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The multi-period vehicle routing problem with refueling decisions: Traveling further to decrease fuel cost?

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ABSTRACT

Most vehicle routing approaches disregard the need to refuel fleets. However, planners search for opportunities to refuel at lower prices even if, counter-intuitively, distant fuel stations need to be visited. We propose a novel mathematical formulation and develop branch-and-cut and matheuristic algorithms to efficiently tackle this problem. Results indicate that, to minimize costs, detour distances may increase up to 6 percentage points when fuel stations with lower prices are farther away from the depot. For practice, these insights imply that current policies disregarding station location and/or fuel prices along with “myopic” planning horizons may lead to sub-optimal decisions.

1. Introduction

Most of the literature on Vehicle Routing Problems (VRPs) does not incorporate the price of fuel when finding the optimal route or routing policies, and the reason being that most transportation companies operate within the same country, where price variability is minimal. Hence, in most cases the nearest gas station is preferred to save time, or refueling is simply ignored from the problem formulation. However, there may be instances when the price differential is so significant that taking into account both the location of fueling stations and the price of gas is not only reasonable but pivotal for achieving efficient operations.

As a real example consider, for instance, the price differential between fuel diesel in Portugal and Spain, which is currently around thirty cents on the dollar, meaning that fuel is considerably more expensive in Portugal. Given the geographical proximity and the degree of market integration between these two countries, it makes sense, from an economic and managerial perspective, to take the cost of refueling into consideration, as significant savings can be obtained. This is particularly relevant since there is a significant number of companies owning truck fleets that operate and cross both countries on a daily basis. Fig. 1 illustrates a real example of a retail company that has delivery locations in both countries.

In the freight transport industry, in particular, fuel costs represent a significant share of the operational costs. Although electric engines may soon displace traditional combustion engines, solving some of these concerns, especially the ones arising from the high volatility of the oil price, the majority of truck fleets still in operation are powered by diesel engines, meaning that these problems are not expected to be gone anytime soon (Unfried, 2018).

Some work in the literature tries to address this in an indirect way, but still does not consider potential savings from choosing the gas station. Typically, the route is planned so that the distance traveled or fuel consumption is minimized. In some cases this objective is further enhanced by guiding the truck payload to be distributed in a fuel-efficient way, so that heavier items are dropped off first

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Fig. 1. In Portugal, there is a high density of delivery locations near the coast and most warehouses are located nearby. A smaller number of delivery locations are also positioned near the border of Spain, representing good opportunities to refuel at a much lower price for a small increment in the travelled distance.

(see, for example, [Suzuki \(2011\)](#)).

Let us emphasize the important fact that minimizing fuel consumption is not the same as minimizing fuel cost when vehicles have the possibility to refuel at different prices. This means that it is possible to have different costs (in monetary units) for the same fuel consumption (in volumetric units). In a context where several fuel prices are available, a distance minimization objective is no longer adequate to improve fuel cost as it disregards the option of refueling at different costs.

Despite the concern of minimizing fuel costs, which is erroneously substituted by a distance minimization objective in several cases, few studies consider price differentials between fuel stations, and when they do, fuel stations are selected *ex post* of the optimal route, i.e. a sub-problem of finding the nearest station next to a previously defined path is solved after generating the route. We argue that, in some regional contexts, the price differential between fuel stations may be relevant enough, such that it should be incorporated in the original optimization problem. For example, Portugal and Spain, where price differentials may rise up to 36 cents per litre, invite many drivers for a detour in order to fill up the tank in Spain ([Atalaia, 2018](#)). When tackling these problems together (i.e. routing and refueling), a trade-off then arises between reducing detours and saving on fuel costs. Considering the difference between fuel prices, which in some cases may go up to 20%, ignoring the fuel costs may trim the operational margin significantly as we will latter illustrate in the computational experiments.

The contributions of this paper are towards solving this integrated problem of routing and refueling and are threefold. Firstly, we introduce a new problem, proposing a respective mathematical formulation to extend the periodic VRP by considering refueling decisions. Secondly, we develop two solution methods to solve it (an exact branch-and-cut algorithm and a matheuristic). Finally, we perform extensive computational experiments and a sensitivity analysis to provide valuable managerial insights on refueling decisions in a finite time horizon with multiple periods.

The remainder of the paper is organized as follows. Section 2 presents a literature review of refueling-related problems, providing a brief description of the gaps to be filled by this research. In Section 3, we describe the problem we aim to tackle and propose a novel mathematical formulation to model it. Section 4 presents the proposed branch-and-cut and the matheuristic approach for solving larger instances. Section 5 provides extensive computational experiments aiming at deriving managerial insights on refueling decisions through a finite time horizon. Finally, Section 6 presents the conclusions and suggests future research avenues concerning refueling decisions.

2. Literature review

The importance of refueling decisions, which is motivated in the former section, has been neglected in most transportation planning activities in practice, which are usually tackled through rules of thumb. The literature provides several mathematical formulations and solution approaches for defining refueling strategies for several types of vehicles. Few, however, consider the

impact of fuel costs on the optimality of the routes, which may be non-negligible if substantial gains can be obtained by selecting the cheaper stations. We review the literature related to our problem, focusing on road transportation problems, which mainly includes vehicle routing problems with fuel consumption (with no refueling option), vehicle routing problems with refueling decisions, and electrical vehicle routing problems.

2.1. Vehicle routing with fuel consumption

To the best of our knowledge, fueling considerations in vehicle routing date back to 1983 where [Ichimori et al. \(1983\)](#) introduced routing problems with the limitation of fuel. Later, other authors proposed routing problems where, for instance, a fuel consumption rate is considered. This means that the fuel consumption between two locations is variable according to some features related to the truck load, road conditions, weather conditions, etc. Note that in these works, the fuel price does not influence the fuel consumption.

[Xiao et al. \(2012\)](#) explore the influence of the vehicle load on the fuel consumption rate and consider it in the Capacitated Vehicle Routing Problem (CVRP). A mathematical model is proposed and solved by means of a Simulated Annealing (SA) algorithm with a hybrid exchange rule. The authors provide interesting insights on how CVRP route structure changes when fuel consumption is considered. This work does not consider refueling decisions and the instances are single periodic. [Sundar et al. \(2016\)](#) present formulations and algorithms for the fuel-constrained VRP with multiple depots acting as refueling stations. The objective is to compute a set of routes able to visit a set of customers without allowing any vehicle to run out of fuel. A set of random generated instances considering a single period is solved with a general-purpose solver. Four different formulations are compared in terms of solution quality, however, the paper does not provide any insight on the solution structure of fuel-constrained VRPs. Similarly to the later paper, [Suzuki \(2011\)](#) focuses on the minimization of fuel consumption and pollutants emissions in a time constrained Traveling Salesman Problem with Time Windows (TSPTW). Heavier items need to be unloaded first so as to travel less kilometers with higher fuel consumption rates. The authors report fuel savings between 4.9% and 6.9% in random generated instances where a set of customers is served by a single tour. Recently, in [Suzuki and Lan \(2018\)](#), the authors propose fuel-saving approaches for the U.S. truckload industry. It is concluded that buying large amounts of fuel before entering uphill or congested road segments is not beneficial to fuel consumption. The authors indicate that the study has limitations particularly concerning the simplicity of the solution approach and the lack of empirical support. To the best of our knowledge, [Cheng et al. \(2018\)](#) is the only work that actually considers multiple periods while minimizing refueling cost in an Inventory-Routing Problem (IRP). However, the authors assume that the fuel price is constant and do not track the fuel quantities in each vehicle. Since the vehicles have unlimited fuel, they do not have the necessity to perform detours to visit fuel stations. The only variable influencing the fuel consumption is the load of each vehicle when traversing an arc, which is not period-dependent.

All the aforementioned works disregard refueling decisions which is a critical feature in the problem we are proposing. Additionally, they generally focus on reducing fuel consumption which is different from reducing fuel cost.

2.2. Vehicle routing with refueling decisions

In this subsection we review the literature considering routing problems where the vehicles need to refuel the fuel tank at some point.

[Suzuki \(2009\)](#) claims that commercial fuel optimizing software usually confiscate the driver's freedom to choose truck stops and propose an approach to increase driver compliance rates. The author explores available commercial fuel optimizing options and indicate that none is able to consider dynamic fuel prices (i.e., prices change on a daily basis). A decision support tool based on a stochastic dynamic programming model is validated with real-world data. The work provides interesting insights on the behavior of the drivers by comparing several strategies to perform stops. However, the proposed methodology requires a large investment on GPS and satellite communication systems and is mostly suited for routes spanning several days. This is, however, one of the few attempts to consider different fuel prices per refueling station. Suzuki has been involved in other decision support tools concerning several refueling problems (e.g., [Suzuki \(2012\)](#), [Suzuki and Jing \(2013\)](#) and [Suzuki et al. \(2014\)](#)).

[Lin et al. \(2007\)](#) adapt the inventory-capacitated lot-sizing problem to solve the Fixed-Route Vehicle Refueling Problem (FRVRP). In this paper, vehicles travel along a fixed route with a series of fuel stations. Between each node, fuel is consumed at different rates due to terrain conditions and it can be purchased at different prices. A linear-time greedy algorithm is proposed for finding optimal refueling policies, but the paper lacks a section with computational experiments. [Khuller et al. \(2011\)](#) state that the significant variance in the price of fuel between fuel stations may have a large impact on the transportation cost. The authors claim that inside the Washington DC area the price amplitude can be as much as 20%. The paper focuses on shortest path problems (the authors define the problem as a Variable-Route Vehicle Refueling Problem (VRVRP)) where refueling decisions need to be addressed, solving them with both exact and approximation algorithms. However, the paper also lacks a section with computational experiments and does not provide further insights on these problems. In a computational-oriented study, [Suzuki \(2014\)](#) proposes a variable reduction technique to increase the efficiency of solution approaches to the FRVRP. The author presents the mathematical formulation for the problem and a methodology to find unattractive refueling points based on optimal conditions that need to be shown by optimal solutions. The results suggest large reductions in the number of considered fuel stations and in the computational time. Note that the problem we are addressing is more complex because routes are not fixed but are jointly defined with the stops at each refueling station. [Bousonville et al. \(2011\)](#) tackle a Vehicle Routing Problem with Time Windows (VRPTW) with refueling decisions by means of a constructive heuristic which verifies solution feasibility using a refueling model. The solution quality is not guaranteed by any means as no local search is applied to the solutions provided by the proposed constructive heuristic.

We consider that the aforementioned articles do not properly integrate routing and refueling decisions. First, most of them deal with the FRVRP which starts with a given routing sequence. Second, when integrated routing and refueling models are presented, they are solved sequentially or they do not guarantee optimality of the integrated problem (see Bousonville et al. (2011)).

2.3. Electric and alternative-fuel vehicles

The recent advances on electric vehicles' autonomy have shifted the interest of researchers towards new transportation problems. Hence, new modelling and refueling strategies are being deployed for electrical and other alternative-fuel vehicles. Yavuz and Çapar (2017) propose a model that is able to consider several alternative-fuel vehicles by adopting different driving ranges, refueling times, and availability of refueling stations. To solve the model, an efficient Variable Neighborhood Search (VNS) heuristic is developed. The approach is flexible enough to consider both alternative-fuel and gasoline or diesel vehicles. Extensive computational experiments show the impact of considering different fleet mixes on four objective functions. It is concluded that minimizing the distance traveled is not always desirable and that increasing range extension and the number of refueling stations may be better than faster refueling capability. Lin et al. (2016) present a general formulation for the Electric Vehicle Routing Problem (EVRP) for finding minimal cost routing strategies using electrical vehicles. The battery consumption is affected both by travel speed and vehicle load, which depend on customer demand and the visiting order. In a case study, the authors conclude that electrical vehicles can translate into a considerable amount of labor cost due to long re-charging times, despite their advantageous zero-emission characteristics. Furthermore, an interesting conclusion of this work is that the relative distribution of charging stations to customer locations largely affects routing strategies. Schneider et al. (2014) introduce the Electric Vehicle Routing Problem with Time Windows and Recharging Stations (E-VRPTW). A novel mathematical formulation that allows vehicles to recharge at available stations using a certain re-charging scheme is proposed. Moreover, a hybrid heuristic combining a VNS with a Tabu Search (TS) is used to solve larger instances of the problem. The computational experiments show that the model can be solved with general-purpose solvers if the instances are small. Furthermore, the benefits of the proposed hybrid approach are also shown in several sets of instances of related VRP problems. From a single period perspective, this paper models battery capacity similarly to our approach to the fuel deposit of each vehicle, as shown later in this paper.

Analyzing the aforementioned papers, it is clear that most models have been focusing on single period problems, and that few consider the financial impact of fuel price differentials between stations. In this regard, most studies assume that price is homogeneous throughout the fuel stations, which may not be the case in reality. Additionally, few papers truly integrate the VRP with refueling decisions as most of them consider routes that are fixed beforehand. In reality, it is common to find transportation activities where a single fuel deposit is able to serve several periods. This means that a certain vehicle can have a large number of opportunities to refuel at different locations, with different prices at different moments in time. To the best of our knowledge, planning multi-priced refueling decisions over a time horizon with multiple periods has not been considered by the VRP literature. Therefore, we aim at filling this literature gap by addressing The Multi-Period Vehicle Routing Problem with Refueling Decisions (mPVRPR). To better position this problem in the literature, we present Table 1, presenting recent works dealing with fuel consumption and refueling decisions.

3. Problem description and mathematical formulation

To introduce the mPVRPR, consider a complete graph $G = (V, E)$ where the set of vertices $V = \{0, 1, \dots, v, v + 1\}$ is partitioned into vertex 0, which is a departing vertex at the depot, vertices $\{1, \dots, v\}$, corresponding to v locations that can be customers belonging to the set N or fuel stations belonging to the set S , and vertex $v + 1$, which is an arrival vertex at the depot. Let K be a set of vehicles, based at the depot, which can be used to serve the demands d_i^t of each customer $i \in N$ in each period $t \in T = \{1, \dots, e\}$. One period corresponds to the time needed to perform one route, usually a day or a work shift. Each vehicle k needs to respect its capacity C^k and the fuel level of its tank needs to be maintained between a lower and upper limit defined by the interval $[L^k, U^k]$. The initial fuel level of each vehicle k is given by I^k . Whenever a vehicle traverses an edge $(i, j) \in E$, its fuel level is reduced by a quantity c_{ij}^k . In order to raise the fuel level of the tank, each vehicle is able to visit a fuel station s and buy a certain quantity of fuel at a unitary price of p_s . The objective is to serve all customers' demand over a finite planning horizon, while deciding when and where to refuel each vehicle, trading-off fuel consumption with different fuel prices provided by each fuel station. The cost structure is considered to be fixed, with a fixed fleet that is to be used as efficiently as possible. Note that with the possibility to refuel at different prices, the typical distance minimization objective function will no longer be a correct measure because the vehicle is consuming a mixture of fuel bought at difference prices. The initial fuel quantity is a sunk cost (purchased in a previous planning period), but if this quantity is not sufficient, an additional quantity needs to be purchased, just like in real life. Hence, total cost of refueling is the goal to be optimized. Fig. 2 presents an example comprising one route and one period of a mPVRPR.

3.1. Mathematical formulation

To model the mPVRPR, we propose a novel mathematical formulation, which models the deposit of each vehicle as an inventory of a certain commodity. The two most common inventory replenishment policies are the Maximum Level (ML) policy and the Order-up-to Level (OU) (Coelho et al., 2014). The replenishment or refueling policy considered in this research is based in the more flexible ML policy, meaning that any amount of fuel can be achieved in each visit to a fuel station. We use binary decision variables x_{ij}^{kt} and z_i^{kt} for defining routing decisions, and continuous variables f_i^{kt} and r_s^{kt} for dealing with refueling decisions. Let x_{ij}^{kt} be the binary

Table 1

The literature regarding integrated vehicle routing with refueling is scarce. Most approaches found in the literature consider a single fuel price, a single period, and generally do not deal with integrated routing problems as the routing variables are fixed beforehand. We are particularly interested in the mPVRP with multiple vehicles, periods and fuel prices.

Authors (year)	Problem Description					Solution Method		
	Routing problem	Integrated routing	Fuel price	Refuel option	Planning horizon	Objective function	Type	Approach
Xiao et al. (2012)	CVRP	✓	Single	×	SP	Vehicle fixed cost and fuel cost	H	Simulated annealing
Sundar et al. (2016)	VRP	✓	Single	×	SP	Fuel consumption	E	General-purpose solver (CPLEX)
Suzuki (2011)	TSPTW	✓	Single	×	SP	Fuel consumption	H	Compressed annealing
Suzuki and Lan (2018)	FRVRP	×	Single	×	SP	Fuel consumption	H	Commercial software and manual procedure
Cheng et al. (2018)	IRP	✓	Single	×	MP	Inventory, fleet, driver, and fuel cost	E	Branch-and-cut algorithm
Suzuki (2009)	FRVRP	×	Multiple	✓	MP	Fuel cost	H	Dynamic programming following Before-after and Min-max rules
Suzuki (2012)	TSPTW	✓	Multiple	✓	SP	Fuel cost	H	Compressed annealing and General-purpose solver (CPLEX)
Suzuki and Jing (2013)	VRVRP	✓	Multiple	✓	SP	Traveled distance and fuel cost	E	General-purpose solver (CPLEX) to create Pareto fronts
Suzuki et al. (2014)	FRVRP	×	Multiple	✓	SP	Fuel cost	E	General-purpose solver (CPLEX)
Lin et al. (2007)	FRVRP	×	Multiple	✓	SP	Fuel cost	E	Greedy algorithm achieves optimal policies
Khuller et al. (2011)	VRVRP	✓	Multiple	✓	SP	Fuel cost	H	Problem-specific algorithms
Suzuki (2014)	FRVRP	×	Multiple	✓	SP	Fuel cost	E	Variable-reduction technique and General-purpose solver (CPLEX)
Bousonville et al. (2011)	VRPTW	✓	Multiple	✓	SP	Fuel cost	H	Solumun II constructive heuristic with fuel feasibility checker MIP
Our Approach	mPVRP	✓	Multiple	✓	Multiple	Fuel cost	E/MH	Branch-and-cut algorithm and Variable MIP neighbourhood descent

Legend: SP - Single period | MP - Multi-period | H - Heuristic | E - Exact | MH - Matheuristic | mPVRP - Multi-period VRP.

Fig. 2. Example of a route and one period where the vehicle visits several customers and one fuel station to fill up the fuel tank before going back to the depot.

variables to indicate whether an edge (i, j) is traversed by vehicle k in period t and z_i^{kt} indicate if location i is visited by vehicle k in period t . Auxiliary continuous variables f_i^{kt} are used to define the level of fuel available in vehicle k when it arrives location i in period t . Finally, we define continuous variables r_s^{kt} to decide the fuel quantities filled in vehicle k at location s in each period t . The proposed formulation reads as follows: mPVRPR:

$$\text{minimize } \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} p_s r_s^{kt} \quad (1)$$

s.t.

$$\sum_{j \in V \setminus \{0\}} x_{ij}^{kt} = z_i^{kt} \quad \forall i \in V \setminus \{v+1\}, k \in K, t \in T \quad (2)$$

$$\sum_{j \in V \setminus \{v+1\}} x_{ji}^{kt} = z_i^{kt} \quad \forall i \in V \setminus \{0\}, k \in K, t \in T \quad (3)$$

$$\sum_{k \in K} z_i^{kt} = \begin{cases} 1, & \text{if } d_i^t > 0 \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N, t \in T \quad (4)$$

$$\sum_{j \in V} x_{0j}^{kt} = 1 \quad \forall k \in K, t \in T \quad (5)$$

$$\sum_{i \in N} d_i^t z_i^{kt} \leq C^k \quad \forall k \in K, t \in T \quad (6)$$

$$\sum_{i \in O} \sum_{j \in O} x_{ij}^{kt} \leq \sum_{i \in O} z_i^{kt} - z_g^{kt} \quad \forall O \subseteq V, k \in K, t \in T, \text{ for some } g \in O \quad (7)$$

$$f_i^{kt} + r_i^{kt} - c_{ij}^k \leq f_j^{kt} + U^k(1 - x_{ij}^{kt}) \quad \forall (i, j) \in E, k \in K, t \in T \quad (8)$$

$$f_i^{kt} + r_i^{kt} - c_{ij}^k \geq f_j^{kt} - U^k(1 - x_{ij}^{kt}) \quad \forall (i, j) \in E, k \in K, t \in T \quad (9)$$

$$r_i^{kt} \leq z_i^{kt}(U^k - L^k) \quad \forall i \in S, k \in K, t \in T \quad (10)$$

$$f_{v+1}^{kt-1} = f_0^{kt} \quad \forall k \in K, t \in T: t > 1 \quad (11)$$

$$f_i^{kt} \geq z_i^{kt} L^k \quad \forall i \in V, k \in K, t \in T \quad (12)$$

$$f_i^{kt} + r_i^{kt} \leq z_i^{kt} U^k \quad \forall i \in V, k \in K, t \in T \quad (13)$$

$$f_{v+1}^{kt} \geq I^k \quad \forall k \in K, t = |T| \quad (14)$$

$$z_i^{kt}, x_{ij}^{kt} \in \{0, 1\}, \quad f_i^{kt}, r_i^{kt} \in \mathbb{R}^+. \quad (15)$$

Objective function (1) minimizes the total cost of refueling the fleet over the considered planning horizon. Again, the typical VRP traveled distance minimization should not be used in this problem as the fuel in the deposit can be obtained at different prices. The vehicle flow conservation in customer locations is ensured by constraints (2) and (3). Constraints (4) force a customer to be visited if it has a positive demand. Each vehicle is allowed to leave the depot at most once, as ensured by constraints (5). If a certain vehicle in a certain period does not leave the depot, the fictitious arc $x_{0,v+1}^{kt}$ is used. Constraints (6) are vehicle capacity constraints and constraints (7) eliminate subtours in the solutions provided by the model. The fuel levels in the deposit of each vehicle are defined by constraints (8) and (9). In order to refuel, vehicles need to visit a fuel station, as imposed by constraints (10). Given that only fuel stations can provide fuel, variables r_i^{kt} are previously set to zero for every customer node. Fuel level continuity among periods is ensured by constraints (11). Note that the initial fuel level of each vehicle k is previously set by imposing $f_0^{k1} = I^k$. Additionally, the lower and upper levels of fuel in each vehicle are defined by constraints (12) and (13), respectively. Constraints (14) impose a final fuel level that is equal or greater than the initial fuel level. Finally, constraints (15) define the type and bounds of each variable.

3.2. Valid inequalities

To strengthen the formulation we propose some valid inequalities. Some of them are based on the VRP literature and adapted to the mPVRPR. Archetti et al. (2007) introduced few inequalities for the IRP that are also applicable to the mPVRPR.

$$x_{ij}^{kt} \leq z_i^{kt} \quad (i, j) \in E, k \in K, t \in T \quad (16)$$

$$z_i^{kt} \leq 1 - x_{0,v+1}^{kt} \quad i \in V \setminus \{0\}, k \in K, t \in T \quad (17)$$

Constraints (16) and (17) are usually referred as logical constraints. Constraints (16) impose that an arc can only be active if the origin node of the arc is visited while constraints (17) ensure that each vehicle can only visit a customer or fuel station node in case it leaves the depot.

In Archetti et al. (2007), the authors also propose a set of valid inequalities to compute a lower bound on the number of visits to each customer node. We adapt these constraints to provide a strengthened lower bound on the number of visits to each fuel station in our formulation.

$$\sum_{t=t1}^{t2} z_i^{kt} \geq \frac{\sum_{t=t1}^{t2} r_i^{kt}}{U^k - L^k} \quad i \in S, k \in K, t1, t2 \in T, t1 < t2 \quad (18)$$

Constraints (18) track the refueled quantities at each fuel station in order to strengthen the relaxation of the z_i^{kt} variables.

Coelho et al. (2012) have also proposed inequalities for the case of homogeneous fleets to cut symmetric solutions due to similarities between vehicles. In the mPVRPR case these inequalities need to be further adapted because the concept of homogeneity respects not only to the capacity for transporting products, but also to the fuel deposit. Therefore, the following inequalities are only valid if the vehicles are homogeneous in respect to their capacity and fuel deposit, meaning that C^k , I^k , L^k , and U^k need to be similar. Furthermore, given that the initial fuel level of each vehicle in the beginning of each period is a variable, these constraints can only be applied to the first period ($t = 1$). These inequalities are written as follows.

$$z_i^{kt} \leq \sum_{j < i} z_j^{k-1,t} \quad i \in N, k \in K, t = 1 \quad (19)$$

Constraints (19) state that if a customer i is assigned to vehicle k in the first period, then vehicle $k - 1$ needs to serve a customer with an index smaller than i .

Finally, we present a set of new inequalities proposed specifically to the mPVRPR.

(a) Instance with a price differential of 10 cents (small) between normal and cheap fuel stations.

(b) Instance with a price differential of 50 cents (large) between normal and cheap fuel stations.

Legend:

Depot
 Customer
 Normal price fuel station
 Cheap price fuel station

Fig. 3. Solutions of an instance with 10 customers, 3 vehicles, and 3 periods. Each vehicle is represented with a different color, arc pattern, and arc identification (i.e. vehicle 1 corresponds to arcs k1). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$\sum_{i \in S} \sum_{t=t_1}^{t_2} z_i^{kt} \geq \frac{\sum_{(i,j) \in E} \sum_{t=t_1}^{t_2} x_{ij}^{kt} c_{ij}^k - (f_0^{k,t_1} - L^k)}{U^k - L^k} \quad k \in K, \quad t_1, t_2 \in T, \quad t_1 < t_2 \quad (20)$$

$$x_{0i}^{kt} \leq \sum_{j \in N: i < j} x_{j,v+1}^{kt} + \sum_{s \in S} z_s^{kt} \quad i \in N, \quad k \in K, \quad t \in T \quad (21)$$

The set of valid inequalities (20) states that if the fuel quantity consumed in the interval $[t_1, t_2]$ is larger than the fuel quantity available in the beginning of t_1 , then at least one visit must have been performed to a fuel station. Inequalities (21) cut the symmetry of routes where fuel stations are not visited by stating that the arc leaving the depot needs to visit a node with a higher index than the one of the node connected to the returning arc. Note that these inequalities are only valid in the case of symmetric distance matrices.

To further define the initial and final fuel levels of each vehicle in between a given pair of periods, we add a new set of constraints.

Table 2

Comparing routing indicators achieved on the instances presented in Fig. 3 a and b.

	Price Differential		Difference $\left(\frac{b-a}{a}\right)$
	Small (a)	Large (b)	
Refueling cost	213.84	152.85	− 28.52%
Traveled distance	594	680	14.48%
Detour distance	97	168	73.20%
Normal Station Visits	3	1	−
Cheap Station Visits	0	2	−

$$f_0^{kt1} + \sum_{t=t1}^{t2} \left(\sum_{i \in S} r_i^{kt} - \sum_{(i,j) \in E} c_{ij}^k x_{ij}^{kt} \right) = f_{v+1}^{kt2} \quad k \in K, \quad t1 \in T, \quad t2 \in T, \quad t1 \leq t2 \quad (22)$$

Constraints (22) capture the relation between the initial and fuel levels with the consumption and refueling operations of each vehicle, in other words, the initial fuel level plus the refueled quantity minus the consumed quantity is equal to the final fuel level. As we show in [Appendix A](#), these constraints are critical to improve the linear relaxation of the formulation.

3.3. Illustrative example

To illustrate the solution structure and the inherent economic trade-offs considered by the mathematical formulation, [Fig. 3](#) presents an example comprising 10 customers, 3 vehicles, 3 periods, and 2 fuel stations with different fuel prices in two scenarios.

Several interesting facts occur in this example, which further motivate this research. In the small price differential instance presented in [Fig. 3a](#), we observe that each used vehicle needs to perform at least one visit to a fuel station. The vehicles try to find good opportunities to perform their refueling task. In each period, the geographical assignment of each vehicle is generally rotated, anticipating good points in time for performing refueling tasks with a small detour. Therefore, vehicles visit good regions to refuel one at a time in each period. This idea enforces the importance of considering multiple periods while planning transportation activities. Note that the cheaper fuel stations are never visited in this case. Since the price differential is low, it is not worth to drive large distances to refuel.

In the large price differential instance presented in [Fig. 3b](#), the solution is slightly changed. The vehicles continue to anticipate the best period to refuel but now, in periods 1 and 2, they prefer to drive a larger distance to refuel at lower price. Therefore, a larger differential between normal and cheap priced fuel stations may induce a substantial impact on the vehicle routes.

Consider [Table 2](#), for an overview over the main routing indicators that are impacted by these fuel price differentials.

The table shows the values for the refueling cost, traveled distance, detour distance, number of visits to normal price stations and number of visits to cheap price stations. When cheaper fuel is available in the example of [Fig. 3b](#), the refueling cost could be reduced by 28.52%. However, these savings are obtained by traveling a larger distance due to larger detours to access cheaper fuel prices. Note that the detour distance increased by 73.20%, which would be very counter-intuitive for manual planners who often search for solutions with small detours.

4. Solution approaches

The mPVRPR is $\mathcal{NP}$ -hard given that it comprises a VRP as a particular case. The mathematical formulation proposed for the problem can be solved by general-purpose solvers only when the size of the instances does not go beyond few customers, periods and vehicles. Note that for medium and large instances, the number of variables and constraints figuring in the formulation renders the problem intractable. For that reason, we propose two solution approaches to tackle the mPVRPR, an exact branch-and-cut algorithm and a matheuristic.

4.1. An exact branch-and-cut algorithm

To solve small to medium-sized instances exactly, we devise a branch-and-cut algorithm, which dynamically adds violated constraints during the search procedure. These constraints are usually complicating constraints that jeopardize the efficiency of mathematical formulations and should be previously removed from the formulation.

In the proposed mPVRPR formulation, constraints (7)–(9), dealing with subtour elimination and fuel flows, introduce a large degree of complexity to the problem. Therefore, in the branch-and-cut algorithm, these complicating constraints will be added dynamically whenever they are violated. [Algorithm 1](#) shows the general steps of our exact branch-and-cut algorithm.

Algorithm 1. Exact Branch-And-Cut

```

1: procedure BC(tlimit)
2:    $solution_{best} \leftarrow$  Generate an initial solution with a general-purpose solver.
3:   At the root node of the search tree, insert all valid inequalities (16)–(21).
4:   Subproblem solution generated by general-purpose solver.
   Solve Linear Program (LP) relaxation of the node.
5:
6:   Check stopping criteria:
7:   if there are no further nodes to evaluate or the runtime tlimit is achieved then
8:     Stop.
9:   else
10:    if the current solution is a new best solution then
11:       $solution_{best} \leftarrow$  Update the solution at branch-and-cut level.
      Select a node from the search tree.
12:    while the solution of the current LP relaxation has violated constraints do
13:      Identify connected components using a heuristic procedure.
14:      Add violated subtour elimination constraints (7).
15:      if no subtour elimination constraints were added in the last step then
16:        Identify violated fuel flows.
17:        Add violated fuel flow constraints (8) and (9).
18:      Subproblem solution generated by general-purpose solver. Solve LP relaxation of the node.
19:      if the solution of the current LP relaxation is integer then
20:        Go to Check stopping criteria.
21:      else
22:        Branch on one of the fractional variables prioritizing the fuel station visits.
23:        Go to Check stopping criteria.
24:    return  $solution_{best}$ 

```

The model is built on a general-purpose solver that manages the search tree and is responsible for finding new feasible upper bounds (Algorithm 1, line 2). At a generic node of the search tree, a LP defined by (1)–(6) and (10)–(14) is solved (Algorithm 1, line 4). A search for violated complicating constraints (7)–(9) is performed (Algorithm 1, line 13) and the cuts found to be violated are added to the current model (Algorithm 1, line 14), which is then re-optimized using the general-purpose solver. This procedure is repeated until a feasible or dominated solution is found, until all the cuts were added or until the maximum runtime *tlimit* is achieved. Afterwards, branching on a fractional variable occurs with priority on the visiting variables z_i^{kt} related to fuel stations (Algorithm 1, line 22).

4.2. A matheuristic approach

To solve larger instances of the mPVRPR, we propose a matheuristic approach based on the ideas of the fix-and-optimize proposed by Helber and Sahling (2010) and the Variable MIP Neighborhood Search (VMNS) presented by Larrain et al. (2017), which follows predefined decomposition strategies to iteratively solve a series of tractable subproblems to find improvements in a large problem. In the following subsections, we first detail all the decomposition strategies that are used during our matheuristic approach and then we detail how these strategies are intertwined to achieve an efficient approach.

4.2.1. Decomposition strategies

We refer to a decomposition strategy as a method for selecting a subset of variables to find local improvements, maintaining the remaining variables fixed. The proposed matheuristic approach focuses on small sets of decisions at a time using the formulation proposed in Section 3.1. Consider the entire set of integer variables Υ in the formulation. Each decomposed subproblem is defined by selecting a subset of variables $\Upsilon^{Opt} \subseteq \Upsilon$ to be re-optimized. The remaining variables $\Upsilon^{Fix} = \Upsilon \setminus \Upsilon^{Opt}$ are fixed with the values obtained in the incumbent solution. Hence, a subproblem mPVRPR-SUB can be stated as follows:

(mPVRPR – SUB): minimize objective function (1) subject to constraints (2)–(15) and the additional constraints:

$$x_{ij}^{kt} = \bar{x}_{ij}^{kt} \quad \forall (i, j, k, t) \mid (i, j, k, t) \in \Upsilon^{Fix} \quad (23)$$

$$z_i^{kt} = \bar{z}_i^{kt} \quad \forall (i, k, t) \mid (i, k, t) \in \Upsilon^{Fix}, \quad (24)$$

where $\bar{x}_{ij}^{kt}$ and $\bar{z}_i^{kt}$ are values coming from the incumbent solution.

The subset Υ^{Opt} can be built based on a combination of dimensions related to the mPVRPR, which are identified by a decomposition strategy $\omega \in \Omega$. In each decomposition strategy there is always a trade-off between computational complexity and the potential for finding improvements. Therefore, a set of decomposition strategies needs to explore subsets of variables that are diverse in their composition (exploration) and size (intensification). To achieve this diversity we propose 8 different decomposition strategies:

Fig. 4. Schematic representation of the proposed set of decomposition strategies Ω .

1. Period-Oriented Decomposition (POD): each subproblem considers the routing and visiting variables related to a single period t (all customers and vehicles considered).
2. Vehicle-Oriented Decomposition (VOD): each subproblem considers the routing and visiting variables related to a single vehicle k (all customers and periods considered).
3. Period and Vehicle-Oriented Decomposition (PVOD): each subproblem considers the routing and visiting variables related to a single period and vehicle (all customers considered).
4. Periods and Vehicle Sets-Oriented Decomposition (PVsOD): each subproblem considers the routing and visiting variables related to a subset of periods and vehicles (all customers considered).
5. Period, Vehicle, and Customer Sets-Oriented Decomposition (PVCsOD): each subproblem considers the routing and visiting variables related to a subset of periods, vehicles, and customers.
6. Period Angle-Oriented Decomposition (PAOD): each subproblem considers all customers contained in an angular interval measured from the depot, for all vehicles in a single period t .
7. Angle-Oriented Decomposition (AOD): each subproblem considers all customers contained in an angular interval measured from the depot, for all vehicles and periods.
8. Arc-Oriented Decomposition (ArcOD): each subproblem considers all the arcs (i, j) used in the incumbent solution for all vehicles and periods. This allows, for example, promising routes to be matched with vehicles with the most adequate have fuel levels, while maintaining a low complexity of the subproblem.

In Fig. 4, we provide a visual representation of the proposed decomposition strategies.

4.2.2. VMNS algorithm

Our matheuristic approach uses the aforementioned decomposition strategies to iteratively solve a set of Mixed-Integer Programs (MIPs) to explore the search space of the mPVRPR. To solve each subproblem, the subset of variables Υ^{Opt} is provided to the general-purpose solver and the iteration time limit is set to *sub_limit*. However, after we select a subset Υ^{Opt} of variables to be optimized, we have no clue about the complexity of the subproblem, nor the potential it has to improve the incumbent solution. In fact, subproblems with large gaps can indicate that we have a large potential for improvement, but to compute the gap we still need to try to solve it, spending some computational time. In order not to completely loose this time and an opportunity for improvement, we propose a procedure for solving subproblems, that fixes the values of a percentage, *tofix*, of variables belonging to Υ^{Opt} and re-optimizes the subproblem. The percentage of variables to be fixed in each re-optimization is gradually increased by a percentage, *step*, until an optimal solution is achieved or until all the variables Υ^{Opt} are fixed. For example, with a fixing step of 20% (*step* = 0.2), six steps can be performed at maximum. If the subproblem is easy to solve, the optimum solution is achieved in the first iteration. Otherwise, 20% of the set Υ^{Opt} is fixed and a new iteration is performed. While the solution is not optimal, new iterations with fixing percentages of 40%, 60%, 80%, and 100% will be gradually performed. Note that in each iteration, the random fixing process is totally reset to add an

additional randomness to the variables that are fixed. We present our subproblem solver in [Algorithm 2](#).

Algorithm 2. Subproblem solver

```

1: procedure SOLVESUBPROBLEM(solution,  $\Upsilon^{Opt}$ , sub_tlimit, step)
2:   gap  $\leftarrow 1$ , tofix  $\leftarrow 0$ .
3:   Set the general-purpose solver time limit to sub_tlimit.
4:   while tofix  $\leq 1$  and gap  $> 0$  do
5:     New_ $\Upsilon^{Opt}$   $\leftarrow$  Randomly exclude tofix of the variables in the set  $\Upsilon^{Opt}$ .
6:     Free all variables belonging to the set New_ $\Upsilon^{Opt}$ .
7:     solution  $\leftarrow$  Solve subproblem using a general-purpose solver.
8:     gap  $\leftarrow$  Evaluate the optimality gap of the subproblem.
9:     tofix  $\leftarrow$  tofix + step.
10:  return solution

```

The inclusion of such methodology to search for improvements hidden in the neighborhood defined by the variables Υ^{Opt} is critical to the success of our matheuristic. Even if the MIP associated with the subset Υ^{Opt} is consistently too complex to be solved, an adequate level of complexity will be found by the process of gradually fixing that is implemented. Now that we presented the novel methodology to solve subproblems, we are in condition to describe how the rest of our matheuristic approach works.

The algorithm starts with an initial solution and then improves it. This kind of sequential approach has been recently applied with substantial success in complex real-world problems (see [Ciancio et al. \(2018\)](#) and [Neves-Moreira et al. \(2019\)](#)). We follow the steps presented in [Algorithm 3](#) to obtain the initial solution.

Algorithm 3. Build Initial Solution Procedure

```

1: procedure BUILDINITIALSOLUTION(sub_tlimit, step)
2:   solution  $\leftarrow \emptyset$ ,  $\Upsilon^{Opt} \leftarrow \emptyset$ .
3:   solution  $\leftarrow$  Solve subproblem with all routing variables  $x_{ij}^{kt}$  and  $z_i^{kt}$  set to zero.
4:   for every period t do
5:     a  $\leftarrow 0$ , b  $\leftarrow \frac{\pi}{2}$ .
6:     while b  $\leq 2\pi$  do
7:        $\Upsilon^{Opt} \leftarrow$  Select subset of variables related to period t and customers in [a, b], using PAOD strategy.
8:       solution  $\leftarrow$  SolveSubproblem(solutionbest,  $\Upsilon^{Opt}$ , sub_tlimit, step).
9:       a  $\leftarrow a + \frac{\pi}{4}$ , b  $\leftarrow b + \frac{\pi}{4}$ .
10:  for every pair of vehicle and period (k, t) do
11:     $\Upsilon^{Opt} \leftarrow$  Select subset of variables related to (k, t) using PVOD strategy.
12:    solution  $\leftarrow$  SolveSubproblem(solutionbest,  $\Upsilon^{Opt}$ , sub_tlimit, step).
13:  return solution

```

To obtain a fast integer solution, we reformulate the problem so that all customers can be served by a subcontracted service. We add auxiliary variables o_i^t which indicate whether a certain demand of a customer *i* on a period *t* is served by a subcontracted service. In order to allow subcontracted services, constraints (4) are substituted by constraints (25) as follows.

$$\sum_{k \in K} z_i^{kt} = \begin{cases} 1 - o_i^t, & \text{if } d_i^t > 0 \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N, t \in T. \quad (25)$$

Then, to obtain a solution to the reformulated problem we simply need to set integer variables o_i^t to one and x_{ij}^{kt} , and z_i^{kt} to zero. Note that variables o_i^t need to be highly penalized in the objective function as they need to become zero to obtain a feasible solution. This acceleration technique allows the general-purpose solver to quickly find an integer solution with subcontracted services ([Algorithm 3](#), line 3).

Afterwards, the algorithm starts “cleaning” these highly penalized subcontracted services using decomposition strategy PAOD. Customers in between angle intervals [*a*, *b*] with an amplitude of $\frac{\pi}{2}$ radians are selected. Using a step of $\frac{\pi}{4}$ radians in an anti-clockwise direction, eight iterations are performed for each period *t* ([Algorithm 3](#), line 7). The structure of the solution obtained by this anti-clockwise solution is similar to the solutions obtained by [Fisher and Jaikumar \(1981\)](#) and [Ryan et al. \(1993\)](#), as customers will be first clustered in petals around the depot. A recent paper by [Rossit et al. \(2019\)](#) where visual attractiveness of routing problems is analysed, suggests that clustered are related to good quality solutions. Since it is still possible to have some positive subcontracted variables o_i^t and to further improve the solution, we use the quick decomposition strategy PVOD to solve a subproblem for each pair of vehicle and period (*k*, *t*) ([Algorithm 3](#), line 11). An example of an initial and final solutions is presented in [B](#). When this series of subproblems is solved, the initial solution is built, hopefully with no subcontracted services.

The next step is the improvement phase of our matheuristic approach as it is shown in the pseudo-code of [Algorithm 4](#), lines 4–12.

Algorithm 4. Matheuristic Approach

```

1: procedure VMNS( $noimp_{max}$ ,  $tlimit$ ,  $sub\_tlimit$ ,  $step$ )
2:    $stop \leftarrow false$ ,  $noimp \leftarrow 0$ ,  $solution \leftarrow 0$ ,  $solution_{best} \leftarrow 0$ 
3:    $solution_{best} \leftarrow BuildInitialSolution(sub\_tlimit, step)$ 
4:   while not  $stop$  do
5:      $\Upsilon^{Opt} \leftarrow$  Select variables based on a random decomposition strategy  $\omega \in \Omega$ 
6:      $solution \leftarrow SolveSubproblem(solution_{best}, \Upsilon^{Opt}, sub\_tlimit, step)$ 
7:     if  $solution < solution_{best}$  then
8:        $solution_{best} \leftarrow solution$ ,  $noimp \leftarrow 0$ 
9:     else
10:       $noimp \leftarrow noimp + 1$ 
11:      if  $noimp > noimp_{max}$  or  $time > tlimit$  then
12:         $stop \leftarrow true$ 
13:   return  $solution_{best}$ 

```

While the stopping criteria are not met, a cyclic procedure begins (Algorithm 4, line 4). In each cycle, a decomposition strategy $\omega \in \Omega$ is randomly selected and a subproblem considering the variable in the correspondent subset of variables Υ^{Opt} is solved (Algorithm 4, line 6). If the incumbent solution improves, it is accepted (Algorithm 4, line 7). The cyclic procedure finishes when a certain number of non-improvements $noimp_{max}$ or a certain maximum time limit $tlimit$ are achieved (Algorithm 4, line 11).

5. Computational Experiments

In this section, we first introduce the instance generator that served as a basis for all the computational experiments. In the following subsections, (1) we assess the computational efficiency of the approaches we adopted to tackle the mPVRPR, (2) we analyze the trade-off between traveled distance and refueling cost that is inherent to the problem, and (3) we explore the advantages of multi-period planning compared to a rolling horizon based single-period planning.

5.1. Instance generation

The instances were generated with the settings presented in Table 3.

The instance generator uses the depot and customer coordinates of the Solomon's random instances (Solomon, 1987) as it is common in the VRP literature. Instances are generated with a different **number of customers** $|N|$ by taking the first 10, 20, 30, and 40 customers of the Solomons's random instances graph (i.e. instances with 20 customers include all the customers in the instances with 10 customers and so on). The **number of vehicles** $|K|$ is 1, 3, or 5 and the **time horizon** $|T|$ can be composed of 3 or 5 periods. Regarding the fuel stations, there are **Normal** and **Cheap** which correspond to two different fuel prices. This division in Normal and Cheap corresponds to the setting observed when different regions have different tax plans for fuels (i.e. Spain fuel taxes are lower than in Portugal) or within markets with regular (i.e. Repsol) and "low cost" fuel stations (i.e. Prio). Fuel stations that can be located according to two different **fuel station layouts** ψ . In the **CheapIsFar** layout, cheap fuel stations are further away from the depot compared to the normal ones. In the **CheapIsNear** layout, cheap fuel stations are closer to the depot. To obtain these scenarios, first, a large radius R is randomly computed. Second, a small radius $r = \frac{1}{2} \cdot R$ is defined. Third, for each fuel station to be located, a random angle of ϕ radians is generated. All the fuel stations are located around the depot using a radius and an angle. For instance, in the

Table 3

Instance data set description. All the combinations between the possible values of the first six parameters in the table are generated.

Number of customers	$ N $	10, 20, 30, 40 (the first $ N $ from Solomon's R instances)
Number of vehicles	$ K $	1, 3, 5
Time horizon	$ T $	3, 5
Fuel station layouts	ψ	CheapIsFar, CheapIsNear
Number of fuel stations	ρ	2, 4, 6 (50% of which with a lower fuel price)
Fuel price differential	δ	0.1, 0.3, 0.5
Customer demand	d_i^l	Randomly generated integer from the interval $[-20; 80]$ (negative gives zero)
Vehicle capacity	C^k	$\frac{3 \sum_{i \in N} \sum_{l \in T} d_i^l}{2 \cdot K \cdot T }$
Normal fuel price	p	1.2 (Cheap fuel price: $p - \delta$)
Vehicle arc consumption	c_{ij}^k	$0.3 \cdot \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$, with (X_i, Y_i) from Solomon's R instances
Maximum deposit level	U^k	600
Initial deposit level	I^k	$\frac{1}{2} \cdot U^k$
Minimum deposit level	L^k	$\frac{1}{4} \cdot U^k$

Fig. 5. Fuel station layouts for an instance with five customers.

CheapIsNear layout, cheap stations are located at a distance of r whereas normal stations are located at a distance of R . An example of each layout is presented in Fig. 5.

Each instance has the same number of normal and cheap fuel stations. The total number of fuel stations is defined according to the **number of fuel stations** ρ which is drawn from the set $\{2, 4, 6\}$. Each instance is also characterized by a **fuel price differential** parameter δ which indicates the difference between the normal fuel price p and the cheap fuel price $p - \delta$. We considered three values for the δ parameter, belonging to the set $\{0.1, 0.3, 0.5\}$. Our instance set comprises one instance for each combination $|N| \cdot |K| \cdot |T| \cdot \psi \cdot \rho \cdot \delta$ totaling 432 instances. For each combination, the remaining parameters are obtained according to the expressions presented in the bottom part of Table 3. The demand of each customer is randomly generated and the capacity of each vehicle corresponds to two thirds of the total demand per vehicle per period. The normal fuel price was set at 1.2 and each vehicle consumes 0.3 fuel units per each distance unit. The deposit of each vehicle has a maximum capacity of 600 fuel units. Each vehicle starts with half of the deposit filled and the fuel level should never be below a quarter of the deposit capacity.

5.2. Efficiency assessment of proposed approaches

The approaches described in Section 4 were implemented in C++ and all tests were performed on Intel Core processors running at 2.4 GHz and 8 GB of RAM. The general-purpose solver adopted to solve all the MIPs was CPLEX 12.8. Note that all the default CPLEX cuts are allowed whenever the solver is needed. To assess the efficiency of the proposed approaches, we solve the complete set of instances generated with the instance generator presented in the latter subsection.

Each run of each solution approach was limited to 1 h ($limit = 3600$ s). The matheuristic approach also stops after 30 subproblems without improving the solution ($noimp_{max} = 30$) and each subproblem has a time limit of 15 s ($sub_limit = 15$ s) with a fixing step of 20% ($step = 0.2$).

We compare the average gap obtained by each approach (i.e. general-purpose solver, branch-and-cut, and matheuristic) as well as the runtime. The results are presented on Table 4. Additional tests on the impact of the proposed valid inequalities are presented in Appendix A.

Each row of the table corresponds to a different combination of number of periods $|T|$, vehicles $|K|$ and customers $|N|$. For each row, we provide the average gap and runtime of each approach computed over 18 instances (all combinations between layout, price differential, and number of fuel stations).

The results show that for the smaller instances, with a single vehicle, there is no significant difference between the three approaches in terms of optimality gap. In terms of computational time while solving these small instances, the proposed branch-and-cut algorithm is the most efficient approach, being able to solve instances with 5 periods and 40 customers in less than 10 min. The proposed matheuristic approach is able to achieve near optimal solutions consistently within reasonable runtimes.

Analyzing the larger instances, with multiple vehicles, we observe that both exact approaches (general-purpose solver and branch-and-cut algorithm) struggle to achieve gaps of less than 10%. In fact, there are 8 instances that cannot be solved when the mathematical formulation is solved by the considered general-purpose solver and 4 instances that are not solved by our branch-and-cut algorithm. Note that the proposed matheuristic approach is able to solve the entire instance set composed of 432 instances.

In total, the general-purpose solver achieves 140 optimal solutions when solving the model, while the branch-and-cut algorithm proves 160 solutions to be optimal. The matheuristic also outperforms the general-purpose solver in the number of optimal solutions,

Table 4

Comparison between the mathematical formulation (MF), the exact branch-and-cut (B&C) algorithm, and the matheuristic approach (MH).

			MF		B&C		MH		
T	K	N	Avg Gap (%) ^a	Avg Runtime (s)	Avg Gap (%)	Avg Runtime (s)	Avg Gap (%) ^b	Avg Runtime (s)	
3	1	10	0.00 (18)	0.83	0.00 (18)	1.11	0.00 (18)	6.94	
		20	0.00 (18)	18.28	0.00 (18)	4.94	0.00 (18)	56.00	
		30	0.00 (18)	124.22	0.00 (18)	5.94	0.00 (18)	170.44	
		40	0.19 (18)	1744.00	0.00 (18)	40.78	0.10 (18)	244.83	
	3	10	1.25 (18)	1469.61	2.70 (18)	1289.78	1.40 (18)	136.28	
		20	7.57 (18)	3600.33	12.46 (18)	3600.44	5.46 (18)	404.44	
		30	26.39 (18)	3600.22	21.04 (18)	3600.72	6.84 (18)	607.61	
		40	46.63 (18)	3600.50	23.60 (18)	3600.83	7.09 (18)	917.44	
	5	10	3.59 (18)	2183.17	10.09 (18)	3443.17	3.53 (18)	245.89	
		20	34.97 (18)	3600.28	42.29 (18)	3600.78	16.00 (18)	597.67	
		30	61.37 (18)	3600.39	43.25 (18)	3601.44	20.75 (18)	1183.44	
		40	66.73 (18)	3601.11	60.51 (18)	3603.28	12.14 (18)	1720.78	
	5	1	10	0.00 (18)	1.56	0.00 (18)	1.55	0.00 (18)	11.89
			20	0.00 (18)	37.83	0.00 (18)	6.22	0.00 (18)	114.72
			30	0.14 (18)	2143.00	0.00 (18)	84.39	0.01 (18)	284.28
			40	1.17 (18)	3600.17	0.02 (18)	611.67	0.08 (18)	688.50
3		10	8.74 (18)	3600.06	17.62 (18)	3600.17	8.20 (18)	433.83	
		20	18.66 (18)	3600.33	21.69 (18)	3600.72	7.59 (18)	727.17	
		30	50.58 (18)	3600.67	28.58 (18)	3601.44	10.46 (18)	1580.22	
		40	64.89 (17)	3601.24	25.73 (18)	3601.67	7.55 (18)	2026.67	
5		10	8.13 (18)	3600.22	17.68 (18)	3600.22	7.22 (18)	493.17	
		20	43.67 (18)	3600.44	43.82 (17)	3601.41	15.09 (18)	1318.06	
		30	66.53 (18)	3600.94	69.40 (18)	3601.61	16.58 (18)	1931.72	
		40	72.52 (11)	3601.55	73.09 (15)	3602.13	13.60 (18)	3063.50	
Avg (Total)			24.32 (424)	2572.12	21.40 (428)	2329.43	6.46 (432)	789.31	

Bold values indicate the best approach (criteria order: instances solved, gap, and runtime).

^a The number in brackets appearing after the relative gap indicates the number of instances solved.^b Relatively to the lower bound provided by the mathematical formulation. **Fig. 6.** Average refueling cost, detour distance, and percentage of cheap station visits for all instances with a single vehicle.

achieving 146 optimal solutions. Furthermore, while both exact approaches achieve an average gap higher than 20%, the matheuristic approach achieves a considerably lower average gap of 6.46% with shorter runtimes on average. It is clear that the matheuristic is achieving near-optimal solutions for small instances and it is more suitable for solving larger problems. Unfortunately, due to the poor performance of both exact approaches, it is not possible to have good lower bounds for a better assessment of the

Table 5

Summary of the average values obtaining for the routing measures used to derive our managerial insights.

Routing Measures	$\frac{ S }{2}$	CheapIsNear			CheapIsFar		
		10	30	50	10	30	50
Refueling cost	1	350.0	286.3	222.7	376.7	309.6	241.0
	2	349.4	285.9	222.3	375.4	308.9	240.7
	3	348.6	285.3	221.9	374.9	308.5	240.4
	Avg	349.3	285.8	222.3	375.7	309.0	240.7
Detour distance	1	31.4	32.5	31.4	75.5	92.0	98.4
	2	29.8	32.6	30.9	77.6	91.6	96.9
	3	23.4	25.3	21.3	72.4	88.4	93.6
	Avg	28.2	30.1	27.8	75.2	90.7	96.3
Percentage of cheap station visits	1	1.0	1.0	1.0	0.7	0.9	1.0
	2	1.0	1.0	1.0	0.7	0.9	1.0
	3	1.0	1.0	1.0	0.7	0.9	1.0
	Avg	1.0	1.0	1.0	0.7	0.9	1.0
Traveled distance	1	1060.5	1060.5	1060.5	1127.3	1141.4	1147.6
	2	1058.8	1058.8	1058.8	1123.8	1139.4	1146.4
	3	1056.5	1056.5	1056.5	1122.1	1137.6	1144.6
	Avg	1058.6	1058.6	1058.6	1124.4	1139.5	1146.2

Header info:

 $\frac{|S|}{2}$ Number of fuel stations of each price (fuel station density).

Columns refer to combinations between graph layout and price differential.

matheuristic in the larger instances. Nonetheless, is it clear that the solution quality is largely improved by the matheuristic when compared to the solutions obtained by the exact approaches.

5.3. Managerial insights on refueling decisions

In the latter subsection, we analyzed the instances using the descriptors that drive the computational complexity of each problem (number of periods, vehicles, and customers). In this subsection we aim at deriving further insights on the structure of the solutions of a mPVRPR by focusing on relevant business-related measures. To do so, we use the remaining instance descriptors, namely, graph layout, number of fuel stations, and price differential, and show their impact on the type of routes and solutions obtained when refueling decisions are considered in the VRP.

To perform this analysis, we will focus on the optimal solutions provided by any of the presented solution approaches. Therefore, according to Table 4, we consider the single vehicle case. In Fig. 6, we present a chart for each number of fuel stations ρ , where we compare the average values of the total refueling cost (dots), the percentage of detour distance in respect to the total distance (columns), and the percentage of cheap fuel station visits (number at base of each column) performed by the vehicle. Each chart details the values obtained for each graph layout ψ and price differential δ .

If we observe each chart individually, the results show that when cheap fuel stations are near the depot (CheapIsNear layout), the vehicles should always refuel at the cheapest price. As the fuel price gets cheaper (i.e. for larger price differentials), the total cost of refueling is directly impacted. Since cheap fuel stations are nearby, there is no real impact on the detour distance. Note that the small differences correspond to degenerate solutions with slight differences in the vehicles routes but still with the same refueling cost.

When the cheap fuel stations are more distant from the depot (CheapIsFar layout), we observe an impact in every routing measure represented. If the price differential is low (i.e 10 and 30 cents), some refueling operations are still performed in normal priced fuel stations. However, it is clear that the larger the price differential, the larger the detour distance. It is rewarding to travel a larger distance to save in refueling cost, which is something that can be counter-intuitive for practitioners planning routing operations by manual processes. Thus, the number of visits to cheap stations increases as the price differential increases.

To evaluate the impact of adding new fuel stations to the graph we provide a different view of the results in Table 5. The average total traveled distance is also presented.

We observe that none of the measures is highly impacted by increasing the fuel station density of the considered instances. Only marginal improvements are achieved when new fuel stations were added to the graph. For practitioners, this means that given that a larger number of fuel stations substantially increases computational complexity, considering several fuel stations with the same price may not be worth when planning distribution activities.

From a public policy perspective, it is interesting to observe that the cheap is far layout contributes constantly to larger traveled distances, which are further boosted by large price differentials. Hence, sustainability concerns come into play in such scenarios. These results may help policy makers to address this externality by avoiding such layouts.

5.4. Comparing single-period and multi-period approaches

In this subsection, we will compare our multi-period approach with a single-period approach. Single-period approaches are a common practice followed by transportation providers. However, the decisions made in different periods may be interconnected. Therefore, considering information from future periods may influence routing plans to be performed in the current period. In the mPVRPR, fuel deposits are a feature, which induces an interdependence between periods. Therefore, a single-period approach may be myopic in terms finding the best refueling opportunities.

To simulate the daily transportation planning of a transportation provider, we devise a rolling horizon approach considering a single period in each planning iteration. The algorithm optimizes one period at a time and fixes everything that is defined in the previous periods.

Due the lack of visibility of the following periods, the multi-period approach would have a large advantage when using the ML refueling policy, providing unfair results. For this reason, we adapted the mathematical formulation considered for these tests:

1. Constraints (14) are removed from the formulation so that no minimum fuel conditions are imposed in the last period, which is not visible in the first planning iterations of the single-period approach.

The lack of visibility of a single period approach will prevent it from using the fuel deposit capacity to anticipate deliveries in future periods. Therefore, to obtain comparable solutions, we force a certain refueling policy to be followed both by the single and multi period approaches. The refueling policy is now defined by an interval $[f_a, f_b]$, which indicates the minimum and maximum quantity of fuel allowed to be purchased in each fuel station visit. To adapt the model to consider this definition of refueling policy, we add the following constraints.

$$r_i^{kt} \geq z_i^{kt} f_a \quad \forall i \in V, k \in K, t \in T \quad (26)$$

$$r_i^{kt} \leq z_i^{kt} f_b \quad \forall i \in V, k \in K, t \in T \quad (27)$$

2. The objective function is now considering the fuel quantity consumed at the cheapest price and the total refueling cost. This is necessary because in the single-period approach there may be planning iterations where no fuel station visits are needed. Since the single period approach is myopic, when there is enough fuel to perform the routes of a certain period, every solution visiting all the customers is feasible and optimal. In fact, if no quantity of fuel is purchased, objective function (1) will be always zero independently of the distance traveled by the fleet. Thus, we also need the distance to be minimized. To convert the fuel quantity c_{ij}^k consumed in each arc into a cost, we multiply it by the lowest fuel price available $(p - \delta)$. The new objective function (28) reads as follows.

$$\text{minimize } \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} p_s r_s^{kt} + (p - \delta) \sum_{(i,j) \in E} \sum_{k \in K} \sum_{t \in T} c_{ij}^k \quad (28)$$

Once again, to maintain our conclusions with maximum legitimacy, we only analyse the single vehicle case. Hence, all the multi-period solutions are optimal, and all the iterations of the single-period approach are optimal.

We tested two different refueling policies. The *Fixed* refueling policy is defined by the interval $[150, 150]$, meaning that if a vehicle visits a fuel station it is mandatory for it to refuel its deposit with 150 fuel units. The *Flexible* refueling policy is defined by the interval $[150, 300]$, meaning that when a fuel station is visited, the refuel quantity should be at least 150 and at 300 fuel units at most.

Table 6

Routing measures are improved when multiple periods are considered in the planning phase.

Refueling Policy	Periods	Refueling Cost	Fuel Station Visits	Detour Distance	Traveled Distance
Fixed [150, 150]	3	0.00%	0.00%	−3.64%	−0.16%
Fixed [150, 150]	5	−1.49%	−3.11%	−11.74%	−0.88%
	Avg	−0.75%	−1.55%	−7.69%	−0.52%
Flexible [150, 300]	3	−20.18%	−25.00%	−26.89%	−0.85%
Flexible [150, 300]	5	−17.81%	−38.51%	−37.80%	−1.68%
	Avg	−18.99%	−31.75%	−32.35%	−1.27%

After solving the new formulation with both approaches, we measured the advantages of adopting a multi-period approach in the total refueling cost, detour distance, traveled distance, and number of visits to fuel stations (independently of the fuel price). In Table 6, we present the results obtained for both refueling policies and for each number of periods considered in the instances, by indicating the magnitude of the aforementioned advantages.

The results show that for the *Fixed* refueling policy, the multi-period approach brings marginal improvements. In the instances considering 3 periods, there were no savings in the refueling cost and the traveled distance is only decreased by 0.16%. This means that in the third period, the quantity of fuel in deposit of the multi-period approach was slightly larger. In the instances considering 5 periods, we observe savings of 1.49% coming from the necessity of visiting less fuel stations.

In the *Flexible* refueling policy, the improvements coming from the multi-period approach are remarkable. The average refueling cost savings was 18.99% and the number of visits to fuel stations decreased by 31.75%. This is due to the larger visibility achieved by the multi-period approach that allows it to foresee future deliveries and make better use of the fuel deposit. Reducing the number of visits to fuel stations is critical as it also has an impact on the total traveled and distance. Average savings of 1.27% are achieved by the multi-period approach in the considered set of instances.

An important aspect demonstrated by these results is that the multi-period approach is able to improve all the routing measures. The substantial cost savings are due to the large reduction in the number of visits to fuel stations which, consequently, reduces the detour distance. This insight is extremely relevant for transportation operators, which can be interested in new and more sustainable transportation planning approaches that are able to conjugate economical and environmental benefits.

6. Conclusions

In this paper we introduce a new vehicle routing problem to determine minimum refueling cost routes in multi-period planning horizons. The mPVRPR considers vehicles with limited fuel deposits which need to manage their fuel levels while satisfying the dynamic demand of a set of customers.

We propose a novel mathematical formulation for the mPVRPR and develop an exact branch-and-cut algorithm and a mathematical heuristic approach to solve instances considering up to 40 customers, 5 vehicles, and 5 periods. Our approaches achieve a substantial number of optimal solutions and provide good quality solutions for large instances in reasonable computational time.

A sensitivity analysis on several graph layouts, price differentials, and fuel station densities revealed interesting insights on the structure of mPVRPRs. The computational experiments showed that larger price differentials can have a large impact on vehicle routing measures such as refueling cost and detour distances. This impact is particularly large when the cheap fuel stations are far from the depot. Considering the variation on the total refueling cost, the price differential may have a large impact than the number of fuel stations available. These results may be valuable for both transportation planners and policy makers.

Additionally, the proposed multi-period approach shows that it is profitable to rotate vehicles on certain customer visits (demanding small detours for refueling) so that every vehicle can take advantage of good opportunities for refueling at some point in time. This is an interesting insight as it may clash with trendy planning strategies in vehicle routing such as transportation activities based on driver consistent routes (e.g., Kovacs et al. (2015)).

Note that the entire approach proposed in this paper is valid regardless of the type of fuel considered. As long as vehicles have a deposit of fuel and need to stop at certain locations to refuel, all the aforementioned conclusions are valid.

For future developments on the VRP with refueling decisions we encourage researchers to develop new mathematical formulations and solution approaches for tackling larger and more realistic instances. Furthermore, we suspect that the impact of the mPVRPR on total transportation cost can be even stronger on transportation problems considering longer planning horizons. For that reason, solving a real-world case study may be an interesting contribution to further motivate the integration of the VRP with refueling decisions.

Another interesting research avenue should be encountered in the development of extensions for this problem. Although we conclude that it can be beneficial to travel longer distances to achieve lower fuel costs, we do not consider and analyse the issues related to traveling longer distances. For instance, larger maintenance costs may be incurred, drivers fatigue may increase or turn unbalanced, and CO₂ emissions may increase. Further analysis is necessary to understand if the fuel cost savings compensate for all the aforementioned disadvantages. Indeed, refueling decisions may lead to conflicting situations if they are considered within the scope of other trendy VRPs such as the Balanced VRP and the Consistent VRP. Exploring the trade-offs inherent to such integration is a very interesting topic that should be addressed in future extensions of the mPVRPR. Finally, since we consider a fleet with a fixed cost structure, exploring enhanced objective functions for this problem could possibly bring interesting insights. For instance, incorporating other costs such as vehicle (fixed per period) or driver (hourly wage) can have a large impact on the type of routing solutions obtained. Seeking for and quantifying these impacts is a challenging scientific problem that can provide valuable managerial insights for practitioners.

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Appendix A. Valid inequalities computational impact

We explored the impact of each valid inequality included in the proposed formulation. To this purpose, we consider a subset of instances containing 20 customers, 1 vehicle and 5 periods (Instance Group 20_1_5) and a subset of instances containing 20 customers, 3 vehicles and 3 periods (Instance Group 20_3_3). First, we use the general-purpose solver, CPLEX, to solve these instances by considering all the proposed valid inequalities. Then, for each valid inequality, we remove it from the formulation and solve the instances again. Note that all the cuts generated by the adopted general-purpose solver are disabled in these experiments. The results are presented in the following tables (each row corresponds to a number of fuel stations ρ , aggregated by graph layout and price differential).

Table A.7 presents the average objective values of the linear relaxation at the root node. We observe that constraints (17) have some impact in the solutions. However, we should emphasize the importance of constraints (22). When these constraints are

Table A.7

Average objective values of the linear relaxation at the root node.

Group	$\frac{ S }{2}$	All	(16)	(17)	(18)	(19)	(20)	(21)	(22)
20_1_5	1	288.70	288.70	271.10	288.70	288.70	288.70	288.70	0.00
	2	282.18	282.18	265.83	282.18	282.18	282.18	282.18	0.00
	3	279.88	279.88	263.94	279.88	279.88	279.88	279.88	0.00
	Avg	283.58	283.58	266.96	283.58	283.58	283.58	283.58	0.00
20_3_3	1	173.93	173.93	160.67	173.93	171.17	173.93	173.93	0.00
	2	169.34	169.34	155.42	169.34	165.87	169.34	169.34	0.00
	3	169.07	169.07	154.32	169.07	165.60	169.07	169.07	0.00
	Avg	170.78	170.78	156.80	170.78	167.55	170.78	170.78	0.00
Total Avg		227.18	227.18	211.88	227.18	225.57	227.18	227.18	0.00

Table A.8

Average final MIP relative gap.

Group	$\frac{ S }{2}$	All	(16)	(17)	(18)	(19)	(20)	(21)	(22)
20_1_5	1	0.00	0.00	1.13	0.00	0.00	0.00	0.00	100.00
	2	0.00	0.00	2.09	0.00	0.00	0.00	0.00	100.00
	3	0.00	0.00	1.81	0.00	0.00	0.00	0.00	100.00
	Avg	0.00	0.00	1.68	0.00	0.00	0.00	0.00	100.00
20_3_3	1	6.93	5.77	19.78	7.63	8.84	6.31	5.91	100.00
	2	8.75	9.16	27.33	9.12	10.58	8.81	10.07	100.00
	3	7.77	9.45	26.74	10.18	12.08	11.00	9.94	100.00
	Avg	7.81	8.12	24.68	8.97	10.50	8.71	8.64	100.00
Total Avg		3.32	3.46	11.85	3.83	4.50	3.72	3.69	100.00

Table A.9

Average runtimes and number of optimal solutions achieved (in brackets).

Group	$\frac{ S }{2}$	All	(16)	(17)	(18)	(19)	(20)	(21)	(22)
20_1_5	1	105 (6)	114 (6)	1969 (3)	166 (6)	138 (6)	111 (6)	168 (6)	3600 (0)
	2	177 (6)	114 (6)	2506 (2)	172 (6)	288 (6)	239 (6)	195 (6)	3600 (0)
	3	337 (6)	306 (6)	2478 (3)	423 (6)	508 (6)	429 (6)	331 (6)	3600 (0)
	Avg	206 (18)	178 (18)	2317 (8)	254 (18)	311 (18)	260 (18)	231 (18)	3600 (0)
20_3_3	1	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)
	2	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)
	3	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)
	Avg	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)	3600 (0)
Total Avg		1905 (18)	1891 (18)	2961 (8)	1930 (18)	1959 (18)	1932 (18)	1919 (18)	3600 (0)

removed, the linear relaxation at the root node turns to zero in every instance. Note that these conclusions apply to both groups of instances.

Table A.8 presents the average MIP relative gap for the considered subset of instances. We observe that the best total average gap, 3.32%, is obtained by the formulation considering all valid inequalities. When constraints (17) are removed, the total average gap increases to 11.85%. Furthermore, we conclude that constraints (22) are critical to the performance of this formulation, given that the lower bounds are always zero if these constraints are not present. Note that these conclusions apply to both groups of instances.

Table A.9 presents the average runtime and the number of optimal solutions obtained in each group of instances. For group 20_1_5, single vehicle instances with 5 periods, the instances are all solved to optimality except when constraints (17) or (22) are removed. When constraints (16) are removed, the model becomes faster when compared with the version where all valid inequalities are used. For group 20_3_3, none of the instances was solved to optimality, therefore the stopping criterion for these runs was the runtime of one hour.

Appendix B. Initial solution structure

Fig. B.7.

Fig. B.7. Solutions for an instance with 40 customers, 3 vehicles and 3 periods. The structure of the initial and final solutions is similar.

Appendix C. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.tre.2019.11.011>.

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