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Evolutionary Game Dynamics Driven by Setting a Ceiling in Payoffs of Defectors

ZHANG Jianlei^{1,2}, ZHU Yuying^{1,2}, CHEN Zengqiang^{1,2}, CAO Ming³

1. Department of Automation, College of Computer and Control Engineering, Nankai University, Tianjin 300071, P. R. China
E-mail: jianleizhang@nankai.edu.cn

2. Tianjin Key Laboratory of Intelligent Robotics, College of Computer and Control Engineering, Nankai University, Tianjin 300071, P. R. China

3. Engineering and Technology Institute Groningen, University of Groningen, The Netherlands

Abstract: The collective cooperation dilemma, where players are involved in a public goods game and must decide whether to make an individual contribution or not, has gained increasing attention from game theorists and control engineers. Different from the traditional definition, defectors here will face the payoff restriction after adopting the strategy of free riding others' contributions. Specially, we introduce *payoff ceilings* applied to the gained benefits of the defectors, which leads to an extensive public goods game model. When the number of cooperators is lower than the threshold value, the current model can be transferred to the setups of the general public goods game. Then further increasing the number of cooperators will bring the restricted payoffs for defectors. Analytical results suggest that the payoff ceilings provide us with more insight into the resolution of cooperative dilemma. Particularly, together with other parameters (e.g., the amplification factor and the size of gaming population) of the introduced model, lower values of the payoff ceilings can better facilitate the domination of cooperation strategy at the steady state.

Key Words: Evolutionary game theory, Replicator dynamics, Public goods games

1 Introduction

Explaining the evolution of cooperative behavior from the perspective of 'survival of the fittest', which are commonplace in the real world, has been actively investigated in many fields [1–3]. Collective cooperation dilemmas are situations in which the optimal strategy of an individual is in conflict with the optimal decision of the group. Plenty of works suggest that evolutionary game theory can be frequently utilized in investigating this problem as the standard framework. The Prisoner's dilemma game (PDG), served as a paradigmatic example of cooperative dilemma, has usually been used as a key model to explore the evolution of cooperation between pairs of players. In a PDG encounter, the two participating individuals have the option to choose between cooperation (C) and defection (D). Besides the two-player game, the public goods game (PGG) [4] captures the group interactions and requires more than two participants to establish the game [5].

The public goods game (PGG), is established as an archetypical context that succinctly describes the cooperative conflict between group interest and individual benefits. In its simplest form, the establishment of PGG requires that players need to independently and simultaneously decide whether to contribute to a common pool or not. Then the collected sum is multiplied by a factor r (a fixed rate of return, $1 < r < N$) and is redistributed to the N players equally, independent of their contributions. Namely, the cooperative contribution will be a loss for the cooperators because only a portion $r/N < 1$ will be repaid. Therefore, rational and selfish players make no contribution and a social dilemma occurs.

Studies in the realm of evolutionary game theory has identified a number of crucial factors for the evolution of altruistic traits, for example, kinship [6, 7], group size [8–10], partner selection [11], obligations [12], volunteering [13–17], positive interactions [18], and probabilistic participation [19]. Moreover, the evolution of cooperation is also possible with a simple model of a population of agents that can move between groups [20]. Other interesting studies include the proposed role of insurance in threshold public goods games [21]. It is not appropriate to talk about the effective mechanisms without discussing the game playing in structured populations. In recent years, the combination of evolutionary game theory and complex network theory opens a new chapter for investigating the possible solutions of the cooperative dilemma [22, 23]. One such line of research that has proven particularly interesting is evolutionary game theory on graphs, in which interaction between agents is determined by a graph [24].

Besides the two classic strategies (i.e. cooperation and defection), there is a growing literature on other strategy choices, each one arising from different scenarios, such as loners [25, 26], punishers [27, 28], or insured players [29]. Recent work has uncovered a remarkable class of extortion strategies that afford one player a disproportionate payoff when facing an unwitting opponent [30, 31]. Moreover, a strong body of empirical evidence points to the behaviors of reward [32–35]. This has been suggested as interesting possibilities that may emerge in real-life systems. Briefly, rich types of options have been shown to have important roles in the dynamics of evolutionary games.

According to the traditional definition of general PGG, there is no special limitation on the payoffs which defectors will gain from the contributions of cooperators. However in many real-life situations, it is hardly realistic. Consider the following example. Nowadays there are some websites which are established for a resource sharing among the in-

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dividuals with the same interest together. In this situation, individuals face two choices: uploading resources for others or downloading resources provided by others. Uploading can be seen as the contributions to the common pool, while downloading shares a common point with the behaviors of free riders. In this sense, it is similar to the cooperative dilemma which occurs in the tragedy of commons. All participants in the community would like to derive benefit from the uploaded resources about some subject. Meanwhile, uploading resources will incur losses, e.g., time and resource sharing, to the individual herself. In contrast, downloading does not need to incur to pay the cost. To avoid this dilemma, many websites set out rules that each one only has limited chances to free ride others' contributions. If these opportunities have been used out, the agents need to gain chances of downloading by uploading resources, i.e. acted as a cooperator.

Though in a great part of previous approaches, the payoff reduction of defectors may be led by the proposed mechanisms, such as punishment, few study is established to model the above mentioned system which imposes restrictions on the free riding behaviors of players. And as a practical matter, this type of systems seems to be valid in inhibiting free riding behaviors which is the optimal strategy for each player. Then in accordance with this natural observation, we believe that multiple models can be proposed and established to capture the individual decisions in this real public goods environment. Here, we put forward a new viewpoint in which the payoff ceilings of defectors are introduced. From a different perspective, this can also be seen as an intrinsic punishment which works on a class of agents with low degrees of altruism. However, this payoff ceilings can effectively avoid the complicated setup of some proposed mechanisms, including when or how to bring this mechanism or rule into force.

This paper argues for a close look at the nature of payoff ceilings of defectors in situations of collective dilemmas, especially how the payoff ceilings will affect the payoff difference and strategy evolution. Reconciling individuals' interests with public interests is a challenge which occurs in many domains of life. According to the reality of collective dilemma and the existing rules which have not been modeled mathematically, it is intriguing to analyze the strategy evolution in the presence of payoff ceilings on defectors. Inspired by the social activities, learning how players can be enticed to forgo individual interests for collective interest is useful to enrich the understanding about social behavior.

The rest of the paper proceeds as follows. Section 2 provides a brief introduction about evolutionary game theory, Section 3 of our paper describes how the traditional PGG is altered to incorporate the payoff ceilings of defectors, which nicely captures some real situations. Analytical results are summarized in Section 4, and conclusions are provided in Section 5.

2 Preliminaries of Evolutionary Game Theory

Evolutionary game model description: Evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the spread and evolution of strategy competition [2]. Strategy is one of the key factor in the game playing and the payoff calculation

[36]. Generally, for any player in the multi-agent system, her provided strategy profile is denoted by $S := \{A, B, \dots\}$. One among these, for example A or B , will be the strategy each player can play within the multi-agent system.

The two-strategy payoff matrix: Here, we consider a game contested by players who can make an option from two strategies, i.e. A and B . Players choose strategies from a binary set $S := \{A, B\}$. In general, an A -player interacting with another A -player receives the benefits of R . If she interacts with a B -player, she obtains the payoff of S . Similarly, the B -player receives T from the A -player and P from other B -players. The payoff gained by each player depends on the following payoff matrix,

Table 1: Payoff matrix of general symmetric two-player games

	C	D
C	R	S
D	T	P

As for the details of the underlying payoff matrix which may affect the strategy switches, here we employ three typical game models, which are: PDG, snowdrift game (SDG), and the Stag-Hunt game (SHG) respectively. To describe the collective cooperation problem, here employing A to represent strategy of cooperation C and B for defection D , the special problem of resolving the cooperative dilemma can be recovered.

For example, in the PDG (described by $T > R > P > S$) whose payoff matrix is given by Table 2, where b and c ($b > c > 0$) indicate the benefits and costs of cooperation action, respectively. Deduced from the payoff matrix of PDG, the only stable evolutionary equilibrium by replicator dynamics equation is the domination state of full defection.

Table 2: Payoff Matrix of PDG

	C	D
C	$b - c$	$-c$
D	b	0

Table 3: Payoff Matrix of SDG

	C	D
C	$b - \frac{c}{2}$	$b - c$
D	b	0

Table 4: Payoff Matrix of SHG

	C	D
C	b	0
D	$b - c$	$b - \frac{c}{2}$

3 Problem description and formulations

As for the game theoretical framework, we consider a two-strategy and multi-player game contested by players who can make an option from two strategies, C (e.g. cooperation) and D (e.g. defection). Cooperators will make a contribution c to the common pool, while defectors will make no contribution. Then the summarized contributions cn_c will be enlarged by the magnification factor r , which produce the public goods rcn_c . Finally, each group member can evenly enjoy $\frac{rcn_c}{N}$ from the common group, irrespective of their contributions. In the following modeling and analysis, we respectively employ P_c and P_d to denote the expected payoffs for a cooperator and defector.

Here in our study, for the public goods game with payoff ceilings on defectors, the payoffs of the corresponding strategies can be described by the following equations. T ($1 \leq T \leq N$) is the threshold number of cooperators, above which the defectors will receive a limited payoff, and it is a positive integer. Specifically, if the number n_c of cooperators is no more than T , the payoffs gained by adopting the two strategies are,

$$\begin{cases} P_c = \frac{rcn_c}{N} - c \\ P_d = \frac{rcn_c}{N} \end{cases} \quad (1)$$

Conversely, when the number of cooperators satisfies $n_c > T$,

$$\begin{cases} P_c = \frac{rcn_c}{N} - c \\ P_d = \frac{rcT}{N} \end{cases} \quad (2)$$

We now focus on the strategy evolution dynamics of the groups composed by these two types of players. Similar to the replicator dynamics, here a strategy's payoff determines the growth rate of its fraction in the population [36].

$$\dot{\rho}_i = \rho_i(P_i - P_{avg}), \quad (3)$$

where $i \in (C, D)$, and $P_{avg} = \rho_c P_c + \rho_d P_d$ is the average payoff of a player in the population.

4 Replicator dynamics analysis

In many situations, the number of players who participate in the evolutionary games is finite. So we consider a scenario as follows. From time to time, N players are chosen randomly from a sufficiently large and well-mixed population, among which the fraction of cooperators is x_c ($0 \leq x_c \leq 1$). Within such a group, n_c denotes the number of cooperators among the players.

The simplest implementation of evolutionary dynamics (well-mixed approximation) for the PGG assumes that: the $N - 1$ agents that form a group with the focal player are randomly sampled without assortment from an infinite population of cooperators and defectors, with an instantaneous fraction x_c of cooperators.

Theorem 1 *Under the dynamics (3), it holds that larger T leads to a smaller value of $P_c - P_d$, i.e. the payoff advantage of cooperators over defectors.*

Proof: In the above dynamics, the probability that the focal player interacts with m cooperators is [26]

$$\binom{N-1}{m} x_c^m (1-x_c)^{N-1-m}, \quad (4)$$

and this probability is independent of whether the agent is a cooperator or defector.

Given this condition, the expected payoff of a defecting focal player is

$$P_d = \sum_{m=0}^T \binom{N-1}{m} x_c^m (1-x_c)^{N-1-m} \frac{rcm}{N} + \sum_{m=T+1}^{N-1} \binom{N-1}{m} x_c^m (1-x_c)^{N-1-m} \frac{rcT}{N}. \quad (5)$$

Here it can be rewritten by

$$P_d = \frac{rc}{N} \sum_{m=T+1}^{N-1} \binom{N-1}{m} x_c^m (1-x_c)^{N-1-m} (T-m) + \frac{rc}{N} (N-1)x_c. \quad (6)$$

While for a cooperator acted by the focal player, the payoff will be

$$P_c = \sum_{m=0}^{T-1} \binom{N-1}{m} x_c^m (1-x_c)^{N-1-m} \left[\frac{rc(m+1)}{N} - c \right] + \sum_{m=T}^{N-1} \binom{N-1}{m} x_c^m (1-x_c)^{N-1-m} \left[\frac{rc(m+1)}{N} - c \right], \quad (7)$$

which is equivalent to

$$P_c = \frac{rc}{N} [(N-1)x_c + 1] - c. \quad (8)$$

It is clear that the strategy evolution will depend on the respective payoffs led by different strategies. Here we first focus on the competition between strategy C and D . The sign of $P_c - P_d$ determines whether it pays to switch from defection to cooperation or vice versa, with $P_c - P_d = 0$ being the equilibrium condition. The payoff difference of the two strategies is

$$P_c - P_d = \frac{rc}{N} \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T) x_c^m (1-x_c)^{N-1-m} + \frac{rc}{N} - c. \quad (9)$$

We are interested in the dependence of game dynamics on the addition of payoff ceilings in the framework of this public goods game model. We further deduce

$$\begin{aligned}
P_c - P_d = & \frac{rc}{N} - c + \frac{rc}{N} \left[\binom{N-1}{T+1} x_c^{T+1} (1-x_c)^{N-T-2} \right. \\
& + \binom{N-1}{T+2} 2x_c^{T+2} (1-x_c)^{N-T-3} + \dots \\
& \left. + \binom{N-1}{N-1} x_c^{N-1} (N-1-T) \right].
\end{aligned} \quad (10)$$

The parameter T is the given threshold value of the number of cooperators in the group, and thus it is a positive integer. Without loss of generality, we assume that $T = k - 1$ where k is positive integer which is smaller than N . In this case, the details of

$$\chi = \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T) x_c^m (1-x_c)^{N-1-m} \quad (11)$$

will be the following

$$\begin{aligned}
\chi = & \binom{N-1}{k} x_c^k (1-x_c)^{N-1-k} \\
& + \binom{N-1}{k+1} 2x_c^{k+1} (1-x_c)^{N-2-k} \\
& + \binom{N-1}{k+2} 3x_c^{k+2} (1-x_c)^{N-3-k} + \dots \\
& + \binom{N-1}{N-1} x_c^{N-1} (N-k).
\end{aligned} \quad (12)$$

If $T = k$, the expression

$$\psi = \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T) x_c^m (1-x_c)^{N-1-m} \quad (13)$$

can be further expanded into

$$\begin{aligned}
\psi = & \binom{N-1}{k+1} x_c^{k+1} (1-x_c)^{N-2-k} \\
& + \binom{N-1}{k+2} 2x_c^{k+2} (1-x_c)^{N-3-k} + \dots \\
& + \binom{N-1}{N-1} x_c^{N-1} (N-k-1).
\end{aligned} \quad (14)$$

From the above calculation, we finally attain the information that larger T will lead to a smaller value of $P_c - P_d$, i.e. the payoff advantage of cooperators over defectors. $\square$

Therefore, we can safely get the conclusion that larger ceiling T of defectors will be harmful for the evolution of cooperation strategy. From the perspective of punishment, it is understandable if we consider the set ceilings on defectors as a type of punishment. Decreasing this ceiling can be regarded as increasing the intensity of punishment, while the increasing ceiling can be seen as a mitigating factor for the punishment.

(a) The relationship between $\dot{x}_c$ and x_c . Here, open dots are unstable equilibrium points and closed dots are stable equilibrium points. Resulting game dynamics depending on the initial state x_c are shown in the three scenarios of T : $T = 1$, $T = 2$ and $T = 3$ respectively. Besides the stable point of pure cooperators and defectors, an unstable border equilibrium consisting of cooperators and defectors also occurs. Moreover, larger ceilings T on defectors is harmful for the payoff advantage of cooperators over defectors, e.g. $T = 3$, where the system will easily evolve into the full defection state, even for a large cooperation fraction (e.g., $x_c = 0.85$) initially. However, smaller ceilings on defectors will help cooperators to obtain a higher payoff, hence they could get more chances to dominate the population.

(b) The correlation between the payoff difference $P_c - P_d$ and x_c . Resulting game dynamics depending on the initial state x_c are illustrated in the four cases of T : $T = 1$, $T = 2$, $T = 3$ and $T = 4$ respectively. When other variables are fixed, the unstable equilibrium point disappears in the wake of higher threshold, e.g., $T = 4$, in which the whole group will evolve into the full defection state regardless of the initial fraction of cooperators.

Fig. 1: The strategy evolution dynamics in this special public goods game characterized by the parameter T , where in the presence of larger T , cooperators are hard to stabilize. The parameters in panels (a) and (b) satisfy $N = 5, r = 3, c = 1$.

Moreover, the evolution dynamics of cooperation strategy is

$$\begin{aligned}
\dot{x}_c = & x_c(1-x_c)(P_c - P_d) \\
= & x_c(1-x_c) \left[\frac{rc}{N} - c + \frac{rc}{N} \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T) \right. \\
& \left. \cdot x_c^m (1-x_c)^{N-1-m} \right].
\end{aligned} \quad (15)$$

As illustrated in Figure 1, we obtain the scenarios concerning the values of T and x_c , where these two parameters play key roles in deterring defection. Besides the stable point of pure cooperators and defectors, there might exist an additional border equilibrium consisting of cooperators and defectors. However, this is an unstable border equilibrium, and any perturbation will drive the system evolves into the stable state which is either full cooperation or full defection. Fur-

ther, results suggest that smaller ceiling of defectors will be beneficial for the survival and domination of cooperators in the system.

The influence of parameter r on the fixation of strategies

The amplification factor in the PGG always play a key role, therefore its impact on affecting the final states cannot be overlooked. As mentioned before, besides the stable equilibrium of $x_c = 0$ and $x_c = 1$, we can obtain other possible equilibria by setting $P_c - P_d = 0$, namely

$$\frac{rc}{N} \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T)x_c^m (1-x_c)^{N-1-m} + \frac{rc}{N} - c = 0, \quad (16)$$

We assume that $P_c = P_d$ when $x_c = x_c^*$, the above equation can be rewritten by

$$\frac{r}{N} + \frac{r}{N} \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m-T)(x_c^*)^m (1-x_c^*)^{N-1-m} = 1. \quad (17)$$

Next we will study the existence of the equilibrium point x_c^* , and the correlation between multiplication factor r and the location of the equilibrium point.

Theorem 2 The inner equilibrium point $x_c^* \in (0, 1)$ exists only if the criterion $r > \frac{N}{N-T}$ is satisfied.

Proof:

As described, we aim to get further understanding about the competition results between strategy C and D by the following analysis. $P_c - P_d$ can be modified to

$$P_c - P_d = \frac{rc}{N} - c + \frac{rcT(N-T)(N-T-1)}{N^2} \binom{N}{T} \cdot \int_0^{x_c} t^{T-1}(1-t)^{N-T-2}(x_c-t)dt. \quad (18)$$

Then we get

$$\frac{\partial(P_c - P_d)}{\partial x_c} = \frac{rcT(N-T)(N-T-1)}{N^2} \binom{N}{T} \cdot \int_0^{x_c} t^{T-1}(1-t)^{N-T-2}dt. \quad (19)$$

And,

$$\frac{\partial^2(P_c - P_d)}{\partial x_c^2} = \frac{rcT(N-T)(N-T-1)}{N^2} \binom{N}{T} \cdot x_c^{T-1}(1-x_c)^{N-T-2}. \quad (20)$$

When $x_c \in (0, 1)$, we can easily find that $\frac{\partial(P_c - P_d)}{\partial x_c} > 0$. Hence $P_c - P_d$ is monotonically increasing while confronting with larger fraction of cooperators. In the open interval $x_c \in (0, 1)$, it is convincing that $P_c - P_d = 0$ has no

root or at most only one unstable root. Besides, the rate of the payoff difference grows more rapidly as x_c increases and achieves its peak value at the point of $x_c = 1$, as a result of $\frac{\partial^2(P_c - P_d)}{\partial x_c^2} > 0$.

According to eqs.(9) and (18), $P_c - P_d|_{x_c=0} = \frac{rc}{N} - c < 0$, $P_c - P_d|_{x_c=1} = c(r-1-\frac{rT}{N})$. Therefore, there are two cases needed to be considered:

Case1: Parameters settings $r \leq \frac{N}{N-T}$. Because of $P_c - P_d|_{x=1} \leq 0$, there is no root for $P_c - P_d = 0$ in the open interval $x_c \in (0, 1)$. In this scenario, the system will evolve into the full defection state despite the initial fraction of cooperators.

Case2: Parameters satisfy $r > \frac{N}{N-T}$. In this case, $P_c - P_d|_{x=1} > 0$, there is only one root x_c^* in the open interval $x_c \in (0, 1)$ for $P_c - P_d = 0$. And the clear information is that if the initial fraction x_c of cooperators in the system is no more than x_c^* , the system will evolve into the full defection state. Conversely, if the initial fraction of cooperators exceeds x_c^* , the system will evolve into a state of full cooperation.

In panel (b) of Figure.1, $r = 3 > \frac{N}{N-T}$ is available when $T = 1$, $T = 2$ and $T = 3$, where there is only one unstable border equilibrium x_c^* below which the population will evolve into full defection state. However, as the ceiling T increasing from 3 to 4, $r = 3 < \frac{N}{N-T} = 5$, $P_c - P_d = 0$ has no root in the open interval $x_c \in (0, 1)$.

Theorem 3 Under the dynamics of (3), the values of x_c^* decreases in the wake of the increasing multiplication factor r .

Proof:

According to eq.(18), if $P_c - P_d = 0$,

$$\int_0^{x_c^*} t^{T-1}(1-t)^{N-T-2}(x_c^*-t)dt = \frac{1}{T \binom{N-1}{T}} \frac{N-r}{r(N-T-1)}. \quad (21)$$

Here we suppose $\psi(x_c^*) = \int_0^{x_c^*} t^{T-1}(1-t)^{N-T-2}(x_c^*-t)dt$. It is clear that $\psi(x_c^*)$ is monotonically increases with x_c^* as a result of $\frac{d\psi(x_c^*)}{dx_c^*} > 0$. And eq.(21) could be modified into

$$r = \frac{N}{\psi(x_c^*) \binom{N-1}{T} T(N-T-1) + 1}. \quad (22)$$

From the above calculation, the values of r is inversely proportional to that of $\psi(x_c^*)$, when facing the fixed parameters N and T . That means, if we increase the value of r , the critical value of x_c^* will decrease. $\square$

The unstable equilibrium of x_c calculated by the payoff difference $P_c - P_d$, above which the system evolves into the full cooperation state and below which the system evolves into the full defection state, is our focus here. In Figure.2, we obtain the results which clearly describe the influence of the amplification factor r on the fraction x_c of cooperators in the population. No equilibrium of x_c exists if $r < \frac{N}{N-T}$, for example, $T = 3$ and $r < 2.5$ in panel(a) of Figure.2. When fixing other parameters in the model, i.e. for a fixed

Fig. 2: The unstable equilibrium of x_c , above which the system evolves into the full cooperation state and below which the system evolves into the full defection state, in dependence on the amplification factor r of the public goods game, with the different values of threshold. Resulting game dynamics depending on the initial state x_c are shown in the three scenarios of T : $T = 1$, $T = 2$ and $T = 3$ respectively and the parameters satisfy $N = 5, c = 1$ in the panel (a) where amplification factor r takes values in $r \in [1, 5]$. As a contrast, parameters in the panel (b) adopt $N = 100, c = 1$. In this situation, r ranges from 1 to 100. There are also three cases accordingly: $T = 5$, $T = 30$ and $T = 50$. Larger ceilings T on defectors facilitates the unstable equilibrium of x_c , which is harmful for the evolution of cooperation. While larger value of r inhibits the unstable equilibrium point and promotes cooperation.

payoff ceiling T , it is easy to conclude that larger r will decrease the unstable equilibrium point existing on the edge of $C-D$ (cooperation to defection). It means that the increment of amplification factor r will benefit the survival and maintaining of cooperators, even at a low initial fraction of cooperators in the population. As for the special case of $r = N$, the evolution dynamics of cooperation strategy is given by

$$\begin{aligned} \dot{x}_c &= x_c(1 - x_c)(P_c - P_d) \\ &= cx_c(1 - x_c) \sum_{m=T+1}^{N-1} \binom{N-1}{m} (m - T) x_c^m (1 - x_c)^{N-1-m}, \end{aligned} \quad (23)$$

which suggests that $\dot{x}_c > 0$ supported by the condition of $x_c \in [0, 1]$. In this case, the number of cooperators will increase till the full-cooperation state.

5 Conclusions

Multi-agent gaming interactions frequently occur in plenty of real systems, and it is of key importance to deal with N -agent interactions as well as to take into account the details of the payoff matrix which decides the cooperative dilemma. A general underlying assumption in the previously reported studies in the evolutionary dynamics of PGG is that there is no restriction on the payoffs of free riders. However, in real social societies with abundant collective dilemmas, setting ceilings on the gained payoffs of defectors is common especially when, e.g., the public goods is the e-resources on web-sites: as a mechanism of encouraging contributive behaviors, limited free loading is provided for the one to attempt that, following which is the paying to download, which can be realized by the contributive behaviors for the common goods as an exchange.

In this paper, by exploiting the models under some cooperative phenomenon that indeed exist, we consider the role that payoff ceilings of defectors has on the emergence of cooperation in the PGG. Our results point out that setting the payoff ceilings on defectors plays a positive role in inhibiting the behaviors of free riding. It is understandable that the payoff ceilings on defectors indicate a limitation of the gained benefits, compared with the unlimited one in the traditional study. For verification, results show that larger payoff ceilings (drive by parameter T) lead to a decline in cooperation. In comparison to this, lower values of payoff ceilings together with the appropriate initial fraction of cooperators, can keep cooperators alive and dominant. Last, the variation of the related parameters, e.g., the amplification factor, the initial fraction of cooperators, helps us to verify that reasonable settings of payoff ceilings is an efficient feature for sustaining contribution levels.

Summarizing, our analysis can offer some novel predictions for the complicated collective cooperation dilemma in real social systems, where many rules play an increasingly important role in sustaining cooperative contributions at the group level, meanwhile mathematical modeling and analysis for this are in urgent need. And, to investigate the competition dynamics of multiple strategies in more realistic collective cooperation dilemma is also of our interest, and further study concerning this topic will be carried out.

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