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A combined nodal and uniform pricing mechanism for congestion management in distribution power networks

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ABSTRACT

We present a price-based approach for congestion management in distribution power systems with renewable generations with the final goal of maximizing a social welfare function which includes net revenues of network users as well as the cost of real power losses. Particularly, we design a combined nodal and uniform pricing mechanism for maximization of the social welfare function while maintaining voltage magnitudes in a desired bound and respecting the grid's actual capacity. Storage systems are considered in order to increase/add elasticity to generations and loads. Simulation results are provided for a modified IEEE-37 bus. A scenario is numerically simulated where the excess of generation in the absence of storage and price signals leads to deviations of voltages from a desired bound. Thereafter, it is shown that the application of the price signals is effective to maintain the voltages within the desired bound while the designed social welfare function is maximized.

1. Introduction

The recent growth of renewable energy sources, i.e. wind and solar sources, in electrical power systems is tremendous. Comparing to fossil-fuel sources, renewable ones are produced and located in a more decentralized fashion and their availability mainly depends on the climate and weather conditions. As a result, the current energy systems are facing new challenges. On the one hand, network operators need to cope with the effects of distributed generations to maintain the desired performance of the physical grid. On the other hand, the economic and societal aspects of energy systems have to be considered which may require changing the design of electricity markets. The injection of renewables particularly affects the distribution grid where the main challenges are to maintain the nodal voltages within an acceptable bound and prevent network bottlenecks despite changes in loads and generations. Generally speaking, network bottlenecks (congestion) occur as a result of a surplus of generation and/or consumption if there is no transmission possibility. So far, congestion management has been mainly addressed in transmission systems. One reason is that the distribution system operators traditionally have not been proactive in the grid management. However, the recent increase in the penetration of renewables and the parallel increase in the uncertainty of loads and productions are stimulating congestion management in the distribution

systems.

One trivial solution to tackle network congestion is to extend the physical grid. As extending the capacity of the grid requires significant amount of investments, giving incentives to network users to adapt their network usage can be an efficient alternative solution. Such incentives can be given by price-based methods. Price-based methods motivate consumers and producers to utilize controllable (price-elastic) components which enable them to react to price signals to obtain and maintain the desired performance of the grid (e.g. [1,2]). Examples of price-based techniques include frequency regulation and congestion management in transmission systems (e.g. [1]), demand-supply co-ordination (e.g. [3,4]), scheduling in a dynamic pricing environment [5], analysis of demand, supply dynamics with real-time pricing [6], and locational marginal pricing (e.g. [7,8]) for resource allocation and congestion management. For distribution systems, congestion problems include both voltage and overloading problems [9]. So far, voltage levels has been mostly controlled (e.g. [10–12]) using only the reactive power, and not using price signals. In fact, the direct effects of voltage deviations on the congestion prices have only been recently considered in the literature, e.g. [13].

Main contribution: We consider a distribution system with renewable generations, storage systems and constant loads. Storage systems add flexibility (elasticity) to both the load and the generation side.

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We first maximize a social welfare function, which includes both the revenues of the prosumers (units which both consume and produce energy) and the cost of real power losses, constrained by the limitations on the nodal voltages and the actual grid capacity. We use the obtained Lagrange multipliers (shadow prices) to design a twofold congestion price. The first component, which depends on both active and reactive power, is a nodal price corresponding to the deviations of the voltages at each busbar. The second component, which depends on the aggregated active power, is a uniform price corresponding to the installed physical capacity of the grid (the capacity of main power lines and transformers). Second, we simulate a realistic scenario based on the real consumption, production and exogenous price data on a modified IEEE 37 benchmark model in order to validate our design. Simulation results are provided for a modified IEEE-37 bus. A scenario is numerically simulated where the excess of generation in the absence of storage and price signals leads to deviations of voltages from a desired bound. Thereafter, it is shown that the application of the price signals is effective to maintain the voltages within the desired bound while the designed social welfare function is maximized. Further, we discuss re-designing of the distribution network tariffs based on the congestion prices.

Compared to the previous works, our contribution is to derive congestion prices depending on both active and reactive power for social welfare maximization and respecting the limitations of the physical grid. The designed price signals are intended to be used in the design of dynamic network tariffs. The potential of dynamic electricity tariffs to reduce costs of energy systems have been analyzed by, e.g. [14]. In this regard, the contribution of our paper is to define congestion prices based on voltages and grid capacity, analyze the impact of renewable generation and storage systems on such prices and how this impact can be translated into dynamic network tariffs by combining insights from both physics and regulatory economics.

Preliminary results were presented in [15]. Compared with [15], this paper presents a social welfare maximization problem which includes power losses while no restrictions on power lines are imposed. In addition, this paper considers the addition of storage systems, discusses the relation between congestion prices and dynamic tariffs, and presents the results of a case study based on an IEEE benchmark model.

The paper is organized as follows. The problem description and physical model of the grid are presented in Section 2. Section 3 presents the distributed pricing mechanism. Simulation results of a case study are discussed in Section 4. Dynamic distribution tariffs are discussed in Section 5. The paper is concluded in Section 6.

2. Problem description and physical model of the grid

We design congestion prices for distribution networks via maximizing a social welfare function. In defining the social welfare function (Section 3), we consider a society composed of ‘ n ’ prosumers and one network operator. Each prosumer corresponds to one node of the physical network and the operator corresponds to all of the nodes, i.e. the cost of the operator is influenced by all nodes.

We define a twofold measure for the congestion considering both nodal voltages and the actual grid capacity. We define the grid capacity as the limited zonal aggregated active power. In distribution systems, for instance, multiple prosumers are connected to one transformer or power line feeder with a specific capacity. Hence, the net power of prosumers connected to one feeder should not exceed its actual capacity (notice that this paper considers only one zone). In addition, maintaining the steady state voltages within an acceptable range is one of the main goals in distribution networks. This paper considers maintaining desired voltage levels by means of giving references for both (nodal) active and reactive power flows using price signals.

2.1. Physical model of the distribution network

Consider a distribution network composed of n busbars and m power lines. The nodal active and reactive power obey the well-known AC power flow model as follows:

$$\begin{aligned} P_i &= \sum_{j=1}^n G_{i,j} V_i^2 - G_{i,j} V_i V_j \cos(\theta_{i,j}) - B_{i,j} V_i V_j \sin(\theta_{i,j}), \\ Q_i &= \sum_{j=1}^n -B_{i,j} V_i^2 + B_{i,j} V_i V_j \cos(\theta_{i,j}) - G_{i,j} V_i V_j \sin(\theta_{i,j}), \end{aligned} \quad (1)$$

where P_i , Q_i , V_i , θ_i are the active power, reactive power, voltage magnitude and voltage angle of busbar i respectively. The variable $\theta_{i,j} = \theta_i - \theta_j$ is the relative angle and $G_{i,j}$ and $B_{i,j}$ are the conductance and susceptance of line i, j . For the purpose of our paper, it is sufficient to assume a three-phase balanced network [16]. The latter allow us to equivalently consider a single-phase network. We define all of the busbars as PQ busbars. The PQ busbars imply that P and Q are known, while V and θ are then indirectly obtained from their relationship with P and Q . We assume that each busbar is connected to one ‘prosumer’ which is a renewable energy source (e.g. photovoltaic panel) with an inverter together with active and reactive loads. That is,

$$P_i = P_{g,i} - P_{l,i} + P_{s,i}, \quad Q_i = Q_{g,i} - Q_{l,i}, \quad (2)$$

where indexes g, l, s represent the generation, load and storage of busbar i (connected to prosumer i). This paper assumes that the loads are constant (price-inelastic) and storage systems only contribute to the active power. Fig. 1 illustrates our model. The distribution system is connected by using a transformer to the HV/MV network. Notice that this paper models the interaction of the HV and the distribution network by imposing constraints on the exchanged power between the two networks based on the physical capacity of the substations.

Storage system. We adopt the following discrete-time model for storage i connected to busbar i

$$E_{s,i}(k+1) = E_{s,i}(k) - \eta_i P_{s,i}(k), \quad (3)$$

where $E_{s,i}(k)$ and $P_{s,i}(k)$ are the stored energy and active power flow at time-instant k , and η_i indicates the efficiency of the storage system. The storage is discharging if $P_{s,i}(k) \geq 0$, otherwise it is charging. We assume that each storage unit is subject to the following constraints:

$$\underline{E}_{s,i} \leq E_{s,i}(k) \leq \bar{E}_{s,i}, \quad -P_{s,i}^{\text{rated}} \leq P_{s,i}(k) \leq P_{s,i}^{\text{rated}}. \quad (4)$$

In the above, $\underline{E}_{s,i}$ and $\bar{E}_{s,i}$ are the bounds on the energy of the storage and $P_{s,i}^{\text{rated}}$ is the bound on the power of the storage. Inequalities in (4) can be combined to obtain

Fig. 1. The illustration of the distribution network, $G_i = (P_{g,i}, Q_{g,i})$, $L_i = (P_{l,i}, Q_{l,i})$ and $C_i = P_{s,i}$.

$$P_{s,i}^{\min} \leq P_{s,i}(k) \leq P_{s,i}^{\max} \quad (5)$$

such that

$$P_{s,i}^{\min} = \max \left\{ -P_{s,i}^{\text{rated}}, -\frac{\bar{E}_{s,i} - E_{s,i}(k)}{\eta_i} \right\} \quad \text{and} \quad P_{s,i}^{\max} = \min \{ P_{s,i}^{\text{rated}}, (E_{s,i}(k) - \underline{E}_{s,i})\eta_i \}.$$

2.2. Power losses in distribution systems

We consider two sources of power losses: real power losses [10] due to the resistance of the power lines (cost of the operator), and power losses related to each inverter (cost of the prosumer).¹

The real power losses for a network with nodal active and reactive power as in (1) is equal to

$$P_{\text{loss}}^r = \sum_{i=1}^n P_i = \sum_{i=1}^n P_{g,i} - P_{l,i} + P_{s,i}. \quad (6)$$

We use (6) in formulating a distributed social welfare optimization problem in Section 3.

In addition to the power line losses (cost of the operator), we also consider the losses for each PV inverter unit as the cost of the production for prosumers. The power loss of inverters includes a constant loss, a voltage-dependent loss over power electronic components of the inverter, and a current-dependent power loss (see [17]). In this paper, we neglect the loss of power electronic devices and estimate the power loss per inverter by its current-dependent component as follows

$$P_{i,\text{inv}} = \varepsilon_i (P_{g,i}^2 + Q_{g,i}^2), \quad (7)$$

where $P_{g,i} = P_i - P_{l,i} + P_{s,i}$, $Q_{g,i} = Q_i + Q_{l,i}$ and ε_i is a coefficient. In designing our market-based objective function, the coefficient ε_i is assumed to be price-dependent (see Eq. (29)).

3. Social welfare maximization

In economic terms, we distinguish the following agents in our model: a network operator and prosumers. We present a problem formulation which involves both the prosumers and the network operator. Consider the following optimization problem²

$$\max_x -f_0(x) + \sum_{i=1}^n f_i(x_i) \quad (8)$$

$$\text{s. t. } x_i \in X_i, \quad (9)$$

where $x = (x_1^T, \dots, x_n^T)^T$, $x_i = (P_{g,i}, P_{s,i}, Q_{g,i})$. Further, $f_i(x_i)$ is the profit of prosumer i and $f_0(x)$ is equal to the cost of power line losses, which is the cost of the operator. In other words, $-f_0(x)$ represents the network utilization. Considering the real power losses in (6), the cost of losses should be equal to $\gamma^p P_{\text{loss}}^r$ where γ^p denotes the exogenous price for active power. However, since the model of losses in (6) is a linear function and the load ($P_{l,i}$) is not controllable in our paper, in order to minimize the difference of generation and consumption, hence losses, we modify $\gamma^p P_{\text{loss}}^r$ as follows

$$f_0 = \gamma^p \frac{P_{\text{loss}}^r}{|\sum_i P_{l,i}|} = \gamma^\ell \left(\sum_i P_{g,i} + P_{s,i} \right) - \gamma^p$$

where $\gamma^\ell = \frac{\gamma^p}{|\sum_i P_{l,i}|}$. Define $P_i^\ell = \gamma^\ell (P_{g,i} + P_{s,i}) - \frac{\gamma^p}{n}$, then

¹ The cost of batteries due to the losses of the power electronic units is assumed to be negligible [17].

² This paper considers static optimization at each time step k . For representation clarity, the term k is omitted for all formulations in the rest of the paper.

$$f_0 = \sum_i P_i^\ell. \quad (10)$$

Now, we characterize f_i in (8) which is the profit of prosumer i . Define

$$f_i(P_{g,i}, P_{s,i}, Q_{g,i}) = U_i(P_{g,i}, P_{s,i}, Q_{g,i}) - C_i(P_{g,i}, P_{s,i}, Q_{g,i}) \quad (11)$$

where U_i is the revenue and C_i is the cost of production and storage for prosumer i . We assign the inverter losses in (7), as the cost of production for each prosumer and assume a strictly convex (e.g. quadratic) cost for the storage system. Hence, C_i is a differentiable and strictly convex function. The term U_i is modeled as the income of prosumer i by trading its generated and stored power, i.e.

$$U_i = U_i^p(P_{g,i}) + U_i^q(Q_{g,i}) + U_i^s(P_{s,i}), \quad (12)$$

where $U_i^p = \gamma^p P_{g,i}$, $U_i^q = \gamma^q Q_{g,i}$, $U_i^s = \gamma^s P_{s,i}$ such that γ^p , γ^q , γ^s are the exogenous price signals for active, reactive power and stored power. The definitions of C_i , U_i and f_0 allow us to assume that the objective function in (8) is strictly concave and differentiable. Considering (8), we have

$$-f_0(x) + \sum_i f_i(x_i) = \sum_i f_i(P_{g,i}, P_{s,i}, Q_{g,i}) - \sum_i P_i^\ell.$$

An example of such objective function is presented in Section 4. Let us define a combined nodal utility function $J_i(P_{g,i}, Q_{g,i}, P_{s,i})$ such that

$$J_i(P_{g,i}, Q_{g,i}, P_{s,i}) = f_i(P_{g,i}, P_{s,i}, Q_{g,i}) - P_i^\ell.$$

The overall optimization problem is

$$\max_{\{P_{g,i}, Q_{g,i}, P_{s,i}\}} \sum_i J_i \quad (13)$$

$$\text{s. t. } 0 \leq P_{g,i} \leq \bar{P}_{g,i}, \quad (14)$$

$$Q_{g,i} \leq Q_{g,i} \leq \bar{Q}_{g,i}, \quad (15)$$

$$P_{s,i}^{\min} \leq P_{s,i} \leq P_{s,i}^{\max}, \quad (16)$$

$$V_m \leq V_i \leq V_M, \quad (17)$$

$$\phi_m \leq \theta_{i,j} \leq \phi_M, \quad (18)$$

$$c_m \leq \sum_i P_{g,i} + P_{s,i} - P_{l,i} \leq c_M. \quad (19)$$

Constraints. The constraints in (14) and (15) refer to the nodal active and reactive power due to the limitations on the generation capacity where $\bar{P}_{g,i}$, $\bar{Q}_{g,i} \geq 0$ and $Q_{g,i} \leq 0$. The storage power is bounded in (16). Constraint (17) relates to maintaining voltage magnitudes within a certain bound, i.e. 5% deviation of the nominal magnitude. The constraint in (18) tightens the relative voltage angle. We assume that the voltage angles belong to an acceptable practical bound (i.e. $[-\frac{\pi}{2}, \frac{\pi}{2}]$). This constraint allows us to include a general representation of power flow equations (i.e. expression of V , θ based on P , Q) in our optimization which allows future extensions for the case of microgrids. Discussion on the limits for tightening the relative voltage angles is beyond the scope of this paper (see [18] for an example). The last constraint reflects the limitations on the grid capacity (as stated in Section 2). Owing to the bidirectional flows in distribution systems, the aggregated active power is constrained by a lower and an upper bound representing the maximum allowable loads and generations respectively. The limits in (19) depend on the physical capacity of the physical network main equipment, for example the capacity of the substation's transformer or the main HV power line which feeds the distribution network (i.e. the zone of interest). Define k_m , $k_M \geq 0$ and denote the exchanged power with the HV/MV network with P^T . We have,

$$P^T - P^{\text{loss}} + \sum_i P_{g,i} + P_{s,i} - P_{l,i} = 0,$$

where $-k_m \leq P^T \leq k_M$ and P^{loss} represents general technical losses. Thus, the limits in (19) are

$$c_m = P^{\text{loss}} - k^M, \quad c_M = k_m + P^{\text{loss}}.$$

Estimations of the bounds of losses (e.g. obtained based on load curves [19]) could be used to adjust the limits.

3.1. Analysis

The convexity of the optimization formulation in (13) cannot be concluded due to the constraints (17) and (18). In fact, the nonlinear AC power flow equations in (1) do not give an explicit representation for V and θ as functions of P and Q . In order to analytically solve the problem in (13), we approximate constraints (17) and (18) by using a general linear approximation of the AC power flow equations with respect to a steady-state solution of (1). This approximation, which is a valid simplification for distribution networks' analysis, preserves the structure of the AC power flow equations and gives an implicit algebraic relations between the nodal voltages and the nodal power injections [20]. Define $x_i = (V_i, \theta_i, P_i, Q_i)$. A general linearized approximation of (1) containing the network topology is expressed by (see [20])

$$A^*(x - x^*) = 0, \quad (20)$$

where x^* is an operating point such that the AC power flow equations has a solution, i.e. a flat voltage operating point with $x_i^* = (1, 0, 0, 0)$. Define $\tilde{V} = V - V^*$, $\tilde{\theta} = \theta - \theta^*$, $\tilde{P} = P - P^*$ and $\tilde{Q} = Q - Q^*$ where $P = (P_1, \dots, P_n)^T$, $Q = (Q_1, \dots, Q_n)^T$, $V = (V_1, \dots, V_n)^T$, and $\theta = (\theta_1, \dots, \theta_n)^T$. Without loss of generality, we assume conditions under which (20) can be written as follows

$$M \begin{bmatrix} \tilde{V} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} \tilde{P} \\ \tilde{Q} \end{bmatrix}, \quad (21)$$

where $\tilde{V}$, $\tilde{\theta}$, $\tilde{P}$, $\tilde{Q}$ refer to the deviations from the operation point x^* and elements of M depend on the line parameters, e.g. admittances (see [20] for examples of M). We assume that matrix M in (21) is invertible. Consider $M^{-1} = \begin{bmatrix} M_v \\ M_\theta \end{bmatrix}$. Then, $\tilde{V}$, $\tilde{\theta}$ can be written as linear functions of $\tilde{P}$ and $\tilde{Q}$

$$\tilde{V} = M_v \begin{bmatrix} \tilde{P} \\ \tilde{Q} \end{bmatrix}, \quad \tilde{\theta} = M_\theta \begin{bmatrix} \tilde{P} \\ \tilde{Q} \end{bmatrix}. \quad (22)$$

Considering the above model in the maximization in (13), we solve the following problem

$$\min_{\{P_{g,i}, Q_{g,i}, P_{s,i}\}} \sum_i -\tilde{f}_i \quad (23)$$

Subject to: constraints (14)–(16)

$$\underline{V} \leq \tilde{V}_i \leq \bar{V}, \quad \underline{\phi} \leq \tilde{\theta}_{i,j} \leq \bar{\phi}, \quad \underline{\epsilon} \leq \sum_i \tilde{P}_i \leq \bar{\epsilon}, \quad (24)$$

where $\tilde{V}_i$ and $\tilde{\theta}_{i,j}$ are the i -element of vectors $\tilde{V}$ and $\tilde{\theta}$ defined in (22) and $\tilde{\theta}_{i,j} = \tilde{\theta}_i - \tilde{\theta}_j$. Furthermore,

$$\tilde{f}_i(P_{g,i}, Q_{g,i}, P_{s,i}) = \tilde{f}_i^*(P_{g,i}, P_{s,i}, Q_{g,i}) - \tilde{P}_i^\ell,$$

$\tilde{P} = P^g + P^s - P^l - P^*$ and $\tilde{Q} = Q - Q^*$ where P^* and Q^* are constant vectors. For the optimization in (23), the Lagrangian dual problem is

$$\begin{aligned} \max_{\gamma, \eta, \beta, \lambda, \mu, \chi} \quad & \inf_{P_g, P_s, Q_g} L(P_g, P_s, Q_g, \gamma, \eta, \beta, \lambda, \mu, \chi), \\ \text{s. t.} \quad & \gamma, \eta, \beta, \lambda, \mu \geq 0, \chi \geq 0, \end{aligned} \quad (25)$$

where $\gamma = (\gamma^+, \gamma^-)$, $\eta = (\eta^+, \eta^-)$, $\beta = (\beta^+, \beta^-)$, $\lambda = (\lambda^+, \lambda^-)$, $\mu = (\mu^+, \mu^-)$, $\chi = (\chi^+, \chi^-)$ are Lagrange multipliers related to the constraints (14)–(16) and (24) respectively. See [21] for a detailed expression of the Lagrangian together with the K.K.T conditions [22].

Lemma 1. Assuming strict inequalities for (23) (i.e. Slater's condition [22]), the K.K.T conditions are necessary and sufficient conditions for the optimality of both the primal and dual problems. Moreover, the duality gap is

zero and the problem (23) has a unique solution.

3.2. Design of price signals

The constraints in (24) are coupling constraints which prevent solving the problem in (23) in a distributed manner. Thus, the optimization formulation in (23) is a centralized problem which is not preferable in large networks. Moreover, the constraints (14) to (16) are nodal which can be handled locally by each prosumer (busbar). The latter can be a critical requirement if the prosumers are not willing to reveal their own constraints to the network operator. This encourages decoupling the original problem into sub-problems coordinated by a master problem. In what follows, we take some steps to decompose the original problem (23) to distributively solvable sub-problems by means of a dual-decomposition approach [23] such that the problem in (23) is decomposed to one master problem and n sub-problems. At lower level, each busbar (prosumer) i solves the following sub-problem which is derived from decomposition of the Lagrangian (25). Given $\rho_{i,p}$ and $\rho_{i,q}$, each busbar i optimizes the cost function $G_i(\rho_{i,p}, \rho_{i,q})$ to obtain P_i^o and Q_i^o , hence $G_i^o(\rho_{i,p}, \rho_{i,q})$, by solving the following problem

$$\min_{P_{g,i}, P_{s,i}, Q_{s,i}} G_i(\rho_{i,p}, \rho_{i,q}) \quad (26)$$

Subject to: constraints (14)–(16)

where $G_i(\rho_{i,p}, \rho_{i,q}) = -\tilde{f}_i(P_{g,i}, Q_{g,i}, P_{s,i}) + \rho_{i,p}(P_{g,i} + P_{s,i}) + \rho_{i,q}Q_{g,i}$.

At the higher level, the operator solves the following master problem to obtain the updated multipliers $\rho_{i,p}$ and $\rho_{i,q}$. The dual master problem is

$$\begin{aligned} \max_{\lambda, \mu, \chi} \quad & \sum_i G_i^o(\rho_{i,p}, \rho_{i,q}) + \lambda^T(\tilde{V} - \bar{V}\mathbf{1}_n) \\ & + \lambda^T(\underline{V}\mathbf{1}_n - \tilde{V}) + \mu^T(\tilde{\theta} - \bar{\phi}\mathbf{1}_m) \\ & + \mu^T(\underline{\phi}\mathbf{1}_m - \tilde{\theta}) + \chi^T(\mathbf{1}_n^T \tilde{P} - \bar{\epsilon}) + \chi^T(\underline{\epsilon} - \mathbf{1}_n^T \tilde{P}) \\ \text{s. t.} \quad & \lambda^+, \lambda^-, \mu^+, \mu^- \geq 0, \quad \chi^+, \chi^- \geq 0, \end{aligned} \quad (27)$$

where $\mathbf{1}_n$ denotes the vector of all one. In what follows, we obtain the multipliers $\rho_{i,p}$, $\rho_{i,q}$ for each busbar i based on the dual variables λ , μ and χ .

Proposition 1. The prices $\rho_{i,p}$, $\rho_{i,q}$ for each sub-problem (busbar/prosumer) i in (26) are obtained from

$$\begin{aligned} \rho_{i,p} &= \chi + \sum_j \lambda_j \frac{\partial \tilde{V}_j}{\partial P_i} + \sum_{\ell \in j} b_{j\ell} \mu_\ell \frac{\partial \tilde{\theta}_j}{\partial P_i}, \\ \rho_{i,q} &= \sum_j \lambda_j \frac{\partial \tilde{V}_j}{\partial Q_i} + \sum_{\ell \in j} b_{j\ell} \mu_\ell \frac{\partial \tilde{\theta}_j}{\partial Q_i}, \end{aligned} \quad (28)$$

where $b_{j\ell}$ is the element of the physical graph incidence matrix, $\lambda_j = \lambda_j^+ - \lambda_j^-$, $\mu_\ell = \mu_\ell^+ - \mu_\ell^-$, $\chi = \chi^+ - \chi^-$.

The proof and hints on the implementation of the distributed algorithm are provided in [21].

Remark 1 (Congestion price). Notice that $\rho_{i,p}$ is composed of a uniform price (χ due to the aggregated capacity) and a nodal price (the share of active power in nodal voltage deviations) while $\rho_{i,q}$ is a nodal price (the share of reactive power in nodal voltage deviations). It is worth mentioning that we assume that the prosumers react to the price signals by adjusting their production and consumption. One of the mechanisms to adjust the load and production is curtailment which is sometimes inevitable.

4. Numerical analysis of a case study

Consider a single phase representation for IEEE-37 distribution bus together with photovoltaics, active and reactive power loads and storage systems as modeled in [24]. Let us consider the original AC power

flow equations as in (1) for simulations and numerical analysis. To model the AC power flow, we use a Newton–Raphson method based on a modified MatPower code in MATLAB. The implementation of the first iteration of the Newton–Raphson method resembles the linearized model used in Section 3 to design the congestion prices. However, an accurate analogy between the two has not been analytically studied in this paper. We refer the interested reader to the literature using Newton methods in linearization, for example [25]. For optimization, we use the Yalmip optimization toolbox in MATLAB. A fixed step size method is used to update each gradient. The physical model and data for the PV inverters, loads and a modified IEEE 37 bus (single phase) are based on [24].

Objective function for the case study. For the optimization of the case study, we define the social welfare function $J(P_g, P_s, Q_g) = \sum_i J_i$ where J_i is defined by

$$J_i = \gamma_p P_{g,i} + \gamma_q Q_{g,i} + (\gamma_p - C_s)(P_{s,i}) - \varepsilon_p (P_{g,i})^2 - \varepsilon_q (Q_{g,i})^2 - (\gamma_p - C_s)((1 - \eta)P_{s,i})^2 - P_i^e. \quad (29)$$

In the above γ_p , γ_q and C_s are exogenous price signals for the active power, reactive power price and storage, respectively. The hourly active power price γ_p is based on the average hourly day-head active power price in the Dutch power market based on APX in 2014. Since the reactive power load in our simulations is about 50% of active power (following IEEE 37 bus data), we choose $\gamma_q < 0.5\gamma_p$. We set the other prices and coefficients as follows: $\gamma_q = 0.36\gamma_p$, $C_s = 0.9\gamma_p$, $\varepsilon_p = 0.0018\gamma_p$, $\varepsilon_q = 0.0018\gamma_q$, $\eta = 0.8$. Notice that the above coefficients are designed for our simulation case. The choice of C_s is very close to the active power price such that the coefficient $\gamma_p - C_s$ is small. The reason is that we simulate a day in summer subject to the excess of generation. Here, the storage systems are used to increase the aggregated loads to deal with the excess of renewable generations. Hence, the prosumers do not pay for storing part of their own production. In our simulations, the relative voltage angles are very small and do not contribute to the congestion prices.

Simulation results. We run the static optimization in (13) at each hour during one day. The simulations results relate to the data of one average day of the month June (see [24]). An example of the plots of total generations and total load is also provided in [21]. The maximum active and reactive power generations are set as $\bar{P}_{g,i} = 0.59\text{irr}(t)$ and $\bar{Q}_{g,i} = 0.32\text{irr}(t)$ where $\text{irr}(t)$ [W/m²] denotes the solar irradiation. When the storage systems are considered, the installed capacity is assumed to be 9000 kW which is uniformly distributed between 36 busbars (250 kW per busbar). The maximum allowable aggregated active power, C_M , is set to 2500 kW which is the capacity of the transformer of IEEE 37 bus. The bound for the voltage magnitudes is set to $\pm 5\%$ of the nominal voltage that is 4.8 kV.

In the plots for active or reactive power, the following are used. The term $P_g = \sum_i P_{g,i}$ (similarly for Q_g) represents the total generated active (reactive) power, $P_l = \sum_i P_{l,i}$ (similarly for Q_l) represents the total active (reactive) load, $P_s = \sum_i P_{s,i}$ represents the total stored active power, $P_{\text{net}} = \sum_i P_i$ represents the net power. The plots of congestion prices for active and reactive power refer to the price signals in Proposition 1.

Fig. 2 shows the case where the installed storage capacity is zero and no constraints exist on the actual grid capacity and voltages. The purpose of this case is to build a scenario such that both the desired limits on voltage and grid capacity are violated. Next, we add the constraint on the actual grid capacity such that the maximum aggregated allowed power is 2500 kW. Fig. 3 shows this case. As shown, the total net active power is now within the acceptable bound. Comparing to Fig. 2, there is a uniform congestion price for active power to keep the net power within the bounds. Before dealing with the voltage magnitudes, we increase the installed storage capacity from zero to 9000 kW. The results of using the storage capacity are shown in Fig. 4. Comparing to the same scenario without storage (Fig. 3), the peak value of the uniform active power price is decreased. In order to keep the

voltage magnitudes within the desired bounds, we now add the voltage constraint. As Fig. 5 shows, both the net power and voltage magnitudes are within the desired bounds. However, there are nodal active and reactive power prices where the peak values of the reactive power prices are higher than the active ones. Simulating the case in Fig. 5 without storage results in higher prices.

The congestion prices refer to the price signals in Proposition 1 and Remark 1. Hence, they include the physics of the network. As a result, the prices are not always positive, for instance the prices of reactive power in Fig. 5 is negative which lead to the increase in generation of reactive power in order to decrease the voltage level.

Price due to the grid capacity. As shown in the simulation results, the uniform congestion price due to the limited capacity is effective in respecting the actual grid capacity and to some extent voltage regulation. Results in Fig. 3 shows improvements in voltage magnitudes compared with Fig. 2 where no price signal is used. This implies that limiting the grid capacity could be used to prevent voltage limit violations. However, respecting a conservative capacity bound may require limiting renewable generations or loads for example by curtailment. By increasing the limit of actual capacity, we will allow more generations but still the voltage can violate the limits. Thus, both of the measures are required to maintain the desired grid performance as in Fig. 5.

Price due to the voltage magnitude. The congestion price due to the voltage magnitude is nodal and depends on both active and reactive power. Based on Fig. 5, the price due to the reactive power is larger which is inline with the results on voltage control by only reactive power [12]. However, in the current market, the implantation of reactive power price in the distribution grid has not yet been practiced (in many countries). In simulations, we set the exogenous reactive power price proportional to the active power one (almost) based on the ratio of the total active power load to the reactive load. Our results suggest the usefulness of reactive power price in maintaining the voltage bounds and dealing with local bottlenecks. Moreover, the plots of the total revenue of prosumers and the cost of real power losses show the effectiveness of voltage-dependent prices in decreasing the losses and improving the revenues. Fig. 6 shows the cost of total losses and the total revenues in three cases: no constraints on the grid capacity and voltages (no prices), only capacity constraint (uniform congestion price), and both the capacity and voltage constraint (uniform and nodal congestion prices). The cost of losses is decreased by the application of prices and is lowest when both of the constraints on the grid capacity and voltages are applied. Application of voltage prices, which motivates generation of more reactive power, will compensate the effect of the limitations on active power generation on the total revenue function such that the joint use of prices will result in less power loss and more revenues.

The effects of the storage systems. Figs. 4 and 5 show the effect of storage in decreasing the congestion prices. Since we assume that storage systems only contribute to the active power, its effect on decreasing the congestion price of active power is direct. However, because of the coupling of the active and reactive power in voltage dynamics, storage also indirectly affects the reactive power congestion price. Notice that in our case study, the variations of daily loads are not large and we also consider a summer day with high solar radiation. Hence, the storage mainly help to deal with the excess of generations.

5. Translating congestion prices to network tariffs

In order to let users of the distribution grids respond to the degree of utilization, the grid tariffs should be related to the congestion prices. Such a relationship, however, is not common practice. Grid tariffs are mainly meant to give network operators compensation for the network costs. Generally, these costs only refer to the fixed costs of the investments in the network as well as the variable costs of network use. The first component is translated into a fixed tariff per connection per period, while the second component is translated into a tariff per size of

Fig. 2. The case with zero storage capacity and no constraint on the grid capacity and voltages. The plots on the left show the voltage magnitudes and angles for all busbars. The plots on the right show the aggregated active and reactive power: P_g solid line, P_l dotted line, P_{net} dotted-dashed line (similar settings for the reactive power).

the network use per period. In this design, the tariffs do have some relationship with the dynamics of energy use. The fixed tariff is related to the maximum level of usage during a period, while the variable tariff may be distinguished in, for instance, peak and off-peak tariffs. Hence, the relationship between the actual usage of network use over time and the level of the network tariffs is weak, which implies that the network users have soft incentives to respond to bottlenecks by reducing or shifting in time actual network usage. Making network tariffs dynamic could improve the incentives for optimal network usage. Considering

changing the design of network tariffs, one should pay attention to two constraints [26]. First, the overall level of revenues coming from the network tariffs should be sufficient to compensate for the costs of the network operator. This is the so-called financeability constraint [27]. Second, the existence of dynamic network tariffs may not give an incentive to the network operator to create or maintain bottlenecks. After all, such an adverse effect may occur if the operator may make more profit by charging higher prices during congestion. This is the so-called *incentive-compatibility* constraint. Both constraints for tariff design can

Fig. 3. The case study with constraint on grid capacity, zero installed storage and no constraint on voltages.

Fig. 4. The case study with installed storage and constraint on grid capacity but no constraint on voltages.

Fig. 5. The case study with installed storage and constraints on grid capacity and voltages.

be jointly met by a) making the revenues for cost recovery independent of the congestion prices (financeability constraint) and b) reserving the revenues coming from the congestion prices for investments reducing network bottlenecks. As a result, we conclude that a tariff structure which gives both revenues for cost recovery as well as incentives to network users to adapt the timing and level of their network usage to the actual utilization rate of the grids includes the following components: As a result, such a tariff structure includes: (a) a fixed part per

size of connection per year, (b) a variable part depending on aggregated network usage per year (can be divided in peak/off periods), (c) a dynamic part depending on hourly congestion price. In this system, only the dynamic, hourly, component of the network tariffs gives incentives to the hourly network usage. The fixed annual component gives incentives to optimize size of the connection, while the variable annual component gives incentives to optimize the aggregated annual level of energy consumption independent of its timing. Although these three

Fig. 6. The plots of the cost of losses and total revenues of prosumers. 'N': no constraint on the grid capacity and voltages, 'C': only constraint on the grid capacity, 'V.C.': constraint on both the grid capacity and voltages.

components have different dimensions, indirectly they are related to each other as changes in the network use in one dimension may affect the level in the other dimensions. For instance, if network users increase the size of their connection to the grid, there peak network usage may increase as well which may result in more or higher bottlenecks. Also the other way round, if network users respond to the dynamic component of the network tariff by load shifting, the capacity of their connection may be reduced. As the purpose of the dynamic component of the network tariffs is to give incentives to network users to reconsider their network usage, the revenues resulting from this tariff component should not affect the overall profitability of the network operator or the overall costs for the network users. This result can be achieved by correcting ex post the fixed tariff component by the annual amount of revenues coming from the dynamic component. Such a correction does not affect the optimal marginal decisions of network users while it prevents the network operator make supra-normal profits.

6. Conclusions

This paper has presented a pricing mechanism for congestion management in distribution systems. We have designed active and reactive power price signals and implemented our result on a modified IEEE benchmark model considering storage systems. This numerical implementation shows that giving incentives to network users to adapt their network usage, e.g. by using storage systems, is effective in maintaining grid's desired operation. We have also discussed how the congestion prices can be used in designing dynamic tariffs. Future lines of research include considering multiple zones (the current paper considered only one zone), and a dynamic pricing framework to design dynamic tariffs and for the adjustment of reactive and storage power exogenous prices.

Conflict of interest

The authors declare no conflict of interest.

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