathbf{q}}_0$, which are bounded as $\|\mathbf{1}_n \otimes \dot{\mathbf{q}}_0\|_2 \leq k_p$ and $\|\mathbf{1}_n \otimes \ddot{\mathbf{q}}_0\|_2 \leq k_q$. The positive constants k_p and k_q are known *a priori*.

Assumption 2: All possible initial conditions lie in some fixed but arbitrarily large set that is known. In particular, $\|\mathbf{q}_i(0)\|_2 \leq k_a/\sqrt{n}$ and $\|\dot{\mathbf{q}}_i(0)\|_2 \leq k_b/\sqrt{n}$, where k_a and k_b are known *a priori*.

These two assumptions are not unreasonable, as many systems will have an expected operating range for $\mathbf{q}_i$ and $\dot{\mathbf{q}}_i$.

The follower agents' capability to sense relative states is captured by the directed graph $\mathcal{G}_A$ with an associated Laplacian $\mathcal{L}_A$. In Section III, we assume $\mathcal{G}_A$ is fixed. Later, in Section IV, it is assumed that $\mathcal{G}_A$ is dynamic, i.e., time varying. Thus, if $a_{ij} > 0$, then agent i can sense $\mathbf{q}_i - \mathbf{q}_j$ and $\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j$. We denote the neighbor set of agent i on $\mathcal{G}_A$ as $\mathcal{N}_{Ai}$. We further assume that agent i can measure its own $\mathbf{q}_i$ and $\dot{\mathbf{q}}_i$. A second weighted and directed time-varying graph $\mathcal{G}_B(t)$, with the associated Laplacian $\mathcal{L}_B(t)$, exists between the followers to *communicate* estimates of the leader's state. Denote the neighbor set of agent i on $\mathcal{G}_B(t)$ at time t as $\mathcal{N}_{Bi}(t)$. Note that $v_j \in \mathcal{N}_{Bi}(t)$, when agent j communicates directly to agent i , its estimates of the leader's state at time t (the precise nature of this estimate is described in Section III-A). Further note that $\mathcal{G}_A$ is not necessarily equal to $\mathcal{G}_B$ and so $\mathcal{N}_{Ai} \neq \mathcal{N}_{Bi}$ in general. However, the node sets of $\mathcal{G}_A$ and $\mathcal{G}_B$ are the same.

Remark 1 (Comparison of this paper to recent leader tracking results): Almost all mechanical systems will have trajectories that satisfy the mild Assumption 1. In comparison, more restrictive assumption are made on the leader trajectory in [13] and [24]. In [13] and [24], the leader trajectory is describable by an LTI system, with system matrix defined as $\mathbf{S}$. In [13], it is assumed that all eigenvalues of $\mathbf{S}$ are purely imaginary. In [24], it is assumed that $\mathbf{S}$ is marginally stable. More importantly, both [13] and [24] assume that $\mathbf{S}$ is *known to all agents*, which is a highly restrictive assumption. As will become apparent in the sequel, we use a distributed observer to allow every agent to obtain $\mathbf{q}_0(t)$ and $\dot{\mathbf{q}}_0(t)$ precisely. Ghapani *et al.* [12] have similar assumptions to this paper, but uses an adaptive algorithm, and is therefore, fundamentally different to the model-independent controller studied in this paper.

III. LEADER TRACKING ON FIXED DIRECTED NETWORKS

A. Finite-Time Distributed Observer

Before we show the main result, we detail a distributed finite-time observer developed in [30], which allows each follower agent to obtain $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$. Let $\hat{\mathbf{r}}_i$ and $\hat{\mathbf{v}}_i$ be the i th agent's

estimated values for the leader position and velocity, respectively. Agent $i \in \{1, \dots, n\}$ runs the observer

$$\dot{\hat{\mathbf{r}}}_i = \hat{\mathbf{v}}_i - \omega_1 \operatorname{sgn} \left(\sum_{j \in \mathcal{N}_{Bi}(t)} b_{ij}(t) (\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j) \right) \quad (7a)$$

$$\dot{\hat{\mathbf{v}}}_i = -\omega_2 \operatorname{sgn} \left(\sum_{j \in \mathcal{N}_{Bi}(t)} b_{ij}(t) (\hat{\mathbf{v}}_i - \hat{\mathbf{v}}_j) \right) \quad (7b)$$

where b_{ij} are the elements of the adjacency matrix associated with graph $\mathcal{G}_B(t)$ and $\omega_1, \omega_2 > 0$ are internal gains of the observer. Clearly, if $a_{i0} > 0$, then agent i can directly sense the leader, $\mathbf{v}_0$, and thus, learns of $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$. For such an agent i , we set $b_{i0} > 0$ and $\hat{\mathbf{r}}_0(t) = \mathbf{q}_0(t)$ and $\hat{\mathbf{v}}_0(t) = \dot{\mathbf{q}}_0(t)$; agent i still runs the distributed observer (7). We now give a theorem for convergence of the observer, and explain later why all followers execute (7) even if they learn of $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ from $\mathcal{G}_A$.

Theorem 1: (See [30, Th. 4.1]). Suppose that the leader trajectory $\mathbf{q}_0(t)$ satisfies Assumption 1. If at every t , $\mathcal{G}_B(t)$ contains a directed spanning tree, and $\omega_2 > k_q/n$, then, for some $T_1 < \infty$, there holds $\hat{\mathbf{r}}_i(t) = \mathbf{q}_0(t)$ and $\hat{\mathbf{v}}_i(t) = \dot{\mathbf{q}}_0(t)$ for all $i \in \{1, \dots, n\}$, for all $t \geq T_1$.

The key reason for agent i to run the distributed observer even if $a_{i0} > 0$ (and thus agent i knows $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$) is to ensure robustness to network changes over time (e.g., switching topology due to loss of connection). We elaborate further. In the case of a fixed $\mathcal{G}_A$, then agent i will know $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ for all t and there will be no need for the observer. However, we explore switching $\mathcal{G}_A(t)$ in Section IV. Consider the case where $a_{i0}(t) = 1$ for $t \in [0, \bar{t}]$, $0 < \bar{t} < \infty$ and $a_{i0}(t) = 0$ for $t \in [\bar{t}, \infty)$. If agent i does not run (7), then for $t \geq \bar{t}$, it would not know $\mathbf{q}_0(t)$ and $\dot{\mathbf{q}}_0(t)$ because $a_{i0}(t) = 0 \Rightarrow b_{i0}(t) = 0$. If agent i runs (7) from $t = 0$, then for all $t \geq T_1$, it is guaranteed that $\hat{\mathbf{r}}_1(t) = \mathbf{q}_0(t)$ and $\hat{\mathbf{v}}_1(t) = \dot{\mathbf{q}}_0(t)$ even if $\mathcal{G}_A(t)$ switches, so long as the connectivity condition in Theorem 1 is satisfied. A second reason is that agent i may acquire states by sensing over $\mathcal{G}_A$; (7) acts as a filter for noisy measurements. While (7) allows each agent to obtain $\mathbf{q}_0(t)$ and $\dot{\mathbf{q}}_0(t)$ in finite time, the leader tracking objective must still be achieved. Toward that end, we now propose a distributed model-independent algorithm and study its stability properties, using analysis that is entirely different from that used in [30] to establish Theorem 1.

B. Model-Independent Control Law

Consider the following algorithm for the i th agent

$$\begin{aligned} \tau_i = & -\eta \sum_{j \in \mathcal{N}_{Ai}} a_{ij} \left((\mathbf{q}_i - \mathbf{q}_j) + \mu(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j) \right) \\ & - \beta \operatorname{sgn} \left((\mathbf{q}_i - \hat{\mathbf{r}}_i) + \mu(\dot{\mathbf{q}}_i - \hat{\mathbf{v}}_i) \right) \end{aligned} \quad (8)$$

where a_{ij} is the weighted (i, j) entry of the adjacency matrix $\mathcal{A}$ associated with the weighted directed graph $\mathcal{G}_A$. The control gains μ, η , and β are positive constants and their design will be specified later. It is assumed that $\eta > 1$. Note that for all i , $\hat{\mathbf{r}}_i$ and $\hat{\mathbf{v}}_i$ are replaced with $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$, respectively, $\forall t > T_1$.

Let us denote the new error variable $\tilde{\mathbf{q}}_i = \mathbf{q}_i - \mathbf{q}_0$. Let $\tilde{\mathbf{q}} = [\tilde{\mathbf{q}}_1^\top, \dots, \tilde{\mathbf{q}}_n^\top]^\top \in \mathbb{R}^{np}$ be the stacked column vector of

all $\tilde{q}_i$. The leader tracking objective is, therefore, achieved if $\tilde{q}(t) = \dot{\tilde{q}}(t) = 0$ as $t \rightarrow \infty$. We denote $\mathbf{g} = [\mathbf{g}_1^\top, \dots, \mathbf{g}_n^\top]^\top$, $\boldsymbol{\zeta} = [\boldsymbol{\zeta}_1^\top, \dots, \boldsymbol{\zeta}_n^\top]^\top$, $\mathbf{q} = [\mathbf{q}_1^\top, \dots, \mathbf{q}_n^\top]^\top$, and $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_1^\top, \dots, \dot{\mathbf{q}}_n^\top]^\top$ as the np -column vectors of all $\mathbf{g}_i$, $\boldsymbol{\zeta}_i$, $\mathbf{q}_i$, and $\dot{\mathbf{q}}_i$, respectively. Let $\mathbf{M}(\mathbf{q}) = \text{diag}[\mathbf{M}_1(\mathbf{q}_1), \dots, \mathbf{M}_n(\mathbf{q}_n)] \in \mathbb{R}^{np \times np}$, and $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \text{diag}[\mathbf{C}_1(\mathbf{q}_1, \dot{\mathbf{q}}_1), \dots, \mathbf{C}_n(\mathbf{q}_n, \dot{\mathbf{q}}_n)] \in \mathbb{R}^{np \times np}$. Since $M_i > 0 \forall i$, then $\mathbf{M}$ is also symmetric positive definite. Define an error vector, $\mathbf{e}_i = \hat{\mathbf{r}}_i - \mathbf{q}_0$ and $\dot{\mathbf{e}}_i = \hat{\mathbf{v}}_i - \dot{\mathbf{q}}_0 \forall i = 1, \dots, n$. Define $\mathbf{e} = [\mathbf{e}_1^\top, \dots, \mathbf{e}_n^\top]^\top \in \mathbb{R}^{np}$, $\dot{\mathbf{e}} = [\dot{\mathbf{e}}_1^\top, \dots, \dot{\mathbf{e}}_n^\top]^\top \in \mathbb{R}^{np}$.

The definition of $\tilde{q}_i$ yields $M_i \ddot{\tilde{q}}_i = M_i \ddot{\mathbf{q}}_i - M_i \ddot{\mathbf{q}}_0$ and combining the agent dynamics (6) and the control law (8), the closed-loop system for the follower network, with nodes $v_1, \dots, v_n$, can be expressed as

$$\ddot{\tilde{\mathbf{q}}} \in^{a.e.} \mathcal{K}[-\mathbf{M}^{-1}[\mathbf{C}\dot{\tilde{\mathbf{q}}} + \eta(\mathcal{L}_{22} \otimes \mathbf{I}_p)(\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}) + \mathbf{g} + \boldsymbol{\zeta} + \beta \text{sgn}(\mathbf{s} + \mu\dot{\mathbf{s}}) + \mathbf{M}(\mathbf{1}_n \otimes \ddot{\mathbf{q}}_0) + \mathbf{C}(\mathbf{1}_n \otimes \dot{\mathbf{q}}_0)] \quad (9)$$

where $\mathcal{K}$ denotes the differential inclusion, *a.e.* stands for “almost everywhere,” and $\mathbf{s} = \tilde{\mathbf{q}} - \mathbf{e}$. Here, $\mathcal{L}_{22}$ is the lower block matrix of $\mathcal{L}_A$ as partitioned in (5). Filippov solutions of $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ for (9) exist because the signum function is measurable and locally essentially bounded, and $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$ are absolutely continuous functions of time [31].

C. Upper Bound Using Initial Conditions

Before proceeding with the main proof, we calculate an upper bound (which may not be tight) on the initial states expressed as $\|\tilde{\mathbf{q}}(0)\|_2 < \mathcal{X}$ and $\|\dot{\tilde{\mathbf{q}}}(0)\|_2 < \mathcal{Y}$ using Assumption 2. In the sequel, we show that these bounds hold for all time, and exponential convergence results. In keeping with the model-independent approach, define

$$\bar{V}_\mu = \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix}^\top \begin{bmatrix} \eta\lambda_{\max}(\mathbf{X})\mathbf{I}_{np} & \frac{1}{2}\mu^{-1}(k_{\overline{M}} + \delta)\mathbf{I}_{np} \\ \frac{1}{2}\mu^{-1}(k_{\overline{M}} + \delta)\mathbf{I}_{np} & \frac{1}{2}(k_{\overline{M}} + \delta)\mathbf{I}_{np} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} \quad (10)$$

where¹ $\mathbf{X} = (\boldsymbol{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\boldsymbol{\Gamma}) \otimes \mathbf{I}_p > 0$ from Lemma 4 and the constant $\delta > 0$ is sufficiently small such that $k_{\underline{m}} - \delta > 0$. Without loss of generality, assume that $\boldsymbol{\Gamma}$ is scaled such that $\bar{\gamma} = 1$. Let the matrix in (10) be $\mathbf{L}_\mu$. By observing that $(k_{\overline{M}} + \delta)\mathbf{I}_{np} > \mathbf{M}$, then according to Lemma 1, $\mathbf{L}_\mu > 0$ if and only if $\eta\lambda_{\max}(\mathbf{X})\mathbf{I}_{np} - \frac{1}{2}\mu^{-2}(k_{\overline{M}} + \delta)\mathbf{I}_{np} > 0$. It follows that $\mathbf{L}_\mu > 0$ for any $\mu \geq \mu_1^*$, where $\mu_1^* > \sqrt{(k_{\overline{M}} + \delta)/2\lambda_{\max}(\mathbf{X})}$. Since $\mathbf{X} > 0$, such a μ_1^* always exists. For convenience, we use $\bar{V}_\mu(t)$ to denote $\bar{V}_\mu(\tilde{\mathbf{q}}(t), \dot{\tilde{\mathbf{q}}}(t))$, and observe that there holds

$$\begin{aligned} \bar{V}_\mu(t) &\leq \eta\lambda_{\max}(\mathbf{X})\|\tilde{\mathbf{q}}\|_2^2 + (k_{\overline{M}} + \delta) \\ &\quad \times \left(\frac{1}{2}\|\dot{\tilde{\mathbf{q}}}\|_2^2 + \mu^{-1}\|\tilde{\mathbf{q}}\|_2\|\dot{\tilde{\mathbf{q}}}\|_2 \right) \end{aligned} \quad (11)$$

for all t . Next, define

$$\underline{V}_\mu = \frac{1}{2} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix}^\top \begin{bmatrix} \frac{1}{2}\eta\lambda_{\min}(\mathbf{X})\mathbf{I}_{np} & \mu^{-1}\underline{\gamma}(k_{\underline{m}} - \delta)\mathbf{I}_{np} \\ \mu^{-1}\underline{\gamma}(k_{\underline{m}} - \delta)\mathbf{I}_{np} & \underline{\gamma}(k_{\underline{m}} - \delta)\mathbf{I}_{np} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} \quad (12)$$

¹Note that $\bar{V}_\mu$ is not a Lyapunov function.

Denote the matrix in (12) as $\mathbf{N}_\mu$. One can use Lemma 1 to show that $\mathbf{N}_\mu > 0$ for any $\mu \geq \mu_2^*$, where $\mu_2^* > \sqrt{2\underline{\gamma}(k_{\underline{m}} - \delta)/\lambda_{\min}(\mathbf{X})}$. Set $\mu_3^* = \max\{\mu_1^*, \mu_2^*\}$. Define

$$\rho_1(\mu) = \eta\lambda_{\max}(\mathbf{X}) - \frac{1}{2}\mu^{-2}(k_{\overline{M}} + \delta) \quad (13a)$$

$$\rho_2(\mu) = \eta\frac{1}{4}\lambda_{\min}(\mathbf{X}) - \frac{1}{2}\mu^{-2}\underline{\gamma}(k_{\underline{m}} - \delta) \quad (13b)$$

and verify that $\rho_1(\mu_3^*) > \rho_2(\mu_3^*)$. Note that for any $\mu \geq \mu_3^*$, there holds $\bar{V}_\mu \leq \bar{V}_{\mu_3^*}$ and $\rho_i(\mu_3^*) \leq \rho_i(\mu)$, $i = 1, 2$. Compute

$$\bar{V}^* = \eta\lambda_{\max}(\mathbf{X})k_a^2 + \frac{1}{2}(k_{\overline{M}} + \delta)k_b^2 + \mu_3^{*-1}(k_{\overline{M}} + \delta)k_a k_b.$$

From Assumption 2, one has that $\|\tilde{\mathbf{q}}(0)\|_2 \leq k_a$ and $\|\dot{\tilde{\mathbf{q}}}(0)\|_2 \leq k_b$. Thus, one concludes from (11) and the equation above that there holds $\bar{V}_\mu(0) \leq \bar{V}^*$ for any $\mu \geq \mu_3^*$. Because we assumed $\eta > 1$, it follows from Lemma 2 and (2a) that

$$\|\tilde{\mathbf{q}}(0)\|_2 \leq \sqrt{\frac{\bar{V}_\mu(0)}{\rho_1(\mu)}} \leq \sqrt{\frac{\bar{V}_\mu(0)}{\rho_1(\mu_3^*)}} < \sqrt{\frac{\bar{V}^*}{\rho_2(\mu_3^*)}} \triangleq \mathcal{X}_1. \quad (14)$$

Following a similar method yields $\mathcal{Y}_1$. Next, compute

$$\begin{aligned} \hat{V}^* &= \eta\lambda_{\max}(\mathbf{X})\mathcal{X}_1^2 + \frac{1}{2}(k_{\overline{M}} + \delta)\mathcal{Y}_1^2 \\ &\quad + (\mu_3^*)^{-1}(k_{\overline{M}} + \delta)\mathcal{X}_1\mathcal{Y}_1 \end{aligned}$$

and observe that $\bar{V}^* \leq \hat{V}^*$. Finally, compute the bound

$$\mathcal{X} = \sqrt{\hat{V}^*/\rho_2(\mu_3^*)} \quad (15)$$

and notice that $\|\tilde{\mathbf{q}}(0)\|_2 \leq \mathcal{X}_1 \leq \mathcal{X}$. Similarly, $\mathcal{Y}$ is obtained using (2b), with the steps omitted due to spatial limitations. Because both sides of (15) are independent of μ , the values $\mathcal{Y}$ and $\mathcal{X}$ do not change for all $\mu \geq \mu_3^*$.

D. Stability Proof

Theorem 2: Suppose that the conditions in Theorem 1 are satisfied. Under Assumptions 1 and 2, the leader tracking is achieved exponentially fast if: 1) the network $\mathcal{G}_A$ contains a directed spanning tree with the leader as the root node, and 2) the control gains μ , η , and β satisfy a set of lower bounding inequalities.² For a given $\mathcal{G}_A$ containing a directed spanning tree, there always exists μ , η , and β , which satisfy the inequalities.

Proof: The proof will be presented in four parts. In *Part 1*, we study a Lyapunov-like candidate function V . In *Part 2*, we analyze $\dot{V}$ and show that it is upper bounded. *Part 3* shows that the system trajectory remains bounded for all time, and exponential convergence is proved in *Part 4*.

Part 1: Consider the Lyapunov-like candidate function

$$\begin{aligned} V &= \frac{1}{2}\eta\tilde{\mathbf{q}}^\top \mathbf{X}\tilde{\mathbf{q}} + \mu^{-1}\tilde{\mathbf{q}}^\top \boldsymbol{\Gamma}_p \mathbf{M}\dot{\tilde{\mathbf{q}}} + \frac{1}{2}\dot{\tilde{\mathbf{q}}}^\top \boldsymbol{\Gamma}_p \mathbf{M}\dot{\tilde{\mathbf{q}}} \\ &= V_1 + V_2 + V_3 \end{aligned}$$

²In Remark 2, we detail an approach for designing the gains.

with $\mathbf{X}$ given after (10), and $\Gamma_p = \Gamma \otimes \mathbf{I}_p$. Observe that

$$V = \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix}^\top \begin{bmatrix} \frac{1}{2}\eta\mathbf{X} & \frac{1}{2}\mu^{-1}\Gamma_p\mathbf{M} \\ \frac{1}{2}\mu^{-1}\Gamma_p\mathbf{M} & \frac{1}{2}\Gamma_p\mathbf{M} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix}. \quad (16)$$

Call the matrix in (16) $\mathbf{H}_\mu$. From Lemma 1, and the assumed properties of $\mathbf{M}_i$, there holds $\mathbf{H}_\mu > 0$ if and only if $\eta\mathbf{X} - \mu^{-2}\Gamma_p\mathbf{M} > 0$, which is implied by $\lambda_{\min}(\mathbf{X}) - \mu^{-2}k_{\overline{\mathbf{M}}} > 0$. This is because $k_{\overline{\mathbf{M}}} \geq \sup_{\mathbf{q}} \lambda_{\max}(\mathbf{M})$, and we assumed that $\eta > 1$ and $\bar{\gamma} = 1$. For any $\mu \geq \mu_4^*$, where

$$\mu_4^* > \sqrt{2k_{\overline{\mathbf{M}}}/\lambda_{\min}(\mathbf{X})} \quad (17)$$

there holds $\mathbf{L}_\mu > \mathbf{H}_\mu > \mathbf{N}_\mu > 0$ because $\mu_4^* \geq \mu_3^*$ as defined after (12). Thus, although the eigenvalues $\lambda_i(\mathbf{H}_\mu)$ depend on $\mathbf{q}(t)$, there holds $\lambda_{\min}(\mathbf{N}_\mu) \leq \lambda_i(\mathbf{H}_\mu) \leq \lambda_{\max}(\mathbf{L}_\mu)$ for all i , and for all $t \geq 0$. Thus, for any $\mu \geq \mu_4^*$, $V > 0$ and is radially unbounded. For simplicity, let $V(t)$ denote $V(\tilde{\mathbf{q}}(t), \dot{\tilde{\mathbf{q}}}(t))$ and observe that $V(t) < \bar{V}_\mu(t) \forall t$ because

$$V(t) \leq \frac{1}{2}\eta\lambda_{\max}(\mathbf{X})\|\tilde{\mathbf{q}}(t)\|_2^2 + \frac{1}{2}k_{\overline{\mathbf{M}}}\|\dot{\tilde{\mathbf{q}}}(t)\|_2^2 + \mu^{-1}k_{\overline{\mathbf{M}}}\|\tilde{\mathbf{q}}(t)\|_2\|\dot{\tilde{\mathbf{q}}}(t)\|_2. \quad (18)$$

Part 2: Let $\dot{V}$ be the set-valued derivative of V with respect to time, along the trajectories of the system (9). We obtain

$$\begin{aligned} \dot{V} \in & -\mu^{-1}\mathcal{K}\left[\frac{1}{2}\eta\tilde{\mathbf{q}}^\top\mathbf{X}\tilde{\mathbf{q}} + \frac{1}{2}\mu^2\eta\dot{\tilde{\mathbf{q}}}^\top\mathbf{X}\dot{\tilde{\mathbf{q}}} - \dot{\tilde{\mathbf{q}}}^\top\Gamma_p\mathbf{M}\dot{\tilde{\mathbf{q}}}\right. \\ & \left. - \dot{\tilde{\mathbf{q}}}^\top\Gamma_p\mathbf{C}^\top\tilde{\mathbf{q}} + \mathbf{x}^\top\Gamma_p\mathbf{C}(\mathbf{1}_n \otimes \dot{\mathbf{q}}_0) - \mathbf{x}^\top\Gamma_p\Delta\right. \\ & \left. - \beta\mathbf{x}^\top\Gamma_p\operatorname{sgn}(\mathbf{x} - \mathbf{y})\right] \end{aligned} \quad (19)$$

where $\Delta = \mathbf{g} + \boldsymbol{\zeta} + \mathbf{M}(\mathbf{1}_n \times \dot{\mathbf{q}}_0)$ and $\mathbf{x} = \tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}$ and $\mathbf{y} = \mathbf{e} + \mu\dot{\mathbf{e}}$ (recall that $\mathbf{s} = \tilde{\mathbf{q}} - \mathbf{e}$). We refer the reader to [28, Th. 2] for the detailed calculations to obtained (19).

Define $\dot{V}_A$ (absolutely continuous) and $\dot{V}_B$ (set valued) as

$$\begin{aligned} \dot{V}_A = & -\mu^{-1}(\varphi(\mu, \eta)\|\dot{\tilde{\mathbf{q}}}\|_2^2 + \frac{1}{2}\eta\lambda_{\min}(\mathbf{X})\|\tilde{\mathbf{q}}\|_2^2 \\ & - 2k_Ck_p\|\tilde{\mathbf{q}}\|_2\|\dot{\tilde{\mathbf{q}}}\|_2 - k_C\|\tilde{\mathbf{q}}\|_2\|\dot{\tilde{\mathbf{q}}}\|_2^2) \\ \triangleq & -\mu^{-1}g(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2) \end{aligned} \quad (20a)$$

$$\dot{V}_B \in \mu^{-1}\mathcal{K}[-\beta\mathbf{x}^\top\Gamma_p\operatorname{sgn}(\mathbf{x} - \mathbf{y}) + k_Ck_p^2\|\mathbf{x}\|_2 + \xi\|\mathbf{x}\|_2] \quad (20b)$$

where $\varphi(\mu, \eta) = \frac{1}{2}\mu^2\eta\lambda_{\min}(\mathbf{X}) - \mu k_Ck_p - k_{\overline{\mathbf{M}}}$. Straightforward calculations (see [28, Th. 2]) show that

$$\dot{V} \leq \dot{V}_A + \dot{V}_B. \quad (21)$$

Part 3: In *Part 3.1*, we study $\dot{V}_A$ and $\dot{V}_B$ separately to establish negative definiteness properties. Then, *Part 3.2* studies $\dot{V}_A + \dot{V}_B$ and proves a boundedness property.

Part 3.1: Consider the region of the state variables given by $\|\tilde{\mathbf{q}}(t)\|_2 \in [0, \mathcal{X}]$ and $\|\dot{\tilde{\mathbf{q}}}(t)\|_2 \in [0, \infty)$, where $\mathcal{X} > 0$ was computed in Section III-C. One can compute a $\mu_5^* \geq \mu_4^*$ and η_1^* such that $\varphi(\mu, \eta) > 0 \forall \mu \geq \mu_5^*, \eta \geq \eta_1^*$. Note that $\mathbf{L}_\mu > \mathbf{H}_\mu > \mathbf{N}_\mu > 0$ continues to hold. Observe that $g(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ in (20a) is of the same form as $g(x, y)$ in Lemma 3 with $x = \|\tilde{\mathbf{q}}\|_2$

and $y = \|\dot{\tilde{\mathbf{q}}}\|_2$. With $b = \varphi(\mu, \eta) > 0$, check if the inequality $\varphi(\mu_5^*, \eta_1^*) > \frac{(2k_Ck_p)^2}{2\eta_1^*\lambda_{\min}(\mathbf{X})} + k_C\mathcal{X}$ holds. If the inequality holds, then $\dot{V}_A$ in (20a) is negative definite in the region and proceed to *Part 3.2*. However, if the inequality does not hold, then there exists a $\mu_6^* \geq \mu_5^*$ and $\eta_2^* \geq \eta_1^*$ such that

$$\varphi(\mu_6^*, \eta_2^*) > \frac{(2k_Ck_p)^2}{2\eta_2^*\lambda_{\min}(\mathbf{X})} + k_C\mathcal{X}. \quad (22)$$

Recall from (10) and (15) that $\hat{V}^*$ is dependent on η , but independent of μ because $\mu_6^* \geq \mu_3^*$. One could leave $\eta_2^* = \eta_1^*$ and find a sufficiently large μ_6^* to satisfy (22). Alternatively, we could increase η . Notice that $\rho_2(\mu_3^*)$ and $\hat{V}^*$ are both of $\mathcal{O}(\eta)$. Thus, as η increases, $\mathcal{X}$ becomes independent of η , whereas $\varphi = \mathcal{O}(\eta)$. We conclude that there exists a sufficiently large η_2^* satisfying (22), and for which $\mathcal{X}_1, \mathcal{Y}_1, \mathcal{X}$, and $\mathcal{Y}$ need not be recomputed. With μ_6^* and η_2^* satisfying (22), $\dot{V}_A < 0$ in the aforementioned region of state variables.

Now consider $\dot{V}_B$ over two time intervals, $t_P = [0, T_1)$ and $t_Q = [T_1, T_2)$, where T_1 is given in Theorem 1 and T_2 is the infimum of those values of t for which one of the inequalities $\|\tilde{\mathbf{q}}(t)\|_2 < \mathcal{X}$, $\|\dot{\tilde{\mathbf{q}}}(t)\|_2 < \mathcal{Y}$ fails. In *Part 3.2*, we argue that without loss of generality, it is possible to take $T_2 > T_1$. In fact, we establish that the inequalities never fail; T_2 does not exist,³ and thus, $t_Q = [T_1, \infty)$.

Consider $t \in t_P$. Observe that the set-valued function $-\beta\mathbf{x}^\top\Gamma_p\operatorname{sgn}(\mathbf{x} - \mathbf{y})$ is upper bounded by the single-valued function $\beta\|\mathbf{x}\|_1$. Recalling $\dot{V}_B$ in (20b) yields

$$\dot{V}_B \leq (\sqrt{n}\beta + k_Ck_p^2 + \xi)(\mu^{-1}\|\tilde{\mathbf{q}}\|_2 + \|\dot{\tilde{\mathbf{q}}}\|_2) := \dot{V}_{\overline{B}} \quad (23)$$

because $\|\cdot\|_2 \leq \|\cdot\|_1 \leq \sqrt{n}\|\cdot\|_2$ [27]. Next, for $t \in t_Q$, Theorem 1 yields that $\mathbf{e}(t) = \dot{\mathbf{e}}(t) = \mathbf{0}$, which implies that $\mathbf{y} = \mathbf{0}$. Thus, the set-valued term $\mathcal{K}[\mathbf{x}^\top\Gamma_p\operatorname{sgn}(\mathbf{x} - \mathbf{y})]$ in (20b) becomes the singleton $\mathcal{K}[\mathbf{x}^\top\Gamma_p\operatorname{sgn}(\mathbf{x})] = \{\|\Gamma_p\mathbf{x}\|_1\}$ (since $\Gamma_p > 0$ is diagonal). It then follows that

$$\dot{V}_B = -\mu^{-1}\beta\|\Gamma_p\mathbf{x}\|_1 + k_Ck_p^2\|\mathbf{x}\|_2 + \xi\|\mathbf{x}\|_2. \quad (24)$$

In other words, $\dot{V}_B$ for $t \in t_Q$ is a continuous, single-valued function in the variables $\tilde{\mathbf{q}}$ and $\dot{\tilde{\mathbf{q}}}$. For $t \in t_Q$, we observe that $\dot{V}_B \leq -\mu^{-1}(\beta\gamma - k_Ck_p^2 - \xi)\|\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}\|_1 < 0$ if

$$\beta > (k_Ck_p^2 + \xi)/\gamma. \quad (25)$$

Part 3.2: To aid in this part of the proof, refer to Fig. 1. Consider first $\dot{V}$ for $t \in t_P$. Specifically, one can show that

$$\dot{V}_{t_P} \triangleq \dot{V}_A + \dot{V}_{\overline{B}} = -\mu^{-1}p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2) \quad (26)$$

where the function $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ is of the form of $h(x, y)$ in Corollary 1 with $x = \|\tilde{\mathbf{q}}\|_2$ and $y = \|\dot{\tilde{\mathbf{q}}}\|_2$. The coefficients of $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ are given by $a = \frac{1}{2}\eta\lambda_{\min}(\mathbf{X})$, $b = \varphi(\mu, \eta)$, $c = k_C$, $d = 2k_Ck_p$, $e = (\sqrt{n}\beta + k_Ck_p^2 + \xi)$, and $f = \mu e$. Note that $\dot{V} \leq \dot{V}_{t_P}$, i.e., $\dot{V}$ for $t \in t_P$ is a differential inclusion, which is upper bounded by a continuous function. Thus, for some given $\vartheta, \varepsilon, \mathcal{X}$, and $\mathcal{Y}$ satisfying the requirements detailed in Corollary 1, one can find μ and η such that $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ is

³Establishing that T_2 does not exist rules out the possibility of finite-time escape for system (6).

Fig. 1. Diagram for *Part 3* of the proof of Theorem 2. The red region is $\mathcal{S}$, in which $\dot{V}(t) < 0$ for all $t \geq 0$. The blue region is $\mathcal{T}$, in which $\dot{V}(t)$ is sign indefinite. A trajectory of (9) is shown with the black curve. We define $t = T_2$, if it exists, as the infimum of all t values for which one of the inequalities $\|\tilde{\mathbf{q}}(T_2)\| < \mathcal{X}$ or $\|\tilde{\mathbf{q}}(T_2)\| < \mathcal{Y}$ fails to hold, i.e., as the time at which the system (9) first leaves $\mathcal{S} \cup \mathcal{T}$. By contradiction, it is shown in *Part 3.2* that the trajectory of (9) satisfies $\|\tilde{\mathbf{q}}(T_2)\| < \mathcal{X}$, $\|\tilde{\mathbf{q}}(T_2)\| < \mathcal{Y}$. That is, T_2 does not exist and the trajectory remains in $\mathcal{T} \cup \mathcal{S} \forall t$. The sign indefiniteness of $\dot{V}$ in $\mathcal{T}$ arises due to terms linear in $\|\tilde{\mathbf{q}}\|$, $\|\tilde{\mathbf{q}}\|$ in (26). These terms disappear at $t = T_1$, when the finite-time observer converges. For all $t > T_1$, $\dot{V} < 0$ in $\mathcal{T} \cup \mathcal{S}$ as shown in *Part 4*. Exponential convergence to the origin follows.

positive definite in the region $\mathcal{R}$. Note that ϑ and ε can be selected by the designer. Choose $\vartheta > \mathcal{X} - \mathcal{X}_1$ and $\varepsilon > \mathcal{Y} - \mathcal{Y}_1$, and ensure that $\mathcal{X} - \vartheta, \mathcal{Y} - \varepsilon > 0$. Note the fact that $\mathcal{X} \geq \mathcal{X}_1$ and $\mathcal{Y} \geq \mathcal{Y}_1$ implies $\vartheta, \varepsilon > 0$.

Define the sets $\mathcal{U}$ and $\mathcal{V}$ and the region $\mathcal{R}$ as in Corollary 1 with $x = \|\tilde{\mathbf{q}}\|_2$ and $y = \|\dot{\tilde{\mathbf{q}}}\|_2$. Define further sets $\bar{\mathcal{U}} = \{\|\tilde{\mathbf{q}}\|_2 : \|\tilde{\mathbf{q}}\|_2 > \mathcal{X}\}$ and $\bar{\mathcal{V}} = \{\|\dot{\tilde{\mathbf{q}}}\|_2 : \|\dot{\tilde{\mathbf{q}}}\|_2 > \mathcal{Y}\}$. Define the compact region $\mathcal{S} = \mathcal{U} \cup \mathcal{V} \setminus \bar{\mathcal{U}} \cup \bar{\mathcal{V}}$, see Fig. 1 for a visualization of $\mathcal{S}$. Note $\mathcal{S} \subset \mathcal{R}$. Using Corollary 1, and with precise calculation details given in [28, Th. 2], one can find a pair of gains η and μ , which ensures that $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ is positive definite in $\mathcal{R}$. This implies $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ is positive definite in $\mathcal{S}$. It follows that $\dot{V}_{t_p}$ is negative definite in $\mathcal{S}$. Further define the region $\|\tilde{\mathbf{q}}(t)\|_2 \in [0, \mathcal{X} - \vartheta]$ and $\|\dot{\tilde{\mathbf{q}}}(t)\|_2 \in [0, \mathcal{Y} - \varepsilon]$ as $\mathcal{T}$, again with visualization in Fig 1.

Now, we justify the fact that we can assume $T_2 > T_1$. In fact, we show that the existence of T_2 creates a contradiction; the trajectories of (9) remain in $\mathcal{T} \cup \mathcal{S}$ for all time. See Fig. 1 for a visualization. Although $\dot{V}$ is sign indefinite in $\mathcal{T}$ (i.e., $V(t)$ can increase), notice from (18) that, in $\mathcal{T}$, there holds

$$V(t) \leq \frac{1}{2}\eta\lambda_{\max}(\mathbf{X})(\mathcal{X} - \vartheta)^2 + \frac{1}{2}k_{\bar{M}}(\mathcal{Y} - \varepsilon)^2 + \mu^{-1}k_{\bar{M}}(\mathcal{X} - \vartheta)(\mathcal{Y} - \varepsilon) := \mathcal{Z}. \quad (27)$$

It can then be shown that all trajectories of (9) beginning⁴ in $\mathcal{T} \cup \mathcal{S}$ satisfy $V(t) \leq \max\{\mathcal{Z}, V(0)\} < \hat{V}^*$ for all $t \leq T_2$ (see [28, Th. 2] for the arguments leading to this conclusion).

On the other hand, and from Lemma 2, there holds at T_2

$$\|\tilde{\mathbf{q}}(T_2)\|_2 \leq \sqrt{\frac{V(T_2)}{\chi}} < \sqrt{\frac{\hat{V}^*}{\chi}} < \sqrt{\frac{\hat{V}^*}{\rho_2(\mu_3^*)}} = \mathcal{X} \quad (28)$$

⁴It is evident from (14) that $\tilde{\mathbf{q}}(0), \dot{\tilde{\mathbf{q}}}(0) \in \mathcal{S} \cup \mathcal{T}$.

where $\chi = \lambda_{\min}(\frac{1}{2}\eta\mathbf{X} - \frac{1}{2}\mu^{-2}\Gamma_p\mathbf{M}) > \rho_2(\mu_3^*)$. One can also show that $\|\dot{\tilde{\mathbf{q}}}(T_2)\|_2 < \mathcal{Y}$ using an argument paralleling the argument leading to (28); we omit this due to spatial limitations. The existence of (28) and a similar inequality for $\|\dot{\tilde{\mathbf{q}}}(T_2)\|_2$ contradicts the definition of T_2 . In other words, T_2 does not exist and $\|\tilde{\mathbf{q}}(t)\|_2 < \mathcal{X}$, $\|\dot{\tilde{\mathbf{q}}}(t)\|_2 < \mathcal{Y}$ hold for all t .

Part 4: Observe that $\dot{V}_B$ changes at $t = T_1$ to become negative definite. Consider now $t \in t_Q = [T_1, T_2)$. Recalling that $\dot{V} \leq \dot{V}_A + \dot{V}_B$, we have

$$\begin{aligned} \dot{V} \leq & -\mu^{-1} \left[\varphi(\mu, \eta) \|\dot{\tilde{\mathbf{q}}}\|_2^2 + \frac{1}{2}\eta\lambda_{\min}(\mathbf{X}) \|\tilde{\mathbf{q}}\|_2^2 \right. \\ & - 2k_C k_p \|\tilde{\mathbf{q}}\|_2 \|\dot{\tilde{\mathbf{q}}}\|_2 - k_C \|\tilde{\mathbf{q}}\|_2 \|\dot{\tilde{\mathbf{q}}}\|_2^2 \\ & \left. - (\beta - k_C k_p^2 - \xi) \|\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}\|_1 \right] < 0 \end{aligned} \quad (29)$$

in the region $\mathcal{D} \triangleq \mathcal{S} \cup \mathcal{T}$. Since $\|\dot{\tilde{\mathbf{q}}}(T_1)\|_2 < \mathcal{Y}$, there holds $\dot{V}(T_1) < 0$. The argument applied to the interval $[0, \min\{T_1, T_2\}]$ previously, culminating in (28), is now applied to the interval t_Q to show that it remains true that T_2 does not exist, implying that the trajectory of (9) remains in $\mathcal{D}$ and $\dot{V} < 0$ for $t \in [T_1, \infty)$.

Recall, as previously mentioned below (17), that the eigenvalues of $\mathbf{H}_\mu$ are uniformly upper and lower bounded.

It follows that there holds $\lambda_{\min}(\mathbf{N}_\mu) \|\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top\|_2^2 \leq V \leq \lambda_{\max}(\mathbf{L}_\mu) \|\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top\|_2^2$. Because $\mathcal{D}$ is compact, one can find a scalar $a_3 > 0$ such that $\dot{V} \leq -a_3 \|\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top\|_2^2$. It follows that $\dot{V} \leq -[a_3/\lambda_{\max}(\mathbf{L}_\mu)]V$ in $\mathcal{D}$. This implies that V decays exponentially fast to zero, with a minimum rate $e^{-a_3/\lambda_{\max}(\mathbf{L}_\mu)t}$ [32]. It follows that $\lim_{t \rightarrow \infty} \tilde{\mathbf{q}}(t) = \mathbf{0}_n$ and $\lim_{t \rightarrow \infty} \dot{\tilde{\mathbf{q}}}(t) = \mathbf{0}_n$ exponentially and the leader tracking objective is achieved. ■

Remark 2 (Designing the gains): We summarize here the process to design μ, η , and β to satisfy inequalities detailed in the proof of Theorem 2. First, one may select β to satisfy (25). Then, μ should be set to satisfy (17). The quantities $\mathcal{X}$ and $\mathcal{Y}$ discussed in Section III-C are then computed with $\eta \gtrsim 1$; we noted below (22) that $\mathcal{X}$ and $\mathcal{Y}$ are independent of η as η increases. Having computed $\mathcal{X}$ and $\mathcal{Y}$, the last step is to adjust η to ensure $g(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ and $p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2)$ are positive definite (see Part 3.1 of Theorem 2 proof). Details of the inequalities to ensure positive definiteness are found in the proofs of Lemma 3 and Corollary 1 in [28, Sect. II-A].

Remark 3 (Additional degree of freedom): In [25], we assumed that $\eta = 1$. There is more flexibility in this paper since we allow $\eta > 1$; one can adjust separately, or simultaneously, μ and η . While the interplay between μ, η , and β , and its effect on performance, is difficult to quantify, we observe from extensive simulations that in general, one should make β and μ as small as possible. Where possible, one should hold μ constant and increase η to satisfy an inequality involving both, e.g., (22). Notice that $\lambda_{\max}(\mathbf{H}_\mu)$ and a_3 are both $\mathcal{O}(\eta)$. As μ increases $\lambda_{\max}(\mathbf{H}_\mu)$ does not increase but a_3 decreases. Thus, the convergence rate $a_3/\lambda_{\max}(\mathbf{L}_\mu)$ is not negatively affected by increasing η but is reduced by increasing μ . If only μ is increased (as in

[25]), then velocity consensus is quickly achieved but position consensus is achieved only after a long time.

Remark 4 (Robustness): The proposed algorithm (8) is robust in several aspects. First, the exponential stability property implies that small amounts of noise produce small departures from the ideal. Moreover, the sigum term in the controller offers robustness to the unknown disturbance $\zeta_i(t)$. In contrast, and as discussed in the introduction, adaptive controllers are not robust to unmodeled agent dynamics.

Remark 5 (Controller structure): Consider the controller (8). The term containing the sigum function ensures exact tracking of the leader's trajectory. Consider Fig. 1. For $t < T_1$, the sigum term results in the region $\mathcal{T}$, where $\dot{V}$ is sign indefinite. This sigum term can in fact drive an agent away from its neighbors due to the nonzero error term $e_i(t)$, $t < T_1$. However, for $t < T_1$, the linear terms of the controller (and in particular, adjustment of the gains η and μ) ensure that $\dot{V} < 0$ in $\mathcal{S}$. This ensures that the followers remain in the bounded region $\mathcal{S}$ centered on the leader. Such a controller gives added robustness. For example, if $\mathcal{G}_B$ becomes temporarily disconnected, all agents will remain close to the leader so long as $\mathcal{G}_A$ has a directed spanning tree. When connectivity of $\mathcal{G}_B$ is restored, perfect tracking follows. This is illustrated in the simulation later.

E. Practical Tracking by Approximating the Sigum Function

Although the sigum function term in (8) allows the leader-tracking objective to be achieved, it carries an offsetting disadvantage. Use of the sigum function can cause mechanical components to fatigue due to the rapid switching of the control input. Moreover, chattering often results, which can excite the natural frequencies of high-order unmodeled dynamics. A modified controller is now proposed using a continuous approximation of the sigum function and we derive an explicit upper bound on the error in tracking of the leader.⁵

Consider the following continuous, model-independent algorithm for the i th agent, replacing (8)

$$\begin{aligned} \tau_i = & -\eta \sum_{j \in \mathcal{N}_{A_i}} a_{ij} ((q_i - q_j) + \mu(\dot{q}_i - \dot{q}_j)) \\ & - \beta z_i((q_i - \hat{r}_i) + \mu(\dot{q}_i - \hat{v}_i)) \end{aligned} \quad (30)$$

where $z_i(x) \triangleq x/(\|x\|_2 + \epsilon)$ with $\epsilon > 0$ being the degree of approximation. The function $z_i(x)$ approximates $\text{sgn}(x)$ via the boundary layer concept [33]. The networked system is

$$\begin{aligned} M\ddot{\mathbf{q}} + C\dot{\mathbf{q}} + \eta(\mathcal{L}_{22} \otimes \mathbf{I}_p)(\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}) + \mathbf{g} + \zeta \\ + \beta z(\mathbf{s} + \mu\dot{\mathbf{s}}) + \mathbf{M}(\mathbf{1}_n \otimes \ddot{\mathbf{q}}_0) + \mathbf{C}(\mathbf{1}_n \otimes \dot{\mathbf{q}}_0) = \mathbf{0} \end{aligned} \quad (31)$$

where $z(\mathbf{s} + \mu\dot{\mathbf{s}}) = [z_1(\mathbf{s}_1 + \mu\dot{\mathbf{s}}_1)^\top, \dots, z_n(\mathbf{s}_n + \mu\dot{\mathbf{s}}_n)^\top]^\top$. Note that $\|z_i(x_i)\|_2 < 1$ for any $\epsilon > 0$. The computation of $\mathcal{X}$ and $\mathcal{Y}$ in Section III-C is unchanged. Because of similarity, we provide only a proof sketch. Consider the same

⁵We do not consider an approximation for (7) because the observer involves computing state estimates, as opposed to the physical control input for (6).

Lyapunov-like function in (16), with μ sufficiently large to ensure $\mathbf{H}_\mu > 0$.

Let t_p and t_Q be defined as in Part 3.1 of the proof of Theorem 2. For $t \in t_p$, the derivative of (16) with respect to time along the trajectories of (31), can be shown to obey

$$\dot{V} \leq -\mu^{-1}p(\|\tilde{\mathbf{q}}\|_2, \|\dot{\tilde{\mathbf{q}}}\|_2) \quad (32)$$

with $p(\cdot, \cdot)$ defined in (26) (we refer the reader to [28, Sec. III-E] for detailed calculations on $\dot{V}$ and its bound). Thus, any μ and η , which ensures that the system (9) remains in $\mathcal{S} \cup \mathcal{T}$, will also ensure that the system (31) remains in $\mathcal{S} \cup \mathcal{T}$ for all time.

Consider now $t \in t_Q$. Using calculations that we detail in [28, Sec. III-E], one can show that

$$\begin{aligned} \dot{V} \leq & -\mu^{-1} \left[\varphi(\mu, \eta) \|\dot{\tilde{\mathbf{q}}}\|_2^2 + \frac{1}{2} \eta \lambda_{\min}(\mathbf{X}) \|\tilde{\mathbf{q}}\|_2^2 \right. \\ & - 2k_C k_p \|\tilde{\mathbf{q}}\|_2 \|\dot{\tilde{\mathbf{q}}}\|_2 - k_C \|\tilde{\mathbf{q}}\|_2 \|\dot{\tilde{\mathbf{q}}}\|_2^2 \\ & \left. - \sum_{i=1}^n (k_C k_p^2 + \xi) \|\mathbf{x}_i\|_2 + \beta \gamma \sum_{i=1}^n \frac{\|\mathbf{x}_i\|_2^2}{\|\mathbf{x}_i\|_2 + \epsilon} \right]. \end{aligned} \quad (33)$$

If β satisfies (25), then there holds

$$\beta \gamma \sum_{i=1}^n \frac{\|\mathbf{x}_i\|_2^2}{\|\mathbf{x}_i\|_2 + \epsilon} - \sum_{i=1}^n (k_C k_p^2 + \xi) \|\mathbf{x}_i\|_2 > -\beta \gamma n \epsilon \quad (34)$$

because $\|\mathbf{x}_i\|_2/(\|\mathbf{x}_i\|_2 + \epsilon) < 1$ for all $\epsilon > 0$ (we refer the reader to [28, Sec. III-E] for detailed calculations used to obtain this inequality Eq. (34)). From this, we conclude that $\dot{V} \leq \dot{V}_A + \beta \gamma n \epsilon$. Recall that any μ and η for which $p(\cdot, \cdot)$ is positive definite in $\mathcal{S}$ also ensures that $\dot{V}_A$ is negative definite in $\mathcal{D}$. Similar to Part 4 of the proof of Theorem 2, one has $\dot{V} \leq \psi V + \beta \gamma n \epsilon$, for some $\psi > 0$. We conclude using [32, Lemma 3.4 (Comparison Lemma)] that $V(t) \leq V(0)e^{-\psi t} + \beta \gamma n \epsilon \int_0^t e^{-\psi(t-\tau)} d\tau$, which is equivalent to $V(t) \leq e^{-\psi t} [V(0) + \beta \gamma n \epsilon / \psi] + \beta \gamma n \epsilon / \psi$. This implies that $V(t)$ goes to the set $\{[\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top]^\top : V \leq \beta \gamma n \epsilon / \psi\}$ exponentially fast. Since $V \geq \lambda_{\min}(\mathbf{N}_\mu) \|\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top\|_2^2$, the trajectories of (31) converge to the bounded set

$$\Omega = \left\{ [\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top]^\top : \|\tilde{\mathbf{q}}^\top, \dot{\tilde{\mathbf{q}}}^\top\|_2 \leq \left(\frac{\beta \gamma n \epsilon}{\psi \lambda_{\min}(\mathbf{N}_\mu)} \right)^{\frac{1}{2}} \right\}. \quad (35)$$

IV. LEADER TRACKING ON DYNAMIC NETWORKS

In this section, we consider the case where the sensing graph $\mathcal{G}_A(t)$ is dynamic, i.e., time varying. We assume that there is a finite set $\mathcal{J}$ of m possible network topologies, given as $\bar{\mathcal{G}}_A = \{\mathcal{G}_{A,j} = (\mathcal{V}, \mathcal{E}_j, \mathcal{A}_j) : j \in \mathcal{J}\}$, where $\mathcal{J} = \{1, \dots, m\}$ is the index set. We assume further that $\mathcal{G}_{A,j} \forall j$, contains a directed spanning tree, with v_0 as the root node and with no edges incoming to v_0 . Define $\sigma(t) : [0, \infty) \mapsto \mathcal{J}$ as the piecewise constant switching signal that determines the switching of $\mathcal{G}_A(t)$, with a finite number of switches. The switching times are indexed as $t_1, t_2, \dots$ and we assume that $\sigma(t)$ is such that $t_{i+1} - t_i > \pi_d > 0$ for all i , where π_d is the dwell time.

The dynamic network is modeled by the graph $\mathcal{G}_A(t) = \mathcal{G}_{A,\sigma(t)}$, which in turn implies that the Laplacian associated

TABLE I
AGENT PARAMETERS USED IN SIMULATION

	m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2	$q_i^{(1)}(0)$	$q_i^{(2)}(0)$	$\dot{q}_i^{(1)}(0)$	$\dot{q}_i^{(2)}(0)$
Agent 1	0.5	0.4	0.4	0.3	0.2	0.15	0.1	0.05	0.1	0.9	-0.5	-0.6
Agent 2	0.2	0.4	0.6	0.1	0.35	0.08	0.15	0.08	-0.4	0.9	0.1	-1.4
Agent 3	0.5	0.4	0.4	0.3	0.2	0.15	0.1	0.05	0.9	-1.2	0.3	0.6
Agent 4	1	0.6	0.45	0.8	0.2	0.4	0.15	0.5	-2.0	-2.0	-1.0	0.2
Agent 5	0.25	0.4	0.8	0.5	0.3	0.1	0.45	0.15	0.3	1.5	1.0	1.2

with $\mathcal{G}_A(t)$ is dynamic, given by $\mathcal{L}_A(t) = \mathcal{L}_{A,\sigma(t)}$. Denote $\mathcal{L}_{22}(t) = \mathcal{L}_{22,\sigma(t)}$ as the lower block matrix of $\mathcal{L}_A(t)$, partitioned as in (5). It is straightforward to show that the follower network dynamics is given by

$$\ddot{\mathbf{q}} \in^{a.e.} \mathcal{K} \left[-\mathbf{M}^{-1} [C\dot{\mathbf{q}} + \eta(\mathcal{L}_{22,\sigma(t)} \otimes \mathbf{I}_p)(\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}) + \mathbf{g} + \boldsymbol{\zeta} + \beta \operatorname{sgn}(\mathbf{s} + \mu\dot{\mathbf{s}}) + \mathbf{M}(\mathbf{1}_n \otimes \ddot{\mathbf{q}}_0) + \mathbf{C}(\mathbf{1}_n \otimes \dot{\mathbf{q}}_0)] \right]. \quad (36)$$

We now seek to exploit an established result that states that a switched system is exponentially stable if the switching is sufficiently slow [34], and its “frozen” versions of the various systems arising between switching instants are all exponentially stable. Specifically, the following result holds.

Theorem 3 ([34, Th. 3.2]): Consider the family of systems $\dot{\mathbf{x}} = \mathbf{f}_j(\mathbf{x}), j \in \mathcal{J}$. Suppose that, in a domain $D \subseteq \mathbb{R}^n$ containing $\mathbf{x} = \mathbf{0}$, $\exists \mathcal{C}^1$ functions $V_j : D \mapsto \mathbb{R}, j \in \mathcal{J}$, and positive constants c_j, d_j, Λ_j such that

$$c_j \|\mathbf{x}\|_2^2 \leq V_j(\mathbf{x}) \leq d_j \|\mathbf{x}\|_2^2 \quad \forall \mathbf{x} \in D \quad \forall j \in \mathcal{J} \quad (37)$$

and $\dot{V}_j(\mathbf{x}) \leq -\Lambda_j V_j(\mathbf{x}) \quad \forall \mathbf{x} \in D \quad \forall j \in \mathcal{J}$. Define $\kappa \triangleq \sup_{p,q \in \mathcal{J}} \{V_p(\mathbf{x})/V_q(\mathbf{x}) : \mathbf{x} \in D\}$, and suppose further that $0 < \kappa < 1$. Then, for $\mathbf{x}(0) \in D, \mathbf{x} = \mathbf{0}$ of the switched system $\dot{\mathbf{x}} = \mathbf{f}_{\sigma(t)}(\mathbf{x})$ is exponentially stable for every switching signal $\sigma(t)$ with dwell time $\pi_d > \log(\kappa)/\Lambda$, where $\Lambda = \min_{j \in \mathcal{J}} \Lambda_j$.

Under Assumptions 1 and 2, we know from the previous Theorem 2 that for each j th subsystem

$$\ddot{\mathbf{q}} \in^{a.e.} \mathcal{K} \left[-\mathbf{M}^{-1} [C\dot{\mathbf{q}} + \eta(\mathcal{L}_{22,j} \otimes \mathbf{I}_p)(\tilde{\mathbf{q}} + \mu\dot{\tilde{\mathbf{q}}}) + \mathbf{g} + \boldsymbol{\zeta} + \beta \operatorname{sgn}(\mathbf{s} + \mu\dot{\mathbf{s}}) + \mathbf{M}(\mathbf{1}_n \otimes \ddot{\mathbf{q}}_0) + \mathbf{C}(\mathbf{1}_n \otimes \dot{\mathbf{q}}_0)] \right] \quad (38)$$

there exist control gains μ_j, η_j , and β_j , which exponentially achieve the leader tracking objective. In seeking to apply Theorem 3 to the system (36), we obtain, for each $j \in \mathcal{J}$ with V_j given in (16), the values $\lambda_{\min}(\mathbf{N}_{\mu,j}) = c_j$, $\lambda_{\max}(\mathbf{L}_{\mu,j}) = d_j$ and $\Lambda_j = a_{3,j}/\lambda_{\max}(\mathbf{L}_{\mu,j})$, where $a_{3,j}$ was computed after (29). It follows that $\Lambda = \min_{j \in \mathcal{J}} a_{3,j}/\lambda_{\max}(\mathbf{L}_{\mu,j})$, and one can also obtain that $\kappa = \max_{j \in \mathcal{J}} \lambda_{\max}(\mathbf{L}_{\mu,j})/\min_{j \in \mathcal{J}} \lambda_{\min}(\mathbf{N}_{\mu,j})$.

Theorem 4: Under Assumptions 1 and 2, with dynamic topology given by $\mathcal{G}_A(t) = \mathcal{G}_{A,\sigma(t)}$, the leader tracking objective is achieved using (8) if: 1) the control gains μ, η , and β satisfy a set of lower bounding inequalities, and 2) the dwell time π_d satisfies the inequality $\pi_d > \log(\kappa)/\Lambda$, where κ and Λ are as defined in the immediately preceding paragraph.

Proof: By selecting $\mu = \max_{j \in \mathcal{J}} \mu_j$, $\eta = \max_{j \in \mathcal{J}} \eta_j$, and $\beta = \max_{j \in \mathcal{J}} \beta_j$, we guarantee each j th subsystem (38) is expo-

Fig. 2. In the simulation, graph $\mathcal{G}_A(t)$ switches between the aforementioned three graphs periodically at a rate of 1 Hz.

Fig. 3. Graph $\mathcal{G}_B(t)$ switches between the aforementioned graphs periodically at a rate of 1 Hz; if $\mathcal{G}_A(t) = \mathcal{G}_{A,i}$, then $\mathcal{G}_B(t) = \mathcal{G}_{B,i}$, for $i = 1, 2$, and 3.

ponentially stable, and also guarantee the boundedness of the trajectories of (36) before the finite-time observer has converged. After convergence of the finite-time observer, application of Theorem 3 using the quantities of κ and Λ outlined previously delivers the conclusion that (36) is exponentially stable, i.e., the leader tracking objective is achieved. ■

V. SIMULATIONS

A simulation is now provided to demonstrate the algorithm (8). Each agent is a two-link robotic arm and there are five follower agents. The equations of motion are given in [26, pp. 259–262]. The generalized coordinates for agent i are $\mathbf{q}_i = [q_i^{(1)}, q_i^{(2)}]^\top$, which are the angles of each link in radians. The agent parameters are given in Table I, and are chosen arbitrarily. Several aspects of the simulation are designed to highlight the robustness of the algorithm. First, the topology is assumed to be switching, with the graph $\mathcal{G}_A(t)$ switching periodically between the three graphs indicated in Fig. 2, at a frequency of 1 Hz. Graph $\mathcal{G}_B(t)$ switches between the three graphs indicated in Fig. 3, also at a frequency of 1 Hz. Moreover, if $\mathcal{G}_A(t) = \mathcal{G}_{A,i}$, then $\mathcal{G}_B(t) = \mathcal{G}_{B,i}$, for $i = 1, 2$, and 3. Additionally, $\mathcal{G}_B(t)$ is entirely disconnected for $t \in [10, 20)$ of the simulation. Last, each agent has a disturbance $\boldsymbol{\zeta}_i(t) = [\sin(i \times 0.1t), \cos(i \times 0.1t)]^\top$, for $i = 1, \dots, 5$. All edges of $\mathcal{G}_A(t)$ and $\mathcal{G}_B(t)$ have edge weights of 5. The control gains are set as $\mu = 1.5$, $\eta = 16$, and $\beta = 25$; they are first computed using the inequalities, and then, adjusted be-

Fig. 4. Plot of generalized coordinates versus time.

Fig. 5. Plot of generalized velocities versus time.

cause the inequalities can lead to conservative gain choices. For the observer, set $\omega_1 = 1$, $\omega_2 = 5$. The leader trajectory is

$$\mathbf{q}_0(t) = \begin{bmatrix} 0.5 \sin(t) - 0.2 \sin(0.5t) \\ 0.4 \left(2 \sin(t) + \frac{\sin(2t)}{(2)} + \frac{\sin(3t)}{(3)} + \frac{\sin(4t)}{(4)} \right) \end{bmatrix}.$$

Fig. 4 shows the generalized coordinates $q^{(1)}$ and $q^{(2)}$. The generalized velocities, $\dot{q}^{(1)}$ and $\dot{q}^{(2)}$, are shown in Fig. 5. The well-studied observer results are omitted. Consider Fig. 4. Clearly, $q_i(t)$ has almost tracked the leader by $t = 10$, but the distributed observer graph $\mathcal{G}_B(t)$ disconnects for $t \in [10, 20)$. As discussed in Remark 5, the controller (8) has robustness to network failure, since the linear term in (8) ensures the trajectories remain bounded as long as $\mathcal{G}_B(t)$ is disconnected (thus, followers do not possess accurate knowledge of $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$). In the simulation, we observe leader tracking is achieved once $\mathcal{G}_B(t)$ reconnects at $t > 20$. In [28], additional simulations show the effect of increasing the gain μ (see Remark 3), and the continuous approximation (30).

VI. CONCLUSION

In this paper, a distributed, discontinuous model-independent algorithm was proposed for a directed network of Euler-Lagrange agents. It was shown that the leader tracking objective is achieved semiglobally exponentially fast if the directed graph contains a directed spanning tree, rooted at the leader, and if

three control gains satisfied a set of lower bounding inequalities. The algorithm was shown to be robust to agent disturbances, unmodeled agent dynamics, and modeling uncertainties. A continuous approximation of the algorithm was proposed to avoid chattering, and we then extended the result to include switching topologies. A numerical simulation illustrated the algorithm's effectiveness.

REFERENCES

- [1] W. Ren and Y. Cao, *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models and Issues*. London, U.K.: Springer, 2011.
- [2] R. Ortega, J. A. L. Perez, P. J. Nicklasson, and H. Sira-Ramirez, *Passivity-Based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications*. London, U.K.: Springer, 2013.
- [3] S.-J. Chung and J.-J. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Trans. Robot.*, vol. 25, no. 3, pp. 686–700, Jun. 2009.
- [4] Q. Hu, B. Xiao, and P. Shi, "Tracking control of uncertain Euler-Lagrange systems with finite-time convergence," *Int. J. Robust Nonlinear Control*, vol. 25, no. 17, pp. 3299–3315, 2015.
- [5] Q. Yang, H. Fang, J. Chen, Z. Jiang, and M. Cao, "Distributed global output-feedback control for a class of Euler-Lagrange systems," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4855–4861, Sep. 2017.
- [6] Z. Meng, Z. Lin, and W. Ren, "Leader-follower swarm tracking for networked Lagrange systems," *Syst. Control Lett.*, vol. 61, no. 1, pp. 117–126, 2012.
- [7] J. Mei, W. Ren, and G. Ma, "Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph," *Automatica*, vol. 48, no. 4, pp. 653–659, 2012.
- [8] A. Abdessameud, I. G. Polushin, and A. Tayebi, "Synchronization of Lagrangian systems with irregular communication delays," *IEEE Trans. Autom. Control*, vol. 59, no. 1, pp. 187–193, Jan. 2014.
- [9] G. Chen and F. L. Lewis, "Distributed adaptive tracking control for synchronization of unknown networked Lagrangian systems," *IEEE Trans. Syst., Man, Cybern., B, Cybern.*, vol. 41, no. 3, pp. 805–816, Jun. 2011.
- [10] E. Nuno, R. Ortega, L. Basanez, and D. Hill, "Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 935–941, Apr. 2011.
- [11] Z. Meng, D. V. Dimarogonas, and K. H. Johansson, "Leader-follower coordinated tracking of multiple heterogeneous Lagrange systems using continuous control," *IEEE Trans. Robot.*, vol. 30, no. 3, pp. 739–745, Jun. 2014.
- [12] S. Ghabani, J. Mei, W. Ren, and Y. Song, "Fully distributed flocking with a moving leader for Lagrange networks with parametric uncertainties," *Automatica*, vol. 67, pp. 67–76, 2016.
- [13] A. Abdessameud, A. Tayebi, and I. Polushin, "Leader-follower synchronization of Euler-Lagrange systems with time-varying leader trajectory and constrained discrete-time communication," *IEEE Trans. Autom. Control*, vol. 62, no. 5, pp. 2539–2545, May 2017.
- [14] W. Ren, "Distributed leaderless consensus algorithms for networked Euler-Lagrange systems," *Int. J. Control*, vol. 82, no. 11, pp. 2137–2149, 2009.
- [15] E. Nuno, I. Sarras, and L. Basanez, "Consensus in networks of nonidentical Euler-Lagrange systems using P+d controllers," *IEEE Trans. Robot.*, vol. 29, no. 6, pp. 1503–1508, Dec. 2013.
- [16] Z. Meng, T. Yang, G. Shi, D. V. Dimarogonas, Y. Hong, and K. H. Johansson, "Set target aggregation of multiple mechanical systems," in *Proc. IEEE 53rd Annu. Conf. Decis. Control*, 2014, pp. 6830–6835.
- [17] J. Mei, W. Ren, and G. Ma, "Distributed coordinated tracking with a dynamic leader for multiple Euler-Lagrange systems," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1415–1421, Jun. 2011.
- [18] Y. Zhao, Z. Duan, and G. Wen, "Distributed finite-time tracking of multiple Euler-Lagrange systems without velocity measurements," *Int. J. Robust Nonlinear Control*, vol. 25, no. 11, pp. 1688–1703, 2015.
- [19] J. R. Klotz, Z. Kan, J. M. Shea, E. L. Pasilião, and W. E. Dixon, "Asymptotic synchronization of a leader-follower network of uncertain Euler-Lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 2, pp. 174–182, Jun. 2015.
- [20] P. F. Hokayem, D. M. Stipanović, and M. W. Spong, "Coordination and collision avoidance for Lagrangian systems with disturbances," *Appl. Math. Comput.*, vol. 217, no. 3, pp. 1085–1094, 2010.

- [21] N. Chopra, "Output synchronization on strongly connected graphs," *IEEE Trans. Autom. Control*, vol. 57, no. 11, pp. 2896–2901, Nov. 2012.
- [22] M. Ye, C. Yu, and B. D. O. Anderson, "Model-independent rendezvous of Euler-Lagrange agents on directed networks," in *Proc. IEEE 54th Annu. Conf. Decis. Control*, Osaka, Japan, 2015, pp. 3499–3505.
- [23] M. Ye, B. D. O. Anderson, and C. Yu, "Distributed model-independent consensus of Euler-Lagrange agents on directed networks," *Int. J. Robust Nonlinear Control*, vol. 27, no. 14, pp. 2428–2450, Sep. 2017.
- [24] Z. Feng, G. Hu, W. Ren, W. E. Dixon, and J. Mei, "Distributed coordination of multiple unknown Euler-Lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 55–66, Mar. 2018.
- [25] M. Ye, B. D. O. Anderson, and C. Yu, "Model-independent trajectory tracking of Euler-Lagrange agents on directed networks," in *Proc. IEEE 55th Annu. Conf. Decis. Control*, Las Vegas, NV, USA, 2016, pp. 6921–6927.
- [26] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, vol. 3. Hoboken, NJ, USA: Wiley, 2006.
- [27] R. A. Horn and C. R. Johnson, *Matrix Analysis*. New York, NY, USA: Cambridge Univ. Press, 2012.
- [28] M. Ye, B. D. O. Anderson, and C. Yu, "Leader tracking of Euler-Lagrange agents on directed switching networks using a model-independent algorithm," arXiv:1802.00906 [cs.SY], 2018. [Online]. Available: <https://arxiv.org/abs/1802.00906>
- [29] H. Zhang, Z. Li, Z. Qu, and F. L. Lewis, "On constructing Lyapunov functions for multi-agent systems," *Automatica*, vol. 58, pp. 39–42, 2015.
- [30] Y. Cao, W. Ren, and Z. Meng, "Decentralized finite-time sliding mode estimators and their applications in decentralized finite-time formation tracking," *Syst. Control Lett.*, vol. 59, no. 9, pp. 522–529, 2010.
- [31] J. Cortes, "Discontinuous dynamical systems," *IEEE Control Syst.*, vol. 28, no. 3, pp. 36–73, Jun. 2008.
- [32] H. Khalil, *Nonlinear Systems*. Englewood Cliffs, NJ, USA: Prentice-Hall, 2002.
- [33] C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications* (Series in Systems and Control). Boca Raton, FL, USA: CRC, 1998.
- [34] D. Liberzon, *Switching in Systems and Control*. New York, NY, USA: Springer, 2012.

Mengbin Ye (M'17) was born in Guangzhou, China. He received the B.E. degree (with First Class Hons.) in mechanical engineering from the University of Auckland, Auckland, New Zealand, in 2013, and the Ph.D. degree in engineering from the Australian National University, Canberra, Australia, in 2018.

He is currently a postdoctoral fellow with the Faculty of Science and Engineering, ENTEG, University of Groningen, The Netherlands. His current research interests include opinion dynamics and social networks, consensus and synchronization of Euler-Lagrange systems, and localization using bearing measurements.

Brian D. O. Anderson (M'66–SM'74–F'75–LF'07) was born in Sydney, Australia. He received the B.Sc. degree in pure mathematics and the B.E. degree in electrical engineering from the Sydney University, Sydney, Australia, in 1962 and 1964, respectively, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, USA, in 1966.

He is an Emeritus Professor with the Australian National University, Canberra, Australia (having retired as a Distinguished Professor in 2016), a Distinguished Professor with Hangzhou Dianzi University, Hangzhou, China, and a Distinguished Researcher with National ICT Australia, Canberra. His current research interests include distributed control, sensor networks, and econometric modeling.

Dr. Anderson was the recipient of the IEEE Control Systems Award of 1997, the 2001 IEEE James H. Mulligan, Jr. Education Medal, and the Bode Prize of the IEEE Control System Society in 1992, as well as several IEEE and other best paper prizes. He is a Fellow of the Australian Academy of Science, the Australian Academy of Technological Sciences and Engineering, and the Royal Society, and a Foreign Member of the US National Academy of Engineering. He holds honorary doctorates from a number of universities, including Universit Catholique de Louvain, Belgium, and ETH, Zrich. He is a past President of the International Federation of Automatic Control and the Australian Academy of Science.

Changbin Yu (M'08–SM'10) was born in Shandong, China. He received the B.Eng. degree with first class honors in computer engineering from Nanyang Technological University, Singapore, in 2004, and the Ph.D. degree in engineering from the Australian National University (ANU), Canberra, Australia, in 2008.

Since 2008, he has been a faculty member with the Research School of Engineering, ANU. In 2017, he became the first engineering faculty member to join the Westlake Institute for Advanced Study, and helped establish the first private research-oriented university in China—Westlake University, Hangzhou, China.

Dr. Yu was a recipient of the 2010 ANU Top Supervisor Award, the 2006–2008 Asian Journal of Control Best Paper Award, the ARC Australian Postdoctoral Fellowship in 2007, the Queen Elizabeth II Fellowship in 2010, the 2006 Chinese Government Award for Outstanding Students Abroad, and the 2005 Australian Government's Endeavor Asia Award. He is a Fellow of the Institution of Engineers Australia and a Member of IFAC Technical Committee on Networked Systems. He served as a Subject Editor for the *International Journal of Robust and Nonlinear Control* and was an Associate Editor for the *System & Control Letters* and *IET Control Theory & Applications*.