Chapter 8

Auctions vs. negotiations with asymmetric or colluding buyers

8.1 Introduction

When an owner of a valuable asset decides to sell his property he may conduct an auction to maximize his expected profits, but there are alternative mechanisms. In practice, a seller (or, in a procurement context, a buyer) often reverts to negotiations with a single buyer. The use of negotiations is puzzling, as Bulow and Klemperer (1996) (BK henceforth) have shown that auctions tend to yield more revenue than negotiations. More precisely, they demonstrate that an English auction without a reserve price and \( n + 1 \) buyers generates more revenue than the optimal mechanism (assuming the seller has superior bargaining skills) with \( n \) buyers. This suggests that revenue-maximizing sellers should direct attention at expanding the set of buyers instead of designing ‘clever’ negotiation procedures.

The aim of this chapter is to understand why some sellers still prefer negotiations over auctions. To do so, the analysis of BK is adjusted in three directions. First, the seller is allowed to have better information than buyers, in the sense that a seller knows the value of each buyer whereas the buyers only know their own value and perceive the value of each other buyer as being randomly drawn from some probability distribution. Then, with a superiorly informed seller, BK’s reve-

1 Examples include public procurement officials or boards of directors involved in the takeover of their company. See Significant B.V. (2005) for an overview of the practice of public procurement in the Netherlands. They report that procurement agencies are reluctant to comply with European procurement guidelines, which stipulate that large projects should be publicly tendered. Boone and Mulherin (2007) study the market for corporate takeovers. They find that about 50% of corporate acquisitions are accomplished through auctions and the other half through negotiations.
nue ranking is shown to be reversed. Second, buyers are often asymmetric. In a European Union-wide public procurement process, as an example, the costs of a domestic firm are likely to be drawn from a very different distribution than the costs of a foreign firm. Third, the buyers are assumed to coordinate their behavior in a bidding ring. Asymmetry and collusion turn out to have similar effects. BK’s revenue ranking remains intact if the buyers are not too asymmetric or the cartel is not too large.

This chapter continues with a quick review of BK’s result in section 8.2.1. BK assume that the seller is uninformed about the realization of the values of the buyers. In section 8.2.2 the effect of an informed buyer in considered. The “auction vs. negotiation” result is reconsidered for asymmetric buyers and colluding buyers in section 8.3 and 8.4, respectively. Section 8.5 offers some final remarks.

8.2 The symmetric benchmark

8.2.1 An uninformed seller

A seller owns an indivisible object and may sell it to one of \( n \geq 1 \) symmetric buyers. The seller’s value for the object is normalized to zero, and the buyers’ values are drawn independently from a common cumulative distribution function \( F \) with support \([0, v]\).\(^2\) Buyers privately observe their value and are uninformed about the realization of the other values. \( F \) is continuously differentiable and has a positive density on its entire support. Additionally, the virtual valuation \( \phi(u) \equiv u - \frac{1-F(u)}{f(u)} \) increases in \( u \). This assumption guarantees that the seller’s mechanism design problem is regular. In the present context, regularity implies that it is optimal to set a reserve price and allocate the object to the highest bidder. Each buyer is privately informed about its value before the auction or mechanism takes place. The seller and the buyers are risk-neutral and aim to maximize expected payoffs.

From the theory of mechanism design, it is well-known that the seller can extract at most

\[ \Pi_N = E[\max\{\phi_1, \phi_2, \ldots, \phi_n, 0\}] \] \hspace{1cm} (8.1)

from the buyers. The subscript \( N \) refers to negotiations and \( \phi_i \) is the virtual valuation of buyer \( i \). See Myerson (1981) for the seminal contribution to mechanism

\(^2\)This chapter restricts attention to the private values framework. BK study auctions and negotiations in a more general interdependent values setting.
design and Krishna (2002) for a recent introduction. This places an upper bound on
the revenue the seller may expect from negotiations. In fact, the seller is likely to
obtain much less revenue from negotiations. This is because the seller is unlikely to
hold all bargaining power, as mechanism design theory presupposes. Furthermore,
he may not be able to commit to actions that are dominated *ex post*.

The seller may also allocate the object by means of an absolute English auction.
This is an auction without a reserve price in which the price continuously increases
until all but one bidder have dropped out. By the assumption that buyers are sym-
metric and that virtual valuations are increasing, the revenue of the auction can be
written as

$$\Pi_{EA} = E[\max\{\phi_1, \phi_2, \ldots, \phi_n\}]$$

where the subscript $EA$ refers to English auction. This is clearly, and by definition,
lower than the revenue from the optimal mechanism. However, BK show that with
just one additional buyer, the revenue of the absolute English auction is strictly
larger than the revenue of negotiating with the former set of buyers.

**Proposition 8.1** (Bulow and Klemperer, 1996). *An absolute English auction with $n + 1$
buyers yields more expected revenue than any negotiation with $n$ buyers.*

**Proof.** The proof follows Krishna (2002). It is obvious that the auction dominates the
optimal mechanism if $\max\{\phi_1, \ldots, \phi_n\} > 0$. Therefore, suppose that $\max\{\phi_1, \ldots, 
\phi_n\} < 0$. Then,

$$E[\max\{\phi_1, \ldots, \phi_n, \phi_{n+1}\} | \max\{\phi_1, \ldots, \phi_n\} < 0]$$

$$> \max\{E[\phi_1 | \phi_1 < 0], \ldots, E[\phi_n | \phi_n < 0], E[\phi_{n+1}]\}$$

$$= \max\{E[\phi_1 | \phi_1 < 0], \ldots, E[\phi_n | \phi_n < 0], 0\}$$

$$= 0$$

$$= E[\max\{\phi_1, \ldots, \phi_n, 0\} | \max\{\phi_1, \ldots, \phi_n\} < 0].$$

The inequality follows from Jensen’s inequality and the fact that “max” is a convex
function.\(^3\)

Hence, the auction is also superior if $\max\{\phi_1, \ldots, \phi_n\} < 0$ and the result follows.

\(^3\)To see this, note first that a function $f(x) = f(x_1, x_2, \ldots, x_n)$ is convex if $\lambda f(x) + (1 - \lambda)f(y) \geq f(\lambda x + (1 - \lambda)y)$ for all $x$ and $y$ in the domain of $f$ and all $\lambda$ in (0, 1). Suppose without loss of generality
that $x_1 \geq x_2 \geq \ldots \geq x_n$. Then, $\max\{x\}$ is convex if $\lambda x_1 + (1 - \lambda) \max\{y\} \geq \max\{\lambda x + (1 - \lambda)y\}$. By
assumption, $\lambda x_1 + (1 - \lambda) \max\{y\} \geq \max\{\lambda x + (1 - \lambda)y\}$ and a sufficient condition for convexity is
therefore $\lambda x_1 + (1 - \lambda) \max\{y\} \geq \lambda x_1 + (1 - \lambda) \max\{y\}$. This weak inequality is exactly met.
The common interpretation of proposition 8.1 is that competition is more valuable than negotiations. A revenue-maximizing seller is better off by spending resources to enlarge the number of buyers, than to find the optimal mechanism.

Krishna’s (2002) proof of proposition 8.1 is used here mainly because of its elegance. However, it may be instructive to discuss an alternative proof. In a recent contribution, Kirkegaard (2006) offers an intuitive interpretation of BK’s result. Consider the revenue of the optimal mechanism, $\Pi_N$, with $n$ buyers. With $n + 1$ buyers, the seller can easily obtain the same revenue by applying the optimal mechanism to the first $n$ buyers, and give the object for free to the $(n + 1)^{th}$ buyer if the object was not allocated under the optimal mechanism. This mechanism always allocates the object to a buyer. However, the seller can do even better by conducting an absolute English auction. This is because the absolute English auction is the optimal mechanism to allocate an object, subject to the condition that the object must be allocated.

### 8.2.2 An informed seller

BK assume that the revenue of negotiations is bounded by the expected revenue of the optimal mechanism. This assumption implicitly supposes that the seller is equally informed about the value of any given buyer as each other buyer. In many important settings, this common prior assumption seems to be justifiable. For instance, buyers and sellers on the market of collectible items frequently interact, regularly change positions (a buyer today could be a seller tomorrow), and have similar sophisticated information about the value of items.

In other settings, however, it may be more realistic to assume that the seller has better information about the values than the buyers. This might apply to the housing market or the market for second-hand cars. In these markets, sellers trade frequently with inexperienced buyers. The large number of trades enables sellers to predict the values of potential buyers. For instance, the value of a potential buyer of a car might be correlated with observable characteristics, like the size of the buyer’s family, the buyer’s current car, or whether the buyer owns a dog. Other potential buyers are less capable of estimating a buyer’s value because they lack the seller’s experience.

The simplest way to incorporate the idea that the seller has superior information, is to suppose that the seller knows the buyers’ values. The buyers still consider
the value of any given buyer as an independent draw from $F$. By making a take-it-or-leave-it offer to the buyer with the highest value, the seller can extract the entire surplus. The (expected) revenue is simply the (expected) highest value. Call this mechanism *perfect negotiations*.

The obvious question is whether BK’s revenue ranking extends to perfect negotiations. The answer is no.

**Lemma 8.1.** *Perfect negotiations with $n$ buyers generate more revenue than an absolute English auction with $n + 1$ buyers.*

**Proof.** Perfect negotiations are superior to auctions for any realization of the values. Let $x$ be the highest value among $n$ buyers, and $y$ the highest value among $n + 1$ buyers. The revenue of perfect negotiations is $x$. First, notice that for $y > x$ the auction yields a revenue of $x$. This is because the winner of the auction, which is the additional buyer, pays the second-highest bid, $x$. For $x > y$, the winner is among the first $n$ buyers and pays less than $x$. Hence, the revenue of the auction is bounded by $x$, and strictly lower with positive probability.

This result turns BK’s revenue ranking around. An informed seller would never find it optimal to invite an additional buyer and let an English auction determine the price.

One could conjecture that lemma 8.1 follows from the fact that in an English auction buyers have a dominant strategy to bid their own value. This implies, as the proof demonstrates, that the revenue of the auction is bounded by the revenue of negotiations for any realization of the values. Alternative auction formats may not have this property. However, a simple application of the Revenue Equivalence Principle shows that lemma 8.1 extends to a comparison of perfect negotiations and any standard auction format.\(^4\), \(^5\)

**Proposition 8.2.** *Perfect negotiations with $n$ buyers generate more revenue than any symmetric and increasing equilibrium of any absolute standard auction with $n + 1$ buyers.*

Proposition 8.2 generalizes lemma 8.1. Given that the seller has perfect information, he prefers negotiations with $n$ buyers over a simple auction with $n + 1$ buyers. Under the assumption that the seller is uninformed, BK obtain the exact opposite

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\(^4\) Krishna (2002) defines a standard auction as an auction whose rules dictate that the buyer who bids the highest amount is awarded the object.

\(^5\) In the discussion of their main result, BK anticipate this result. However, they do not derive it formally and do not discuss its implications.
result. BK derive an upper bound on the value of mechanism design, whereas proposition 8.2 provides an upper bound on the value of competition. This helps to understand not only the limitations of BK’s result, but also institutional characteristics of particular markets. For instance, proposition 8.2 may explain why houses and second-hand cars are predominantly sold via negotiations, and government bonds and art via auctions.

Admittedly, proposition 8.2 applies only to environments where buyers have private values. An open question is how this result can be generalized to buyers with interdependent values.

8.3 Asymmetric buyers

Introducing an informational asymmetry between the seller and buyers upturns BK’s result. Ex ante asymmetry may also arise between buyers.\(^6\) In many important settings, such as public procurement or corporate takeovers, asymmetry appears to play a crucial role. The first firm to enter a procurement process, for instance, is likely to have a lower cost of providing the good than the second firm. This section discusses the effects of asymmetry between buyers on BK’s revenue ranking.

To keep the analysis tractable, restrict the number of buyers to two and assume that values of the buyers are drawn from the power function distribution with support \([0, 1]\). The value of the first buyer is drawn from \(x^\alpha\) and the second buyer’s value is drawn from \(y^\beta\). The exponent of the distribution functions indicates the strength of a buyer. Buyer 1 is called strong if \(\alpha > \beta\) and weak otherwise. Suppose that \(\alpha \geq 1\). This restriction ensures that the mechanism design problem is regular.

As before, the revenue of the optimal mechanism is \(E[\max\{\phi_1, 0\}]\). This can be written as

\[
\Pi_N = \Pr \left( x - \frac{1-x^\alpha}{\alpha x^{\alpha-1}} > 0 \right) E \left[ x - \frac{1-x^\alpha}{\alpha x^{\alpha-1}} \middle| x > \frac{1-x^\alpha}{\alpha x^{\alpha-1}} \right] \\
= \frac{\alpha}{\alpha + 1} E \left[ x - \frac{1-x^\alpha}{\alpha x^{\alpha-1}} \middle| x > \left( \frac{1}{1+\alpha} \right)^{1/\alpha} \right] \\
= \alpha (\alpha + 1)^{\frac{\alpha+1}{\alpha}}. \tag{8.3}
\]

Suppose the seller has the opportunity to invite another potential buyer and conduct an absolute English auction. Let \(F\) and \(G\) be the cumulative distribution function.
tions of the values of the first and second buyer. Then, expected revenue can be written as

\[ \Pi_{EA} = \int_0^v (1 - F(u))ug(u)du + \int_0^v (1 - G(u))uf(u)du. \]

For the power function distribution, this simplifies to

\[ \Pi_{EA} = \frac{\alpha \beta (2 + \alpha + \beta)}{(1 + \beta)(1 + \alpha + \beta)}. \quad (8.4) \]

Suppose \( \alpha = 1 \). Then, the auction is revenue superior if and only if \( \beta > -\frac{3}{2} + \frac{1}{2}\sqrt{17} \approx 0.562 \). Hence, BK’s result holds if the additional buyer is not too weak.

This insight applies more generally. The English auction is revenue superior if and only if

\[ \frac{\beta (2 + \alpha + \beta)}{(1 + \beta)(1 + \alpha + \beta)} - (1 + \alpha)^{-1/\alpha} > 0. \quad (8.5) \]

It is not hard to show the following.

**Proposition 8.3.** There exists a unique \( \beta \) such that for any finite \( \alpha \), the absolute English auction with two buyers yields more expected revenue than any negotiation with one buyer.

**Proof.** For \( \beta = 0 \), a non-serious additional buyer, the optimal mechanism yields a higher revenue than the auction. As \( \beta \) increases, the expected revenue of the auction increases monotonically. In the limit, as \( \beta \) becomes arbitrarily large, the auction dominates negotiations because the price paid by the winner of the auction is the value of the first buyer. As a result, there is a unique \( \beta \) for which the auction and negotiations yield the same revenue.

The seller prefers the English auction over any negotiation if the second buyer is sufficiently strong. This extends BK’s revenue ranking to asymmetric buyers and uncovers an important restriction of the validity of BK’s analysis. In the concluding remarks, BK argue that “a firm that refused to negotiate with a potential buyer, and instead put itself up for auction, should be presumed to have exercised reasonable business judgment.”. The result of proposition 8.3 substantially weakens this assertion.

One could object, however, that proposition 8.3 states the obvious. BK’s result is valid for symmetric buyers and, by continuity, remains valid if the additional buyer is not too weak. It may be more important to understand how weak the additional buyer is allowed to be.
To address this critique, notice first that the explicit solution for $\Pi_N$ and $\Pi_{EA}$ can be used to check whether the additional buyer is strong enough for any $\alpha$ and $\beta$. Second, $\bar{\beta}(\alpha)$, which is the lowest $\beta$ for which (8.5) holds, is strictly lower than $\alpha$. This is because (8.5) holds for $\alpha = \beta$ and, by continuity, also if $\beta$ is slightly lower. Third, it is easy to show that $\frac{\partial}{\partial \alpha} \bar{\beta}(\alpha) < 1$. This implies that, as the first buyer becomes stronger, the additional buyer needs to be relatively less strong.

8.4 Colluding buyers

Buyers may cooperate and form a cartel of buyers, also known as a bidding ring. The presence of bidding rings has been documented in various industries and, according to some observers, bidding rings almost always plague auctions. Even if firms are \textit{ex ante} symmetric, the formation of an incomplete bidding ring creates asymmetries between ring-members and outsiders. This section studies the effect of a bidding ring on the value of competition.

Suppose the initial $n \geq 1$ buyers have formed a bidding ring, of the type described in Graham and Marshall (1987). Then, the bidding ring sends the buyer with the highest value to the negotiations. This implies that the seller faces a buyer whose value is drawn from $F(u)^n$. For tractability, assume that individual buyers draw their value from the uniform distribution on the unit interval. Then, by relabeling (8.3), the revenue of the optimal mechanism can be written as

$$\Pi_N = n(n + 1)^{n+1}. \tag{8.6}$$

Instead of inviting one additional buyer, as BK assume, suppose that the seller may invite $k$ additional buyers and run an absolute English auction. Assume that the new buyers do not join the ring or collude themselves. (Otherwise, the auction yields zero revenue to the seller.) To derive the revenue of the English auction, note that if the cartel’s nominee wins the auction, he expects to pay the maximum value among the non-colluding buyers. This implies that, for a general common distribution function $F$, the expected payment of the nominee when his value is $x$ is

$$M(x) = \int_0^x u dF(u)^k. \tag{8.7}$$
The *ex ante* expected payment, i.e. before the nominee learns his value, is simply

\[
E[M] = \int_0^v u(1 - F(u)^n)dF(u)^k. \tag{8.8}
\]

Given that the nominee’s value is \(x\), the representative new buyer’s value is \(y_i\), and \(y_i > x\), buyer \(i\) expects to pay

\[
m_i(x, y_i) = \Pr(x > \max\{y_{-i}\}) x \\
+ \Pr(x < \max\{y_{-i}\} < y_i) E[\max\{y_{-i}\} < y_i] \\
= F(x)^{k-1}x + \int_x^{y_i} udF(u)^{k-1}.
\]

By integrating over \(x\) and \(y\) one obtains the new buyer’s *ex ante* expected payment

\[
E[m] = \int_0^v (1 - F(u))F(u)^{k-1}udF(u)^n + \int_0^v (1 - F(u))F(u)^nudF(u)^{k-1}. \tag{8.9}
\]

The revenue of the English auction is simply \(E[M] + kE[m]\). For the uniform distribution, this becomes

\[
\Pi_{EA} = \frac{k(n + k^2 + 2kn + n^2 - 1)}{(k + 1)(n + k)(n + k + 1)}. \tag{8.10}
\]

Does the presence of collusion destroy BK’s revenue ranking? To answer this question, consider first a bidding ring of two buyers \((n = 2)\). In that case, as is straightforward to verify by comparing \(\Pi_N\) and \(\Pi_{EA}\), just one additional buyer is sufficient to ensure that the auction yields more revenue. So, even if a seller faces an all-inclusive cartel in negotiations, inviting just one extra buyer and hold an auction instead more than offsets the loss in bargaining power.

Just as with asymmetric non-cooperative buyers, the English auction generates more revenue if the number of additional (non-cooperative) buyers is sufficiently large.

**Proposition 8.4.** There is a unique number of buyers \(k\) such that for every bidding ring with \(n\) buyers the absolute English auction with \(n + k\) members raises more revenue than any negotiation with the bidding ring.

**Proof.** For \(k = 0\), the English auction yields zero revenue, and is therefore dominated by negotiations. As \(k\) goes to infinity, the revenue of the English auction converges to 1, which is strictly above \(\Pi_N\). Finally, since the revenue of the English auction with \(n + k\) buyers increases in \(k\), there is a unique \(k\) such that \(\Pi_N = \Pi_{EA}\). \(\blacksquare\)
Similar to the case of asymmetric buyers, collusion among buyers does not imply that additional buyers are less valuable than the optimal mechanism. Negotiations with a small bidding ring are less profitable to the seller than an English auction with a relatively large group of outsiders. To understand how large the number of outsiders should be, consider the ratio \( k(n)/n \), where \( k(n) \) is the (unique) solution of \( \Pi_N(k) = \Pi_EA(k) \). It is straightforward to show that \( k(1)/1 \) is 2/3. Moreover, \( k(n)/n \) decreases in \( n \). Hence, for any bidding ring, the number of outsiders is at most 2/3 of the number of cartel members.

Two caveats. When a bidding ring exists, foregoing the ability to negotiate may be very costly, irrespective of the number of additional buyers. First, the new buyers may collectively join the ring, extracting all surplus from the seller. Second, switching from negotiations to an auction may actually induce collusion, because, as Robinson (1985) noted, collusion in an English auction is one-shot stable.

### 8.5 Concluding remarks

This chapter complements BK’s upper bound on the value of mechanism design with an upper bound on the value of competition. This new upper bound helps to explain why in many environments sellers prefer negotiations over auctions. In particular, a seller who possesses better information about the value of a buyer than any other buyer is better off by relying on negotiations to maximize revenue.

There are more reasons why a seller may prefer negotiations over auctions. Large asymmetries between buyers tilts the scale in favor of negotiations, as do bidding rings. The underlying reason why BK’s revenue ranking of auctions and negotiations fails in these two cases is the same. In negotiations, the seller is able to extract (through a take-it-or-leave-it-offer) the buyer’s or ring’s large expected value. In a simple auction without a reserve price he is unable to exploit his knowledge of one buyer’s or ring’s large willingness-to-pay.

In view of the general theme of this thesis, collusion, this chapter’s main insight is that in the presence of a cartel a seller may respond by changing the selling format. This is more drastic than the policy recommendations in chapters 6 and 7, which concluded that an auctioneer may adapt the auction rules in response to collusion. The analysis in this chapter advises to abandon the auction altogether in certain special cases.