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Unstable vortices do not confine

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Abstract

Recently, a geometric model for the confinement of magnetic charges in the context of type II string compactifications was constructed by Greene, Morrison and Vafa [Nucl. Phys. B 481 (1996) 513]. This model assumes the existence of stable magnetic vortices with quantized flux in the low energy theory. However, quantization of flux alone does not imply that the vortex is stable, since the flux may not be confined to a tube of definite size. We show that in the field theoretical model which underlies the geometric model of confinement, static, cylindrically symmetric magnetic vortices do not exist. While our results do not preclude the existence of confinement in a different low-energy regime of string theory, they show that confinement is not a universal outcome of the string picture, and its origin in the low energy theory remains to be understood. © 1998 Elsevier Science B.V.

1. Introduction

In a recent paper a mechanism for confinement of magnetic flux was put forward in type II string compactifications on Calabi-Yau manifolds, where magnetic states can arise from D branes wrapped around non-trivial chains [1]. In this picture, chains must be attached to other chains in order to make a closed 3-cycle on which to wrap the brane. In four dimensions, the configuration would look like pairs of oppositely charged magnetic monopoles joined by flux tubes (sometimes known as “dumbbells”), thus providing a mechanism for confinement.

From the field theory point of view, the low energy theory contains sixteen hypermultiplets charged under fifteen U(1) gauge groups, in such a way that the condition for finite energy per unit length of a vortex-like configuration translates into a correlation between the various windings at infinity

which ultimately leads to the quantization of magnetic flux. Taken together with the fact that the Higgs mechanism is operating in this model one would be tempted to conclude that there are stable vortices with a width given by the inverse vector mass, similar to those found in the Abelian Higgs model [2,3].

It is the purpose of this letter to show that this assumption may not be justified. We will see that the low energy theory discussed in [1] does not admit stable static axisymmetric magnetic vortices of a fixed width. Any such configuration immediately decays by expanding its core radius indefinitely. While our result does not preclude the existence of non-axisymmetric flux tubes, it is very unlikely that such stable structures exist. The physical origin of the instability can be traced back to the repulsion of magnetic field lines which, unlike for the Abelian Higgs model, is not compensated here by an increase

in potential energy during the expansion because the potential has flat directions. It is difficult to see how non-axisymmetric configurations could circumvent this problem.

It is well known that the quantization of magnetic flux does not guarantee its confinement into flux tubes, even when the gauge bosons are made massive by the Higgs mechanism. Consider, for instance, the so-called semilocal strings [4,5] which arise in the Weinberg-Salam model in the limit of zero $SU(2)$ coupling, a model that has several features in common with the one analysed here. In particular, the relative shortage of gauge field degrees of freedom also introduces a correlation between the winding of the scalars at infinity. As a result, magnetic flux is quantized, with the various sectors separated by infinite energy barriers. However, it has been shown that the stability of vortex solutions in this model depends on the ratio of the scalar and vector masses. If $m_{\text{scalar}} < m_{\text{vector}}$ there are stable vortices whose width is related to the inverse vector mass (they are in fact identical to the Nielsen-Olesen vortices in the Abelian Higgs model). However, when $m_{\text{scalar}} > m_{\text{vector}}$, there are no stable vortices; the magnetic core tends to expand indefinitely while conserving magnetic flux. Thus, magnetic flux is quantized but not confined in this case. (This is not so surprising; the quantization of magnetic flux has to do with the behaviour of the fields far away from the vortex, whereas stability depends on their behaviour at the core, and unless there is a topological reason to link these two distance scales, they will be independent of each other). Moreover, when $m_{\text{scalar}} = m_{\text{vector}}$, the vortices saturate a Bogomol'nyi bound (which automatically ensures their stability) but this is still not enough to guarantee the confinement of magnetic flux to tubes of a definite size (see [5,6]), because there is an entire family of solutions with the same energy and different core sizes.

What this example illustrates is that, in general, identifying a topological invariant such as a conserved magnetic flux is not enough to guarantee the existence of stable solutions carrying this topological charge. Experience shows that this is a particularly dangerous assumption when the theory contains both global and local symmetries linked in a non-trivial way [7]. In the semilocal model of the previous paragraph, the existence of such solutions depends

on the parameters of the theory (the scalar and vector masses); in the model analysed here, the situation is even more dramatic, since it seems that there are *no* values of the parameters for which stable vortices exist, even in the lowest non-zero energy sector.

Thus, as long as the string theory regime is such that its low energy behaviour is the one discussed in [1], confinement of magnetic charges is very unlikely. Needless to say, this does not preclude the existence of confinement in other low-energy regimes; it merely shows that confinement is not a *necessary* outcome of the string picture and remains to be understood.

In [1], the existence of these confining flux tubes was argued in the simpler example of a field theory with two hypermultiplets with opposite charges under a single $U(1)$ gauge field. Since the physics in the discussion is essentially the same as in the sixteen multiplet case, we will also consider this simplified model. In order to study vortex solutions we will impose translational symmetry in the z direction and reduce the model to $2 + 1$ dimensions. We will then prove the non-existence of static axisymmetric vortices, and discuss the implications for the $3 + 1$ dimensional “dumbell” configurations and confinement.

2. The model

The simplified field theory model of [1] contains two $N = 2$ hypermultiplets, each containing two physical and two auxiliary scalar fields (all complex) and a Dirac spinor, coupled to the $N = 2$ Abelian vector multiplet. The Lagrangian (in Wess-Zumino gauge) reads [8]

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{interaction}}, \quad (1)$$

where (implicit summation over a , which counts the hypermultiplets)

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & \frac{1}{2} (\partial_\mu M)^2 + \frac{1}{2} (\partial_\mu N)^2 + \frac{i}{2} \bar{\lambda}_i \gamma^\mu \partial_\mu \lambda^i \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} D^2, \end{aligned}$$

$$\mathcal{L}_{\text{matter}} = \frac{1}{2} D^\mu h_a^i D_\mu h_{ai} + i \bar{\psi}_a \gamma^\mu D_\mu \psi_a + F_a^i F_{ai},$$

$$\begin{aligned}
\mathcal{L}_{\text{interaction}} = & i q_a h_a^i \bar{\lambda}_i \psi_a - i q_a \bar{\psi}_a \lambda^i h_{ai} \\
& - q_a \bar{\psi}_a (M - \gamma^5 N) \psi_a \\
& - \frac{1}{2} h_a^i (M^2 + N^2) h_{ai} \\
& + \frac{1}{2} q_a h_a^i \tau_i^j D h_{aj}.
\end{aligned} \quad (2)$$

The q_a are the charges of the hypermultiplets, so that

$$D_\mu h_{ai} = (\partial_\mu + i q_a A_\mu) h_{ai}. \quad (3)$$

The supersymmetry transformations of the fields of the hypermultiplets take on the form

$$\begin{aligned}
\delta h_{ai} &= 2 \bar{\epsilon}_i \psi_a, \\
\delta \psi_a &= -i \epsilon^i F_{ai} - (i \gamma^\mu D_\mu + M + \gamma^5 N) \epsilon^i h_{ai}, \\
\delta F_{ai} &= 2 \bar{\epsilon}_i (\gamma^\mu D_\mu + i M - i \gamma^5 N) \psi_a - 2 \bar{\epsilon}_j \lambda^j h_{ai},
\end{aligned} \quad (4)$$

while the fields of the gauge multiplet transform as

$$\begin{aligned}
\delta A_\mu &= i \bar{\epsilon}_i \gamma_\mu \lambda^i, \\
\delta M &= i \bar{\epsilon}_i \lambda^i, \\
\delta N &= i \bar{\epsilon}_i \gamma^5 \lambda^i, \\
\delta \lambda^i &= -\frac{i}{2} \sigma^{\mu\nu} \epsilon^i F_{\mu\nu} - \gamma^\mu \partial_\mu (M + \gamma^5 N) \epsilon^i \\
&\quad - i \epsilon^j \tau_j^i D, \\
\delta D &= \epsilon_i \tau_j^i \gamma^\mu \partial_\mu \lambda^j.
\end{aligned} \quad (5)$$

Furthermore, the Lagrangian has a global $SU(2)$ symmetry that rotates the two scalar fields of the multiplets. Note that there is no continuous symmetry that mixes the hypermultiplets, unless they have the same $U(1)$ -charge.

We can eliminate the auxiliary fields and get a self-interaction term for the scalars h_{ai} . The equations of motion for the auxiliary fields are

$$F_{ai} = 0, \quad D = H_j^i \tau_i^j, \quad (6)$$

where $H_j^i = -\frac{1}{2} q_a h_a^i h_{aj}$ ($h_a^i = h_{ai}^*$). When we substitute these equations back into the Lagrangian, a term $-V(h_{ai})$ arises, with

$$\begin{aligned}
V(h_{ai}) &= \frac{1}{2} D^2 \\
&= \frac{1}{2} \left[(H_2^1 + H_1^2)^2 + (i H_2^1 - i H_1^2)^2 \right. \\
&\quad \left. + (H_1^1 - H_2^2)^2 \right].
\end{aligned} \quad (7)$$

Note that $(H_j^i)^* = H_i^j$, so the potential is a sum of three positive terms. In what follows we will limit ourselves to the case where the two hypermultiplets have $U(1)$ -charges $q_a = (1, -1)$, as is done in [1]. The minimum of the potential, $V(h_{ai}) = 0$, is obtained for

$$\begin{aligned}
H_2^1 &= -\frac{1}{2} [h_{11}^* h_{12} - h_{21}^* h_{22}] = 0, \\
H_2^1 &= -\frac{1}{2} [h_{12}^* h_{11} - h_{22}^* h_{21}] = 0, \\
H_1^1 - H_2^2 &= -\frac{1}{2} [|h_{11}|^2 + |h_{22}|^2 - |h_{12}|^2 - |h_{21}|^2] \\
&= 0.
\end{aligned} \quad (8)$$

In order to find the vacuum manifold, we follow [1] and parametrize the complex scalar fields as:

$$h_{ai} = r_{ai} e^{i\theta_{ai}}. \quad (9)$$

Eqs. (8) now become

$$\begin{aligned}
e^{i(\theta_{11} - \theta_{12})} &= e^{i(\theta_{21} - \theta_{22})}, \\
r_{11} r_{12} &= r_{21} r_{22}, \\
(r_{11})^2 - (r_{21})^2 &= (r_{12})^2 - (r_{22})^2.
\end{aligned} \quad (10)$$

These equations are solved (up to a factor $2k\pi$ in the angles) by

$$\theta_{11} - \theta_{12} = \theta_{21} - \theta_{22}, \quad (11)$$

$$r_{1i} = r_{2i}. \quad (12)$$

Consider the bosonic sector. The contribution to the energy from the scalars M and N is positive definite, and they must tend to zero at infinity, so we set $M = N = 0$. The model then contains four complex scalars and one $U(1)$ gauge field. Since we are interested in the possible existence of magnetic vortex solutions, we will now reduce the problem to $2 + 1$ dimensions by imposing translational symmetry in the z -direction, and consider static solutions. Specifically, we require all fields to be independent of t and z (note that, in principle, the scalar fields could have a non-trivial dependence on these coordinates which would lead to spinning or electrically charged configurations, but they all have higher energy). For the same reason we take $A_t = A_z = 0$. The electric field is zero, and the only component of the magnetic field, B , is in the z -direction.

The energy per unit length becomes ($m, n = 1, 2$):

$$\mathcal{E} = \int d^2x \left[\frac{1}{2} |D_m h_{ai}|^2 + \frac{1}{4} F_{mn}^2 + V(h_{ai}) \right] \quad (13)$$

where $V(h_{ai})$ is given in (7). In order to have a finite energy string, we require

$$D_m h_{ai} \rightarrow 0, \quad F_{mn} \rightarrow 0, \quad V(h_{ai}) \rightarrow 0, \quad (14)$$

as $r \rightarrow \infty$ which, together with (11) and (12), lead to the following asymptotic behaviour for the fields:

$$\begin{aligned} h_1 &\equiv \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} \rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} e^{-i n \theta}, \\ h_2 &\equiv \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} \rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} e^{i \Delta} e^{i n \theta}, \\ A_\theta &\rightarrow \frac{n}{r}, \quad A_r \rightarrow 0 \end{aligned} \quad (15)$$

for $r \rightarrow \infty$. Here c_i are arbitrary complex constants, Δ is real.

These boundary conditions are analogous to those of Nielsen-Olesen and semilocal vortices in that they correspond to a winding of the hypermultiplets around a gauge orbit at infinity (n is the winding number of the configuration). Due to the relative shortage of degrees of freedom, the windings are correlated, not only within each hypermultiplet but also between h_1 and h_2 . Note that when the winding numbers are not correlated like this, the energy diverges, so the various winding sectors are separated by infinite energy barriers. Indeed, magnetic flux is quantized,

$$\Phi = \int d^2 x B = \oint r d\theta A_\theta = 2 \pi n, \quad (16)$$

but we will see in the next section that, unlike for the Abelian Higgs and its semilocal extensions, there are no cylindrically symmetric vortex solutions in this case.

3. Static, axisymmetric configurations

In what follows we will restrict ourselves to $n = 1$, but our results generalize trivially to any winding number.

The condition of axial symmetry means that, when the solution is rotated around the z -axis, the rotated configuration is related by symmetry to the original one. Consider first the gauge fields. Under infinitesimal rotations one should find $\partial_\theta A_m(r, \theta) =$

$\partial_m \alpha(r, \theta)$. We can analyse these conditions by choosing the gauge $A_r = 0$, in which case the function α is restricted to be independent of r , and therefore $A_\theta = \alpha(\theta)/r + v(r)$ (where $v(r)$ is an arbitrary function of r). We can gauge-transform to $A_\theta = v(r)$, which determines the function α up to an arbitrary constant. The residual invariance is a global $U(1)$. Now we turn to the scalar fields. It is straightforward to show that the most general configuration for the scalar hypermultiplets compatible with cylindrical symmetry is

$$h_1 = \begin{pmatrix} g_1(r) \\ f_1(r) \end{pmatrix} e^{-i \theta}, \quad h_2 = \begin{pmatrix} g_2(r) \\ f_2(r) \end{pmatrix} e^{i \Delta} e^{i \theta}, \quad (17)$$

with boundary conditions $f_a, g_a \rightarrow 0$ as $r \rightarrow 0$ and $g_a \rightarrow c_1, f_a \rightarrow c_2$ as $r \rightarrow \infty$.

We now prove that there are no stable axisymmetric solutions to the equations of motion by showing that the energy of such a configuration can always be lowered by a continuous deformation which respects the boundary conditions. Indeed, the energy of the family

$$\begin{aligned} h_1(\xi) &\equiv \begin{pmatrix} (1 - \xi) g_1 + \xi g_2 \\ (1 - \xi) f_1 + \xi f_2 \end{pmatrix} e^{-i \theta}, \\ h_2(\xi) &\equiv \begin{pmatrix} \xi g_1 + (1 - \xi) g_2 \\ \xi f_1 + (1 - \xi) f_2 \end{pmatrix} e^{i \Delta} e^{i \theta}, \end{aligned}$$

(with the same A_μ) is given by $\mathcal{E}_g(\xi) + \mathcal{E}_p(\xi) + \mathcal{E}_m(\xi)$, where

$$\mathcal{E}_g(\xi) = \mathcal{E}_g(0) + \xi(\xi - 1)\mathcal{A},$$

$$\mathcal{E}_p(\xi) = (1 - 2\xi)^2 \mathcal{E}_p(0),$$

$$\mathcal{E}_m(\xi) = \mathcal{E}_m(0), \quad (18)$$

are the energy contributions from the scalar gradients, scalar potential and magnetic field respectively, and $\mathcal{A}$ is some positive constant. The energy decreases monotonically from $\xi = 0$ (our starting configuration) until it reaches a minimum at $\xi = 1/2$. Moreover, at $\xi = 1/2$ the potential energy is zero and therefore the energy can be lowered even further by letting the magnetic core expand. Consider the

family of expanding configurations for fixed magnetic flux ($\hat{h}_a \equiv h_a(\xi = 1/2)$, $\hat{\mathcal{E}} \equiv \mathcal{E}(\xi = 1/2)$)

$$h_{a,\lambda}(r, \theta) \equiv \hat{h}_a\left(\frac{r}{\lambda}, \theta\right), \quad (19)$$

$$A_{m,\lambda}(r, \theta) \equiv \frac{1}{\lambda} A_m\left(\frac{r}{\lambda}, \theta\right), \quad (20)$$

whose energy per unit length is

$$\mathcal{E}_\lambda = \hat{\mathcal{E}}_g + \frac{\hat{\mathcal{E}}_m}{\lambda^2}. \quad (21)$$

The energy decreases monotonically as $\lambda \rightarrow \infty$ (this is of course a variation of the scaling argument used to prove Derrick's theorem [9]). This implies that, for a cylindrically symmetric configuration, the magnetic flux can never be confined. The magnetic field lines tend to spread to infinity, since this lowers the energy.

It is straightforward to check that the instability would still be there if we had allowed for non-zero scalars M and N . Finally, it should be obvious that our results apply unchanged to the low energy approximation of the model discussed in [1], a model containing sixteen hypermultiplets charged under fifteen $U(1)$'s.

4. Discussion

We have argued that stable, infinitely long magnetic vortices almost certainly do not exist in this supersymmetric model. While our proof only concerns axisymmetric vortices, it is hard to see how non-axisymmetric solutions could avoid the instability described in the previous section. It should be stressed that the total magnetic flux in each sector is topologically conserved. Thus, in general, identifying a topological invariant of the theory is not enough to guarantee the existence of stable solutions carrying this topological charge, particularly if the theory contains both global and local symmetries [7], as is the case in many supersymmetric models. Finding a higher dimensional origin for such a topological invariant, while mathematically very compelling, may be of limited assistance in understanding the low energy spectrum of the compactified theory.

If we now consider $3 + 1$ dimensional “dumbell”

configurations consisting of a monopole-antimonopole pair joined by a finite segment of this putative string the expectation is that the configuration will decay into a diffuse and more-or-less spherical lump. The magnetic charges will still be linked, as the string picture suggests, but the confining character of the potential will be lost.

Of course it is possible that quantum corrections could modify our results, but we have to stress that the stability of the solution will, in general, depend on the *specific* details of the potential [5]. Neither is it sufficient to prove that the configurations saturate a Bogomol'nyi bound. While BPS states are indeed stable, saturation of a BPS bound does not preclude the existence of zero modes or flat directions, which sometimes may result in an expansion of the core of the string [5,6]. Whether this can be considered to lead to confinement is, at best, open to discussion.

After completion of this work, it was brought to our attention that [10] reaches a similar conclusion. We thank the authors pointing this out.

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