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Tunable formation realization for nonholonomic mobile robots using the stress matrix

Qingkai Yang¹, Hao Fang¹, Ming Cao², Chengsi Shang¹, Yue Wei¹

1. School of Automation, Beijing Institute of Technology, Beijing 100081, P. R. China
E-mail: qingkai.yang@bit.edu.cn, fangh@bit.edu.cn, chengsi.shang@gmail.com, griffith0079@gmail.com

2. Faculty of Science and Engineering, University of Groningen, Groningen 9747 AG, The Netherlands
E-mail: m.cao@rug.nl

Abstract: In this paper, we investigate the formation control problem for a team of nonholonomic mobile robots, in which the overall formation shape needs to be adapted to the change in its surroundings. To cope with the limited accessibility to the desired formation pattern, as well as the inherent nonholonomic constraints, we propose a distributed controller by invoking the stress-matrix-based technique, which gives extra freedom in controlling formations. Theoretical analysis is provided in the context of Lyapunov stability to show that tunable formations can be achieved using our proposed control strategy. Besides, it is also proved that collision avoidance can be ensured in the process of formation deformation. Simulations are carried out to validate the theoretical results.

Key Words: Formation Control, Stress Matrix, Nonholonomic Mobile Robots, Multi-agent Systems

1 Introduction

Formation control has been regarded as one fundamental problem in the field of coordinated control of multi-agent systems due to its broad applications, such as search and pursuit [1, 2], environment monitoring [3], and localization of sensor networks [4, 5]. The surge of research interests on multi-agent systems is partly due to the fact that the tasks nowadays we are faced are beyond one single agent's capability. To overcome this barrier, a networked system is constructed by teaming up the simple individuals, rendering a more powerful but low-cost collective.

There have been a considerable number of results reported on formation control. A review is made in the recent survey [6] based on the sensed/controlled variables: position, displacement, and distance, and presents an introduction to the control algorithms designed for linear systems. After continuous development, the last several years have seen many significant breakthroughs both in theoretical study and practical applications. From the perspective of control frameworks, many advanced techniques have been proposed, such as bearing- [7, 8], complex Laplacian- [9, 10], mismatch- [11], limit-cycle- [12], angle/distance rigidity- [13–15], and stress-matrix-inspired control schemes [16, 17], just to name a few.

Apart from stabilizing the fixed formation, it is also essential to change the formation geometric pattern in dynamic environments. One typical problem is controlling the size of the overall formation, see [17, 18] for example. In addition to scaling the formation size, it

is intriguing to reconfigure the formation shape when encountering obstacles as well. In [19], the formation is interpreted as a morphology of the modular self-reconfigurable robotic systems, and coordination rules are built based on local interactions. By taking the system model into consideration, traditional consensus-based dynamic formation control algorithms are designed for unmanned aerial vehicles and general linear systems in [20] and [21], respectively. The idea of assigning different formation patterns in the context of affine transformations is employed in [22–24]. To be specific, the formation transformation is modelled as a homogeneous deformation in [22], where evolution control laws are designed based on the principles of continuum mechanics. A type of circular formations is considered in [23], and some comprehensive analysis on localization and affine formation control is given for double integrator dynamics in [24].

Note that most of the aforementioned results are established based on the cooperative relationships between agents, while the introduction of antagonistic interaction might lead to bipartite consensus [25], where a graph with negative weights is termed as a signed graph. In this paper, with the aim to confine the system to the solely isolated equilibrium, whilst realizing tunable formations, we make use of stress matrices. Mathematically speaking, the stress matrix can be regarded as a special class of Laplacian matrices with rank deficiency, which allows for flexible motions. Partially inspired by [24, 26], we design distributed control laws for a set of nonholonomic mobile robots such that the synchronization on the formation shape and heading angles is achieved. In addition, it is shown that inter-robot collision can be avoided as well.

The rest of the paper is organized as follows. Section 2 introduces basic background knowledge and the problem formulation. In Section 3, we give the coordinated control laws based on the stress matrix, followed by the stability analysis. Simulation results are then presented

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in Section 4. Section 5 gives concluding remarks.

2 Preliminaries and problem formulation

In this section, we introduce the basic concepts on graph theory, stress matrix and relevant knowledge, followed by the problem to be addressed.

2.1 Preliminaries

Suppose we are given a team of n mobile robots, each of which is modelled by

$$\begin{aligned}\dot{x}_i &= v_i \cos(\theta_i) \\ \dot{y}_i &= v_i \sin(\theta_i), \\ \dot{\theta}_i &= w_i,\end{aligned}\quad (1)$$

where $q_i = [x_i, y_i]^T \in \mathbb{R}^2$ denotes the position of agent i , θ_i is used to represent agent i 's orientation, and v_i and w_i are the linear and angular velocities (as well as the control inputs). Throughout this paper, we use $q = [q_1^T, \dots, q_n^T] \in \mathbb{R}^{2n}$ to denote the configuration (i.e., coordinates) of the robots.

Let $\mathcal{V} = \{1, 2, \dots, n\}$ be the vertex set corresponding to the group of robots. The interaction relationships between the robots are represented by the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where an edge (i, j) exists only when robot i and j are neighbours of each other. Consequently, an undirected graph $\mathcal{G}$ is generated with the node set $\mathcal{V}$ and the edge set $\mathcal{E}$, i.e., $\mathcal{G}(\mathcal{V}, \mathcal{E})$. The neighbour set of robot i is denoted by $\mathcal{N}_i = \{j | (i, j) \in \mathcal{E}\}$. With these edges, one can define the adjacency matrix as $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ such that $a_{ij} = 1$, if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The graph studied in this paper is assumed to be finite and simple, i.e., without loops or multiple edges. Then the Laplacian matrix $L = [L_{ij}] \in \mathbb{R}^{n \times n}$ can be derived following the convention that $L_{ij} = -a_{ij}$, if $i \neq j$, and $L_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$ for the diagonal elements.

Note that the aforementioned adjacency matrix and the Laplacian matrix only indicate connectivity. To enrich the interactions, such as the behavior types (cooperative or antagonistic) and weights, we introduce the stress ω_{ij} , which is a standard concept in the fields of physics and material science to characterize the stiffness of an object. Mathematically, a stress ω_{ij} is a scalar assigned to each edge (i, j) , and there holds $\omega_{ij} = \omega_{ji}$. We use $\omega \in \mathbb{R}^m$ to denote the concatenated vector $\omega = [\dots, \omega_{ij}, \dots]^T$ where m denotes the number of edges. In parallel to the definition of the Laplacian matrix L , we define the stress matrix $\Omega = [\Omega_{ij}] \in \mathbb{R}^{n \times n}$ as

$$\Omega_{ij} = \begin{cases} -\omega_{ij}, & i \neq j, \\ \sum_{j \in \mathcal{N}_i} \omega_{ij}, & i = j. \end{cases} \quad (2)$$

From the definition (2), it is straightforward to check 0 is an eigenvalue of Ω associated with the eigenvector $\mathbf{1}_n$. However, it is worth noting that a stress ω_{ij} is not constrained to be positive. Thus, different from Laplacian matrices, extra conditions have to be introduced to ensure the positive semidefiniteness of a stress matrix Ω . Since the construction of the stress matrix subject to certain properties is out of the scope of this paper, we omit the detailed discussion. The interested reader is referred to recent results [10, 16].

Suppose we are given an admissible nominal ‘‘desired’’ configuration $q^* = [(q_1^*)^T, \dots, (q_n^*)^T] \in \mathbb{R}^{2n}$. Note that q^* is not necessarily the specific nominal configuration that robots are driven to. Instead, the aim is to describe the prescribed geometric shape through the pairwise displacement. Correspondingly, it is assumed that the stress matrix Ω can be designed with the maximal rank $n - 3$ satisfying

$$(\Omega \otimes I_2)q^* = \mathbf{0}. \quad (3)$$

For additional information on the existence of Ω , a comprehensive explanation from the perspective of rigidity of the so-called tensegrity framework can be found in [27].

To formulate the problem, a connected subgraph $\mathcal{G}_l(\mathcal{V}_l, \mathcal{E}_l)$ needs to be first constructed. Let $\mathcal{V}_l$ be the minimal node set whose dimension of the spanned convex hull is 2, i.e., $|\mathcal{V}_l| = 3$. For the sake of simplicity in stability analysis, $\mathcal{V}_l$ is assigned to be $\{1, 2, 3\}$ without loss of generality. The edges incident to these nodes are denoted by $\mathcal{E}_l = \{(i, j) | i \in \mathcal{V}_l, j \in \mathcal{V}_l\}$. For the subgraph $\mathcal{G}_l(\mathcal{V}_l, \mathcal{E}_l)$, denote $A = [a_{ij}^l] \in \mathbb{R}^{3 \times 3}$ and $L_l \in \mathbb{R}^{3 \times 3}$ as the corresponding adjacency matrix and Laplacian matrix, respectively. Then we have the following results.

Lemma 1 [28] The square matrix $\Omega + \bar{L}$ is positive semi-definite with rank $n - 1$, where the matrix $\bar{L}$ is defined by

$$\bar{L} = \begin{bmatrix} L_l & \mathbf{0}_{3 \times (n-3)} \\ \mathbf{0}_{(n-3) \times 3} & \mathbf{0}_{(n-3) \times (n-3)} \end{bmatrix}.$$

Furthermore, one eigenvector associated with the zero eigenvalue is $\mathbf{1}_n$.

2.2 Problem formulation

Consider a group of nonholonomic mobile robots with kinematics model (1). The control task is to steer the robots to realize different formation patterns in accordance with the constraints from the environment. In this paper, the formation is specified by the displacement variable $q_i^* - q_j^*, \forall i, j \in \mathcal{V}$. We use the transformation matrix $T \in \mathbb{R}^{2 \times 2}$ to regulate the formation, representing the combinations of affine transformation motions. However, it is assumed that only the robots in the set $\mathcal{V}_l$ are informed to know the message of $T(q_i^* - q_j^*)$. In spite of this constraint, all the robots are aware of the displacements with respect to their neighbours, i.e., $q_i - q_j, j \in \mathcal{N}_i$.

Then the problem to be solved is how to design the distributed control inputs (u_i, ω_i) for each robot such that

$$\lim_{t \rightarrow \infty} (q_i - q_j) = T(q_i^* - q_j^*), \quad \forall i \in \mathcal{V}, \quad (4)$$

where the matrix T can be arbitrarily chosen to keep consistent with different situations.

3 Main results

In this section, the control laws assuring the prescribed formations will be presented, and the theoretical analysis on the equilibrium points is then given.

3.1 Control law design

With the objective (4) in mind, we first define an auxiliary variable as

$$q_{ri} = \sum_{j \in \mathcal{N}_i} \omega_{ij} (q_i - q_j) + \sum_{j \in \mathcal{N}_i^l} a_{ij}^l ((q_i - q_j) - T(q_i^* - q_j^*)) \quad (5)$$

where $T \in \mathbb{R}^{2 \times 2}$ is the transformation matrix. The orientation error is defined by

$$\tilde{\theta}_i = \theta_i - \theta_d, \quad (6)$$

where θ_d is the desired orientation for each robot.

Then the control inputs for each robot i are constructed to be

$$\begin{aligned} v_i &= -k_1 [\cos(\theta_i), \sin(\theta_i)] q_{ri} + v_d \cos(\tilde{\theta}_i), \\ w_i &= -k_2 \sin(\tilde{\theta}_i) + v_d [\sin(\theta_i), -\cos(\theta_i)] q_{ri}, \end{aligned} \quad (7)$$

where k_1 and k_2 are positive gains, and $v_d > 0$ is the desired linear velocity. The main idea behind the control law is to drive q_{ri} to zero as long as $\tilde{\theta}_i \rightarrow 0$ holds. To implement such a control law, it can be seen that only a few robots know where they should go, and the rest moves via local observation. This is especially meaningful in the scenarios where the communication is partially denied.

3.2 Stability analysis

We present the main results as follows.

Theorem 1 For a group of mobile agents with kinematics model (1), the formation can be freely tuned guaranteeing (4) under the control law (7). Meanwhile, the orientations of each robot reach synchronization.

Proof Consider the following Lyapunov candidate

$$V = V_1 + V_2, \quad (8)$$

where V_1 and V_2 are respectively defined by

$$\begin{aligned} V_1 &= \frac{1}{2} \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \omega_{ij} \|q_i - q_j\|^2 \\ &\quad + \frac{1}{2} \sum_{i=1}^n \sum_{j \in \mathcal{N}_i^l} a_{ij} \| (q_i - q_j) - T(q_i^* - q_j^*) \|^2, \end{aligned} \quad (9)$$

and

$$V_2 = \sum_{i=1}^n (1 - \cos(\tilde{\theta}_i)). \quad (10)$$

Note that V_1 can be rewritten as

$$V_1 = \frac{1}{2} (q - (I_n \otimes T)q^*)^T ((\Omega + \bar{L}) \otimes I_2) (q - (I_n \otimes T)q^*). \quad (11)$$

From Lemma 1, we know the symmetric matrix $\Omega + \bar{L}$ is positive semi-definite, and thus V_1 is positive semi-definite. In light of the system dynamics (1), the deriva-

tive of V gives

$$\begin{aligned} \dot{V} &= (q - (I_n \otimes T)q^*)^T (\Omega + \bar{L}) \dot{q} + \sum_{i=1}^n \sin(\tilde{\theta}_i) \dot{\theta}_i \\ &= (q - (I_n \otimes T)q^*)^T (\Omega + \bar{L}) ((\text{diag}(v) \otimes I_2) CS(\theta)) \\ &\quad + \sum_{i=1}^n \sin(\tilde{\theta}_i) \dot{\theta}_i, \end{aligned} \quad (12)$$

where $\text{diag}(v) \in \mathbb{R}^{n \times n}$ is defined as $\text{diag}(v) \triangleq \text{diag}(v_1, v_2, \dots, v_n)$, and $CS(\theta) \in \mathbb{R}^{2n}$ is denoted by $CS(\theta) = [\cos(\theta_1), \sin(\theta_1), \dots, \cos(\theta_n), \sin(\theta_n)]^T$. Substituting (7) into (12), it follows

$$\begin{aligned} \dot{V} &= -k_1 \sum_{i=1}^n ([\cos(\theta_i), \sin(\theta_i)] q_{ri})^2 - k_2 \sum_{i=1}^n \sin^2(\tilde{\theta}_i) \\ &\quad + \sum_{i=1}^n v_d [\cos(\theta_i), \sin(\theta_i)] q_{ri} \cos(\tilde{\theta}_i) \\ &\quad + \sum_{i=1}^n v_d \sin(\tilde{\theta}_i) [\sin(\theta_i), -\cos(\theta_i)] q_{ri} \\ &= -k_1 \sum_{i=1}^n ([\cos(\theta_i), \sin(\theta_i)] q_{ri})^2 - k_2 \sum_{i=1}^n \sin^2(\tilde{\theta}_i) \\ &\quad + \sum_{i=1}^n v_d [\cos(\theta_d), \sin(\theta_d)] q_{ri}. \end{aligned} \quad (13)$$

Note that $\mathbf{1}_n$ lies in the null space of $(\Omega + \bar{L})$, and hence we know

$$\sum_{i=1}^n v_d [\cos(\theta_d), \sin(\theta_d)] q_{ri} = 0. \quad (14)$$

Therefore, it follows

$$\dot{V} = -k_1 \sum_{i=1}^n ([\cos(\theta_i), \sin(\theta_i)] q_{ri})^2 - k_2 \sum_{i=1}^n \sin^2(\tilde{\theta}_i). \quad (15)$$

It is easy to see that $\dot{V} \leq 0$. Define the set $\mathbb{E} = \{q_i, \theta_i | \dot{V} = 0\}$, satisfying

$$\mathbb{E} = \{q_i, \theta_i | [\cos(\theta_i), \sin(\theta_i)] q_{ri} = 0, \sin(\tilde{\theta}_i) = 0\}, \quad (16)$$

namely,

$$\mathbb{E} = \{q_i, \theta_i | [\cos(\theta_i), \sin(\theta_i)] q_{ri} = 0, \tilde{\theta}_i = k\pi, k \in \mathbb{Z}\}, \quad (17)$$

where the set $\mathbb{Z}$ denotes the set of integers. It follows from (17) that $\dot{\tilde{\theta}}_i = 0$, i.e., $\dot{\theta}_i = w_i = 0$. Hence in light of (7), we have

$$\begin{cases} [\cos(\theta_i), \sin(\theta_i)] q_{ri} = 0 \\ [\sin(\theta_i), -\cos(\theta_i)] q_{ri} = 0 \end{cases} \quad (18)$$

Then one can derive from (18) that $q_{ri} = \mathbf{0}$. Equivalently, the set $\mathbb{E}$ can be written as

$$\mathbb{E} = \{q_i, \theta_i | q_{ri} = \mathbf{0}, \theta_i = \theta_d + k\pi, k \in \mathbb{Z}\}. \quad (19)$$

It can be seen that θ_i takes can take infinitely many values with periodic 2π . Physically speaking, the robots

points to the identical direction (with the same orientation) under such constraints. So from LaSalle's invariance principle, a compact set, as well as an equilibrium point set, can be constructed as

$$\mathbb{S} = \{q_i, \theta_i | q_{ri} = \mathbf{0}, \theta_i = \{\theta_d, \theta_d + \pi\}\}. \quad (20)$$

Now, let us consider the two equilibrium points $(\mathbf{0}, \theta_d)$ and $(\mathbf{0}, \theta_d + \pi)$, respectively. In view of the fact that $\tilde{\theta}_i = \pi$ is one of the equilibrium points of the dynamic system $\dot{\tilde{\theta}}_i := f(\tilde{\theta}_i) = -k_2 \sin(\tilde{\theta}_i)$. In addition, one knows $f'(\tilde{\theta}_i)|_{\tilde{\theta}_i=\pi} = -k_2 \cos(\tilde{\theta}_i)|_{\tilde{\theta}_i=\pi} = k_2 > 0$. This implies that the repulsiveness of the undesired equilibrium point. In combination with the system dynamics (7), we arrive at the conclusion that the agents are stabilized at the equilibrium point $(\mathbf{0}, \theta_d)$ with respect to $(q_{ri}, \theta_i), i = 1, \dots, n$. Then it can be concluded from [17, 28] that

$$q_i - q_j = T(q_i^* - q_j^*), \quad \forall i, j \in \mathcal{V}.$$

This completes the proof. $\blacksquare$

Remark 1 Note that the reference v_d and θ_d is employed in the proposed control law (7). We want to emphasize that this "limitation" can be easily overcome theoretically through common estimation or network broadcasting in the form of

$$\dot{\hat{v}}_i = - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{v}_i - \hat{v}_j) - a_{i0}(\hat{v}_i - v_d), \quad (21)$$

where $\hat{v}_i$ is the estimation of v_d of agent i . If one wants faster convergence speed, the signum function can be used to achieve the finite time convergence [29]. The estimation $\hat{\theta}_i$ of θ_d can be obtained in a similar manner. To highlight our contributions, we use v_d and θ_d to replace $\hat{v}_i$ and $\hat{\theta}_i$ for clarity. Analogously, by using this technique, the (piecewise) constant transformation matrix T can also be extended to the time-varying form $T(t)$.

4 Discussion on collision avoidance

In addition to the tunable formation realization, it is also expected to guarantee the collision avoidance during the evolution in practical applications. Towards this end, we have the following results.

Proposition 1 Suppose the initial configuration is close to the desired formation, then inter-robot collision avoidance can be guaranteed.

Proof Taking the system dynamics (1) and control inputs (7) into consideration, we have

$$\dot{q}_i = -k_1 q_{ri} + v_d \cos(\tilde{\theta}_i) \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}. \quad (22)$$

Before moving on, we define an auxiliary variable $\tilde{q}_i$ as

$$\tilde{q}_i = q_i - T q_i^* - v_d \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} t, \quad (23)$$

where t denotes the time. Recalling q_{ri} defined in (5), it follows

$$q_{ri} = \sum_{j \in \mathcal{N}_i} \omega_{ij}(\tilde{q}_i - \tilde{q}_j) + \sum_{j \in \mathcal{N}_i^l} a_{ij}^l(\tilde{q}_i - \tilde{q}_j), \quad (24)$$

where we have used the facts that

$$\begin{cases} (\Omega \otimes I_2) q^* = \mathbf{0} \\ (\Omega \otimes I_2) \mathbf{1}_{2n} = \mathbf{0} \\ \bar{L} \mathbf{1}_n = \mathbf{0} \end{cases}$$

In light of (22), taking the derivative of $\tilde{q}_i$, it gives

$$\begin{aligned} \dot{\tilde{q}}_i &= \dot{q}_i - v_d \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} \\ &= -k_1 q_{ri} + v_d \cos(\tilde{\theta}_i) \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} - v_d \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} \\ &= -k_1 \left(\sum_{j \in \mathcal{N}_i} \omega_{ij}(\tilde{q}_i - \tilde{q}_j) + \sum_{j \in \mathcal{N}_i^l} a_{ij}^l(\tilde{q}_i - \tilde{q}_j) \right) \\ &\quad + v_d \left(\cos(\tilde{\theta}_i) \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} - \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} \right). \end{aligned} \quad (25)$$

Then (25) can be written in a compact form as

$$\begin{aligned} \dot{\tilde{q}} &= -k_1 ((\Omega + \bar{L}) \otimes I_2) \tilde{q} + v_d \left((D_{cs}(\tilde{\theta}) \otimes I_2) CS(\theta) \right. \\ &\quad \left. - CS(\theta_d) \right), \end{aligned} \quad (26)$$

where $D_{cs}(\tilde{\theta}) = \text{diag}(\cos(\tilde{\theta}_1), \dots, \cos(\tilde{\theta}_n))$ is a diagonal matrix.

Note that the second term of right-hand side of (26) asymptotically converges to zero. In combination with Lemma 1, we know (26) is stable. Thus there exists a function $\delta_i(\tilde{q}(0), \tilde{\theta}(0))$ such that $\tilde{q}(t)$ is bounded for all $t > 0$, namely,

$$\|\tilde{q}_i\| \leq \delta_i(q_j(0), \theta_j(0)) \triangleq \bar{\delta}_i, \quad j \in \mathcal{N}_i \cup i \quad (27)$$

where $\bar{\delta}_i$ is a constant scalar determined by agent i 's initial states as well as its neighbours'.

Then for any two agents i and k , we have

$$\begin{aligned} \|q_i(t) - q_k(t)\| &= \|q_i(t) - q_k(t) - T(q_i^* - q_k^*) - v_d \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} \\ &\quad + v_d \begin{bmatrix} \cos(\theta_d) \\ \sin(\theta_d) \end{bmatrix} + T(q_i^* - q_k^*)\| \\ &= \|\tilde{q}_i(t) - \tilde{q}_k(t) + T(q_i^* - q_k^*)\| \\ &\geq \|T(q_i^* - q_k^*)\| - \|\tilde{q}_i(t)\| - \|\tilde{q}_k(t)\| \\ &\geq \|T(q_i^* - q_k^*)\| - (\bar{\delta}_i + \bar{\delta}_k) \end{aligned} \quad (28)$$

Hence we draw the conclusion that the collision avoidance (i.e., $\|q_i(t) - q_k(t)\| \geq 0$) can always be guaranteed when the following condition is satisfied

$$\|T(q_i^* - q_k^*)\| > \bar{\delta}_i + \bar{\delta}_k. \quad (29)$$

This completes the proof. $\blacksquare$

Remark 2 We conjecture that the property of collision avoidance holds globally, due to the inherent 'repulsive' interplay introduced by the negative stress.

5 Simulations

In this section, simulation is conducted in a scenario where a group of mobile robots travels in a square formation. To adapt to changing environments, the robots need to alter their formation to pass through narrow pathways that are otherwise impossible.

As an illustrative example, the prescribed formation shape is set as a square with edge length being 1 unit, shown in Fig. 1. The corresponding interaction graph is depicted in Fig. 2 with the assigned stress for each edge. Note that the four-robot case with a resultant complete underlying graph is chosen as a demonstration; our proposed control scheme can be applied to more general situations. The subgraph $\mathcal{G}_l(\mathcal{V}_l, \mathcal{E}_l)$ is chosen with $\mathcal{V} = \{1, 2, 3\}$ (marked in blue) and $\mathcal{E} = \{(1, 2), (1, 3), (2, 3)\}$. The desired linear velocity v_d and orientation θ_d are chosen as 2 m/s and 0 rad , respectively. That means the group of robots moves along the X-axis direction.

To validate the effectiveness of the control laws, we initialize the robots with random state values, i.e., $q_i(0) = \text{rands}(2, 1)$, $v_i(0) = \text{rands}(1)$, and $\theta_i(0) = \text{rands}(1)$, $i = 1, \dots, 4$. The piecewise transformation matrix T is chosen as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, when $t \leq 10$; $R(-\frac{\pi}{5}) \begin{bmatrix} 2 & 1.5 \\ 0.5 & 2 \end{bmatrix}$, when $10 < t \leq 12$; $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, when $12 < t \leq 22$.

Fig. 1: The prescribed formation shape

Fig. 2: Interacting relationships with indicated stress.

Fig. 3 indicates the formation shape change at different time instants. It can be clearly seen that the formation makes a quick response to the change of environment, which is consistent with the setup of the transformation matrix. This is further verified by the errors in terms of distance and orientation presented in Fig. 4(a) and Fig. 4(b), respectively.

To verify the collision avoidance property of the stress-matrix-based control strategy, we also present the errors $\|\tilde{q}_i\|$ in Fig. 5(a), from which we can see that the upper bound $\bar{\delta}_i$ given in (27) approaches to the same value, partially resulting from Lemma 1. It can be observed from Fig. 5(b) that the distance between any pair of agents is larger than zero, which implies that the collision can be eliminated during the whole evolution. The experimental results are presented in Fig. 6.

6 Conclusion

This paper has addressed the problem of flexible formation control for mobile robots with nonholonomic

Fig. 3: Formation evolution

Fig. 4: (a). Distance errors with $e_{ij} = \|q_i - q_j\| - \|T(q_i^* - q_j^*)\|$; (b). Orientation errors

Fig. 5: (a). Change of $\|\tilde{q}_i\|$ with respect to different transformation matrix T ; (b). Distance between robot pairs

constraints. We have designed coordinated control laws by making use of the stress matrix to realize prescribed formations through local interactions. The main idea is to restrict the robotic system's collective behavior from the affine transformations via controlling a small portion of robots. We have shown that not only the formation shape but also the orientation of each robot can be stabilized to the desired ones under an appropriate set of stress assignment. In addition, it has been proven that the collision avoidance property can be inferred. Future works will focus on designing more intelligent control strategies under more mild conditions that less robots have the knowledge of the desired formation.

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Fig. 6: Experimental results of 4 mobile robots using our proposed control law (7). The initial positions are shown in Fig. 6(a). In Fig. 6(b), the formation achieves the desired size associated with the transformation matrix T in the first stage. The formation shrinking is shown in Fig. 6(c) in response to the narrow path. The group recovers to the square pattern after travelling through the constraint area, as shown in Fig. 6(d).

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