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Multi-agent formation control using angle measurements

Liangming Chen, Ming Cao, Chuanjiang Li, Xiaodong Cheng and Yuri Kapitanjuk

Abstract—This paper investigates the triangular and polygonal formation control problem for mobile multi-agent systems under the constraint that each agent can only take angle measurements. For triangular formations, due to the fact that the sum of three interior angles always equals π , the desired triangular shape can be obtained when any two agents achieve desired angles for which they are the corresponding vertices of the triangle. So to achieve the desired shape of a triangular formation, we propose to let one agent remain fixed and the other two agents move along their bisectors respectively with respect to their two neighbors. For convex polygonal formations, since the sum of all interior angles is constant, we are able to use a similar control strategy to achieve the desired polygonal shape. The stability of the closed-loop multi-agent systems is proved using Lyapunov theory. Finally, simulation examples illustrate the validity of the theoretic results.

I. INTRODUCTION

Formations of multi-agent systems have broad applications, e.g., Unmanned Aerial Vehicles (UAVs) formations in the near-Earth space [1], Autonomous Underwater Vehicles (AUVs) formations in oceans [2], and Spacecraft Formation Flying (SFF) in deep space [3]. Formation control has become a hot topic in the research community of multi-agent systems [4], [5]. The sensors used in formation control include GPS, radar, and camera, which produce the measurements of positions, inter-agent distances, and bearings [5], [6]. Many results have concentrated on the position-based or displacement-based formation control problem, partly due to the convenience to apply control theories for linear systems or gradient systems [7]–[10]. The disadvantage of position-based formation control is that a global coordinate system is usually required to be shared among the agents [6], [11]. In distance-based formation control, relative positions measured in each agent's local coordinate system are usually required [12], [13]. However, in many practical applications, it is hard for each agent to gain knowledge of the global coordinate system or measure relative positions with respect to neighboring agents [14], [15]. For example, in deep space, it is difficult for satellites to get the GPS information because there is usually no global coordinate system with respect to the earth [16].

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With the rapid development of sensor technologies, angle measurements are becoming cheaper, more reliable and accessible than relative position or inter-agent distance measurements [6], [11]. Angle information can be easily obtained by the full-range radar or vision-based camera [17]. Thus, angle sensors are more applicable in engineering practices, which motivates us to study the problem of multi-agent formation control using only angle measurements. For angle-based formation control, one agent's interior angle is usually calculated by the relative “bearings” with respect to its neighboring agents. More precisely, there are mainly three different definitions for *bearing*. The first is the angle between a sensing beam and an axis of the global coordinate system [15], [18]. The second refers to the angle between the sensing beam and the x -axis of the corresponding agent's local coordinate system [19], [20]. The third is represented in a global coordinate system by the unit vector starting from the corresponding sensing agent along the sensing beam [21], [22]. Correspondingly, there are research results studying the angle-measurement based formation control problem using each of the above definitions of bearing.

Eren *et al.* [18] used the second definition and established direction-based bearing rigidity using the fact that a bearing constraint can be transformed into a constraint on the direction that an agent is confined to move along once all agents know the global coordinate system. In a follow-up work [14], the triangular formation control problem was investigated, in which some distance measurements were needed. Also using the second definition, Bishop *et al.* [19], [20] studied the triangular formation control problem using only angle measurements; more specifically in [20], Poincare-Bendixson theorem was used to prove the asymptotic convergence of the closed-loop formation control system. The results in [23] extended those in [19], [20] to four-agent formations. The distributed control of angle-constrained cyclic formations was studied in [24] using bearing-only measurements, where local stability was proved. In [15], by using the first definition, stiff rigidity with mixed bearing and distance constraints was defined and a formation control algorithm was designed. Zhao *et al.* [21] established bearing rigidity using the third definition, in which a bearing-only formation control algorithm was constructed to guarantee almost global convergence. In [22], translational and scaling formation maneuvering control was realized via a bearing-based approach, in which both the bearing information and relative positions were required.

Motivated by the existing results, in this study we aim at solving the formation control problem using only angle measurements, in which bearing takes its meaning according

to the second definition. Two classes of formations are considered, i.e., triangular and convex polygonal formations. Since the sum of the three interior angles of any triangle is π , the desired triangular formation is realized when any two agents achieve their desired angles. Thus, our control strategy stipulates that one agent keeps static and the other two are guided to reach their desired angles by moving along the bisector of the corresponding interior angle. Similarly, the sum of the interior angles of a convex polygon is constant. Therefore, to realize the convex polygonal formation control, one can also let one agent keep static and the other agents move to reach their desired angles.

The main contributions of this study can be summarized as follows. The triangular and convex polygonal formations are realized using only angle measurements. Compared with other proposed formation control algorithms which require the measurements of bearings, distances or relative positions [14], [15], only angle measurements are required in this study, which can be realized by only installing on-board camera. Angle-based Lyapunov functions are designed to prove the stability of the closed-loop formation systems. Compared with using Poincare-Bendixson theorem as is done in [20], Lyapunov theorem offers a more general analysis framework. Different from [21], [22], the global coordinate system is not needed in our approach.

The rest of this paper is organized as follows. Section II introduces basic background knowledge and the problem formulation. In Section III, we define the triangular formation control problem, followed by the stability analysis of the closed-loop system. In Section IV, the convex polygonal formation control and the corresponding stability analysis are given. The theoretical results are illustrated by numerical simulations in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

For an N -agent formation, we first define the agents' dynamics and the bearing measurements. Then, the research problem is formally formulated.

A. Agents' dynamics

For the N -agent system moving in the plane, the motion dynamics of an agent i is governed by [20]

$$\dot{p}_i = v_i \begin{bmatrix} \cos \beta_i \\ \sin \beta_i \end{bmatrix}, i = 1, \dots, N \quad (1)$$

where $p_i = [x_i, y_i]^T \in \mathbb{R}^2$ denotes the position of agent i , and $v_i \in \mathbb{R}$ is the speed, and the heading angle β_i is defined counter-clockwise with respect to agent i 's local x -direction and always takes its value from $(-\pi, \pi]$. Both v_i and β_i are the control inputs to be determined.

B. Bearing measurements

Agent i measures only the bearing $\phi_{ij} \in (-\pi, \pi]$, $\forall j \in \mathcal{N}_i$ towards agent j evaluated counter-clockwise from its local x_i -direction, and here $\mathcal{N}_i$ denotes the set of the neighbors of agent i .

First, we define the auxiliary angle

$$\theta_i = |\phi_{i(i+1)} - \phi_{i(i-1)}| \in [0, 2\pi) \quad (2)$$

which is the angle measured counter-clockwise from $\min\{\phi_{i(i+1)}, \phi_{i(i-1)}\}$ to $\max\{\phi_{i(i+1)}, \phi_{i(i-1)}\}$ with respect to agent i 's local x -direction.

Then, we define the interior angle α_i to be

$$\alpha_i = \begin{cases} \theta_i, & \text{if } \theta_i \leq \pi \\ 2\pi - \theta_i, & \text{otherwise} \end{cases} \quad (3)$$

where $\alpha_i \in [0, \pi]$ represents agent i 's interior angle in the triangle formed by $(i-1)$, i and $(i+1)$, see Fig.1.

Fig. 1: The angle measurements.

C. Problem formulation

The goal is to design for each agent i formation control algorithms for v_i and β_i such that the closed-loop system can achieve:

- (1) For a triangular formation ($N = 3$)

$$\lim_{t \rightarrow \infty} (\alpha_i - \alpha_i^*) = 0, \forall i = 1, 2, 3 \quad (4)$$

where α_i^* denote agent i 's desired interior angle, and naturally $\alpha_1^* + \alpha_2^* + \alpha_3^* = \pi$.

- (2) Convex polygonal formation ($N > 3$)

$$\lim_{t \rightarrow \infty} (\alpha_i - \alpha_i^*) = 0, \forall i = 1, \dots, N \quad (5)$$

where $\sum_{i=1}^N \alpha_i^* = (N-2)\pi$ and the given desired formation polygon is convex.

III. TRIANGULAR FORMATION CONTROL

In this section, we aim at solving the triangular formation control problem using only angle measurements. First, we will introduce two related lemmas. Then, for two representative triangles, the formation control algorithms and the corresponding stability analysis will be given respectively.

A. Related lemmas

Define the initial angle error $\tilde{\alpha}_i(0)$ by

$$\tilde{\alpha}_i(0) = \begin{cases} 0, & \text{if } \alpha_i^* = \alpha_i(0) \\ +1, & \text{if } \alpha_i^* - \alpha_i(0) > 0 \\ -1, & \text{if } \alpha_i^* - \alpha_i(0) < 0 \end{cases} \quad (6)$$

The following lemma shows the relationship between the initial triangle and the desired triangle.

Lemma 1: For any triangular formations, if initially there exists one agent i with $\tilde{\alpha}_i(0) \neq 0$, then there always exists a pair of distinct k and j satisfying $\tilde{\alpha}_k(0) > 0$ and $\tilde{\alpha}_j(0) < 0$, $i, j, k \in \{1, 2, 3\}$.

The above lemma holds because $\alpha_1^* + \alpha_2^* + \alpha_3^* = \alpha_1(0) + \alpha_2(0) + \alpha_3(0) = \pi$. The following lemma is on how the angles change when agents move.

Lemma 2: For the triangle formed by agents i , j and k , if $v_j = 0$, $v_k = 0$, and $v_i \neq 0$, then α_i will increase when i moves towards the inside of the triangle and α_i will decrease when i moves towards the outside of the triangle along the bisector of α_i .

Using the above lemmas, we obtain the following table summarizing a set of triangular formation motion strategies for different initial formations.

Triangle	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	Formation motion strategies
Case 1	0	+1	-1	agents 1, 3 keep static, agent 2 moves
Case 2	+1	+1	-1	agent 1 keeps static, agents 2, 3 move
Case 3	+1	-1	-1	agent 2 keeps static, agents 1, 3 move

TABLE I: Triangular formation motion strategies.

Note that the above listed cases represent all the possible scenarios by relabelling the indices of the agents. Based on the above lemmas and table, in the following, we will describe how to realize a triangular formation in Case 1 and Case 2 in detail. For Case 3, one can use the same formation control algorithms for agents 1, 3 as those given in Case 2.

B. Triangular formation control for Case 1

In this case, $\alpha_1(0) = \alpha_1^*$, $\alpha_2(0) < \alpha_2^*$, and $\alpha_3(0) > \alpha_3^*$. In a triangle, the third angle equals the desired value as long as the other two angles equal their desired values. Thus, in the motion strategy of the triangular formation system in Case 1 we let agents 1, 3 keep static and agent 2 move along the line connecting agent 2 and agent 1, which makes $\alpha_1 = \alpha_1^*$ and α_2 converge to α_2^* .

Fig. 2: The triangular formation motion strategy in Case 1.

Specifically, the speed and heading control algorithms for agent 2 become

$$v_2 = -k_2(\alpha_2 - \alpha_2^*) \quad (7)$$

$$\beta_2 = \phi_{21} \quad (8)$$

where k_2 is a positive constant as a design parameter to adjust the convergence speed of the closed-loop system. The speed control law (7) represents the feedback of the relative angle error. The heading control law (8) is used to guide agent 2 to move along the line connecting agents 1 and 2. In the triangular formation control algorithms (7), (8), only the angle information α_2 , ϕ_{21} measured in agent 2's local coordinate system is used.

Now we are ready to prove the main result.

Theorem 1: For the triangular formation system in Case 1, with the formation control algorithms (7), (8), the three-agent formation will achieve its desired shape asymptotically.

Proof: The angular velocity of agent 2 with respect to agent 3 is determined by the tangent component of the velocity vector with respect to the line connecting agent 2 and agent 3. Thus, one can get

$$\dot{\alpha}_3 = -\frac{v_2}{r_{23}} \sin(\alpha_2) = \frac{\alpha_2 - \alpha_2^*}{r_{23}} \sin(\alpha_2) \quad (9)$$

where the sign is negative because α_3 will decrease when $v_2 > 0$, and $r_{ij} = \|p_i - p_j\|$. Since k_2 has no effect on the system's stability, without loss of generality, we use $k_i = 1$, $\forall i = 1, \dots, 3$ in the following proof.

For the triangle in Case 1, because $\dot{\alpha}_1 = 0$ and $\sum_{i=1}^3 \dot{\alpha}_i = 0$, one gets

$$\dot{\alpha}_2 = -\dot{\alpha}_3 = -\frac{\alpha_2 - \alpha_2^*}{r_{23}} \sin(\alpha_2). \quad (10)$$

Denote the angle error by $e_i = \alpha_i - \alpha_i^*$, $i = 1, 2, 3$. Then, we have

$$\dot{e}_2 = -A(e)e_2 \quad (11)$$

where $A(e) = \frac{\sin(\alpha_2)}{r_{23}}$. Note that $\sin(\alpha_2) \neq 0$, $r_{23} \neq 0$, $\forall t > 0$. Thus, we have $A(e) > 0$, $\forall t > 0$.

The only equilibrium of the closed-loop system (11) is $\{e_2 | \dot{e}_2 = 0\} = \{e_2 | e_2 = 0\}$ because $A(e) > 0$, which corresponds to the desired triangular formation shape.

Consider the following candidate Lyapunov function

$$V_1 = \frac{1}{2}e_2^2. \quad (12)$$

Taking the time derivative of V_1 gives

$$\dot{V}_1 = -A(e)e_2^2 < 0. \quad (13)$$

As $e_2 = 0$ is the only equilibrium of the closed-loop system (11), one has

$$\lim_{t \rightarrow \infty} (\alpha_2 - \alpha_2^*) = 0 \quad (14)$$

which implies that $\lim_{t \rightarrow \infty} (\alpha_3 - \alpha_3^*) = 0$ because $e_1 = 0$. ■

C. Triangular formation control for Case 2

In this case, $\alpha_1(0) < \alpha_1^*$, $\alpha_2(0) < \alpha_2^*$, and $\alpha_3(0) > \alpha_3^*$. In the motion strategy for triangular formations in Case 2, we let agent 1 keep static, agent 2 move towards the inside of the triangle along the bisector of α_2 , and agent 3 move towards the outside of the triangle along the bisector of α_3 . The desired triangular formation can be realized when $\alpha_2 = \alpha_2^*$ and $\alpha_3 = \alpha_3^*$.

Fig. 3: The triangular formation motion strategy in Case 2.

Specifically, the triangular formation control algorithms for agents 2 and 3 are

$$v_i = -k_i \frac{\alpha_i - \alpha_i^*}{\sin(\alpha_i/2)}, i = 2, 3 \quad (15)$$

$$\beta_i = \begin{cases} \alpha_i/2 + \min\{\phi_{i(i+1)}, \phi_{i(i-1)}\}, & \text{if } \theta_i \leq \pi \\ \alpha_i/2 + \max\{\phi_{i(i+1)}, \phi_{i(i-1)}\}, & \text{if } \theta_i > \pi \end{cases} \quad (16)$$

Note that different from [20], the denominator $\sin(\alpha_i/2)$ in the speed control law (15) is used in this paper to make the system matrix of the relative angle error dynamics become symmetric, with which we can design Lyapunov functions to analyze the stability. The heading control law (16) implies that agents 2, 3 will move along the bisector of α_2, α_3 . In the triangular formation control algorithms (15), (16), only the angle information α_i , $\phi_{i(i+1)}$, and $\phi_{i(i-1)}$ measured in the agent i 's local coordinate system is used.

Now we are ready to prove the following result.

Theorem 2: For the triangular formation system in Case 2, with the formation control algorithms (15), (16), the three formation agents will achieve the desired formation shape.

Proof: First, we aim at getting the dynamics of the three relative angle errors, i.e., $\dot{e}_1, \dot{e}_2, \dot{e}_3$. As has been shown in [20], the three relative angle error dynamics can be obtained by considering each agent's motion separately. Therefore, we first consider the scenario when agent 2 moves towards the inside of the triangle and agents 1 and 3 are static. Then, we consider the scenario when agent 3 moves towards the outside of the triangle and agents 1 and 2 are static. When agents 2 and 3 are static, the formation system will keep static because agent 1 is always static.

Step 1: We consider that agent 2 moves towards the inside of the triangle, and agents 1 and 3 are static. Following the method used in the proof of Theorem 1, one gets

$$\dot{\alpha}_3 = -\frac{v_2}{r_{23}} \sin(\alpha_2/2) = \frac{\alpha_2 - \alpha_2^*}{r_{23}} \quad (17)$$

$$\dot{\alpha}_1 = -\frac{v_2}{r_{12}} \sin(\alpha_2/2) = \frac{\alpha_2 - \alpha_2^*}{r_{12}}. \quad (18)$$

Using $\sum_{i=1}^3 \dot{\alpha}_i = 0$, it follows that

$$\dot{\alpha}_2 = -\dot{\alpha}_1 - \dot{\alpha}_3 = -\left(\frac{1}{r_{12}} + \frac{1}{r_{23}}\right)(\alpha_2 - \alpha_2^*). \quad (19)$$

Step 2: We consider that agent 3 moves towards the outside of the triangle, and agents 1 and 2 are static.

$$\dot{\alpha}_2 = -\frac{v_3}{r_{23}} \sin(\alpha_3/2) = \frac{\alpha_3 - \alpha_3^*}{r_{23}} \quad (20)$$

$$\dot{\alpha}_1 = -\frac{v_3}{r_{13}} \sin(\alpha_3/2) = \frac{\alpha_3 - \alpha_3^*}{r_{13}} \quad (21)$$

$$\dot{\alpha}_3 = -\dot{\alpha}_1 - \dot{\alpha}_2 = -\left(\frac{1}{r_{32}} + \frac{1}{r_{13}}\right)(\alpha_3 - \alpha_3^*). \quad (22)$$

Combining Step 1 and Step 2, we get the closed-loop dynamics of the triangular formation system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{r_{21}} & \frac{1}{r_{31}} \\ 0 & -\frac{1}{r_{12}} - \frac{1}{r_{23}} & \frac{1}{r_{23}} \\ 0 & \frac{1}{r_{32}} & -\frac{1}{r_{32}} - \frac{1}{r_{13}} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \quad (23)$$

Note that in the above closed-loop dynamics, agent 1's angle error e_1 has no effect on agent 2's angle error e_2 and agent 3's angle error e_3 . Thus, we can obtain the dynamics of the subsystem

$$\begin{aligned} \dot{e} = \begin{bmatrix} \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} &= -\begin{bmatrix} \frac{1}{r_{12}} + \frac{1}{r_{23}} & -\frac{1}{r_{23}} \\ -\frac{1}{r_{32}} & \frac{1}{r_{32}} + \frac{1}{r_{13}} \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \end{bmatrix} \\ &= -A(e)e. \end{aligned} \quad (24)$$

Now we analyze the stability of the above subsystem of angle errors e_2 and e_3 . Note that the three formation agents will not collide into each other or become collinear. Thus, we have $r_{ij} \neq 0, \sin(\alpha_i/2) \neq 0, \forall i, j = 1, 2, 3$. Then, $A(e)^T = A(e)$, $\text{tr}(A(e)) > 0$, and $\det(A(e)) > 0$. Simple calculation shows the positive definiteness of $A(e)$.

Because $A(e) > 0$, one has that the only equilibrium of the closed-loop system (24) is $\{e | \dot{e} = 0\} = \{e | e_2 = e_3 = 0\}$, which corresponds to the desired triangular formation shape.

Consider the following candidate Lyapunov function

$$V_2 = \frac{1}{2} e^T e. \quad (25)$$

Taking the time derivative of it gives

$$\dot{V}_2 = -e^T A(e) e < 0. \quad (26)$$

Then, we have

$$\lim_{t \rightarrow \infty} e = 0 \quad (27)$$

which implies that $\lim_{t \rightarrow \infty} (\alpha_i - \alpha_i^*) = 0, i = 2, 3$ and $\lim_{t \rightarrow \infty} (\alpha_1 - \alpha_1^*) = 0$. ■

Remark 1: For Case 3 in Table 1, by letting agent 2 keep static and agents 1, 3 move with the control algorithms (15), (16), the desired triangular formation shape can also be achieved. As stated in Lemma 1 and Table 1, there always exists a pair “+1” and “-1” in a triangular formation, so one can realize the desired triangular formation shape by letting

one agent keep static and the agent with “+1” move towards the inside of the triangle and the agent with “−1” move towards the outside of the triangle. Although two agents in Case 1 keep static, the triangular formation in Case 1 can also be achieved when only one agent keeps static. Therefore, (15) and (16) can stabilize the triangular formations in Cases 1, 2, and 3.

IV. CONVEX POLYGONAL FORMATION CONTROL

In this section, we extend the results in the previous section to solve the convex polygonal formation control problem using only angle measurements. If $\sum_{i=1}^N \alpha_i^* = (N - 2)\pi$ is the only constraint on the interior angles, the resulting polygon may have multiple realizations. Therefore, we make the following assumption.

Assumption 1: The desired polygonal formation shape for the N agents is convex.

Fig. 4: The convex polygonal formation shape.

The sum of all interior angles is constant, i.e., $\sum_{i=1}^N \alpha_i = (N - 2)\pi$. The motion strategy for the polygonal formation system is to let one agent be static and the other agents move towards the inside of the polygon when $\hat{\alpha}_i > 0$ or the outside when $\hat{\alpha}_i < 0$. Without loss of generality, agent 1 is considered to be static, i.e., $v_1 = 0$. For $i = 2, \dots, N$, the polygonal formation control algorithms are

$$v_i = -k_i \frac{\alpha_i - \alpha_i^*}{\sin(\alpha_i/2)}, i = 2, \dots, N \quad (28)$$

$$\beta_i = \begin{cases} \alpha_i/2 + \min\{\phi_{i(i+1)}, \phi_{i(i-1)}\}, & \text{if } \theta_i \leq \pi \\ \alpha_i/2 + \max\{\phi_{i(i+1)}, \phi_{i(i-1)}\}, & \text{if } \theta_i > \pi \end{cases} \quad (29)$$

Then, we have the following main result.

Theorem 3: For the convex polygonal formation system, with the formation control algorithms (28), (29), if $e_i(0)$, $i = 2, \dots, N$ are sufficiently small, the polygonal formation will achieve its desired shape asymptotically.

Proof: The proof is omitted here due to page limit. ■

Remark 2: Since the sum of all interior angles is constant, one agent is required to keep fixed. Consequently, the Lyapunov functions are successfully designed to analyze stability. In multi-level control architectures, the static agent can act as an agent responsible for trajectory tracking in a higher level of control.

Remark 3: In this work, the formation size is not controlled, which may be a shortcoming in some practical tasks. Note that the formation size is completely determined when just one distance constraint between two arbitrary agents is enforced.

V. SIMULATION EXAMPLES

In this section, we present numerical simulation examples to verify the effectiveness of the formation control algorithms proposed in this study.

(1) Triangular formation

Case 1: 0 +1 −1

Case 1	agent 1	agent 2	agent 3
Initial positions	$[0, 1]^T$	$[0, 0]^T$	$[\sqrt{3}, 0]^T$
Initial angles	$\pi/3$	$\pi/2$	$\pi/6$
Desired angles	$\pi/3$	$\pi/3$	$\pi/3$

TABLE II: Simulation example for triangular formation.

Fig. 6 shows the triangular formation trajectories and the evolution of angle errors.

Fig. 5: Case 1 of Triangular formation

Case 2: +1 +1 −1

Case 2	agent 1	agent 2	agent 3
Initial positions	$[0, 1]^T$	$[\sqrt{3}, 0]^T$	$[0, 0]^T$
Initial angles	$\pi/3$	$\pi/6$	$\pi/2$
Desired angles	$\pi/3 + \pi/20$	$\pi/3 - \pi/20$	$\pi/3$

TABLE III: Simulation example for triangular formation.

Fig. 6 shows the triangular formation trajectories and the evolution of angle errors.

Fig. 6: Case 2 of Triangular formation

(2) Convex polygonal formation

Fig. 7 shows the polygonal formation trajectories and the evolution of angle errors.

Polygonal formation	agent 1	agent 2	agent 3	agent 4
Initial positions	$[0, 1]^T$	$[0, 0]^T$	$[\sqrt{3}, 0]^T$	$[1, 1]^T$
Initial angles	$\pi/3$	$\pi/3$	0.94	2.20
Desired angles	$\frac{2\pi}{3}$	$\pi/3$	$\frac{2\pi}{3}$	$\frac{\pi}{3}$

TABLE IV: Simulation example for polygonal formation.

Fig. 7: Convex polynomial formation

According to Figs. 5-7, one can see that the desired triangular and convex polygonal formations are achieved under the proposed formation control algorithms. Note that in Figs. 5, 6, the angle errors of the controlled agents decrease monotonously.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This study focuses on the problem of controlling triangular and convex polygonal formations using only angle measurements. As the sum of all interior angles in a triangle or convex polygon is always constant, it is unnecessary to enforce each agent to move to achieve the desired formation shape. By letting one agent be static and the other agents move according to the proposed control algorithms regulating their speed and heading, the desired formation shape has been realized. To analyze the stability of the closed-loop formation system, the Lyapunov functions have been constructed. Simulation examples have illustrated the validity of the theoretic results.

B. Future Works

Many studies on formation control using only angle measurements focused on triangular and polygonal shapes. It is still a challenging problem to study general formations using only angle measurements. We are also interested in taking the proposed formation control strategy as a lower-level component of a hierarchical multi-agent control strategies, in which the higher level may correspond to trajectory tracking, cooperative transportation, and formation maneuvering.

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