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Wang, L.; Maschke, B.; Van Der Schaft, A. J.

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Stabilization of Control Contact Systems

Li Wang * B. Maschke** A.J. van der Schaft ***

* Université de Lyon, Lyon, F-69003, France; Université Lyon 1, Faculté Sciences et Technologie, France; CNRS, UMR5007, Laboratoire d’Automatique et Génie des Procédés, Villeurbanne, F-69622, France (Email: lwang@lagep.univ-lyon1.fr)

** Université de Lyon, Lyon, F-69003, France; Université Lyon 1, Faculté Sciences et Technologie, France; CNRS, UMR5007, Laboratoire d’Automatique et Génie des Procédés, Villeurbanne, F-69622, France (Email: maschke@lagep.univ-lyon1.fr)

*** Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, PO Box 407, 9700 AK, the Netherlands (Email: A.J.van.der.Schaft@rug.nl)

Abstract: The control input-output contact system is a comprehensive representation of the irreversible Thermodynamic systems following the Gibbs’ fundamental equations. In this paper, some syntheses of a preserving structure state feedback, which allows stabilization of the closed-loop contact system, will be analysed. For the partial stabilization of the closed-loop contact system: at equilibrium point, the analyses are based on the technical linearisation; on the invariant Legendre submanifold, the analyses are based on the center manifold theory. For the global stabilization of the closed-loop contact system, the possibility for defining an availability function as Lyapunov function will be investigated as well. In order to demonstrate the effectiveness of these concepts, they are further applied to the heat exchanger as an experimental example.

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1. INTRODUCTION

It is generally known that the geometric structure of the irreversible Thermodynamic systems is based on an important thermodynamic equation, namely Gibbs’ fundamental equation. The control input-output contact systems which have been proposed (Eberard et al. [2007]), are one of the representations of those Thermodynamic systems. With a canonical differential-geometric structure called contact structure (Arnold [1989], Eberard et al. [2007], Libermann and Marle [1987]), the researchers attempted to use the control input-output contact systems to solve some control problems. To date, some necessary conditions for the stability of the linearisation of contact vector fields are given (Favache et al. [2009]) and a new framework of conservative controlled contact systems has been recently proposed (Ramírez et al. [2013]), where the state and co-state variables are considered as independent variables. However, after adding the structure preserving feedback, we failed to preserve the contact structure, hence the closed-loop contact system is no longer a contact system. To cope with this challenge, Ramírez et al. proposed a modified contact form in his thesis (Ramírez [2012]). By adding the exterior derivative of a function F which fulfils the condition that it does not depend on the coordinate associated to the Reeb vector field, it turns out to be possible to keep the contact structure preserved, as stated in the IDA-PBC method (Ortega et al. [2002]).

The problem remains in the study of stability and stabilization for the control input-output contact system. Favache analysed the stability of the restriction of contact vector fields to certain invariant Legendre submanifolds at equilibrium points in her thesis and later, Ramírez proposed one class of preserving structure state feedback to keep the closed-loop contact system conservative and proved that the linearisation of a contact vector field at equilibrium points has at most n stable eigenvalues when the manifold has dimension 2n + 1. Therefore, in this paper, a series of control syntheses will be proposed in order to provide limiting conditions while choosing the preserving structure state feedback and to determine the stable invariant Legendre submanifold on which the closed-loop contact system is partially stable. Using an analogue methodology with (Kotyczka [2013]), the control synthesis of the preserving structure state feedback will be analysed through the local study of the linearisation of closed-loop contact system at equilibrium point, through the discussion on the restrictions of the desired Hamiltonian function which generates the stable invariant Legendre submanifold and through the study of asymptotic stability based on Lyapunov’s direct method.

The paper is organized as follows: In Section 2, we recall the definition of contact systems; In Section 3, the control synthesis is proposed; In Section 4, we apply all the previous concepts on a simple thermodynamic model, the
heat exchanger. Some closing remarks and lines of future work are given in Section 5.

2. CONTROL CONTACT SYSTEMS

Control contact systems are generalizations of Hamiltonian systems associated to mechanical systems, to adapted to irreversible Thermodynamical systems and have been defined and their system-theoretical and some control properties have been studied in details (Eberard [2006], Favache et al. [2009], Ramirez [2012]). In the sequel we recall briefly some notations and definitions that are used in this paper and refer to (Libermann and Marle [1987]) for the precise mathematical definitions.

Contact systems are defined on state spaces that are contact manifolds, that are $(2n+1)$-dimensional differential manifolds endowed with a contact form, denoted by $\theta$. We recall here only that, according to Darboux’s theorem, in a set of canonical coordinates $(x_0, x, p) \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n$ the contact form is defined by

$$\theta = dx_0 - \sum_{i=1}^n p_i dx_i$$

Contact vector fields are the vector fields $X$ which leave the contact form $\theta$ invariant. Such a contact vector field is uniquely defined by a contact Hamiltonian function $K \in C^\infty$. Again we only recall its definition in coordinates the contact vector field generated by the Hamiltonian function $K$ in canonical coordinates $(x_0, x, p)$:

$$X_K = \left[\begin{array}{c} K \\ 0 \\ 0 \\ \frac{\partial K}{\partial x_0} \\ \frac{\partial K}{\partial x} \\ \frac{\partial K}{\partial p} \end{array}\right]$$

where $I_n$ denotes the identity matrix of order $n$. Using the definition of contact vector fields (1), the control contact systems can be defined as follows (Ramirez [2012]): A controlled contact system affine in the scalar input $u(t) \in L^1_{loc}(\mathbb{R}_+)$ is defined by two functions $K_0 \in C^\infty(M)$ and $K_c \in C^\infty(M)$, and the state equation is

$$\frac{d\tilde{x}}{dt} = X_{K_0} + X_{K_c}u$$

where the contact vector field $X_{K_0}$ is generated by $K_0$ which is called internal contact Hamiltonian. And the contact vector field $X_{K_c}$ is generated by $K_c$, called the interaction contact Hamiltonian. In the sequel we shall only consider Hamiltonian functions that are invariant of the Reeb vector field that is, in canonical coordinates, do not depend on the $x_0$-coordinate.

2.1 Structure preserving feedback

In this subsection we will recall some results on the structure preserving state feedbacks $u = \alpha(x)$ (Ramirez et al. [2011]). It has been proven that there exist a class of state feedback such that the closed-loop system is again a contact system, however with respect to a modified contact structure. This contact structure is defined with respect to modified contact form which may be expressed by

$$\theta_d = \theta + dF$$

where $dF$ is the differential of a function which is an invariant of the Reeb vector field. The associated feedback $\alpha \in C^\infty(M)$ may be written

$$\alpha(x, p) = \Phi' \circ K_c$$

where $\Phi \in C^\infty(\mathbb{R})$ is an arbitrary function. The closed-loop system is defined by vector field $\dot{X}_K$

$$\dot{X}_K = X_{K_0} + \alpha X_{K_c} = X_{K_0} + (\Phi \circ K_c) X_{K_c}$$

which is a the contact vector field generated with respect to the modified contact form $\theta_d$ and the shaped contact Hamiltonian $\dot{K}$

$$\dot{K} = K_0 + \Phi \circ K_c + c_F$$

where $c_F \in \mathbb{R}$ is a constant.

3. STABILIZATION OF THE CLOSED-LOOP CONTACT SYSTEMS

It has been shown in [Ramirez 2012, chap. 4], that a structure preserving feedback may only achieve partial stability with respect to a Legendre submanifold of the closed-loop contact form $\theta_d$. Here we elaborate on this partial stabilization problem and discuss the choice of the function $\Phi$ defining the structure preserving feedback (3) in order to stabilize the closed-loop system.

3.1 Equilibrium

In this subsection, we analyse the equilibrium conditions of the closed-loop contact vector field (4) with the shaped Hamiltonian (5) and with respect to the function $\Phi$ defining feedback in equation (3). Let us first recall the characterization of an equilibrium points for the closed-loop contact vector field $\dot{X}_K$. A state $(x_0^*, x^*, p^*)$, is an equilibrium point of the closed-loop contact vector field $\dot{X}_K$ if and only if the three following conditions are fulfilled (Favache et al. [2009]):

$$\dot{K} \big|_{(x_0^*, x^*, p^*)} = 0$$

$$\frac{\partial \dot{K}}{\partial p} \big|_{(x_0^*, x^*, p^*)} = 0$$

$$\frac{\partial \dot{K}}{\partial x} \big|_{(x_0^*, x^*, p^*)} = -p^* \frac{\partial \dot{K}}{\partial x_0} \big|_{(x_0^*, x^*, p^*)}$$

Using the expression (5) of the contact Hamiltonian function in closed-loop and the fact that the internal Hamiltonian $K_0$ and the control Hamiltonian function $K_c$ are invariant of the Reeb vector field, these conditions are equivalent to

$$(K_0 + (\Phi \circ K_c) K_c) \big|_{(x_0^*, x^*, p^*)} = 0$$

$$(\frac{\partial K_0}{\partial p} + \frac{\partial K_c}{\partial p} (\Phi \circ K_c) \big|_{(x_0^*, x^*, p^*)} = 0$$

$$(\frac{\partial K_0}{\partial x} + \frac{\partial K_c}{\partial x} (\Phi \circ K_c) \big|_{(x_0^*, x^*, p^*)} = 0$$

Note firstly that the equation (9) implies that that, if $K_c(x_0^*, x^*, p^*) \neq 0$, $\Phi \circ K_c \big|_{(x_0^*, x^*, p^*)} = -K_0 \ast (K_c)^{-1} \big|_{(x_0^*, x^*, p^*)}$

Note secondly that, if the closed-loop equilibrium is also an equilibrium for the open-loop system, that is the
Hamiltonian function $K_0$ satisfies the equations (6) (7) and (8), then the equilibrium conditions in closed-loop are satisfied if $\Phi \circ K_c \mid (x_0^*, p^*) = 0$ and $\Phi' \circ K_c \mid (x_0^*, x^*, p^*) = 0$.

3.2 Local stability of the closed-loop contact system

It has been proven in ([Ramirez 2012, chap. 4]) that the Jacobian matrix $D\hat{X}_K$ of the strict contact vector field $\hat{X}_K$ in closed-loop has 1 zero eigenvalue and that the remaining $2n$ non-zero eigenvalues are symmetrical with respect to the imaginary axis. Thereby the justification of partial stability on some invariant Legendre submanifold had been formulated and justified.

In this subsection, we shall express the Jacobian matrix in terms of the function $\Phi$ defining the structure preserving feedback (3).

The Jacobian matrix of the closed-loop contact vector field in canonical coordinates $(x_0, x, p)$ is

$$ D\hat{X}_k = \frac{\partial \hat{X}_K}{\partial (x_0, x, p)} = DX_{K_0} + D\left(\Phi' \circ K_c \right)X_{K_c} \tag{12} $$

where

$$ D\left(\Phi' \circ K_c \right)X_{K_c} = DX_{K_c} \cdot (\Phi' \circ K_c) + \Phi'' \cdot \frac{\partial K_c}{\partial x}^T \cdot X_{K_c} $$

One may observe that the Jacobian matrix depends on $(\Phi' \circ K_c) (x_0^*, x^*, p^*)$ and $(\Phi'' \circ K_c) (x_0^*, x^*, p^*)$. However in equation (10), the value of $(\Phi' \circ K_c) (x_0^*, x^*, p^*)$ is calculated by the equilibrium condition (9, 10, 11). Hence the poles will be merely determined by the value of $\Phi'' (K_c(x_0^*, x^*, p^*))$. This resembles to the Root Locus method to determine the dynamic behaviour of closed-loop contact system by changing the output gain, that is to say, the value of $(\Phi'' \circ K_c) (x_0^*, x^*, p^*)$.

3.3 Computation of the stable submanifold Legendre

In this subsection, a stable submanifold Legendre will be built for the partial stabilization of the closed-loop contact systems. Indeed, according to the stable manifold theorem (Marle [2003]), there exists an unique submanifold $L$ of the contact manifold, which is invariant with respect to the closed-loop vector field $\hat{X}_K$, tangent to a stable linear subspace, noted as $\Pi$ at the equilibrium point. By considering under which condition the obtained contact system in closed-loop is also conservative, one ask for the closed-loop contact vector field $X$ leaving invariant some Legendre submanifold $L_{\Phi}$ which is satisfied if and only if ([Ramirez 2012, Page 56])

$$ K_0(x, p) \mid L_{\Phi} = c_F \tag{13} $$

It is of interest that the closed-loop contact system leaves the Legendre submanifold $L_{\Phi}$ invariant, which is derived from a desired function $U_d(x)$. In canonical coordinates $(x_0, x, p)$, this Legendre submanifold is shown as

$$ L_{\Phi} : \begin{cases} x_0^d = U_d(x), \\ x = x, \\ p^d = \frac{\partial U_d}{\partial x}(x) \end{cases} \tag{14} $$

With this expression, the equation (13) turns into

$$ K_0(x, \frac{\partial U_d}{\partial x}(x)) + \Phi \circ K_c(x, \frac{\partial U_d}{\partial x}(x)) = -c_F $$

The condition turns out to be an equation of $U_d(x)$ which is a first order multi-variables partial differential equation (PDE). To solve this PDE of $U_d(x)$, the initial condition of $U_d(x)$ is required which is provided by the closed-loop equilibrium point:

$$ U_d(x^*) = x_0^* \tag{15} $$

Now we have to investigate the uniqueness of the solution of the PDE (14). Since the first-order multi-variable PDE (14) is generally non-linear, there could exist multiple solutions if it is solvable. Note that by the center manifold theorem, it could be proven that for a hyperbolic equilibrium point, there exists one stable and one unstable submanifolds of dimension $n$ which are both Legendre submanifolds. Moreover, at the equilibrium point, the invariant Legendre submanifold is tangent with $\Pi^-$. This tangent relation could offer us some restrictions on the $\frac{\partial U_d}{\partial x}(x^*)$ which enable the removal of the false solutions for the PDE (14).

3.4 Lyapunov function and availability function

In this subsection, we will analyse the global asymptotic stability of closed-loop contact system on the invariant Legendre submanifold. So far we have found the invariant Legendre submanifold on which the closed-loop contact system is partially stable. To realize a performance of control on this submanifold, the Lyapunov’s direct method will exploited.

Recall the conditions of Lyapunov function under the assumption that the Lyapunov function depends only on $x$ (Ramirez [2012]): Consider a contact manifold $(M, \theta_d)$ and a set of canonical coordinates $(x_0, x, p)$, a contact vector field $\hat{X}_K = X_{K_0} + (\Phi' \circ K_c)X_{K_c}$, with strict contact Hamiltonian $\hat{K} = K_0 + \Phi \circ K_c$, a Legendre submanifold $L \subset M$ solution to the Pfaffian equation $\theta_d = 0$, an initial state $(x_0(0), x(0), p(0)) \in L$ and the equilibrium state $(x_0^*, x^*, p^*) \in L$ of $X$. Let $V(x) : M \to \mathbb{R}$ be a continuously differentiable function on $M$ such that

$$ V(x^*) = 0 \tag{16} $$

$$ \frac{dV}{dt} = -\frac{\partial K_0}{\partial p} \frac{\partial V}{\partial x} \mid_{L} \leq 0, \quad \forall (x_0, x, p) \mid_{L} = \{x \geq (x_0^*, x^*, p^*)\} \tag{17} $$

Then $(x_0, x, p) = (x_0^*, x^*, p^*)$ is asymptotically stable on $L$.

Remark 1. By reason of the restrictions of the term $\frac{\partial U_d}{\partial p}$ found in the previous subsection, the desired Hamiltonian function $U_d$ can hardly satisfy the equation (18) and be considered as Lyapunov function candidate. Nevertheless we can define an availability function based on the desired Hamiltonian function $U_d$ as a Lyapunov function candidate. According to the articles (Ramirez et al. [2013], Hoang et al. [2012]), in order to make the availability function satisfy the conditions of Lyapunov (16, 17, 18), the desired Hamiltonian function must be convex. This condition allows us to evaluate if the preserving structure
state feedback has been well chosen while solving the PDE (14) in the previous subsection.

Assume that the preserving structure state feedback has been well chosen and the desired Hamiltonian function has been convex. Then an availability function could be defined:

$$A(x) = U_d(x) - U_d(x^*) - k^T(x - x^*)$$  \hspace{1cm} (19)

where $k = \frac{\partial U_d}{\partial x}(x^*)$. $A(x)$ is convex and the time derivative of $A$ is given by

$$\frac{dA}{dt} = \left(\frac{\partial U_d}{\partial x}(x) - \frac{\partial U_d}{\partial x}(x^*)\right)^T \frac{dx}{dt} = \left(\frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*)\right)^T \left[ \begin{array}{c} R_{J} \frac{\partial U}{\partial x}(x) + g(x, u) \\ -\gamma \frac{\partial U}{\partial x}(x^*) J \frac{\partial U}{\partial x}(x) \{S, U\}_T \\ + \left(\frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*)\right)^T g(x, u) \end{array} \right]$$

Recalling the equilibrium conditions of closed-loop contact vector field (6, 7 and 8), for the equilibrium point $(x_0^*, x^*, p^*)$, we have

$$\begin{cases} 
T_1^* = T_2^* = T = p^* = p_1^* = p_2^* = \Phi'(K_C^*) = \Phi(0) \\
\Phi(K_C^*) = 0
\end{cases}$$  \hspace{1cm} (23)

Then, by recalling the expression (12) and adding the equilibrium condition (23), we can achieve the expression of the Jacobian matrix of closed-loop contact vector field $\frac{dX}{dt}$ in canonical coordinates $(x_0, x, p)$, at equilibrium point $(x_0^*, x^*, p^*)$, as shown in (24), where $\Phi''(0)$ represents $\Phi''(0)$.

Afterwards we calculate the determinant of the Jacobian matrix $\frac{dX}{dt}$ and the following symbols are proposed to simplify the expression:

$$A = \frac{\lambda_c}{T^*} \frac{\partial T_1}{\partial S_1}(T_1)$$  \hspace{1cm} (25)

$$B = \lambda_c^2 \frac{\partial^2 T_2}{\partial S_2^2} - \frac{\lambda_c}{T^*}^2 \frac{\partial^2 T_2}{\partial S_2^2} - \frac{\lambda_c^2}{T^*} \frac{\partial T_2}{\partial S_2} \Phi''$$  \hspace{1cm} (26)

$$C = \frac{\lambda_c}{T^*} \frac{\partial T_2}{\partial S_2} - \frac{\lambda_c}{T^*}^2 \frac{\partial T_2}{\partial S_2} \Phi''$$  \hspace{1cm} (27)

$$D = \frac{\lambda_c}{T^*} \frac{\partial T_2}{\partial S_2} \Phi''$$  \hspace{1cm} (28)

Then the expression of the determinant of the Jacobian matrix $\frac{dX}{dt}$ becomes

$$det(SI_n - \frac{dX}{dt}) = S \left[ -S^4 + \left( \frac{\lambda_c}{T^*} \frac{\partial T_2}{\partial S_2} \Phi'' + \right) A^2 + C^2 + 2D \right]$$

As mentioned in the subsection 3.2, the point $(0, 0)$ is one of the eigenvalues of the Jacobian matrix $\frac{dX}{dt}$ and the other 2n eigenvalues are symmetrical with respect to the imaginary axis. Therefore we assume that the remaining eigenvalues of the Jacobian matrix $\frac{dX}{dt}$ are denoted as
Afterwards we continue to analyse the stability of the closed-loop contact system, we only consider the stability of the contact Hamiltonian, 

\[ H = \dot{Q}^T \mathbf{Q} \] 

so there exists one stable and one unstable submanifolds of dimension 2 which are Legendre submanifolds. Based on the contact Hamiltonian and the expression of the feedback Φ, we can find that once the feedback Φ has been chosen, the expression of \( r \) and \( \theta \) can be determined and consequently the 2n non-zero poles will be placed properly in the complex plane.

Afterwards we continue to analyse the stability of the heat exchanger, using the center manifold theorem shown in the subsection 3.3. Obviously, there should be 2n + 1 eigenvalues of the Jacobian matrix \( D\dot{X}_K(x^*) \) which implies that there should be 2n + 1 poles in the complex plane. Among all the poles, there is a zero pole and the other 2n poles are symmetrical about the imaginary axis. Besides, \( -2n \) and \( +2n \) are independent of the function of \( \Phi(x) \), thus they do not influence the feedback on the non-zero poles. On the contrary, owing to the existence of \( \Phi(0) \), if \( \Phi(x) \) is the expression of the feedback \( \Phi(x) \), then these \( 2n \) poles will depend also on \( \Phi(0)^2 \) except for \( \Phi(0) \). In addition, we find that once the feedback \( \Phi \) has been chosen, the expression of \( r \) and \( \theta \) can be determined and consequently the 2n non-zero poles (n stable and n unstable) will be placed properly in the complex plane.

As the contact Hamiltonian is invariant in the Legendre submanifold,

\[ K_{Ud} = (K_0 + \Phi \circ K_C)|_{Ud} = 0 \]

Then the equation becomes a first order quadratic PDE with two variables:

\[ K_0(x, \frac{\partial U_d}{\partial x}(x)) + \Phi \circ K_0(x, \frac{\partial U_d}{\partial x}(x)) = -c_F \]

Assume that \( \Phi(\lambda) = K_0 \) with \( K_0 \) constant and the equation above becomes

\[ \lambda - \lambda T_1 \left( \frac{1}{T_2} - \frac{T_2}{T_1} \right) - \frac{\lambda e T_0}{T_2} \partial U_d \left( \frac{\partial U_d}{\partial x} \right) = \lambda e T_0 - k e \]

This PDE can be solved by the method of characteristics (Myint-U and Deb Nath [2007]) with the initial condition (15),

\[ U_d(x_1, x_2) = -\frac{\lambda e x_1}{\lambda} - \frac{\lambda e T_2}{\lambda} \ln(T_1 - T_2) + \frac{\lambda e T_0^2}{\lambda T_1} \ln(T_1 - T_2) + \frac{\lambda e T_0^3}{\lambda T_1 T_2} \ln(T_1 - T_2) + \frac{\lambda e T_0^4}{\lambda T_1 T_2} \ln(T_1 - T_2) \]

So there exists one stable and one unstable submanifolds of dimension n which are Legendre submanifolds. Based on the center manifold theory, we can construct the characteristic subspace associated with stable poles, noted as \( \Pi^- \),

\[ \Pi^- = ker(s_3 I_5 - D\dot{X}_K) \oplus ker(s_4 I_5 - D\dot{X}_K) \]

Assume that the kernel of the matrix \( s_3 I_5 - D\dot{X}_K \) is a vector of dimension \( \mathbb{R}^{5 \times 1} \), noted as \( v = [v_0, v_1, v_2, v_3, v_4]^T \) which fulfills

\[ (s_3 I_5 - D\dot{X}_K)v = 0 \]

In the same way, we consider the kernel of the matrix \( s_4 I_5 - D\dot{X}_K \) as a vector of dimension \( \mathbb{R}^{5 \times 1} \), noted as \( v' = [v'_0, v'_1, v'_2, v'_3, v'_4]^T \) and we obtain the expression of the characteristic subspace associated with stable poles \( \Pi^- \):

\[ \Pi^- = \{ u \in \mathbb{R}^{5 \times 1} \mid u = k_1 v + k_2 v', \forall k_1, k_2 \in \mathbb{R} \} \]

In respect to the invariant Legendre submanifold is tangent with the subspace \( \Pi^- \) at equilibrium point,

\[ T_x L_{Ud} = \Pi^- \]
So we have

\[
\begin{pmatrix}
\frac{dU_d(x^*)}{dx_1} \\
\frac{dU_d(x^*)}{dx_2}
\end{pmatrix} = w_1 dx_1 + w_2 dx_2
\]

Obviously, the rank of the matrix \( Y = [v \quad v \quad w_1 \quad w_2]^T \) is 2. Through the calculus of the determinant of partial matrix of \( Y \), we can finally obtain the expression of \( \frac{\partial U_d}{\partial x_1}(x^*) \) and \( \frac{\partial U_d}{\partial x_2}(x^*) \):

\[
\frac{\partial U_d}{\partial x_1}(x^*) = -\frac{K\lambda e}{\lambda} - \frac{\lambda + \lambda e}{\lambda} f' (x^*_2 - \frac{\lambda + \lambda e}{\lambda} x^*_1)
\]

\[
\frac{\partial U_d}{\partial x_2}(x^*) = f' (x^*_2 - \frac{\lambda + \lambda e}{\lambda} x^*_1)
\]

At equilibrium point, we have \( \frac{\partial U_d}{\partial x_1}(x^*) = T^* \) and \( \frac{\partial U_d}{\partial x_2}(x^*) = T^* \). Therefore we could infer that

\[ K = T^* \]

It should be remarked that \( U_d \) depends linearly on the preserving structure feedback \( \Phi_0 \). Once the feedback \( \Phi \) has been chosen, the expression \( dU_d(x^*) \) could be determined. These two equations above provide us a general idea while choosing the desired invariant Legendre submanifolds \( L_{U_d} \) and the preserving structure feedback.

Then we compute the availability function (19) as

\[ A(x) = U_d(x) - U_d(x^*) - K^T (x - x^*) \]

Moreover, the conditions (16, 17) are satisfied in the equation (33) and the time derivative of \( A \) could be formulated as

\[
\frac{dA}{dt} = (\frac{\partial U_d}{\partial x}(x) - \frac{\partial U_d}{\partial x}(x^*)) \cdot \frac{dx}{dt}
\]

\[
= -\gamma (x^*) J \frac{\partial U_d}{\partial x}(x) \{S, U \} J + \left( \frac{\partial U}{\partial x}(x) - \frac{\partial U}{\partial x}(x^*) \right)^T g(x, u)
\]

\[
= -\frac{\lambda T^*}{T_1 - T_2} (T_1 - T_2)^2 + (T_2 - T^*) \frac{u - T_2}{u T_2}
\]

\[
= -\frac{\lambda T^*}{T_1 - T_2} (T_1 - T_2)^2 + (T_2 - T^*) \frac{K - T_2}{K T_2}
\]

The equation (34) equals zero at equilibrium point and besides the equilibrium point, the first term is always less than zero. Hence the closed-loop system will converge asymptotically to the equilibrium point if the second term is less than zero,

\[
(T_2 - T^*) \frac{K - T_2}{K T_2} \leq 0
\]

The easiest way is to let \( K = T^* \) which matches the previous results (32).

5. CONCLUSION

In this paper, we conduct several control syntheses of a structure preserving state feedback as follows: After recalling the previous work (Kotyczka [2013]), the local stability of the closed-loop contact system has been studied; The invariant stable submanifold Legendre has been formulated; The possibility of constructing an availability function as Lyapunov function candidates on the Legendre submanifold has been discussed.

Our future work will concentrate on the application of the stabilization theory of closed-loop contact system to more complex thermodynamic process or biologic systems.

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