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Nonlocality amplifies echoes

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In this paper we will provide distinctive signatures of nonlocal interactions while studying reflection and transmission of waves bouncing through two Dirac delta potentials. In particular, we will show that the transmission of waves is less damped compared to the local case, due to the fact that nonlocality weakens the interaction. As a consequence the echoes are amplified. These signatures can be potentially detectable in the context of gravitational waves, where two Dirac delta potentials can mimic the two potential barriers at the surface and at the photon sphere of an ultra compact object, or, at the two photon spheres of a wormhole, experiencing nonlocal interactions.

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I. INTRODUCTION

The nonlocal interactions have been widely studied since the days of Yukawa [1], Pais and Uhlenbeck [2]. It has been studied widely in the context of quantum field theory in order to ameliorate the ultraviolet (UV) (or, in other words, short-distance) behavior of loop integrals and scattering amplitudes [3–6]. One of the striking features of nonlocality is that it weakens the interaction, and as a direct consequence it can regularize singularities through smearing out a point-like Dirac delta distribution [7–10]. The simplest nonlocal interaction can be captured by infinite derivative field theories, where the kinetic operator is generalized by means exponentials of *entire* functions, which do not introduce any new dynamical degrees of freedom, and modifies the UV behavior of the theory. The latter criteria guarantees perturbative unitarity and classical stability which, in general, are absent in any finite higher derivative theory, see [4,9–11]. All these interesting results have inspired the generalization of the Einstein-Hilbert action for gravity to infinite derivative gravity (IDG), with form factors containing nonlocal operators in order to make the theory well defined both at classical and quantum level [4,7–10,12]. In a recent generalization which contains all possible curvature corrections [10], it was shown that IDG can resolve the pointlike singularity in a static [10,13–16], and the ring singularity in a rotating spacetime metric [17], and with the presence of torsion as well [18]. In a dynamical collapse also there are no formation of horizon and singularities [19]. Furthermore, such gravitational actions also help in order to understand the cosmological big bang singularity problem [9,20], and potentially resolving the event horizon in the context of astrophysical compact objects [21].

The aim of this paper is to provide a smoking gun signature for nonlocal interactions. In Sec. II we will study the propagation of waves in nonlocal massless scalar field theory in presence of two Dirac delta potential barriers in one spatial dimension. In Sec. III we will work out the quasi-normal modes (QNMs) and in Sec. IV the echoes of an initial incoming pulse and make the comparison with the local case. In Sec. V we will draw the conclusions.

Our analysis allows us to make a very powerful analogy in astrophysics. First of all, the two Dirac deltas can be used to model the two symmetric photon spheres of a wormhole [22]. In the case of a static spacetime metric, the double delta potential can be also used as a rough toy model to mimic the two potential barriers at the surface and at the photon sphere of an ultra compact object (UCO), located at $2GM(1 + \epsilon)$ and $3GM$, respectively, with $\epsilon < 0.5$, where G is Newton’s constant and M the mass of the UCO [23]. It is worthwhile mentioning that in Refs. [24] it was shown for the first time that merging of horizonless compact objects can show a unique signature of the absence of a horizon through echoes’ production. Interestingly, in Ref. [25] it was performed a data analysis and claimed to be found statistical evidence for late time echoes in some of the LIGO events [26]. However, such a result is still under debate [27] and more progress is still needed. UCO-scenarios have also appeared in the literature in many string inspired realizations in order to avoid the information-loss paradox [21,28–30]; see also Refs. [23,31] for several models of black hole mimickers.

Hence, despite the simplicity of our model, by modifying the dynamics of propagating waves through infinite derivative order operators, we might be able to capture some salient features of nonlocality, which is otherwise extremely elusive to test.

II. NONLOCAL SCALAR FIELD IN DOUBLE DELTA POTENTIAL

Let us consider a nonlocal massless scalar field Φ interacting with a potential V in $1+1$ dimensions with a nonlocal operator given by an exponential of an *entire* function

$$[F(\square)\square - V]\Phi = 0, \quad F(\square) = e^{-\ell_s^2 \square}; \quad (1)$$

we will work with the simplest kinetic operator above but other choices are also possible [4,15,16,32]. In Eq. (1), ℓ_s is the fundamental length scale of nonlocality below which new physics should manifest and $\square = -\partial_t^2 + \partial_x^2$ is the flat d'Alembertian operator. Note that we adopt the metric signature; $\eta = \text{diag}(-1, 1, 1, 1)$, and use the natural units $\hbar = 1 = c$. By separating the time and spatial variables with the ansatz $\Phi(t, x) = e^{-i\omega t}\psi(x)$, Eq. (1) becomes [33]

$$[e^{-\ell_s^2(\partial_x^2 + \omega^2)}(\partial_x^2 + \omega^2) - V(x)]\psi(x) = 0, \quad (2)$$

which is a nonlocal version of the Schrödinger equation. For general potential barriers, $V(x)$ is assumed to be positive and to satisfy $V(x) \rightarrow 0$ as $x \rightarrow \pm\infty$, so that the general solution of $\psi(x)$ has the form

$$\psi(x) = \begin{cases} Ae^{i\omega x} + Be^{-i\omega x}, & x \rightarrow -\infty, \\ Ce^{i\omega x}, & x \rightarrow +\infty. \end{cases} \quad (3)$$

The reflection and transmission coefficients, $\mathcal{R}$ and $\mathcal{T}$ respectively, are defined as

$$\mathcal{R} = \frac{B}{A}, \quad \mathcal{T} = \frac{C}{A}, \quad |\mathcal{R}|^2 + |\mathcal{T}|^2 = 1. \quad (4)$$

For a general potential Eq. (2) may not be easily solvable, but there exist analytic solution in some cases. Here, we assume a symmetric double Dirac delta potentials

$$V(x) = \lambda\delta(x+a) + \lambda\delta(x-a). \quad (5)$$

Note that a single Dirac delta potential in nonlocal theory was first studied in [33]. The nonlocal Schrödinger equation (2) with the potential (5) can be solved analytically by using the Lippmann-Schwinger method [34]

$$\psi(x) = \psi_0(x) - \int_{-\infty}^{\infty} dx' G(x, x') V(x') \psi(x') \quad (6)$$

where $G(x, x')$ is the Green function defined through the equation $e^{-\ell_s^2(\partial_x^2 + \omega^2)}(\partial_x^2 + \omega^2)G(x, x') = -\delta(x - x')$, and reads [33]

$$\mathcal{G}(z) = \frac{i}{4\omega} [e^{i\omega z} Y(z) + e^{-i\omega z} Y(-z)]$$

with $Y(z) = 1 + \text{erf}\left(i\omega\ell_s + \frac{z}{2\ell_s}\right), \quad (7)$

which in the limit $\ell_s \rightarrow 0$ reduces to the local Green function $\mathcal{G}_0(z) = \frac{i}{2\omega} e^{i\omega|z|}$, as expected. The Green function turns out to be symmetric, $G(x, x') = G(x', x)$, and depends only on the variable $z = x - x'$: $G(x, x') \equiv \mathcal{G}(z)$, $\mathcal{G}(z) = \mathcal{G}(-z)$. Hence, by using the Lippmann-Schwinger equation (6), the solution for the symmetric double delta in (5) can be shown to be

$$\psi(x) = e^{i\omega x} - \lambda \sum_{j,k=1}^2 \mathcal{G}(x - a_k) \Phi_{kj}^{-1} e^{i\omega a_j}, \quad (8)$$

where

$$\Phi_{kj} = \begin{cases} 1 + \frac{i\lambda_j}{2\omega} [1 + \text{erf}(i\omega\ell_s)], & k = j; \\ \frac{i\lambda_j}{4\omega} [e^{i2\omega a} Y(2a) + e^{-i2\omega a} Y(-2a)], & k \neq j, \end{cases} \quad (9)$$

and we have defined $a_1 = a$, $a_2 = -a$. By assuming that the incoming wave comes from the left, we can find the reflection and transmission coefficients for the solution in Eq. (9) by identifying the coefficients in front of $e^{-i\omega x}$ when $x \ll a$ and of $e^{i\omega x}$ when $x \gg a$, respectively; they are given by

$$\mathcal{R} = -\frac{2[-2\frac{\omega}{\lambda}i + 1 + \text{erf}(i\omega\ell_s)]\cos(2\omega a) - [1 + \text{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} - [1 + \text{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}}{[-2\frac{\omega}{\lambda}i + 1 + \text{erf}(i\omega\ell_s)]^2 - \frac{1}{4}\{[1 + \text{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} + [1 + \text{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}\}^2}, \quad (10)$$

$$\mathcal{T} = 1 - \frac{2[-2\frac{\omega}{\lambda}i + 1 + \text{erf}(i\omega\ell_s)] - \{[1 + \text{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} + [1 + \text{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}\}\cos(2\omega a)}{[-2\frac{\omega}{\lambda}i + 1 + \text{erf}(i\omega\ell_s)]^2 - \frac{1}{4}\{[1 + \text{erf}(i\omega\ell_s + \frac{a}{\ell_s})]e^{2i\omega a} + [1 + \text{erf}(i\omega\ell_s - \frac{a}{\ell_s})]e^{-2i\omega a}\}^2}, \quad (11)$$

and one can easily check that $|\mathcal{R}|^2 + |\mathcal{T}|^2 = 1$.

III. QUASINORMAL MODES

The poles of $\mathcal{R}$ and $\mathcal{T}$ describe the quasinormal modes (QNMs) of the system, which satisfy the boundary condition such that there is no incoming wave at infinity,

i.e., $\psi \rightarrow e^{-i\omega x}$ as $x \rightarrow -\infty$ and $\psi \rightarrow e^{i\omega x}$ as $x \rightarrow +\infty$ [35]. In a realistic astrophysical framework, the quantities in (10) and (11) are crucial for understanding the effect of nonlocality in the UCOs. We can check that the poles of the reflection and transmission coefficients are given by the equations

$$2 \left[-2 \frac{\omega}{\lambda} i + 1 + \operatorname{erf}(i\omega\ell_s) \right] = \pm \left\{ \left[1 + \operatorname{erf} \left(i\omega\ell_s + \frac{a}{\ell_s} \right) \right] e^{2i\omega a} + \left[1 + \operatorname{erf} \left(i\omega\ell_s - \frac{a}{\ell_s} \right) \right] e^{-2i\omega a} \right\}, \quad (12)$$

whose roots of the type $\omega_n = \mathcal{W}_n - i\omega'_n$ are called quasinormal frequencies (QNFs). Note that for ω with a negative imaginary part, the time dependent piece $e^{-i\omega t}$ behaves like a damped oscillation: the real part gives the energy carried by the wave and the imaginary part (inverse) gives the timescale over which the wave decays after interacting with the potential barrier. It is worthwhile mentioning that in the limit, $a = 0$, we recover the quasinormal condition for a single Dirac delta potential with strength 2λ , i.e., $1 - i\omega/\lambda + \operatorname{erf}(i\omega\ell_s) = 0$ [33]; while in the limit $\ell_s = 0$, Eq. (12) reduces to the well-known local case, $1 - i2\omega/\lambda = \pm e^{2i\omega a}$.

Generally, Eq. (12) cannot be solved, but we can simplify it and obtain an analytic solution by working in a specific regime which turns out to be the most interesting one from a physical point of view. Since we expect the fundamental scale of nonlocality ℓ_s to be always smaller than the distance between the surface and the photon sphere of a UCO, see [21,31], it is sensible to require $a \gg \ell_s$.¹ Moreover, when studying the QNMs and the echoes produced by an initial pulse (see also below) the low frequencies are the most relevant ones as they correspond to longer damping times, therefore we can also impose the inequality $|\omega|\ell_s < a/\ell_s$. In such limits, $\operatorname{erf}(i\omega\ell_s + a/\ell_s) \rightarrow 1$ and $\operatorname{erf}(i\omega\ell_s - a/\ell_s) \rightarrow -1$, so that Eq. (12) reduces to

$$1 - i2\frac{\omega}{\lambda} + \operatorname{erf}(i\omega\ell_s) = \pm e^{2i\omega a}, \quad (13)$$

which in the limit $|\omega|\ell_s \ll 1$ can be further simplified since $\operatorname{erf}(i\omega\ell_s) \approx \frac{2}{\sqrt{\pi}} i\omega\ell_s$. Indeed, in this regime we can find an analytic expression for the QNMs in terms of the Lambert W function:

$$\omega_n = i \left[\frac{\mathcal{W}_n(\pm \alpha a e^{\alpha a})}{2a} - \frac{\alpha}{2} \right], \quad \frac{1}{\alpha} = \frac{1}{\lambda} - \frac{\ell_s}{\sqrt{\pi}}. \quad (14)$$

Furthermore, we can solve numerically Eq. (13) and study the behavior of the real part of the QNMs, $\operatorname{Re}(\omega) = \mathcal{W}_n$, as a function of the negative imaginary part $-\operatorname{Im}(\omega) = \omega'_n$.

¹Note that in this paper we only consider a toy model to understand the qualitative features of nonlocal effects on the propagation of waves, indeed we do not specify any physical value for a and λ which are compatible with some astrophysical scenario. However, a quantitative analysis will require a more realistic model in which the potential in Eq. (2) has to take into account the presence of a horizonless curved spacetime and of a centrifugal contribution.

From Fig. 1, we can see that nonlocal effects make the absolute value of imaginary part of the QNFs smaller, i.e., the damping time is longer. The curve corresponding to the nonlocal case is always below the local one in the low frequency regime as shown in the Fig. 1. The closer the ring-down energy $\mathcal{W}_n$ is to the energy scale $1/\ell_s \equiv M_s$, the more relevant nonlocality is. This means the more obvious becomes the difference between the imaginary parts between local and nonlocal. Only for $\ell_s \rightarrow 0$ the two curves coincide. Moreover, when the strength of the delta potential $\lambda \gtrsim 1/\ell_s \equiv M_s$, the damping is smaller. Therefore, a crucial implication due to nonlocality is that low frequency waves survive a *longer* lifetime as compared to the local case. This feature was expected since it is known that nonlocality weakens the interaction [6,36]. Furthermore, when $\mathcal{W}_n \ell_s \sim \mathcal{O}(1)$, we can also notice the presence of a turning point after which the damping time

FIG. 1. We have shown the behavior of the real part of the QNFs $\mathcal{W}_n$ as a function of the negative imaginary part ω'_n for both local (black line) and nonlocal (orange line) cases; the latter was obtained by solving numerically Eq. (13). In the upper panel we have set $a = 1$, $\ell_s = 0.01$ and $\lambda = 10$, while in the bottom panel $a = 1$, $\ell_s = 0.01$ and $\lambda = 50$.

decreases. This property is mathematically related to the structure of the error function with complex argument but it is not relevant from a physical point of view since it will only describe short-lived waves with energies $\mathcal{W}_n \ell_s \gg 1$.

IV. ECHOES

So far we have learned that nonlocal effects weaken the interaction between wave and potential barriers, and as a consequence the timescales over which a transmitted wave decays turn out to be longer than the local case. We now wish to use the spectral features of the QNMs and study the consequences induced by nonlocality on the echoes production. In order to do so we need to work in the time domain where the form of the wave can be cast as an infinite linear combination of QNMs [22,37]:

$$\Psi(t) = \sum_{n=-\infty}^{\infty} c_n e^{-i\omega_n t}, \quad (15)$$

which in a more realistic scenario can represent the signal produced at the photon sphere after relaxation. Because the oscillation period of the real parts of the QNMs is $\text{Re}(\omega_{n+1} - \omega_n) \simeq \pi/2a$, which is the same also in the local case, and since $\text{Im}(\omega_n) \ll \text{Re}(\omega_n)$, the function $\Psi(t)$ is in good approximation periodic, with period $T = 4a$. This means that the primary signal is reproduced periodically, as it can be reflected by the second potential barrier (UCO's surface), thus giving rise to echoes.

This feature allows us to express the coefficients c_n as a Fourier transform of the first echo, $\Psi_{1^{\text{st echo}}}^{(0)}(t)$,

$$c_n = \frac{1}{4a} \int_0^{4a} dt \Psi_{1^{\text{st echo}}}^{(0)}(t) e^{i\omega_n t}. \quad (16)$$

We can choose the first echo as a Gaussian wave packet of the form

$$\Psi_{1^{\text{st echo}}}^{(0)}(t) = e^{-i\omega_0(t-t_0)} e^{-(t-t_0)^2/2\tau^2}, \quad (17)$$

where ω_0 is the leading frequency of the primary signal. Assuming that $\tau \ll 2a$ we can obtain an approximation for the coefficients c_n of the subsequent echoes by computing the integral in Eq. (16) [22]

$$c_n \simeq \sqrt{\frac{\pi}{2}} \frac{\tau}{2a} \exp \left[i\omega_n t_0 - \frac{\tau^2}{2} (\omega_n - \omega_0)^2 \right]. \quad (18)$$

Hence, we can now use Eq. (15) with the coefficients given in Eq. (18) to plot the time dependent profile of the signal, see Fig. 2. Note that, because only the long-lived QNMs (QNFs with small negative imaginary part) led to the

FIG. 2. We have plotted the behavior of the signal in Eq. (15), with coefficients (18), which describes the evolution of the echoes produced at a fixed spatial point, for both local (black line) and nonlocal (orange line) cases. We have chosen $t_0 = 1$, $\tau = 0.25$, and set $a = 1$, $\lambda = 50$, $\ell_s = 0.01$.

echoes in the late time [38], thus it is also sufficient to analyze these results using Eq. (14).

V. CONCLUSIONS

Before we conclude, let us capture the salient features of our analysis. In the regime $\lambda \ll 1/\ell_s \equiv M_s$, we have already seen that the QNMs are very similar to the local case, therefore the same applies for the echoes. It means that when the height of the potential barriers is below the energy scale $1/\ell_s \equiv M_s$, nonlocality does not play a relevant role, i.e., the echoes are almost the same as the local case. However, when the height of the potential barrier is either comparable or larger than the nonlocal energy scale, i.e., $\lambda \gtrsim M_s$, the damping time of the nonlocal case is much longer at low frequency. So a very distinct difference arises between local and nonlocal cases. As shown in Fig. 2, the amplitudes of the late time echoes are amplified with respect to the local ones, meaning that nonlocality acts as an *amplifier*. We would like to clarify that the echoes are less damped at late time due to nonlocality with respect to the same first echo. The first echo is constructed by the primary signal which is determined by the leading frequency of the quasinormal spectrum, as it was shown in Eq. (17). For any fixed potential configuration, i.e., the height of the delta barriers in our toy model, the leading frequencies are almost the same in local and nonlocal case. Actually, we can adjust the degree of amplification by adjusting the height of the potential λ .² Such a behavior is

²Certainly as pointed out by the referee, the degree of amplification is adjusted by the different value of λ with respect to their own primary signal. This sort of amplification should be distinguished from the amplification due to nonlocality. The amplification is always present in the nonlocal case with respect to local case for any configuration of the double delta potential. Indeed, even for an asymmetric configuration with two different heights of the barriers, i.e., λ_1 and λ_2 , the echoes are still amplified due to nonlocality.

consistent with the fact that the nonlocal interaction makes the life-times of the waves longer [21], which means they are less damped in time, therefore the echo signal is more amplified. The presence of a smaller damping scale also implies that in the process of transmission less energy is dissipated when the interaction between the wave and the potential barrier is nonlocal, therefore the echoes are louder.

Let us conclude the paper by emphasizing that nonlocal effects can be amplified in presence of double delta potential barriers, which can be experimentally probed. In our opinion, gravitational waves in the merger of two UCOs, or wormholes, might yield the secret of nonlocality, which goes in conjunction with the avoidance of an event horizon to explain that blackhole information loss paradox. It is worthwhile mentioning that this kind of nonlocal effects could be captured in models of condensate matter system, in which the reflection and transmission properties of quantum mechanical barriers can be potentially tested in

a table-top experiment, or nonlocal effects on quantum mechanical oscillators have been studied previously [39].

Our analysis has been made in the simple case of flat background. Future studies are needed to take into account the presence of a curvature and of a more realistic effective potential, but we believe that the essence will be very similar to what we have presented here. The key signature will be the presence of a smaller damping scale and the amplification of echoes, which is a distinctive feature of nonlocality.

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