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Robust trajectory tracking for incrementally passive nonlinear systems[☆]

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ABSTRACT

In this paper, we study the robust trajectory tracking problem for a class of nonlinear systems with incremental passivity. The velocity of the desired trajectory and parts of the model information are unknown apart from boundedness assumptions. A velocity observer based method and a sliding mode controller are proposed while the asymptotic tracking result is guaranteed by a zero-state detectability condition for both cases. Unlike previous results, the studied systems are not necessarily feedback linearizable nor in a strict feedback form. The ball and beam system is utilized to illustrate the implementation of the proposed tracking control laws.

1. Introduction

Trajectory tracking has been extensively studied for nonlinear systems of particular forms, such as chains of n-order integrators (corresponds to the normal form of the feedback linearizable nonlinear systems) (Chen, Behal, & Dawson, 2008; Xian, Dawson, de Queiroz, & Chen, 2004), and the strict feedback systems (Brogliato, Ortega, & Lozano, 1995; Lozano & Brogliato, 1992). Recently, trajectory tracking for nonlinear systems is addressed by using the concept of incremental stability (Reyes-Báez, van der Schaft, & Jayawardhana, 2016; Yaghmaei & Yazdanpanah, 2017). Incremental stability of nonlinear systems is first studied in Demidovich (1961) where a condition is given to characterize a class of nonlinear system known as *convergent systems*. In Pavlov and Marconi (2008), the Demidovich condition is weakened to specify the property of incremental passivity (van der Schaft, 2017, Def. 4.7.1).

The aim of the present work is to study the trajectory tracking problem for nonlinear systems by exploiting the property of incremental passivity. The nonlinear systems studied in this paper are assumed to be incrementally passive in the sense of Pavlov and Marconi (2008). Furthermore, we consider a robust tracking problem where the velocity of the desired trajectory as well as part of the model information is unknown. Two state feedback tracking controllers (a velocity observer based method and a sliding mode controller) are proposed resulting in asymptotic tracking. As a major difference compared to the existing works, the studied system is underactuated and is not required to be feedback linearizable.

Notation: For $x \in \mathbb{R}^n$, define a vector function

$$\mathrm{Sgn}(x) \triangleq [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^T$$

where x_i denotes the ith element of x, and $\mathrm{sgn}(x_i) = 1$ if $x_i > 0$, $[-1, 1]$ if $x_i = 0$, -1 if $x_i < 0$.

2. Problem formulation

Consider the following nonlinear system

$$\dot{x} = f(x, t) + Gu \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input with $n \geq m$. The mapping $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is uniformly continuous in t and locally Lipschitz in $x(t)$, and $G \in \mathbb{R}^{n\times m}$ is a constant matrix of full rank. Furthermore, $f(x, t)$ satisfies the following Assumption 1 (Pavlov & Marconi, 2008).

Assumption 1. There exists a constant positive definite symmetric matrix $P \in \mathbb{R}^{n\times n}$, such that the following condition holds for all $x \in \mathbb{R}^n$ and all $t \geq 0$

$$P\frac{\partial f}{\partial x}(x, t) + \frac{\partial^T f}{\partial x}(x, t)P \leq 0. \tag{2}$$

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As noted in Pavlov and Marconi (2008), Assumption 1 implies that the system (1) with the output $y = G^T Px$ is incrementally passive.

The control objective is to let the system (1) asymptotically track a desired trajectory $x_d(t) \in \mathbb{R}^n$. Here, we assume that $x_d(t)$ is a solution of (1), that is, there exists a $u_d(t) \in \mathbb{R}^m$ that satisfies

$$\dot{x}_d = f(x_d, t) + Gu_d(t). \tag{3}$$

In addition, it is assumed that $x(t)$, $x_d(t)$, G, P are known and $u_d(t)$ is unknown. Note from (3) that the unavailability of $u_d(t)$ is equivalent to unavailability of $\dot{x}_d(t)$, which is a scenario often seen in robotic applications (Nakaoka, Nakazawa, Yokoi, Hirukawa, & Ikeuchi, 2003; Pao & Speeter, 1989).

Remark 1. To simplify the presentation, the condition (2) is assumed to hold globally. If it is only satisfied for all $x \in \mathcal{D} \subset \mathbb{R}^n$ and $t \geq 0$ with $\mathcal{D}$ denoting a compact domain, then a locally asymptotic tracking result can be achieved with the subsequently proposed methods provided that $\mathcal{D}$ is positively invariant and $x_d(t)$ is contained in $\mathcal{D}$.

Remark 2. The condition (3) is necessary for an asymptotic tracking result, and it can be viewed as a target system which generates the desired trajectories. Compared to the conditions used in some of the related classic problems such as output tracking (Khalil, 2002) and output regulation (Isidori, 2013), (3) is rather strong since it requires the existence of $u_d(t)$. For certain practical applications, such a desired trajectory satisfying (3) is straightforward to obtain. For example, in Pao and Speeter (1989), the desired trajectory for a multi-figured robotic hand is generated by the human hand because they have the similar kinematic structure and constraints. Furthermore, if the controlled system is strictly incrementally passive, i.e., $P\frac{\partial f}{\partial x}(x, t) + \frac{\partial^T f}{\partial x}(x, t)P < -Q$ for a constant positive definite matrix Q, then a bounded tracking result can still be achieved without requiring the condition (3).

To facilitate the argument, we define the tracking error as

$$z(t) \triangleq x(t) - x_d(t). \tag{4}$$

The dynamics of $z(t)$, i.e., the error system, is given in the following theorem, which will be instrumental for the rest of the paper.

Theorem 1. *Suppose Assumption 1 holds. Then the dynamics of $z(t)$ can be described in a port-Hamiltonian form (van der Schaft & Jeltsema, 2014) with the new control input $u - u_d$ and the Hamiltonian*

$$H(z) = \frac{1}{2} z^T Pz. \tag{5}$$

Proof. According to the Mean Value theorem for vector-valued mappings, we have

$$f(x, t) - f(x_d, t) = \int_0^1 \frac{\partial f}{\partial x}(x_d(t) + \lambda z, t) \mathrm{d}\lambda \cdot z. \tag{6}$$

By viewing $x_d(t)$ as a time-dependent signal, (6) can be rewritten as

$$f(x, t) - f(x_d, t) = A(z, t)z \tag{7}$$

where $A(z, t) \triangleq \int_0^1 \frac{\partial f}{\partial x}(x_d(t) + \lambda z, t)\mathrm{d}\lambda$. Then, we define the following matrices

$$\begin{aligned} &\triangle(z, t) = A(z, t)P^{-1}, \quad J(z, t) = (\triangle - \triangle^T)/2 \\ &R(z, t) = -(\triangle + \triangle^T)/2 \end{aligned} \tag{8}$$

with P given in Assumption 1. Based on (1), (3), (7), and (8), we obtain that

$$\dot{z} = [J(z, t) - R(z, t)]\nabla H(z) + G(u - u_d). \tag{9}$$

From (8), $J(z, t) = -J^T(z, t)$. According to Assumption 1, P is positive definite, thus, $H(z) = \frac{1}{2} z^T Pz$ is strictly convex. Furthermore, (2) implies that

$$\frac{\partial f}{\partial x}(x, t)P^{-1} + P^{-1}\frac{\partial^T f}{\partial x}(x, t) \leq 0 \tag{10}$$

which shows that $R(z, t) = R^T(z, t) \geq 0$. Then, it can be concluded that (9) is a port-Hamiltonian system, satisfying

$$\begin{aligned} \dot{H} &= -\nabla^T HR(z, t)\nabla H + \nabla^T HG(u - u_d) \\ &\leq \nabla^T HG(u - u_d) \end{aligned} \tag{11}$$

which also implies that the system (1) is incrementally passive with respect to the output $y = G^T Px$. ■

Remark 3. It is worth pointing out that the above analysis corrects a similar statement in Pavlov and Marconi (2008), which utilizes the equation $f(x_a, t) - f(x_b, t) = \frac{\partial f}{\partial x}(\xi, t)(x_a - x_b)$ with ξ denoting some point lying on the line segment $[x_a, x_b]$. However, this equation only holds when $f(\cdot)$ is a scalar function. For this reason, Eq. (6) is utilized in our analysis.

Remark 4 (*Incremental Passivity Via Feedback*)**.** In the scenario that Assumption 1 is not satisfied, one can consider designing a feedback that renders the corresponding closed-loop system incrementally passive. Specifically, if there exists a mapping $\phi : \mathbb{R}^n \to \mathbb{R}^m$ such that $F(x, t) \triangleq f(x, t) + G\phi(x)$ satisfies $P\frac{\partial F}{\partial x}(x, t) + \frac{\partial^T F}{\partial x}(x, t)P \leq 0$, then the control input can be designed as $u = \phi(x) + v$, with $v(t) \in \mathbb{R}^m$ being a virtual input, and we can equivalently consider the tracking problem of the new system $\dot{x} = F(x, t) + Gv$.

From (4), the tracking problem is equivalent to stabilizing the origin of (9). To motivate and prepare the development in the next section, we first consider the tracking problem when $u_d(t)$ is known, and the following Assumption 2 is required for an asymptotic tracking result.

Assumption 2. The system (9) is zero-state detectable (Hill & Moylan, 1976) with respect to the output

$$w(z, t) \triangleq C(z, t)\nabla H(z), \quad w \in \mathbb{R}^n \tag{12}$$

where $C(z, t) \in \mathbb{R}^{n \times n}$ satisfies $C^T C = R(z, t) + GBG^T$, and $B \in \mathbb{R}^{m \times m}$ is a constant positive definite symmetric matrix.

Theorem 2. *If Assumptions 1 and 2 are satisfied, then the system (1) in closed-loop with the control input*

$$u = u_d - BG^T \nabla H(z) \tag{13}$$

asymptotically tracks the desired trajectory $x_d(t)$.

The proof of Theorem 2 is omitted here for concision, and it is based on the condition that $f(x, t)$ is uniformly continuous in t and locally Lipschitz in $x(t)$, and a direct application of *Barbalat's lemma* (Khalil, 2002, Lemma 8.2).

3. Tracking control design

Based on Theorem 2, this section proposes two robust tracking controllers for the scenario that $\dot{x}_d$ is unknown. In this case, the term $Gu_d(t)$ in (9) can be viewed as a matched disturbance since G is constant.

3.1. Velocity observer based method

Assumption 3. There exists a constant, diagonal, positive-definite matrix $K_0 \in \mathbb{R}^{n\times n}$ such that

$$K_{0i} > \|\ddot{x}_{di}(t)\|_\infty + \|\dddot{x}_{di}(t)\|_\infty, \ \forall i \in \{1, \dots, n\} \tag{14}$$

where K_{0i} denotes the ith diagonal element of K_0 and $x_{di}(t)$ denotes the ith element of the vector $x_d(t)$.

Define the following control law

$$u = (G^TG)^{-1}G^T(\dot{\hat{x}}_d - f(x_d, t) - \widetilde{x}_d) - BG^T\nabla H \tag{15}$$

where $\hat{x}_d(t)$ denotes the estimate for $x_d(t)$, and $\widetilde{x}_d(t) \triangleq x_d(t) - \hat{x}_d(t)$ denotes the estimation error for $x_d(t)$. Here, note that the matrix G^TG is invertible because G is of full rank. By modifying the velocity observer designed in Xian, de Queiroz, Dawson and McIntyre (2004), the online update law for $\hat{x}_d(t)$ is designed as

$$\dot{\hat{x}}_d = l + (K_1 + I_n)\widetilde{x}_d \tag{16}$$

$$\dot{l} \in K_1\widetilde{x}_d + K_0\mathrm{Sgn}(\widetilde{x}_d) - G(G^TG)^{-T}G^T\nabla H(z) \tag{17}$$

where $K_1 \in \mathbb{R}^{n\times n}$ is a positive-definite matrix, and I_n denotes the identity matrix in $\mathbb{R}^n$. To facilitate the subsequent stability analysis, a filtered error $e_d(t) \in \mathbb{R}^n$ is defined as

$$e_d(t) \triangleq \dot{\widetilde{x}}_d(t) + \widetilde{x}_d(t). \tag{18}$$

By using $u_d = (G^TG)^{-1}G^T(\dot{x}_d - f(x_d, t))$, (3), (9), and (15), the following extended closed-loop system is obtained

$$\begin{aligned} \dot{z} =& [J - R - GBG^T]\nabla H - G(G^TG)^{-1}G^Te_d \\ \dot{e}_d =& - K_1e_d - K_0\eta(t) + \ddot{x}_d + G(G^TG)^{-T}G^T\nabla H \end{aligned} \tag{19}$$

where the variable $\eta(t) \in \mathrm{Sgn}(\widetilde{x}_d(t))$ is introduced to facilitate the subsequent analysis, which is based on the Filippov's solution concept (Filippov, 2013).

Lemma 1. *Define a function $L(t) \in \mathbb{R}$ as*

$$L(t) \triangleq e_d^T(t)[\ddot{x}_d(t) - K_0\eta(t)]. \tag{20}$$

Provided Assumption 3 holds, then $\int_{t_0}^t L(\tau)\mathrm{d}\tau \leq c$ with $c \triangleq \sum_{i=1}^n K_{0i}|\widetilde{x}_{di}(t_0)| - \widetilde{x}_d(t_0)^T\ddot{x}_d(t_0)$ being a constant.

The proof of Lemma 1 is omitted here for concision, and it is based on a similar procedure given in Xian, Dawson et al. (2004, Appendix A).

Theorem 3. *Provided that Assumptions 1–3 are satisfied, then the control design in (15)–(17) yields asymptotic tracking of $x_d(t)$.*

Proof. Define a function $N(t) \in \mathbb{R}$ as

$$N(t) \triangleq c - \int_{t_0}^t L(\tau)\mathrm{d}\tau. \tag{21}$$

According to Lemma 1, $N(t) \geq 0$. Therefore, the following non-negative function can be defined

$$V(z, e_d, t) = H(z) + \frac{1}{2}e_d^Te_d + N(t). \tag{22}$$

Note that $V(t)$ is positive definite and radially unbounded with respect to $[z, e_d, \sqrt{N(t)}]^T$. By using (11)–(13), (19), (21), and according to the chain rule for nonsmooth systems (Paden & Sastry, 1987), $\dot{V}(\cdot)$ exists almost everywhere (a.e.), i.e., for almost all $t \in [t_0, \infty)$, and satisfies

$$\dot{V} \overset{a.e.}{=} \nabla^THż + e_d^T\dot{e}_d - L(t) = -w^Tw - e_d^TK_1e_d. \tag{23}$$

According to LaSalle–Yoshizawa corollary for nonsmooth systems (Fischer, Kamalapurkar, & Dixon, 2013), (22) and (23) imply that $w(t), e_d(t) \to 0$, as $t \to \infty$. Then the zero-state detectability in Assumption 2 implies asymptotic convergence towards the desired trajectory $x_d(t)$. ■

3.2. Sliding mode control

Assumption 4. There exists a constant, diagonal, positive definite matrix $\Gamma_1 \in \mathbb{R}^{m\times m}$ such that

$$\Gamma_{1i} > \|u_{di}(t)\|_\infty, \ \forall i \in \{1, \dots, m\} \tag{24}$$

where Γ_{1i} denotes the ith diagonal element of Γ_1 and $u_{di}(t)$ denotes the ith element of $u_d(t)$.

Theorem 4. *Suppose Assumptions 1, 2 and 4 hold. The desired trajectory $x_d(t)$ for (1) is asymptotically tracked by the control input*

$$u \in -BG^T\nabla H(z) - \Gamma_1 Sgn(G^T\nabla H(z)). \tag{25}$$

Proof. From (9) and (25), the z-dynamics is derived as

$$\dot{z} \in (J - R - GBG^T)\nabla H - Gu_d - G\Gamma_1\mathrm{Sgn}(G^T\nabla H). \tag{26}$$

From (26) and according to the chain rule for nonsmooth systems (Paden & Sastry, 1987), the derivative of $H(z)$ in (5) exists almost everywhere (a.e.), and $\dot{H}(z) \overset{a.e.}{\in} \dot{\tilde{H}}(z)$, where $\dot{\tilde{H}}(z)$ denotes a set satisfying

$$\begin{aligned} \dot{\tilde{H}}(z) \subset& - w^Tw - \nabla^THGu_d - \nabla^THG\Gamma_1\mathrm{Sgn}(G^T\nabla H) \\ \leq& - w^Tw + \sum_{i=1}^n \left|[\nabla^TH(z)G]_i\right| (|u_{di}(t)| - \Gamma_{1i}) . \end{aligned} \tag{27}$$

From (24), the second term on the right side of (27) is negative. Therefore, we have $\dot{\tilde{H}}(z) \leq -w^Tw$, which implies that $w(t) \to 0$, as $t \to \infty$ according to LaSalle–Yoshizawa Corollary for Nonsmooth Systems (Fischer et al., 2013). Hence, the asymptotic stability of $z = 0$ follows from the zero-state detectability in Assumption 2. ■

In contrast to the velocity observer based method, the sliding mode controller (SMC) in (25) does not require the knowledge of $f(x, t)$ and Assumption 4 is weaker than Assumption 3. However, it is worth pointing out that the proposed SMC can result in the chattering phenomenon.

Remark 5. For chattering suppression of SMC, Corless and Leitmann (1981) simply propose to replace the signum function with a smooth approximation. However, it is worth pointing out that this replacement may destabilize the closed-loop system in our case since w^Tw in (27) is only ensured to be positive semi-definite with respect to $z(t)$. If the controlled system is strictly incrementally passive, this approximation method can be applied with a deterioration in the control performance, that is, it only ensures bounded tracking rather than asymptotic tracking. Furthermore, other advanced methods for chattering suppression can be considered for better control performance. For example, Galias and Yu (2007) propose an Euler's discretization method which yields only numerical chattering, and Acary and Brogliato (2010) show that an implicit Euler time-discretization leads to a chattering-free stabilization.

Remark 6. It is well-known that higher-order SMC is effective for chattering avoidance. However, a standard second-order SMC (Fridman & Levant, 1996) is not applicable for the studied tracking problem, because it depends on a variable structure term $\mathrm{Sgn}(\dot{s} + \alpha s)$ with α being a positive constant, and $s = G^T\nabla H$ in this case. Note that $\dot{s}(t)$ is not available since it is related to the unknown $\dot{x}_d(t)$. To avoid this problem, an alternative second-order SMC (Bartolini, Ferrara, & Usai, 1998) can be defined as

$$\dot{u} \in -(G^TPG)^{-1}\Gamma_2(x, t)\mathrm{Sgn}(s) \tag{28}$$

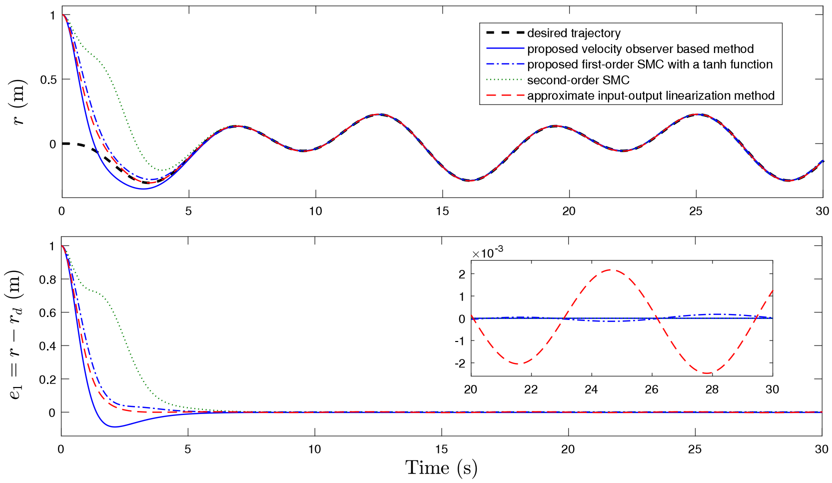

Fig. 1. Tracking results of the ball position.

where $\Gamma_2(x,t) \in \mathbb{R}^{m\times m}$ is a diagonal positive definite matrix satisfying $\Gamma_{2i}(x,t) \geq \|[G^T P(\dot{f}(x,t) - \ddot{x}_d)]_i\|$, $\forall i \in \{1,\ldots,m\}$, $x \in \mathbb{R}^n$, $t \geq 0$. This inequality shows that $\ddot{x}_d(t) \in \mathcal{L}_\infty$ is a necessary condition for the existence of $\Gamma_2(x,t)$. Note that the controller (28) only ensures that $s(t) \to 0$ in a finite time. Therefore, the zero-state detectability from the output $s(t)$ is required for an asymptotic full state tracking result.

4. Example: The ball and beam system

In this section, we apply the proposed controllers developed in Section 3 to the ball and beam system, which is not feedback linearizable as pointed out in Hauser, Sastry, and Kokotovic (1992). Its dynamics is given by

$$\dot{x} = \underbrace{\begin{bmatrix} x_2 \\ \frac{M(x_1x_4^2 - g\sin x_3)}{\mathcal{J}_b/\mathcal{R}^2 + M} \\ x_4 \\ -\frac{M(2x_1x_2x_4 + gx_1\cos x_3)}{Mx_1^2 + \mathcal{J} + \mathcal{J}_b} \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{G} u. \quad (29)$$

where $x = [x_1, x_2, x_3, x_4]^T \triangleq [r, \dot{r}, \theta, \dot{\theta}]$, and $u \triangleq \frac{\tau_b}{Mx_1^2 + \mathcal{J} + \mathcal{J}_b}$. Here, θ denotes the beam angle, r is the ball position, $\mathcal{J}$ is the beam's moment of inertia, and τ_b is the torque applied to the beam. M, $\mathcal{R}$, and $\mathcal{J}_b$ denote the mass, radius, and moment of inertia of the ball, respectively. g denotes the acceleration of gravity. Here, we utilize the same model parameters as in Hauser et al. (1992): $\mathcal{J} = 0.02$ kg m^2, $M = 0.05$ kg, $\mathcal{J}_b = 2\times 10^{-6}$ kg m^2, $\mathcal{R} = 0.01$ m, and $g = 9.81$ m/s^2.

Remark 7. To the best of our knowledge, most of the existing tracking controllers for the ball and beam system address an output tracking problem, i.e., tracking a desired trajectory of the ball position (Hauser et al., 1992; Hirschorn, 2002), in which bounded tracking results are achieved. As a contrast, a state tracking problem is studied in this work, and the proposed controllers ensure a locally asymptotic tracking result for the ball and beam system under the condition that $\dot{x}_d(t)$ is unknown.

Since Assumption 1 is not satisfied in this case, we invoke the incremental passivity via feedback given in Remark 4. Specifically, the mapping $\phi(x)$ is designed as

$$\phi(x) = \frac{M(2x_1x_2x_4 + gx_1\cos x_3)}{Mx_1^2 + \mathcal{J} + \mathcal{J}_b} + \frac{24(\mathcal{J}_b/R^2 + M)x_1}{Mg} + \frac{50(\mathcal{J}_b/R^2 + M)x_2}{Mg} - 35x_3 - 10x_4.$$

It is verified that the condition $P\frac{\partial F}{\partial x}(x) + \frac{\partial^T F}{\partial x}(x)P < 0$ with the constant positive definite symmetric matrix

$$P = \begin{bmatrix} 3.6087 & 2.1779 & -4.2801 & -0.292 \\ 2.1779 & 3.1127 & -6.9958 & -0.4454 \\ -4.2801 & -6.9958 & 25.5017 & 1.4720 \\ -0.292 & -0.4454 & 1.4720 & 0.3972 \end{bmatrix}$$

holds for $|x_1| \leq 3$ m, $x_2 \in \mathbb{R}$, $|x_3| \leq 0.65$ rad, $|x_4| \leq 0.19$ rad/s. That is, the closed-loop system with $u = \phi(x)$ is locally strictly incrementally passive, and thus, $C(z,t)$ in (12) is of full rank and Assumption 2 holds. Assumptions 3 and 4 can be satisfied by appropriately selecting the desired trajectory $x_d(t)$. After verifying all the assumptions, the explicit form of the proposed controllers is given as

- Proposed velocity observer based controller

$$u = (G^TG)^{-1}G^T(\dot{\hat{x}}_d - f(x_d,t) - \tilde{x}_d) + \phi(x) - \phi(x_d) - BG^TP(x - x_d) \quad (30)$$

where $\dot{\hat{x}}_d$ is obtained from (16)–(17).

- Proposed first-order SMC

$$u \in \phi(x) - BG^TP(x - x_d) - \Gamma_1 \mathrm{Sgn}(G^TP(x - x_d)). \quad (31)$$

In the simulation, the desired bounded trajectories are generated by a target system $\dot{x}_d = f(x_d) + Gu_d(t)$ where $\dot{x}_d(t)$ is assumed to be unknown for the control design. Therefore, it is similar to a synchronization problem, i.e., letting the controlled ball and beam system track another one. The control parameters are: $K_0 = K_1 = 2I_4$, $B = 0$, $\Gamma_1 = 40$. According to Remark 5, the term $\mathrm{sgn}(s)$ in (31) is replaced by the smooth approximation $\tanh(10^3 s)$ in order to facilitate the simulation analysis since the ball and beam system with $u = \phi(x)$ is locally strictly incrementally passive. For the comparison purpose, simulations are also conducted by applying the second-order SMC discussed in Remark 6 and an approximate input–output linearization controller given by Hauser et al. (1992, Eq. (3.1) and (3.3)). Since Hauser et al. (1992) study an output tracking problem, the controller relies on the information of $y_d(t) = x_{1d}$, $\dot{y}_d(t) = x_{2d}$, and the unknown $y_d^{(i)}(t)$, $i = 2, 3, 4$, which are estimated from $\dot{y}_d(t) = x_{2d}$ by a high-gain observer in this case. The simulation results with the initial conditions $x(0) = [1, 0, 0.5, 0]$ are given in Figs. 1 and 2. Note that the proposed SMC has a small steady-state tracking error due to using the smooth approximation. Compared to the other three methods, the proposed velocity observer controller achieves a better control performance in terms of the tracking error and the control energy.

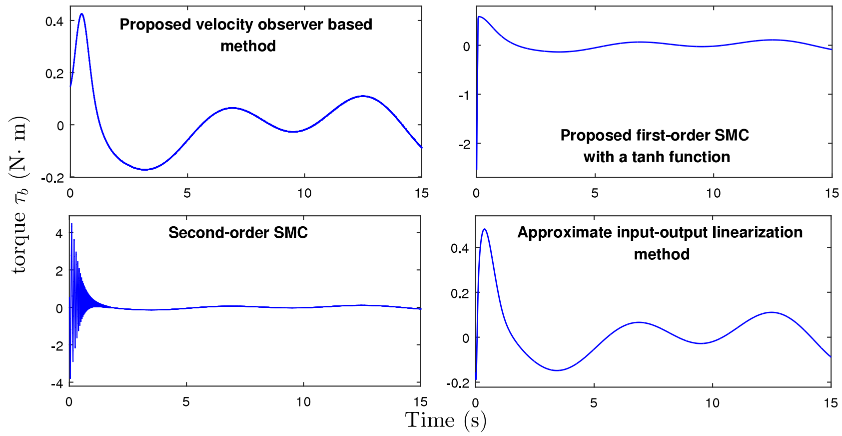

Fig. 2. Control inputs of the different methods.

5. Conclusions

In this paper, we investigate the trajectory tracking problem for nonlinear systems by exploiting the property of incremental passivity. For a desired trajectory with unknown velocity, we propose two robust control designs (a velocity observer based method and a sliding mode controller). Both controllers ensure an asymptotic tracking result provided a condition of zero-state detectability holds. The main contribution is related to a new framework of robust tracking control for underactuated nonlinear systems that are not necessarily feedback linearizable.

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