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Adaptive Fuzzy Behavioral Control of Second-Order Autonomous Agents With Prioritized Missions: Theory and Experiments

Jie Huang , Member, IEEE, Ning Zhou , Member, IEEE, and Ming Cao , Senior Member, IEEE

Abstract—In this paper, we study the adaptive fuzzy formation control of multiple autonomous agents with prioritized missions. For a platoon of autonomous agents in an unknown environment containing multiple obstacles, formation control is investigated, where each agent is modeled by a second-order nonlinear system under unknown external disturbance in the Brunovsky form. We introduce the systematic procedure of null-space-based projection to convert the prioritized multimission control problem into a behavioral control problem. Then, we further develop a class of nonlinear-fast-terminal-sliding-mode-based adaptive control strategies that combine the fuzzy logic systems by jointly considering both kinematic and dynamic levels of the agents. The proposed controllers can guarantee each individual agent to achieve the predesigned desired pattern and drive the entire systems to achieve the prescribed missions. A simulation example with five agents demonstrates the effectiveness of the algorithm. Finally, the strategies are experimentally validated using a platoon of Pioneer 3AT and 3DX mobile robots.

Index Terms—Autonomous agents, behavioral control, Brunovsky system, multiagent systems (MASs), Pioneer mobile robot, prioritized mission.

I. INTRODUCTION

OVER the past decade, coordination of platooning autonomous vehicles and multiagent systems (MAS) has

attracted huge attention, since a group of intelligent agents can be used to solve the problems that are difficult or impossible for a single agent to deal with [1]–[5]. So, in general, a group of agents can be used to increase the effectiveness of the collective systems. Thanks to the versatility and redundancy achieved by taking advantage of the different capabilities of the agents, such systems are usually required to achieve several missions simultaneously. Its broad applications include many practical tasks, e.g., formation, coverage, and flocking control of a platoon of autonomous vehicles.

One of the challenges on the formation control of autonomous agents is how to dynamically balance the constraints of multiple missions and further to control the agents based on the composition result of the prioritized missions. In this context, mission conflict may occur when executing different control goals. Hence, a regulatory mechanism for conflicting missions is needed, and the priorities of the missions need to be configured depending on the relative important goal among the activated missions. Behavioral control is one of the possible solutions to this challenge, which has been studied for more than two decades. However, the traditional methodology of behavioral control, the layered control approach [6], and the motor schema control approach [7], [8] cannot easily combine the missions, and the convergence of the corresponding algorithms cannot be analyzed easily using a mathematical model. In this line of research, more recently, Antonelli and Chiaverini [9] have proposed a cooperative approach named null-space-based behavioral control (NSBC) for a platoon of autonomous vehicles. Later, this framework was validated via experimental results in [10]. Gans *et al.* [11] studied how to track multiple noncooperative moving targets with vision-related control missions. García *et al.* [12] investigated the fusion of different behaviors, including those that are individual, collective, or social. Rahimi *et al.* [13] addressed a cooperative formation control problem for a MAS that appears in rescue and surveillance missions. Saska *et al.* [14] used model-predictive control to realize several simultaneous missions. Schlanbusch *et al.* [15] considered the prioritized multimission control of spacecraft formation configurations at the dynamic level. In [16] and [17], the authors presented the stability analysis of behavioral control under consideration of a group of Euler–Lagrange systems to achieve escorting task and formation mission.

Although the behavioral control scheme has been already used in several case studies in the past decades, most of the

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works mentioned above only considered the prioritized multi-mission control problem with the kinematic model, and few of the papers study the more complex nonlinear dynamics. For theoretical development, the behavioral control methodology still has some open questions, e.g., how to achieve precise control under the behavioral control framework, or how to execute centralized tasks using only decentralized agents, and few of the existing literature studies consider the finite-time convergence requirement under the framework of behavioral control. Motivated by the above discussion, in this paper, we study the adaptive behavioral control for a platoon of second-order uncertain nonlinear autonomous agents under the constraint of prioritized missions. The contributions of this paper are as follows.

- 1) We further develop the adaptive fuzzy behavioral control from only kinematic level to the combination of both kinematic and dynamic levels. And the prioritized mission composition mechanism is carefully addressed.
- 2) The NSBC approach is extended to cover more complicated nonlinear agents. The second-order nonlinear agent in the Brunovsky form is studied in this paper.
- 3) The proposed control laws can guarantee the finite-time convergence of both mission errors and tracking errors. This implies that the control objective of performing mission in finite time can be achieved by rigorous theoretical analysis.
- 4) The theoretical results are successfully implemented on practical platooning Pioneer 3AT series mobile robots.

The rest of this paper is organized as follows. In Sections II and III, we formulate the research problem and present the main methodology of the prioritized multi-mission composition. In Section IV, the main results are presented. In Sections V and VI, a simulation example and an experimental case are studied. Discussion is provided in Section VII. Section VIII concludes this paper. The Appendix gives the proof of Theorem 1.

II. PROBLEM FORMULATION

A. Modeling

Consider a group of $n(n \geq 2)$ second-order uncertain nonlinear agents in the Brunovsky form [18], where the model of the i th ($i = 1, \dots, n$) agent is described by

$$\dot{x}_i = v_i \quad (1a)$$

$$\dot{v}_i = u_i + f_i(x_i) + d_i(t) \quad (1b)$$

where $x_i \in \mathbb{R}^2$ is the position state of the i th agent, $v_i \in \mathbb{R}^2$ is the velocity state of the i th agent, $u_i \in \mathbb{R}^2$ is the control input, and $x_i = [x_i^T, v_i^T]^T \in \mathbb{R}^4$. Following [18], one has the following.

Assumption 1: $f_i(x_i) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is locally Lipschitz with $f_i(0) = 0$ and assumed to be unknown.

Assumption 2: $d_i(t) \in \mathbb{R}^2$ is the time-varying external disturbance, which is unknown but bounded.

B. Control Objective

The control objective is to design control laws for a group of autonomous agents (1a), (1b) to form a desired formation with

prioritized missions under the behavioral control framework, in particular avoiding obstacles in an unknown environment. The motion commands to each agent are elaborated by the supervisor making use of only local information relative to the individual agent, e.g., the sensed distances from its neighbors. All the agents can reconfigure and maintain the formation while meeting precision and robustness control requirements. The objective is twofold: the mechanism for the mission fusion and controller design based on the output of the mission fusion.

III. PRIORITIZED MULTIMISSION COMPOSITION DESIGN

For most instances, the priorities of the missions can be well-established according to the practical situations. For example, when a mobile agent is moving toward the desired target, avoiding the nearby obstacle is apparently a mission of higher priority [19]. Following this idea, we define $\rho_k \in \mathbb{R}^{m_k}$ as the k th mission function, $\forall 1 \leq k \leq r$, where m_k is the dimension of the mission space. And we further define the mission hierarchy, which follows the following rules.

- 1) Assume that $k = 1$ is the top priority. Here, $k_a < k_b$ implies that k_a is the index of a higher priority than k_b ; a mission of priority k_b may not disturb another mission of priority k_a . The lower priority missions are executed in the null space of all higher priority missions.
- 2) The mappings from the velocities to the mission velocities are captured by the mission Jacobian matrix $J_k \in \mathbb{R}^{m_k \times n} \forall 1 \leq k \leq r$.
- 3) The dimension m_r of the lowest level mission may be greater than $n - \sum_{k=1}^{r-1} m_k$ so that n is ensured to be greater than the total dimension of all missions.

Following the aforementioned rules, we define the mission function ρ as

$$\rho = g(x) \quad (2)$$

where $x = [x_1, \dots, x_n]^T$ and $v = [v_1, \dots, v_n]^T$ are the stacked vectors of all the agents' positions and velocities, respectively. Then, the time derivative of the mission function (2) is

$$\dot{\rho} = \sum_{j=1}^n \frac{\partial g(x)}{\partial x_j} v_j = J(x)v. \quad (3)$$

We invert the locally linear mapping (3) to arrive at the least-squares solution as

$$v_d = J^\dagger \dot{\rho}_d = J^T (J J^T)^{-1} \dot{\rho}_d \quad (4)$$

where $J^\dagger = J^T (J J^T)^{-1}$ is the pseudoinverse of the mission Jacobian matrix J .

The practical system is, in general, running under a discrete-time scheme. Hence, the desired *velocity command* will cause a certain drift of the reconstructed agents' positions. And the following closed-loop inverse kinematics (CLIK) form can be used to compensate the drift

$$v_d = J^\dagger (\dot{\rho}_d + \Lambda \tilde{\rho}) \quad (5)$$

where Λ is a positive-definite matrix of constant gains, and $\tilde{\rho} = \rho_d - \rho$ is the *mission error*.

Fig. 1. Sketch of NSBC in a k -mission example.

Fig. 2. Velocity command composition of two conflicting missions.

From (5), the desired value of the k th mission velocity $v_{d,k}$ can be computed as

$$v_{d,k} = J_k^\dagger (\dot{\rho}_{d,k} + \Lambda_k \tilde{\rho}_k). \quad (6)$$

By balancing each mission individually, all the missions can be merged together using

$$v_d = v_{d,k} + (I - J_k^\dagger J_k) v_{d,k+1} \quad (7)$$

where v_d is the current agents' velocity vector including the missions with the priorities from r to k .

For the general case, the sketch of the NSBC including k missions is shown in Fig. 1, in which the supervisor is used to orchestrate the priority of every individual mission. And the dotted lines in the supervisor block denote the swap among different missions under the administration of the mission supervisor. In this case, mission #B (v_B) is given the highest priority v_1 , mission #A (v_A) is given the second highest priority v_2 , ..., and mission #K (v_K) is given the lowest priority v_k . It is worth noticing that the motion commands to each agent are elaborately generated by the mission supervisor and controller together making use of only local information.

Now, consider the formation control of the agents in the unknown environments with obstacles at the kinematic level. First, the behavior that drives a agent move along the desired trajectory can be considered as the basic behavior, named *motion-mission*. On the other hand, the obstacles must be avoided all the time obviously, and this behavior can be defined as the *obstacle-avoidance-mission*. When the agents detect the obstacles in their motion trajectories, the distance between the agents and the obstacles should be increased by generating an extra velocity command. This command should be zero once the agents are out of the regions of obstacles. When the obstacles are somewhere along the lines of the motion direction of the desired positions, the two mission velocity commands will be in conflict. Thus, the NSB scenario takes place to transform the individual mission velocity commands to an integrated one. As shown in Fig. 2, v_1 denotes the desired velocity of the higher priority mission,

and v_2 represents the desired velocity of the lower priority mission. Since v_2 would conflict with v_1 , in order to eliminate the conflicting components, v_2 needs to be projected onto the null space of v_1 . It means that each mission's effective element is combined to construct the integrated velocity command to drive the agent.

Then, the CLIK solution (5) for the two-mission case can be rewritten as

$$v_d = v_1 + (I - J_1^\dagger J_1) v_2 \quad (8)$$

where $I - J_1^\dagger J_1$ is the null-space projector of the higher priority mission.

Remark 1: We remark that a full-dimensional highest priority mission would be with a full-rank J_1 matrix, its null space would be empty, and it will result in that the whole v_2 would be filtered out.

Remark 2: The proposed mechanism further develops the control scheme based on the velocity command (mission function). Thus, it has the possibility to apply the behavioral control result to the practical robots, which are driven by velocity signals, e.g., the Pioneer series mobile robot.

IV. MAIN RESULTS

A. Mission Design and Composition

In this section, we present the main results in the case of formation control with two missions. Before the controller design, a mission supervisor is developed to orchestrate the relative priorities among all the missions. In this case, the priority of the obstacle-avoidance-mission is higher than that of the motion-mission, obviously. First, the motion-mission is considered, which drives the group to form a desired configuration relative to the barycenter. This mission function is defined by the following function:

$$\rho_\varsigma = [(x_1 - x_b)^T, \dots, (x_n - x_b)^T]^T \quad (9)$$

where $x_b = \frac{1}{n} \sum_{i=1}^n x_i$ is the vector of the coordinate of the barycenter.

Then, we define the mission error as

$$\tilde{\rho}_\varsigma = \rho_{\varsigma,r} - \rho_\varsigma \quad (10)$$

where $\rho_{\varsigma,r} = [\rho_{\varsigma,r1}, \dots, \rho_{\varsigma,rn}]^T$ represents the coordinates of all the agents in the predefined configuration. $J_\varsigma = \text{block diag}\{A, A\} \in \mathbb{R}^{2n \times 2n}$ is the corresponding Jacobian matrix with

$$A = (I - \frac{1}{n} \otimes \mathbf{1}_{n \times n}) \in \mathbb{R}^{n \times n} \quad (11)$$

where $\mathbf{1}_{n \times n}$ is a square matrix with all elements 1, $\rho_{\varsigma,r}$ denotes the desired formation configuration, and the elements of $\rho_{\varsigma,r}$ describe the coordinates of each agent. The output of ρ_ς is

$$v_\varsigma = J_\varsigma^\dagger (\dot{\rho}_{\varsigma,r} + \Delta_\varsigma \tilde{\rho}_\varsigma) \quad (12)$$

where Δ_ς is a suitable constant positive-definite matrix of gains, and $J_\varsigma^\dagger$ is the pseudoinverse of J_ς . Since J_ς is symmetric and idempotent, $J_\varsigma^\dagger = J_\varsigma$.

Fig. 3. Technical diagram of the main results.

The desired velocity can be given by

$$v_r = v_o + (I - J_o^\dagger J_o) v_\zeta \quad (13)$$

where $v_r = [v_{1,r}^T, \dots, v_{n,r}^T]^T$, and $v_o = [v_{1,o}^T, \dots, v_{n,o}^T]^T \in \mathbb{R}^{2n}$ is the desired velocity for the obstacle-avoidance-mission, $v_{i,o} = J_{i,o}^\dagger \Delta_{i,o} \tilde{\rho}_{i,o} = (d_i - \|x_i - x_{i,o}\|) \times \Delta_{i,o} (\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) \hat{r}_i$, $x_{i,o}$ is the position of the obstacle with respect to agent i , $v_{i,o}$ is the relative velocity concerning the obstacle and agent i , and $I \in \mathbb{R}^{2n \times 2n}$ denotes the identity matrix. Here, $\mathcal{B}_{i,o} = \{x_i, x_{i,o} \in \mathbb{R}^2 : \|x_i - x_{i,o}\| \leq d_i\}$, and it marks a region $\rho_{i,or} = d_i$, where d_i is the minimum allowed safe distance between the i th agent and an obstacle. $\rho_{i,o} = \|x - x_0\|$ is the mission function to achieve obstacle avoidance, where x_0 is the nearest obstacle's position. If the mission is active, $\tilde{\rho}_{i,o} = \rho_{i,or} - \rho_{i,o} > 0$; otherwise, $\tilde{\rho}_{i,o} = 0$. $\tilde{\rho}_o = [\tilde{\rho}_{1,o}, \tilde{\rho}_{2,o}]^T$. $J_{i,o} = \hat{r}_i^T$ is the Jacobian matrix, $\hat{r}_i = (x_i - x_{i,o}) / \|x_i - x_{i,o}\|$ is a unit vector pointing at the nearest obstacle, and we denote $J_o = [J_{1,o}, \dots, J_{n,o}] \in \mathbb{R}^{1 \times 2n}$. $\Delta_{i,o}(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) > 0$ is a state-dependent gain to be defined. $v_\zeta = [v_{1,\zeta}^T, \dots, v_{n,\zeta}^T]^T = J_\zeta^\dagger \Delta_\zeta \tilde{\rho}_\zeta$ is the desired velocity for formation mission. If the desired formation is achieved, $\dot{\rho}_{\zeta r} = 0$ holds.

B. Controller Design

The technical diagram of the main results of the controller design is shown in Fig. 3. In this paper, combining with the NSBC scheme, a second-order nonlinear fast terminal sliding-mode (NFTSM) is introduced as

$$\sigma = \tilde{v} + c_1 \tilde{x} + c_2 \beta(\tilde{x}) \quad (14)$$

where c_1 and c_2 are two positive constant parameters to be designed, $\tilde{x}$ is the estimation error of the positions with $\tilde{x} = [\tilde{x}_1^T, \dots, \tilde{x}_n^T]^T = x - x_r$, and $\tilde{v} = [\tilde{v}_1^T, \dots, \tilde{v}_n^T]^T = v - v_r$. $x_r = [x_{1,r}^T, \dots, x_{n,r}^T]^T$ is the desired trajectory, which is bounded and can be calculated by integrating v_r in (13). $\sigma = [\sigma_1^T, \dots, \sigma_n^T]^T$, $\sigma_i = [\sigma_{i,1}, \dots, \sigma_{i,m}]^T$, $\beta(\tilde{x}) = [\beta_1(\tilde{x}_1)^T, \dots, \beta_n(\tilde{x}_n)^T]^T$, $\beta_i(\tilde{x}_i) = [\beta_{i,1}(\tilde{x}_{i,1}), \beta_{i,2}(\tilde{x}_{i,2})]^T$, and

$$\beta_{i,j}(\tilde{x}_{i,j}) \triangleq \begin{cases} \tilde{x}_{i,j}, & \text{for } \bar{\sigma}_{i,j} = 0 \text{ or } \bar{\sigma}_{i,j} \neq 0, |\tilde{x}_{i,j}| > \epsilon \\ \zeta_1 \tilde{x}_{i,j} + \zeta_2 \text{sig}^2(\tilde{x}_{i,j}), & \text{for } \bar{\sigma}_{i,j} \neq 0, |\tilde{x}_{i,j}| \leq \epsilon \end{cases}$$

for $i = 1, \dots, n$, $j = 1, \dots, m$, and $r = \frac{r_1}{r_2}$, where r_1 and r_2 are positive odd integers, $\frac{1}{2} < r < 1$, ϵ is a small positive constant, $\zeta_1 = (2 - r)\epsilon^{r-1}$, $\zeta_2 = (r - 1)\epsilon^{r-2}$, and

$\bar{\sigma}_{i,j} = \tilde{v}_{i,j} + c_1 \tilde{x}_{i,j} + c_2 \tilde{x}_{i,j}^r$. The time derivative of $\beta_{i,j}(\tilde{x}_{i,j})$ is

$$\dot{\beta}_{i,j}(\tilde{v}_{i,j}) \triangleq \begin{cases} r \tilde{x}_{i,j}^{r-1} \tilde{v}_{i,j}, & \text{for } \bar{\sigma}_{i,j} = 0 \text{ or } \bar{\sigma}_{i,j} \neq 0, |\tilde{x}_{i,j}| > \epsilon \\ \zeta_1 \tilde{v}_{i,j} + 2\zeta_2 |\tilde{x}_{i,j}| \tilde{v}_{i,j}, & \text{for } \bar{\sigma}_{i,j} \neq 0, |\tilde{x}_{i,j}| \leq \epsilon \end{cases}$$

$\dot{\beta}(\tilde{v}) = [\dot{\beta}_1(\tilde{v}_1)^T, \dots, \dot{\beta}_n(\tilde{v}_n)^T]^T$, and $\dot{\beta}_i(\tilde{v}_i) = [\dot{\beta}_{i,1}(\tilde{v}_{i,1}), \dot{\beta}_{i,2}(\tilde{v}_{i,2})]^T$. Denote $v_\nu = [v_{\nu,1}^T, \dots, v_{\nu,n}^T]^T = v_r - c_1 \tilde{x} - c_2 \beta(\tilde{x})$ and $\dot{v}_\nu = [\dot{v}_{\nu,1}^T, \dots, \dot{v}_{\nu,n}^T]^T = \dot{v}_r - c_1 \tilde{v} - c_2 \dot{\beta}(\tilde{v})$, where $\dot{v}_r$ is bounded; then, (14) becomes $\sigma = v - v_\nu$.

Next, the fuzzy logic system (FLS)-based approximation scheme is applied to approximate the unknown nonlinear function $f_i(\mathbf{x}_i)$ in (1b) [20]–[22]. Define the minimal approximation error $\varepsilon_j = f_i(\mathbf{x}_i) - f_i(\mathbf{x}_i|\theta_i^*)$, where $f_i(\mathbf{x}_i|\theta_i^*) = \theta_i^{*T} \phi_i(\mathbf{x}_i)$, $\theta_i^* = [\theta_{i,1}^*, \dots, \theta_{i,k_i}^*]^T \in \mathbb{R}^{k_i}$ is the optimal fuzzy parameter vector which is only used for analysis and $\phi_i(\mathbf{x}_i) = [\phi_{i,1}(\mathbf{x}_i), \dots, \phi_{i,k_i}(\mathbf{x}_i)]^T \in \mathbb{R}^{k_i}$ is a regression vector. Then, $f_i(\mathbf{x}_i)$ can be approximated by $\hat{f}_i(\mathbf{x}_i) = \hat{\theta}_i^T \phi_i(\mathbf{x}_i)$, where $\hat{\theta}_i$ is the estimation of θ_i^* .

Lemma 1 (see[20]): Let $f_i(\mathbf{x}_i)$ be a continuous function defined on a compact set Ω . Then, for any constant $\varepsilon_i > 0$, there exists an FLS

$$y_i(x_i) = \theta_i^T \phi_i(\mathbf{x}_i) \quad (15)$$

such that

$$\sup_{x_i \in \Omega} |f_i(\mathbf{x}_i) - \theta_i^T \phi_i(\mathbf{x}_i)| \leq \varepsilon_i. \quad (16)$$

Assumption 3 (see[23]): There exists a known positive constant δ_i , such that $\|\varepsilon_i + d_i(t)\| \leq \delta_i$.

We propose the following control laws:

$$u_i = -K_{1i} \sigma_i - K_{2i} \text{sig}^{\frac{1}{2}}(\sigma_i) - \hat{\theta}_i^T \phi_i(\mathbf{x}_i) - \chi_i - \hat{\delta}_i \text{sign}(\sigma_i) \quad (17)$$

$$\dot{\hat{\theta}}_i = -\Gamma_{1i}^{-1} \left(\phi_i(\mathbf{x}_i) \sigma_i + \Gamma_{2i} \hat{\theta}_i + \frac{\Gamma_{2i}}{4} \text{sign}(\hat{\theta}_i) \right) \quad (18)$$

$$\dot{\hat{\delta}}_i = \gamma_{1i} \left(\|\sigma_i\| - \gamma_{2i} \hat{\delta}_i - \frac{\gamma_{2i}}{4} \text{sign}(\hat{\delta}_i) \right) \quad (19)$$

where $\chi_i = -\dot{v}_{ri} + c_1 \tilde{v}_i + c_2 \dot{\beta}_i(\tilde{v}_i)$; $K_{1i} = k_{1i} I$ and $K_{2i} = k_{2i} I$ are positive-definite and constant matrices; Γ_{1i} and Γ_{2i} are positive-definite matrices; k_{1i} , k_{2i} , γ_{1i} , and γ_{2i} are positive design parameters; and $\hat{\theta}_i$ and $\hat{\delta}_i$ are the estimations of θ_i^* and δ_i , respectively. The function $\text{sig}^{\frac{1}{2}}(\cdot)$ is defined as $\text{sig}^{\frac{1}{2}}(\mathbf{x}) = |\mathbf{x}|^{\frac{1}{2}} \text{sgn}(\mathbf{x})$ for $\frac{1}{2} > 0$ and $\text{sig}^{\frac{1}{2}}(\mathbf{x}) = [\text{sig}^{\frac{1}{2}}(x_i)]^T$; it follows that $\frac{d \text{sig}^{\frac{1}{2}}(\mathbf{x})}{d\mathbf{x}} = \frac{1}{2} |\mathbf{x}|^{\frac{1}{2}-1}$.

Theorem 1: Using the control algorithms (17)–(19) for the second-order uncertain nonlinear agent system (1a), (1b), the generalized system errors $\tilde{v}_i$ and $\tilde{x}_i$ will become sufficiently small in finite time, and the entire systems can achieve the prescribed missions if the design parameter satisfies the constraint $\Delta_{i,o}(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) = \Delta_{i,o}^*(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) + \varrho_i$, where $\varrho_i > 0$ is the term guaranteeing robustness.

Proof: See the Appendix. ■

Remark 3: In the presence of the classic local minima problem [24], i.e., the agents are going to frontally collide with

Fig. 4. Trajectories of the five agents in the environment with four obstacles.

the obstacles, the velocity elaborated by the missions is in the agent–obstacle direction. This particular situation makes the agents stop somewhere. In this paper, the method of adding a small measurement noise is utilized to avoid the local minima problem. In addition, the minimum related distance d_i between agent i and the obstacle satisfies $d_i \geq \underline{d}_i + d_{\delta i}$, where $\underline{d}_i$ is the ideal minimum related distance, and $d_{\delta i}$ is a robustness term. That is, the minimum related distance d_i designed in this paper should be greater than $\underline{d}_i$.

V. SIMULATION

Consider a group consisting of five agents moving in the plane, and each agent is modeled by a second-order nonlinear dynamical system. The control objective is to design control laws to let a platoon of agents form a desired formation in an unknown multiobstacle environments. The agents are assumed to have the ability to detect their nearby environment by sensors, and the ability is well-defined by an exact range measurement. The Gaussian membership functions are designed for the FLS to approximate the nonlinear unknown function of the dynamics. In the simulation, the fuzzy input of each FLS is constructed by five membership functions, which are described by $\mu_{F_j}^i(x_j) = e^{-((x_j+1.5)/2)^2}$, for $j = 1, \dots, 5$. The initial positions of the agents are $x_1 = [-14; 24]$, $x_2 = [11; 12]$, $x_3 = [39; -4]$, $x_4 = [9; -16]$, and $x_5 = [-18; -31]$, respectively, and the obstacles' positions are $O_1 = [105; -5]$, $O_2 = [40; -10]$, $O_3 = [35; -18]$, and $O_4 = [85; -32]$. The mission functions for the agents are $\rho_{s,r1} = [-14 + 2t; 28]$, $\rho_{s,r2} = [14 + 2t; 14]$, $\rho_{s,r3} = [42 + 2t; 0]$, $\rho_{s,r4} = [14 + 2t; -14]$, and $\rho_{s,r5} = [-14 + 2t; -28]$, respectively. In the simulation, the minimum safe distance is assumed to be 7 m.

Fig. 4 shows the trajectories of the five agents, which demonstrate the validity of Theorem 1. In this instance, the five agents can achieve a prescribed formation in the environment with four obstacles. The agents are using sensors to detect the locations of the obstacles since they do not know the positions of the obstacles. The symbol “o” in Fig. 4 denotes the obstacles’

Fig. 5. Mission priority statuses of the agents.

Fig. 6. Responses of the distances between agents and obstacles.

positions. Fig. 5 is the mission priority statuses of the agents. MM means that the motion-mission is active, and OAM denotes that the obstacle-avoidance mission is active. Fig. 6 shows the distances between the agents and their nearby obstacles. Once the agent is moving into the range of the obstacle or another agent, the obstacle-avoidance-mission is activated in the higher priority to avoid collision. Fig. 7 shows the tracking errors between agents and the corresponding desired formation trajectories. In the beginning, the control laws drive the agent to the desired trajectories. As the time goes by, when the agent comes into the range of the obstacle or the other agents, the tracking error becomes larger because of the higher prioritized mission component to drag the agent to a safer space. In Figs. 8 and 9, the position values of agents 1 and 2 in their x and y components are shown, respectively. For example, p_{11} is the

Fig. 7. Responses of the tracking errors of the agents.

 Fig. 8. Position trajectories of agent 1 in x -axis (p_{11}) and y -axis (p_{12}), respectively.

 Fig. 9. Position trajectories of agent 2 in x -axis (p_{21}) and y -axis (p_{22}), respectively.

Fig. 10. Pioneer-3AT mobile robot experimental setup.

coordinate of agent 1 and p_{12} is the y coordinate of agent 1. The trajectories match the desired formation requirement.

VI. EXPERIMENT

In this section, the proposed control algorithm is validated by experiments on real differential-drive robots. To be more specific, the experiment is performed on four Pioneer-3AT four-wheel mobile robots and one Pioneer-3DX two-wheel compact mobile robot [25]. The experimental setup of Pioneer-3AT is shown in Fig. 10. It is a small four-wheel four-motor skid-steer robot for all-terrain operation or laboratory experimentation. The Pioneer-3 robot comes complete with one battery, an emergency stop switch, wheel encoders, and a controller (on-board microcontroller or vehicle-mounted computer) with an advanced mobile robotics software development package. In the following experiment, five Lenovo X24 laptops are used as the vehicle-mounted controllers of the corresponding five mobile robots, and the controllers are programmed by VC++ software. Under consideration of the vehicle's kinematic equations and Theorem 1, a practical control algorithm is designed to follow a reference formation configuration with predefined velocity commands. Algorithm 1 is the pseudocode for the controller of each robot.

To be more specific, the kinematic equation of the Pioneer robot is a typical second-order system, which can be described by the formulations (1a), (1b). Thus, the main theoretical result of this paper can be experimented by using a team of Pioneer 3AT/3DX mobile robots. In the experiment, the safe range is set to 40 cm. The agents do not exchange the full set of information and instruction and only use sensors to detect the local environment. All the agents can send and receive their local information to their own supervisor to make decision of the mission priorities. In this experimental case, two missions are defined according to the main result of this paper, which includes motion-mission and obstacle-avoidance-mission. Obviously, the priority of obstacle-avoidance-mission is higher than

Fig. 11. Snapshots of two-mission execution with Pioneer mobile robots at time 14, 28, 42, and 57 s, respectively.

Algorithm 1: Algorithm for Each Agent/Robot j .

Initialization: initialize the positions of the robots, set safety distance $d = 40$ cm, set the orientations of all the robots.

Ensure: keep the sensors work all the time, keep all the robots can send and receive local information to their supervisor.

- 1: **While** The desired overall mission is uncompleted. **do**
 - 2: **if** robot j is out of range of the obstacles or other robots.
 - then**
 - 3: execute motion-mission.
 - 4: **else**
 - 5: activate the obstacle-avoidance-mission and the projection from the lower mission to the null space of higher mission.
 - 6: **end if**
 - 7: **end While**
-

motion-mission in the design. Fig. 11 shows a two-mission formation execution with a team of Pioneer-3AT/3DX robots at time 14, 28, 42, and 57 s, respectively. Fig. 12 shows the experimental curve of the Pioneer-3AT/3DX robots, which is the trajectory of the whole experimental process shown in Fig. 11. It can be seen that the mobile robots execute the motion mission, while avoiding obstacles by the proposed control strategy.

VII. DISCUSSION

We make a comparison of the approach developed in this paper to other techniques here. Before further discussion, first, we briefly summarize three typical formation control schemes: leader-following approach, the behavioral approach, and the virtual structure approach. In the leader-following approach, coordination is achieved through sharing the knowledge of the leader's states. In the behavioral approach, coordination is achieved through sharing the knowledge of the relative configuration states. In the virtual structure approach, coordination is

Fig. 12. Trajectories of the five Pioneer mobile robots recorded by MobileSim software, where the gray lines denote the ideal sensor detection ranges of the robots.

achieved through sharing the knowledge of the states of the virtual structure. All the three approaches have their advantages and disadvantages. The biggest drawback of the leader-following approach is poor robustness of the whole system. For the virtual structure approach, it is easy to describe the overall behavior of the agents and natural to develop a formation controller via feedback loop, but the formation can only be designed based on the virtual underlying rigid structure, which limits the application of this approach. The advantage of behavior-based formation is that it naturally integrates multiple competing goals in a MAS; in a behavior-based approach, a single agent requires only the information about its neighboring agents, which is essentially a distributed control method that scales with the number of the agents. Therefore, the main drawback of this method is that it generally cannot explicitly define the behavior of the group, so the collective behavior usually emerges from local interactions. From the aspect of mathematical quantitative description, it is difficult to analyze many characteristics of the formation, such as stability and convergence speed. Besides the above main research directions, some recent publications show the interesting new research trends, i.e., one parameter estimation asymptotic control [26], and the control of nonlinearities on actuators [27].

In this paper, the proposed compound control makes use of not only behavioral control, but also other nonlinear control strategies. For example, a null-space projection is applied and embedded in the design of the adaptive fuzzy controller. The results can be analyzed by rigorous mathematic proofs, which overcome the disadvantage of the traditional behavioral approach. In addition, the existing multiagent formation control problem usually models the agent using simple linear dynamics; however, most of the real systems are essentially nonlinear. In practical applications, the dynamics of robots or aircraft generally have complex nonlinearities and are difficult to be simply described by single- or second-order integrators. Accordingly, when the nonlinear term is unknown, the traditional linear control method cannot be directly extended to control the MAS with complex nonlinear dynamics. Therefore, it is necessary to study the

multiagent formation problem of general nonlinear dynamics by means of nonlinear control tools and theories. In this paper, we further extend the result on second-order nonlinear dynamics with external disturbances and unmolded dynamics. However, the controller contains many nonlinear system control methods, while considering the complex dynamics of robots to reach task requirements. Therefore, the controller for the complex nonlinear system has to face the increasing of computational load.

VIII. CONCLUSION

In this paper, we presented the adaptive fuzzy behavioral control design for a group of agents to execute prioritized missions. In particular, a case of multiple second-order uncertain nonlinear systems in unknown surroundings containing obstacles was investigated. Combining FLS, NSBC, and NFTSM control, a set of adaptive fuzzy cooperative controllers was elaborately designed for each agent such that all the agents can converge to a desired formation ultimately while avoiding obstacles, and error signals become sufficiently small in finite time. The proposed adaptive fuzzy formation controller is suitable for the second-order nonlinear systems in the Brunovsky form (1a), (1b). In addition, the proposed control strategy was further implemented in practical autonomous vehicles on a platoon of Pioneer-3AT/3DX mobile robots. In the future, we will test our theoretical results for more complicated missions that may involve different conflicts between missions.

APPENDIX PROOF OF THEOREM 1

Substituting (17) into (1b), we obtain

$$\dot{v}_i = f(\mathbf{x}_i) + d_i - k_{1i}\sigma_i - k_{2i}\text{sig}^{\frac{1}{2}}(\sigma_i) - \tilde{\theta}_i^T \phi_i(\mathbf{x}_i) - \chi_i - \hat{\delta}_i \text{sign}(\sigma_i). \quad (20)$$

Consider the Lyapunov function candidate V_1

$$V_1 = V_{\sigma 1} + V_{\rho 1} \quad (21)$$

where $V_{\sigma 1}$ is used to prove the stability of the system for the control purpose, and $V_{\rho 1}$ is constructed to test the stability for the mission purpose. We choose

$$V_{\sigma 1} = V_{\sigma 11} + V_{\sigma 12} \quad (22)$$

where

$$V_{\sigma 11} = \sum_{i=1}^n \frac{1}{2} \sigma_i^2 \quad (23)$$

$$V_{\sigma 12} = \sum_{i=1}^n \frac{1}{2} \tilde{\theta}_i^T \Gamma_{1i} \tilde{\theta}_i + \sum_{i=1}^n \frac{1}{2\gamma_{1i}} \tilde{\delta}_i^2. \quad (24)$$

Compute the derivative of $V_{\sigma 11}$

$$\begin{aligned} \dot{V}_{\sigma 11} &= \sum_{i=1}^n \sigma_i \dot{\sigma}_i = \sum_{i=1}^n \sigma_i [\dot{v}_i - \dot{v}_{i,r} + c_1 \tilde{v}_i + c_2 \dot{\beta}_i(\tilde{v}_i)] \\ &= \sum_{i=1}^n \sigma_i [f(\mathbf{x}_i) + u_i + \chi_i] \end{aligned} \quad (25)$$

where $\chi_i = -\dot{v}_{i,r} + c_1 \tilde{v}_i + c_2 \dot{\beta}_i(\tilde{v}_i)$ and $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$.

Substituting u_i into (25), we obtain

$$\begin{aligned} \dot{V}_{\sigma 11} &= \sum_{i=1}^n \sigma_i \left[\theta_i^{*T} \phi_i(\mathbf{x}_i) + \varepsilon_i + d_i - k_{2i} \text{sig}^{\frac{1}{2}}(\sigma_i) \right. \\ &\quad \left. - k_{1i} \sigma_i - \hat{\theta}_i^T \phi_i(\mathbf{x}_i) - \chi_i - \hat{\delta}_i \text{sign}(\sigma_i) + \chi_i \right] \\ &= \sum_{i=1}^n \sigma_i \left[\tilde{\theta}_i^T \phi_i(\mathbf{x}_i) + \varepsilon_i + d_i - K_{2i} \text{sig}^{\frac{1}{2}}(\sigma_i) \right. \\ &\quad \left. - K_{1i} \sigma_i - \hat{\delta}_i \text{sign}(\sigma_i) \right]. \end{aligned} \quad (26)$$

Because of Assumption 1 that $\|\varepsilon_i + d_i\| \leq \delta_i$, we have

$$\begin{aligned} \dot{V}_{\sigma 11} &\leq \sum_{i=1}^n \sigma_i \tilde{\theta}_i^T \phi_i(\mathbf{x}_i) + \sum_{i=1}^n \delta_i \|\sigma_i\|_1 - \sum_{i=1}^n k_{1i} \sigma_i^2 \\ &\quad - \sum_{i=1}^n k_{2i} \sigma_i \text{sig}^{\frac{1}{2}}(\sigma_i) - \sum_{i=1}^n \hat{\delta}_i \|\sigma_i\|_1 \\ &= \sum_{i=1}^n \sigma_i \tilde{\theta}_i^T \phi_i(\mathbf{x}_i) + \sum_{i=1}^n \tilde{\delta}_i \|\sigma_i\|_1 - \sum_{i=1}^n k_{1i} \sigma_i^2 \\ &\quad - \sum_{i=1}^n k_{2i} \sigma_i \text{sig}^{\frac{1}{2}}(\sigma_i) \end{aligned} \quad (27)$$

$$\begin{aligned} \dot{V}_{\sigma 12} &= - \sum_{i=1}^n \tilde{\theta}_i^T \Gamma_{1i} \dot{\tilde{\theta}}_i - \sum_{i=1}^n \gamma_{1i}^{-1} \tilde{\delta}_i \dot{\tilde{\delta}}_i \\ &= - \sum_{i=1}^n \tilde{\theta}_i^T \phi_i(\mathbf{x}_i) \sigma_i + \sum_{i=1}^n \tilde{\theta}_i^T \Gamma_{2i} \hat{\theta}_i - \sum_{i=1}^n \tilde{\delta}_i \|\sigma_i\|_1 \\ &\quad + \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i \dot{\tilde{\delta}}_i + \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i \text{sign}(\hat{\delta}_i) \\ &\quad + \frac{1}{4} \sum_{i=1}^n \tilde{\theta}_i^T \Gamma_{2i} \text{sign}(\hat{\theta}_i). \end{aligned} \quad (28)$$

Adding and subtracting $\frac{1}{2} \sum_{i=1}^n \Gamma_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}}$ on the right side of $\dot{V}_{\sigma 12}$, we have

$$\begin{aligned} \dot{V}_{\sigma 1} &\leq - \sum_{i=1}^n k_{1i} \sigma_i^2 - \sum_{i=1}^n k_{2i} \sigma_i \text{sig}^{\frac{1}{2}}(\sigma_i) - \frac{1}{2} \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 \\ &\quad + \frac{1}{4} \sum_{i=1}^n \tilde{\theta}_i^T \Gamma_{2i} \text{sign}(\hat{\theta}_i) + \frac{1}{2} \sum_{i=1}^n \Gamma_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 \\ &\quad + \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i \text{sign}(\hat{\delta}_i) + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} \\ &\quad - \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} + \sum_{i=1}^n \tilde{\theta}_i^T \Gamma_{2i} \hat{\theta}_i + \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i \hat{\delta}_i \end{aligned}$$

$$\begin{aligned}
&\leq \Xi_1 - \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \theta_i^{*T} \theta_i^* \\
&\quad + \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \|\theta_i^*\|_1 + \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 - \frac{1}{2} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i^2 \\
&\quad - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} |\hat{\delta}_i| + \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \delta_i + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} \\
&\quad - \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \|\hat{\theta}_i\|_1 + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} \delta_i^2 \\
&\quad - \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 \\
&\leq \Xi_1 - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \tilde{\theta}_i^T \tilde{\theta}_i - \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 \\
&\quad + \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \theta_i^{*T} \theta_i^* + \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \|\theta_i^*\|_1 + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} \delta_i^2 \\
&\quad - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i^2 \\
&\quad - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \|\hat{\theta}_i\|_1 - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} |\hat{\delta}_i| - \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} \\
&\quad + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}} + \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \delta_i - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i^2 \\
&\leq \Xi_1 + \Xi_2 + \Xi_3 - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \left(\tilde{\delta}_i^2 - 2|\tilde{\delta}_i|^{\frac{3}{2}} + |\tilde{\delta}_i| \right) \\
&\quad - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \left(\tilde{\theta}_i^T \tilde{\theta}_i - 2\|\tilde{\theta}_i^{\frac{3}{2}}\|_1 + \|\tilde{\theta}_i\|_1 \right) \\
&\leq \Xi_1 + \Xi_2 + \Xi_3 - \frac{1}{4} \sum_{i=1}^n \lambda_{2i} \left(\left| \tilde{\theta}_i \right| - \left| \tilde{\theta}_i^{\frac{1}{2}} \right| \right)^2 \|\tilde{\theta}_i\|_1 \\
&\quad - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \left(\left| \tilde{\delta}_i \right| - \left| \tilde{\delta}_i^{\frac{1}{2}} \right| \right)^2 \|\tilde{\delta}_i\|_1 \quad (29)
\end{aligned}$$

where λ_{2i} denotes the minimum eigenvalue of the positive-definite matrix Γ_{2i} , $\Xi_1 = -\sum_{i=1}^n k_{1i} \sigma_i^2 - \sum_{i=1}^n k_{2i} \sigma_i \text{sig}^{\frac{1}{2}}(\sigma_i)$, $\Xi_2 = -\frac{1}{4} \sum_{i=1}^n \lambda_{2i} \tilde{\theta}_i^T \tilde{\theta}_i - \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\tilde{\theta}_i^{\frac{3}{2}}\|_1 - \frac{1}{4} \sum_{i=1}^n \gamma_{2i} \tilde{\delta}_i^2 - \frac{1}{2} \sum_{i=1}^n \gamma_{2i} |\tilde{\delta}_i|^{\frac{3}{2}}$, and $\Xi_3 = \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \theta_i^{*T} \theta_i^* + \frac{1}{2} \sum_{i=1}^n \lambda_{2i} \|\theta_i^*\|_1 + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} \delta_i^2 + \frac{1}{2} \sum_{i=1}^n \gamma_{2i} \delta_i$. Let $\xi_{11} = \min_i \{2k_{1i}, \frac{\lambda_{2i}}{2}, \frac{\gamma_{2i}}{2}\}$ and $\xi_{12} = \min_i \{2^{\frac{3}{4}} k_{2i}, \lambda_{2i}/2^{\frac{1}{4}}, \gamma_{2i}/2^{\frac{1}{4}}\}$. Then, (29) can be rewritten as

$$\dot{V}_{\sigma 1} \leq -\xi_{11} V_{\sigma 1} - \xi_{12} V_{\sigma 1}^{\frac{3}{4}} + \Xi_3. \quad (30)$$

We can further reformulate (30) in the following two forms:

$$\dot{V}_{\sigma 1} + \xi_{13} V_{\sigma 1} + \xi_{12} V_{\sigma 1}^{\frac{3}{4}} \leq 0 \quad (31)$$

$$\dot{V}_{\sigma 1} + \xi_{11} V_{\sigma 1} + \xi_{14} V_{\sigma 1}^{\frac{3}{4}} \leq 0 \quad (32)$$

where $\xi_{13} = \xi_{11} - \frac{\Xi_3}{V_{\sigma 1}}$, $\xi_{14} = \xi_{12} - \frac{\Xi_3}{V_{\sigma 1}^{\frac{3}{4}}}$. From (31), (32), and [28, Lemma 3], σ_i will converge to the small region $\underline{\psi}_1 = \min\{\bar{\psi}_{11}, \bar{\psi}_{12}\}$ in finite time, where $\bar{\psi}_{11} = \sqrt{2\Xi_3/\xi_{11}}$ and $\bar{\psi}_{12} = \sqrt[2/3]{2\Xi_3/\xi_{12}}$. Furthermore, $\tilde{x}_i$ and $\tilde{v}_i$ will converge to $\bar{\psi}_{13} = \max\{\epsilon, \min\{\psi_1/c_1, (\psi_1/c_2)^2\}\}$ and $\bar{\psi}_{14} = \bar{\psi}_{11} + c_1 \bar{\psi}_{13} + c_2 \sqrt{\bar{\psi}_{13}}$ in finite time, respectively.

Following the similar analysis in [28], one can conclude that σ_i will become sufficiently small in finite time and further to obtain the finite-time convergence of $\tilde{v}_i$ and $\tilde{x}_i$.

Second, we prove the stability of the mission performance. The Lyapunov function $V_{\rho 1}$ for the missions is designed as

$$V_{\rho 1} = \frac{1}{2} \gamma_o \tilde{\rho}_o^T \tilde{\rho}_o + \frac{1}{2} \gamma_{\varsigma} \tilde{\rho}_{\varsigma}^T \tilde{\rho}_{\varsigma} \quad (33)$$

where γ_o and γ_{ς} are positive parameters; they satisfy $\gamma_o > \sqrt{\frac{\gamma_{\varsigma}}{\Delta_{o,m}}}$, $\Delta_{o,m} = \min_i \{\Delta_{i,o}\}$, and $\tilde{\rho}_o = [\tilde{\rho}_{i,o}]$. The term $J_o^\dagger J_o$ in (13) equals 0; if there is no obstacle in the immediate surroundings of the agents, then the derivative of $V_{\rho 1}$ is

$$\begin{aligned}
\dot{V}_{\rho 1} &= -\gamma_o \tilde{\rho}_o^T [J_o J_o^\dagger \Delta_o \tilde{\rho}_o + J_o (I - J_o^\dagger J_o) J_{\varsigma}^\dagger \Delta_{\varsigma} \tilde{\rho}_{\varsigma}] \\
&\quad - \tilde{\rho}_{\varsigma}^T \gamma_{\varsigma} [J_{\varsigma} J_o^\dagger \Delta_o \tilde{\rho}_o + J_{\varsigma} (I - J_o^\dagger J_o) J_{\varsigma}^\dagger \Delta_{\varsigma} \tilde{\rho}_{\varsigma}] \\
&= -\gamma_o \Delta_o \tilde{\rho}_o^T \tilde{\rho}_o - \tilde{\rho}_{\varsigma}^T \gamma_{\varsigma} J_{\varsigma} J_{\varsigma}^\dagger \Delta_{\varsigma} \tilde{\rho}_{\varsigma} \leq 0 \quad (34)
\end{aligned}$$

where $\Delta_o = \text{black diag}\{\Delta_{i,o} \otimes I\}$. Equation (34) implies that the overall mission is stable. And $\tilde{x}_{i,j}$ and $\tilde{q}_{i,j}$ will converge to the regions η_1 and η_2 in finite time. This implies that all the agents can form and maintain the desired formation.

In the case that there are some obstacles in the environment, the derivative of $V_{\rho 1}$ can be written in the following form:

$$\dot{V}_{\rho 1} = -\tilde{\rho}^T Q \tilde{\rho} \quad (35)$$

where

$$\tilde{\rho} = \begin{bmatrix} \tilde{\rho}_o \\ \tilde{\rho}_{\varsigma} \end{bmatrix}, \quad Q = \begin{bmatrix} \gamma_o \Delta_o & \frac{1}{2} \gamma_{\varsigma} (J_{\varsigma} J_o^\dagger \Delta_o)^T \\ \frac{1}{2} \gamma_{\varsigma} (J_{\varsigma} J_o^\dagger \Delta_o) & \gamma_{\varsigma} J_{\varsigma} (I - J_o^\dagger J_o) J_{\varsigma}^\dagger \Delta_{\varsigma} \end{bmatrix}$$

with $Q_{11} = \gamma_o \Delta_o$, $Q_{21} = Q_{12}^T = \frac{1}{2} \gamma_{\varsigma} (J_{\varsigma} J_o^\dagger \Delta_o)^T$, $Q_{22} = \gamma_{\varsigma} J_{\varsigma} (I - J_o^\dagger J_o) J_{\varsigma}^\dagger \Delta_{\varsigma}$. Using [16, Lemma 2.3], it is shown that

$$\begin{aligned}
\dot{V}_{\rho 1} &\leq -Q_{11,m} \|\tilde{\rho}_o\|^2 - \|Q_{22,m}\| \|\tilde{\rho}_{\varsigma}\|^2 \\
&\quad + 2\|Q_{21,M}\| \|\tilde{\rho}_o\| \|\tilde{\rho}_{\varsigma}\| \\
&= -\left[\|\tilde{\rho}_o\| \right]^T \begin{bmatrix} \gamma_o \Delta_{o,m} & \frac{\gamma_{\varsigma} \Delta_{o,m}}{-2} \\ \frac{\gamma_{\varsigma} \Delta_{o,m}}{-2} & 0 \end{bmatrix} \begin{bmatrix} \|\tilde{\rho}_o\| \\ \|\tilde{\rho}_{\varsigma}\| \end{bmatrix} \\
&= -\tilde{\rho}^T P \tilde{\rho} \quad (36)
\end{aligned}$$

where $Q_{11,m} = \gamma_o \Delta_{o,m}$, $\Delta_{o,m} = \min_i \{\Delta_{i,o}\}$, $Q_{22,m} = 0$ is the lower bound on the induced norm of Q_{22} , $Q_{21,M} = \gamma_{\varsigma} \Delta_{o,m}$ is the upper bound on the induced norm of Q_{21} , $\|J_{\varsigma}\| = \|J_o\| = 1$, $\hat{\rho} = [\|\tilde{\rho}_o\|, \|\tilde{\rho}_{\varsigma}\|]^T$, and

$$P = \begin{bmatrix} \gamma_o \Delta_{o,m} & -\frac{1}{2} \gamma_{\varsigma} \Delta_{o,m} \\ -\frac{1}{2} \gamma_{\varsigma} \Delta_{o,m} & 0 \end{bmatrix}.$$

From $\gamma_o > \sqrt{\frac{\gamma_s}{\Delta_{o,m}}}$, it is obvious that P is a positive-definite symmetric matrix. After that, (36) can be rewritten as

$$\dot{V}_{\rho 1} \leq -\lambda_{\min}\{P\}\hat{\rho}^T \hat{\rho} \leq 0 \quad (37)$$

where $\lambda_{\min}\{P\}$ is the minimum eigenvalue of P .

Here, the maximum of $\Delta_{i,o}$ is calculated by

$$\Delta_{i,o}^*(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) = \frac{-B_i + \sqrt{B_i^2 + 4A_i C_i}}{2A_i} \quad (38)$$

where $A_i = \tilde{\rho}_{i,o}^2$, $B_i = -2\tilde{\rho}_{i,o}(\|v_i\| + c_1\|\tilde{x}_i\| + c_2\|\beta_i(\tilde{x}_i)\|)$, and $C_i = -\|v_i\|^2 + c_1^2\|\tilde{x}_i\|^2 + c_2^2\|\beta_i(\tilde{x}_i)\|^2 + 2c_1\|v_i\|\|\tilde{x}_i\| + 2c_2\|v_i\|\|\beta_i(\tilde{x}_i)\| + 2c_1c_2\|\tilde{x}_i\|\|\beta_i(\tilde{x}_i)\|$ for $i = 1, \dots, n$. We use the fact $\sigma_i^T K_{2i} \text{sig}^{\frac{1}{2}}(\sigma_i) = k_{2i}(\sigma_i^T \sigma_i)^{\frac{3}{4}}$, and then, (38) is also applied to the term $K_{2i} \text{sig}^{\frac{1}{2}}(\sigma_i)$ in the control law. We choose $\Delta_{i,o}(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) = \Delta_{i,o}^*(\|\tilde{x}_i\|, \|\beta_i(\tilde{x}_i)\|, \|v_i\|) + \varrho_i$, where the robust term ϱ_i is positive constant to be designed. This constraint ensures that the minimum related distances between agent i and obstacle remain at d_i .

The proof is complete.

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