Essays on the theory of collusion
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Chapter 4

Price fixing and non-price competition

When a uniform price is agreed upon, or agreed to by, an industry, some or all of the other terms of the sale are left unregulated. [...]n the absence of free entry [...] the question arises: Will any monopoly profit achieved by suppressing price competition be eliminated by non-price competition?

George J. Stigler (1968, p. 149)

4.1 Introduction

The typical industrial organization model of collusion considers the case where firms collude fully on every single aspect of their products. In practice, however, firms are often only able to collude on a limited number of dimensions. Shops that have agreed on a high price for their merchandise will be inclined to exert more effort to secure a sale from a customer that has entered their shop. Construction firms that collude on prices are still able to unilaterally decide on the amount of effort they put in proposing a plan to suit the tastes of the customer they are facing. In these examples, price fixing firms conspire against the consumer, but still compete against each other in the quality dimension.

This chapter is based on joint work with Marco Haan.
Empirical research suggests that, indeed, firms collude primarily by fixing prices rather than coordinating on other aspects of their product. For instance, in his study of post-war British cartels, Symeonidis (2002, p. 35) argues that “The British cartels were primarily price fixing bodies. Regulation of non-price decisions was uncommon”. Another example is the American cigarette industry in the 1920s and 1930s. There, according to Fershtman and Gandal (1994), the three major firms colluded on price but competed on advertising to garner more sales.

It is unclear why most cartels restrict themselves to price agreements. A possible explanation could be that agreements about quality require laborious product descriptions. These descriptions are not only costly to write, but may also be used by the antitrust authority as evidence of collusive behavior. An alternative explanation is offered by Deltas and Serfes (2002). They show that semicollusion may be more profitable than full collusion when demand is stochastic and the collusive agreement inflexible.

The focus of this chapter is not to provide an explanation of price fixing. Instead, price fixing behavior is taken as given and firms are assumed to compete on quality. This chapter studies the effect of this behavior on prices, product quality, profits, and welfare. One immediate question that arises is to what extent such price fixing still enables firms to increase their joint profits. Indeed, as the opening quotation reveals, this very question was already posed by Stigler in 1968.

A related question concerns the effect of price fixing on quality and welfare. One may argue that a high fixed price gives firms an incentive to provide higher quality, which may ultimately benefit consumers.

This chapter examines a model in which consumers search for a product, and firms exert consumer-specific effort to try to secure a sale to a consumer that has entered their store. Each consumer has firm-specific matching values, which reflect the extent to which this consumer likes the products that a firm has on offer. These matching values are unobservable to firms, and only observable to a consumer after it has visited a firm. Firms may form a price fixing cartel, but are not able to collude on effort.

As an example, consider a home-owner who wishes her bathroom to be resty-

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1 As an aside, cartels do exist that make a quality-fixing agreement, but compete in prices. Consider, for example, the Sugar Institute. This cartel of American sugar-refining firms, that operated between 1927 and 1936, did not make price agreements, but instead forced its members to obey extensive quality regulations, see Genesove and Mullin (2001). A quality-fixing cartel in the context of the model in this chapter would simply exert zero effort because raising effort only increases the costs for the winning firm, without increasing the probability of a sale.
She may ask a specialized firm to propose a design, tailored to her specific needs and desires. The attractiveness of the offer will in part depend on how well the design matches the taste of the home-owner, something that is unobservable for the firm. It will also depend on the amount of effort that the firm exerts. Higher effort implies higher quality. Upon learning the price and details of the offer, the home-owner is free to contract with the current firm, to contact another firm, or to quit being active in this market altogether. In the last two cases, the firm incurs a loss, because the costs of preparing the offer is buyer-specific and cannot be recouped. The general features of this market (a buyer searches for potential sellers and sellers exert buyer-specific effort) are shared by many real-world markets. This chapter studies price fixing in such markets.

The analysis reveals that price fixing does allow firms to increase profits. However, the ability to raise profits is partially offset by an increase in competition along the quality dimension. Price fixing may indeed lead to higher quality. Unfortunately, both consumer welfare and social welfare are lower if firms are allowed to fix prices. Most surprisingly, collusion may yield prices that are lower than the competitive price. The intuition is that, by setting a low price, the cartel effectively discourages competition in costly effort.

Admittedly, the model in this chapter is not the first that studies a case in which firms can collude in one dimension, but have to compete in another. Stigler (1968) studies a perfectly competitive industry with persuasive advertising, where firms can either collude on prices or on advertising levels. He shows that, in the context of his model, any benefits from price fixing are competed away provided that there are constant returns to scale.

Jehiel (1992) and Friedman and Thisse (1993) consider collusion in a duopoly model with horizontal product differentiation à la Hotelling. They find that if firms first choose their locations non-cooperatively, and then collude in prices, the equilibrium features minimum product differentiation. Fershtman and Gandal (1994) and Brod and Shivakumar (1999) analyze how collusion in the product market affects profits in a model of R&D along the lines of d’Aspremont and Jacquemin (1988). They show that if firms decide non-cooperatively on R&D investments, and then expect to collude in the product market, profits may be even lower than if they act non-cooperatively in both stages. This depends on the size of the technological spillovers.

These papers have in common that firms first choose some investment before

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2 This example is due to Wolinsky (2005).
they compete on the product market. Current investments then become a bargaining chip in future cartel negotiations. The analysis in this chapter is not focused on such bargaining. In the model presented below, firms first choose collusive prices, and only then decide on some quality aspects of their product. This set-up captures more naturally the issue that seems most interesting: how does a market operate where, after having fixed prices, firms are still able to compete on other dimensions. The only paper that also assumes this timing of events is Fox (1994). Her main finding is that the level of advertising increases in the collusive price. However, her model does not easily lend itself for a welfare analysis, as she focuses on persuasive advertising. Additionally, she does not study the effects of semicollusion on prices.

The remainder of this chapter is structured as follows. Section 4.2 introduces the model. In section 4.3 the model is solved for the case that firms compete in all dimensions, while section 4.4 studies the equilibrium with price fixing. Section 4.5 examines the welfare effects of price fixing behavior. Section 4.6 presents an extension of the model in which industry demand is inelastic. Some brief conclusions are offered in 4.7.

### 4.2 The model

The benchmark model is based on Anderson and Renault (1999) and Wolinsky (2005). Anderson and Renault (1999) show that a generalized version of Perloff and Salop’s (1985) model nests several well-known standard models such as the Diamond search model, monopolistic competition and Bertrand competition. Despite this generality, the model still yields analytic solutions and allows a direct comparison of the non-cooperative and collusive prices.

In the model, a finite number of firms compete by announcing prices and exerting buyer-specific effort. Buyers enjoy a firm-specific matching value, that is a priori unknown to both the firm and the buyer. More specifically, the model is as follows.\(^3\)

#### 4.2.1 Buyers

There is a continuum of buyers with a measure normalized to one. Each buyer has unit demand (e.g. a bathroom) and may approach a firm at cost \(s > 0\). These costs

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\(^3\)The main difference with Wolinsky (2005) is the assumption of a finite number of firms. This allows a comparative statics exercise with respect to the number of firms. Anderson and Renault (1999) do not allow for endogenous quality, which excludes the possibility of semicollusion.
include the costs of finding a firm and the costs of transferring detailed information about her demand. If a buyer approaches a firm $i$, she learns the price $p_i$ of the good, and her total valuation for the product of the firm, which equals $v + e_i + u_i$. Here, $v > 0$ is the intrinsic utility the buyer derives from obtaining the product, $e_i$ is the buyer-specific effort exerted by the firm, and $u_i$ is the matching value between the buyer and the firm. It is assumed that $v$ is sufficiently large so as to guarantee that buyers will always obtain the good.\(^4\) The matching value $u_i$ is a random draw, IID across firms and buyers, from a cumulative density function $F$, which has support $[0, 1]$. For convenience, it is also supposed that $F$ is twice differentiable and has positive marginal probability on its domain. The value $u_i$ is only observable to buyers, and only after she has visited firm $i$. To guarantee that buyers have a positive reservation value, it is assumed that $s < \int_0^1 u dF(u)$.

After any visit to a firm, a buyer may grant the project to a firm from which she has received an offer, or search for another firm. Her payoff if she buys from firm $i$ after $k \geq 1$ visits is given by $U(e_i, u_i, p_i, k) = v + e_i + u_i - p_i - ks$. A buyer is allowed to come back to a previously visited firm at zero costs. It is assumed that the firm commits to a price once it is announced.

Consumers are assumed to search sequentially, which implies that buyers do not commit in advance to the number of searches. The optimal strategy for a buyer is thus some stopping rule that indicates that she should stop searching once she has found an offer that gives her net utility of at least $\hat{x}$, where $\hat{x}$ is to be determined endogenously.

### 4.2.2 Firms

There are $n$ firms. For ease of exposition, the marginal costs of production are equal to zero for all firms. If a firm is not contacted by the buyer, it obtains zero profits. If a firm $i$ is approached by the buyer, it announces its price $p_i$ and exerts effort $e_i$ to formulate a plan. This effort may also include e.g. time to inform the consumer regarding all aspects of the product.

The cost of effort is given by the function $c(e_i)$. This function satisfies the standard regularity assumptions $c(0) = 0$, $c’(e_i) > 0$ for $e_i > 0$, $c’(0) = 0$ and $c''(e) \geq 0$. Additionally, $c(e)$ is such that the following technical condition $\frac{\partial}{\partial e} \frac{c(e)}{c''(e)} > 0$ is met. Effectively, this condition requires that the marginal costs of effort are not

\(^4\) In section 4.6, it is shown that the main results continue to hold when the demand curve is downward-sloping.
too convex. It is always satisfied for any quadratic cost function, for instance.

Effort is sunk. Hence the profit of the firm conditional on being approached by the buyer is $p_i - c(e_i)$ if the buyer buys from this firm and $-c(e_i)$ if she does not. For simplicity, firms are not able to charge diagnosis costs, i.e. firms cannot charge buyers a price solely for formulating a plan. In a slightly different set-up, Wolinsky (2005) does allow for this possibility, but he finds that firms charge zero diagnosis costs in equilibrium because of Bertrand competition in diagnosis fees.

4.2.3 Timing

This chapter considers two scenarios as to how firms compete on this market. First, they may act purely non-cooperatively. Second, they may form a price fixing cartel. In that case, the cartel announces a collusive price that maximizes expected joint profit conditional on every firm adhering to the agreement. Firms still exert effort in a non-cooperative fashion.

In the non-cooperative benchmark, the timing of the model is as follows. First, the representative buyer decides whether to search. If so, she picks a firm at random. Upon being contacted, the firm announces price $p_i$ and exerts effort level $e_i$. After having observed this offer, the buyer may decide to search another firm, buy from the current firm, or quit searching altogether.

In the price fixing model, the timing is as follows. First, the firms collectively decide which uniform price $p$ to set. Second, the buyer decides whether to search. If so, she picks a firm at random. Upon being contacted, the firm sets effort level $e_i$ and just as before the buyer may buy, visit more firms, or quit. For simplicity, assume that firms are patient enough for the cartel to be stable. Calculating the exact value of the discount rate for which this is satisfied would greatly complicate the analysis without adding much insight.

4.3 The non-cooperative benchmark

In this section, the Nash equilibrium for the case that firms non-cooperatively set both prices and efforts is derived. The focus is on the symmetric pure strategy Nash equilibrium that consists of price $p^N$ and effort $e^N$. The analysis here largely follows Wolinsky (2005).

5 Denote marginal costs of effort as $m(e) \equiv c'(e)$. The condition then requires that $(m'm'' - mm''') / m'm' > 0$, or $m'' < m'm'/m$. With $m' \geq 0$ and $m > 0$, this implies that $m''$ should not be too high, hence that marginal costs should not be too convex.
4.3.1 The search rule

Suppose that the buyer has not yet approached all \( n \) firms and that her current best option yields utility \( v + e_i + u_i - p_i \). In the Nash equilibrium, a visit to a new firm \( j \) yields \( v + e^N + E[u_j] - p^N \). The consumer prefers firm \( j \) over firm \( i \) if

\[
    u_j > p^N - p_i - e^N + e_i + u_i \equiv x.
\]

Therefore, the expected benefit of one more search is \( \int_1^{u(x)} (u - x) f(u) du \). Such a search is worthwhile if and only if these benefits exceed the costs on one more search, \( s \). Thus, the buyer is exactly indifferent between searching and stopping if \( x \geq \hat{x} \), with \( \hat{x} \) implicitly defined by

\[
    \int_1^{\hat{x}} (u - \hat{x}) f(u) du = s. \tag{4.1}
\]

Hence, the buyer stops searching as soon as the current best offer is such that \( x > \hat{x} \), where \( \hat{x} \) is the unique solution to \( g(\hat{x}) = s \). In the event that the buyer approached all \( n \) firms, it is clearly optimal for her to return to the firm that offered her the best offer.

4.3.2 Equilibrium effort and price

Consider the decision of firm \( i \). Suppose that all other firms charge \( p^N \) and set effort level \( e^N \). In the following, the best reply of firm \( i \) is derived, which in turn allows one to derive the equilibrium values \( p^N \) and \( e^N \).

The probability that the buyer samples firm \( i \) is given by

\[
    Pr\{\text{firm } i \text{ is sampled}\} = \frac{1}{n} + \frac{F(\hat{x})}{n} + \frac{F(\hat{x})^2}{n} + \ldots + \frac{F(\hat{x})^{n-1}}{n},
\]

which simplifies to

\[
    Pr\{\text{firm } i \text{ is sampled}\} = \frac{1}{n} \cdot \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})}.
\]

as \( F(\hat{x}) \in (0, 1) \).

The probability that the buyer stops at firm \( i \) (i.e. makes an immediate purchase), given that firm \( i \) is sampled, is equal to \( Pr(x > \hat{x}) \), or

\[
    Pr(e_i - e^N + p^N - p_i + u_i > \hat{x}) = Pr(u_i - \Delta > \hat{x}) = 1 - F(\hat{x} + \Delta),
\]
where $\Delta \equiv p_i - p^N - e_i + e^N$. The buyer returns to firm $i$ after having visited all firms if $\max_{j \neq i} u_j < u_i - \Delta$, which occurs with probability $\int_0^{\hat{x} + \Delta} F(u - \Delta)^{n-1} f(u) du$.

Combining these elements, the expected profits of firm $i$, conditionally on being sampled, are given by

$$\pi(\Delta) = p_i \left[ 1 - F(\hat{x} + \Delta) + n \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \int_0^{\hat{x} + \Delta} F(u - \Delta)^{n-1} f(u) du \right] - c(e_i).$$

Following Anderson and Renault (1999), the profit function is assumed to be concave. Straightforward calculations show that

**Proposition 4.1.** The unique symmetric equilibrium has

$$p^N = \frac{1}{1-F(\hat{x})^n} \left[ 1 - F(\hat{x}) - n \int_0^{\hat{x}} F(u)^{n-1} f(u) du \right],$$

$$c'(e^N) = \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n}.$$  

The buyer searches until she encounters a firm $i$ with $p^N - p_i - e^N + e_i + u_i > \hat{x}$, with $\hat{x}$ given by (4.1), or until she has visited all firms. In the latter case, she buys from the firm that offers her the highest net utility.

**Proof.** The expressions follow directly from solving the first-order conditions and imposing symmetry. Existence and uniqueness then follows from the concavity of the profit function and convexity of the feasible set. 

The equilibrium price $p^N$ coincides with Anderson and Renault’s (1999) result. This should not come as a surprise. Given that effort is set optimally, firms in this model face the exact same decision problem regarding price as the firms in their model do. The comparative statics are as follows:

**Proposition 4.2.** If $1 - F(x)$ is log-concave on $[0,1]$, then price $p^N$ is (i) increasing in search costs $s$ and (ii) decreasing in the number of firms $n$. Furthermore, effort $e^N$ is (iii) increasing in $s$ and (iv) decreasing in $n$.

**Proof.** Part (i) and (ii) follow directly from Anderson and Renault (1999). For (iii), first note from (4.1) that $\hat{x}$ is decreasing in $s$. Hence, to show that $e^N$ is decreasing in $s$, it is sufficient to establish that $\frac{\partial}{\partial \hat{x}} \left( \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \right) < 0$. This derivative equals

$$\frac{\partial}{\partial \hat{x}} \left( \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \right) = \frac{f(\hat{x}) \left[ nF(\hat{x})^n - F(\hat{x}) - (n - 1)F(\hat{x})^{n+1} \right]}{F(\hat{x})(1 - F(\hat{x})^n)^2}.$$
Its sign is strictly negative if \( nF(\hat{x})^{n-1} - (n-1)F(\hat{x})^n < 1 \), which clearly holds for \( n = 2 \). Now suppose it holds for some \( n > 1 \). Then it will also hold for \( n+1 \) if \( nF(\hat{x})^n - (n-1)F(\hat{x})^{n+1} > (n+1)F(\hat{x})^{n+1} - nF(\hat{x})^{n+2} \). This inequality can be reduced to \( F(\hat{x})^2 - 2F(\hat{x}) + 1 > 0 \), which is satisfied for all \( F(\hat{x}) \in (0,1) \). Hence, by induction, the result follows. Finally, to derive result (iv), note that

\[
\frac{\partial}{\partial n} \left( \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \right) = \frac{(1 - F(\hat{x}))F(\hat{x})^n \ln(F(\hat{x}))}{(1 - F(\hat{x))^2}
\]

which is strictly negative, as \( \ln(F(\hat{x})) < 0 \).

Proposition 4.2 shows that the equilibrium properties of the model are intuitive. For well-behaved distribution functions, equilibrium price and effort are decreasing in the number of firms. As competition intensifies, firms lower their price to attract the buyer. At the same time, firms exert less effort as the probability that the buyer eventually buys at a given firm decreases. As search costs increase, firm effectively have more market power once they are visited by a consumer. This allows them to set a higher price. At the same time, firms exert more effort as the probability that the buyer eventually buys at a given firm increases.

### 4.4 The price fixing outcome

Under price fixing, each cartel member has agreed to post a fixed price, but is free to compete as vigorously in effort as it pleases.

The profit-maximizing cartel price \( p^* \) can be derived as follows. Given that it is finite (this is confirmed below), the assumption that the market is covered implies that the probability that the buyer will ultimately obtain the good is equal to 1. The expected total cost for the cartel is the expected sum of effort, which is \( \sum_{i=0}^{n-1} c(e)F(\hat{x})^i \). Hence, the problem of the cartel is to set a price \( p^* \) as to maximize

\[
\Pi = p^* - c(e^*(p)) \cdot \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})}.
\]

Effort \( e^*(p) \) is implicitly defined by the first-order condition for individual firm optimality. Thus, the cartel explicitly takes the effect of the fixed cartel price on the firms’ equilibrium effort into account. Under the assumption that all other firms set \( e^*(p) \), the analysis below derives the best reply of one firm, which enables a derivation of \( e^*(p) \). It will be convenient to work with a short-hand expression of
per-firm profit:
\[
\pi(\Delta) = p^*H(\Delta) - c(e_i).
\]

where
\[
H(\Delta) \equiv 1 - F(\hat{x} + \Delta) + n \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \int_{0}^{\hat{x} + \Delta} F(u - \Delta)^{n-1} f(u) du.
\]
is the probability that the buyer obtains the good from firm \(i\), conditional on firm \(i\) being sampled by the buyer. Now, per-firm effort can be written as
\[
c'(e) = p^*h_{e_i}|_{\Delta=0}. \tag{4.4}
\]

where \(h_{e_i}|_{\Delta=0}\) denotes the partial derivative of \(H(\Delta)\) with respect to \(e_i\), evaluated in \(\Delta = 0\). This marginal probability of sale is evaluated in \(\Delta = 0\) as in equilibrium all firms charge the same (collusive) price and provide the same level of effort. For notational convenience, this is simply written as \(h_e\).

Equation (4.4) implicitly defines effort as a function of price. Totally differentiating, one obtains that

\textbf{Lemma 4.1.} Under semicollusion, equilibrium effort \(e^*(p)\) is an increasing function of price;
\[
e'(p) = \frac{h_e}{c''(e)} > 0. \tag{4.5}
\]

\textit{Proof.} Recall that \(c''(e_i) > 0\). Therefore, it is sufficient to show that the marginal probability of sale \(h_e > 0\). Observe that
\[
h_e = f(\hat{x}) + \frac{n [1 - F(\hat{x})] \left[ (n - 1) \int_{0}^{\hat{x}} F(u)^{n-2} f(u)^2 du - F(\hat{x})^{n-1} f(\hat{x}) \right]}{1 - F(\hat{x})^n}
\]
\[
= f(\hat{x}) - \frac{n [1 - F(\hat{x})] \left[ \int_{0}^{\hat{x}} F(u)^{n-1} f'(u) du \right]}{1 - F(\hat{x})^n}. \tag{4.6}
\]

From 4.2, this implies that
\[
h_e = \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \frac{1}{p^N} > 0. \tag{4.7}
\]
Lemma 4.1 is intuitive; as a firm increases its effort, it increases the probability that it trades with the buyer. Therefore, as price increases, individual firms try to win the contract by exerting more effort. A bigger trophy results in more effort. Assuming that $\Pi$ is concave, it can now be shown that:

**Proposition 4.3.** With price fixing, the optimal cartel price is

$$
p^* = \frac{c''(e^*)}{h_e} \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n}.
$$

The effort exerted by each firm in the cartel is given by

$$
c'(e^*) = \frac{c''(e^*)}{h_e} \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n}.
$$

**Proof.** Given concavity of $\Pi$, the sufficient condition for the unique optimal price $p^*$ is $\frac{\partial \Pi}{\partial p} = 0$, or

$$
\frac{\partial \Pi}{\partial p} = 1 - c'(e) \cdot e'(p) \cdot \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})}.
$$

This derivative can be simplified by inserting the expressions for $c'(e)$ and $e'(p)$ in (4.4) and lemma 4.1. Rewriting gives $p^*$.

The level of effort exerted by each firm in the cartel is found by substituting the expression for $p^*$ into equation (4.4). Given that the regularity condition $\frac{\partial}{\partial e} c'(e) > 0$ is satisfied, a unique positive solution to this equation exists.

The cartel sets a fixed price $p^*$, taking the opportunistic behavior of its own members into account. Agreeing to post a very high collusive price is not in the cartel’s interest. When firms have a strong incentive to compete along the effort dimension, the cartel’s profit may be competed away through vigorous competition in effort. Therefore, it is sometimes optimal for the colluding firms to set a relatively low price, as the following result shows.

**Corollary 4.1.** The optimal cartel price $p^*$ is lower than the Nash equilibrium price $p^N$ if and only if

$$
c''(e^*) < h_e.
$$

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6 A sufficient condition for concavity of $\Pi$ is $c'''(e) < 0$. 
Proof. The Nash price $p^N$ is strictly higher than the collusive price $p^*$ if and only if

$$\frac{1}{1-F(\hat{x})} \frac{1}{1-F(\hat{x})} f(\hat{x}) - n \int_0^{\hat{x}} F(u)^{n-1} f'(u) du > \frac{c''(e^*)}{h_e^2} \frac{1-F(\hat{x})}{1-F(\hat{x})^n}.$$ 

This is equivalent to

$$-\frac{H}{h_p} > \frac{c''(e^*)}{h_e^2} \frac{1-F(\hat{x})}{1-F(\hat{x})^n}.$$ 

Note that $h_p = -h_e$: a unit decrease in effort has the same effect on the probability that the firm is selected by the buyer as a unit increase in price. This allows one to simplify the above inequality to

$$c''(e^*) < \frac{1-F(\hat{x})^n}{1-F(\hat{x})} \cdot H \cdot h_e.$$ 

If all firms charge the same price and provide the same level of effort, $H = \frac{1-F(\hat{x})}{1-F(\hat{x})^n}$, and therefore a collusive firm charges a lower price (and exerts less effort) than a non-cooperative firm if

$$c''(e^*) < h_e.$$ 

Thus, if the costs of effort respond sluggishly to an increase in effort (i.e. $c''(e)$ is sufficiently small), the cartel chooses to set a low price in order to prevent a costly war in effort between the firms. For instance, if the cost of effort function is given by $c(e) = ce^2$ and the matching value is drawn from the uniform distribution, a cartel agrees on a price below the competitive price if $c < 1/2$.

Corollary 4.1 can be considered to be the main result of this chapter. The result admits at least two interpretations. First, given that price fixing adequately describes actual collusive behavior, antitrust authorities should also investigate industries with ‘suspicious’ low prices. Those industries keep prices low in order to forestall fierce competition in effort. Another interpretation is that collusion is often not a very profitable strategy, as the collusive price is close to the non-cooperative price. Only if competition along the effort dimension is sufficiently costly, firms may be willing to form a cartel.

Note that concavity of $\Pi$, and hence an equilibrium in pure strategies, requires $c > 1/4$. 

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7 Note that concavity of $\Pi$, and hence an equilibrium in pure strategies, requires $c > 1/4$. 

With price fixing, the following comparative statics can be derived:

**Proposition 4.4.** If $1 - F(x)$ is log-concave on $[0, 1]$, then effort increases in search costs. Furthermore, if $f'(x) \geq 0 \ \forall x$, price increases in search costs as well. If additionally $n > 1/(1 - F(\hat{x}))$, price and effort decrease in the number of firms.

**Proof.** As $\hat{x}$ decreases in $s$, it is sufficient to show that effort decreases in $\hat{x}$ to prove that effort increases in $s$. Note that $e^*$ is uniquely determined by $\frac{c''(e)}{c'(e)} = h_e \frac{1-F(\hat{x})^n}{1-F(\hat{x})} = 1/p^N$, using (4.7). As $\frac{c''(e)}{c'(e)}$ decreases in $e$, $e^*$ increases in $s$ if $1/p^N$ decreases in $\hat{x}$ and, from proposition 4.2, this is known to be true.

To demonstrate that price increases in search costs, note that $p^*$ can be written as $p^* = \frac{c'(e^*)}{h_e}$. As $e^*$ is increasing in $s$, it is sufficient to have $h_e$ non-increasing in $s$, and thus to have $h_e$ non-decreasing in $\hat{x}$. This is true if and only if

$$f'(\hat{x})(1 - F(\hat{x})^n) + nf(\hat{x}) \int_0^{\hat{x}} F(u)^{n-1} f'(u)du \geq 0,$$

This is clearly satisfied if $f' \geq 0$.

For the effect of $n$, first derive the conditions under which effort decreases in the number of firms. This holds if the RHS of

$$\frac{c''(e^*)}{c'(e^*)} = h_e \frac{1-F(\hat{x})^n}{1-F(\hat{x})}$$

increases in $n$. Using (4.6), this is the case if

$$f(\hat{x}) \left( \frac{1-F(\hat{x})^n}{1-F(\hat{x})} \right) - n \int_0^{\hat{x}} F(u)^{n-1} f'(u)du > f(\hat{x}) \left( \frac{1-F(\hat{x})^{n-1}}{1-F(\hat{x})} \right) - (n-1) \int_0^{\hat{x}} F(u)^{n-2} f'(u)du$$

or

$$F(\hat{x})^{n-1} f(\hat{x}) + \int_0^{\hat{x}} F(u)^{n-2} f'(u) (n - 1 - nF(u)) du > 0.$$ Clearly, the inequality holds if $f' \geq 0$ and $n > 1/(1 - F(\hat{x}))$.

Now, examine the effect of an increase in $n$ on the collusive price. Note from (4.3) that one can write

$$p^*(n) = \frac{c'(e^*(n))}{h_e(n)}.$$
The equilibrium price is decreasing in $n$ iff $p^*(n) < p^*(n - 1)$, which holds if

$$
\frac{c'(e^*(n))}{h_e(n)} < \frac{c'(e^*(n - 1))}{h_e(n - 1)}.
$$

From concavity of $c$ and given that $f' \geq 0$ and $n > 1/(1 - F(\hat{x}))$, follows the condition that $c'(e^*(n)) > c'(e^*(n - 1))$. Sufficient for the inequality to be satisfied then is $h_e(n) > h_e(n - 1)$. Using (4.6), this is the case if

$$
\int_0^{\hat{x}} F(u)^{n-2} f'(u) \left[ (n - 1) (1 - F(\hat{x})^n) - n \left( 1 - F(\hat{x})^{n-1} \right) F(u) \right] du > 0
$$

As $f' \geq 0$, sufficient for this to hold is

$$(n - 1) (1 - F(\hat{x})^n) - n \left( 1 - F(\hat{x})^{n-1} \right) F(\hat{x}) > 0,$$

which simplifies to $n (1 - F(\hat{x})) - 1 + F(\hat{x})^n > 0$, which is satisfied for $n > 1/(1 - F(\hat{x}))$.

The sufficient conditions for monotone comparative statics under price fixing are stronger than under non-cooperative behavior. Under non-cooperative behavior, log-concavity guarantees that the competitive price and quality decrease in the number of firms. This result no longer holds in the presence of a price fixing cartel. A possible explanation is the following. An increase in the number of firms has two opposite effects. On the one hand, it intensifies competition between firms. For a given price, this gives them a stronger incentive to compete in the quality dimension, which induces the cartel to charge a lower price. On the other hand, an increase in the number of firms increases the expected matching value of the consumer. This increases the market power of the cartel, and allows it to charge a higher price and hence the cartel members to offer a higher quality. For some distribution functions, the latter effect could outweigh the first effect. However, note that the conditions derived in proposition 4.4 are merely sufficient conditions. The monotone comparative statics are expected to hold for a wider range of parameters.

### 4.5 Social welfare

This section studies the welfare effects of price fixing. The natural measure of social welfare $W$ is the sum of expected buyer’s utility and expected industry profit.
Formally,

\[ W = E[v + e + u - k(c(e) + s)]. \]

Using the facts that \( s \equiv \int_{1}^{1}(u - \hat{x}) f(u) du \) and the expected number of visits \( E[k] \) equals \( \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} \), welfare boils down to

\[ W = v + e - \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} \cdot c(e) + \hat{x} \left[ 1 - F(\hat{x})^n \right] + \int_{0}^{\hat{x}} udF(u)^n. \]

Since the third and fourth terms of this expression are independent of effort, a social planner who can dictate effort would set effort \( e^W \) such that

\[ c'(e^W) = \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} = c'(e^N). \]

Hence, the socially optimal level of effort equals the level of effort under Nash play. This is a notable result in itself. Wolinsky (2005) finds that, as buyers do not fully bear the costs of search themselves, they visit too many firms in equilibrium. This leads to an inefficient level of effort, as compared to the first-best social welfare optimum. This result shows that, if the social planner is unable to meddle with the buyer’s search strategy, social welfare under non-cooperative play is optimal in a second-best sense.

The finding that in the competitive mode effort is set (second-best) optimally implies that a price fixing industry always reduces social welfare, as

\[ c'(e^*) = \frac{c''(e^*)}{h_{e_i}} \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} \neq \frac{1 - F(\hat{x})}{1 - F(\hat{x})^n} = c'(e^W). \]

Thus, effort is distorted away from the optimal competitive level. Moreover, the search strategy of the buyer is invariant under both modes of competition and therefore price fixing firms necessarily yield lower welfare levels than non-cooperative firms. By construction, firms must be better off if they choose to set a price \( p^* \neq p^N \). With total welfare decreasing, and firm profits increasing, consumers are necessarily worse off. The welfare effects of price fixing in our model can thus be summarized as follows:

**Proposition 4.5.** Regardless of its effect on price, price fixing is bad for welfare and bad for consumers. Firms are better off if they can fix prices.

Returning to the question of Stigler (1968) it is clear that firms are strictly better
off with price fixing than they are when competing in prices: the cartel can still choose to fix prices at the competitive level \( p^N \). The fact that it chooses not to do so implies that firms are strictly better off under price fixing.\(^8\) It is also clear that firms could do even better under full collusion, i.e. if they were also able to collude on effort. Thus, in the words of Stigler, the monopoly profit achieved by suppressing price competition will be lowered, but not be eliminated by non-price competition.

### 4.6 Elastic demand

Though the model is quite general, it does assume that buyers always buy. This implies that prices are just a transfer from buyers to firms and greatly simplifies the welfare analysis. However, one may worry that the assumption of inelastic demand is driving the welfare results. To investigate this issue, adjust the model in the following direction. Buyers still have unit demand, but buy if and only if the most attractive offer yields at least a positive utility. That is, \( v + u_i + e_i - p_i > 0 \) is required to hold if the buyer selects firm \( i \). This introduces a downward-sloping demand curve, as the fraction of buyers who purchase good \( i \) decreases in \( p_i - e_i \).

For convenience, normalize \( v \) to zero and suppose that buyers do not face search costs.

As before, suppose that a symmetric equilibrium exists where firms exert effort \( e^N \) and charge a price \( p^N \). To characterize the equilibrium, suppose first that \( e^N - p^N \geq 0 \) and firm \( i \) deviates from the proposed equilibrium. A consumer buys from firm \( i \) if two conditions hold. First, firm \( i \) should offer more utility than any other firm: \( u_i + e_i - p_i > \max\{u_{-i}\} + e^N - p^N \), or \( u_i - \Delta > z \), where \( \Delta \equiv p_i - p^N - e_i + e^N \) and \( \max\{u_{-i}\} \equiv z \). Second, firm \( i \) should offer positive utility and this requires \( u_i > p_i - e_i \). Firm \( i \) chooses \( e_i \) and \( p_i \) as to maximize

\[
\pi(\Delta) = p_i \int_{p_i-e_i}^{1} F(u-\Delta)^{n-1} f(u)du - c(e_i).
\]

Unfortunately, there is no general closed-form solution to this maximization problem. Without an expression for \( p^N \) in closed-form, one cannot compare it with the collusive price. To circumvent this problem, assume that \( u \) is uniformly distributed, \( n = 2 \), and \( c(e) = ce^2 \). Then, in the non-cooperative equilibrium,

\[
e^N = \frac{1}{2c + \sqrt{1 + 4c(2c - 1)}}.
\]

\(^8\) Except, that is, in the knife-edge case \( c''(e^*) = h_\varepsilon \) where there is no effect of semicollusion on welfare.
\[ p^N = \frac{2c}{2c + \sqrt{1 + 4c(2c - 1)}}. \]

It can be verified that this solution is valid only for \( c \geq \frac{1}{2} \). For \( c \) below \( \frac{1}{2} \), \( p^N - e^N < 0 \) and the constraint that the firm should offer a positive payoff no longer binds. In this case, the relevant profit function of a firm that contemplates to deviate from the symmetric equilibrium is

\[ \pi(\Delta) = p_i \int_0^1 F(u - \Delta) F_u f(u) du - c(e). \]

For the special case of a uniform distribution and quadratic costs, the optimal effort and price are

\[ e^N = \frac{1}{4c'}, \quad p^N = \frac{1}{2}. \]

In both cases, effort is given by \( p^N \). Hence, under the price fixing regime, the cartel maximizes

\[ \Pi = \Pr\{\text{sale}\} p - 2ce^2, \]

subject to the constraint \( e = \frac{p}{2c} \). The optimal collusive price is

\[ p^* = \begin{cases} \frac{c}{2c} & \text{if } c \in \left[\frac{1}{4}, \frac{1}{2}\right], \\ \frac{2c}{1 + 2\sqrt{1 + 3c(c - 1)}} & \text{if } c \geq \frac{1}{2}. \end{cases} \]

Clearly, \( p^N > p^* \) if \( c \in \left[\frac{1}{4}, \frac{1}{2}\right] \). So, even with downward-sloping demand, the collusive price may be below the non-cooperative price. Finally, it is easy to show that welfare under price fixing is strictly below welfare under non-cooperative behavior, unless \( c = \frac{1}{2} \), in which case welfare is unchanged.

### 4.7 Concluding remarks

In the real world, fully collusive industries in which all firms can collude on every single aspect of their product, are hard if not impossible to find. This chapter explored the effects of a price fixing agreement in a model with vertical product differentiation and sequential search. In a model that builds on Anderson and Renault...
(1999) and Wolinsky (2005), it was showed that if firms collude in prices but not in effort, they may prefer to set prices below the competitive price to avoid a costly war in effort. Still, social welfare is always lower in the case of price fixing. Firms are strictly better off with price fixing.

These results have important implications for antitrust policy. To find evidence of collusion, antitrust authorities tend to look for industries where prices are relatively high. The analysis in this chapter suggests that in industries where firms can collude on prices but not on other aspects of their product, prices may actually be lower than competitive prices. An alternative interpretation is that a cartel is not very profitable if firms can easily evade the collusive agreement by competing in effort. Finally, a change from non-cooperative behavior to semicollusive behavior induces a change in product design. Antitrust authorities should take this into account when calculating the ‘but for’ price to determine the damages of collusion.

As a suggestion for future research, it would be very interesting to study collusion in a dynamic framework. This may help to understand why many cartels restrict themselves to fixing prices.